Transport properties and thermodynamics in an effective field theory framework

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Outline

- **Effective field theories**

  why we use them? for what can we use them?

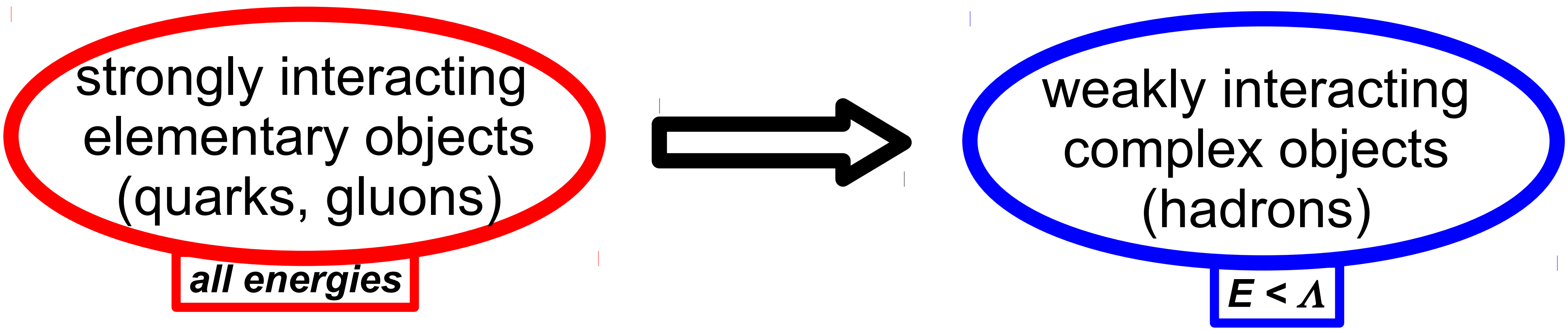
- **Melting particles**
  - non-interacting objects beyond quasi-particle picture
  - motivation & formalism, toy models for calculating transport coefficients

- **Possible applications**

  fitting thermal lattice QCD observables

Effective model building

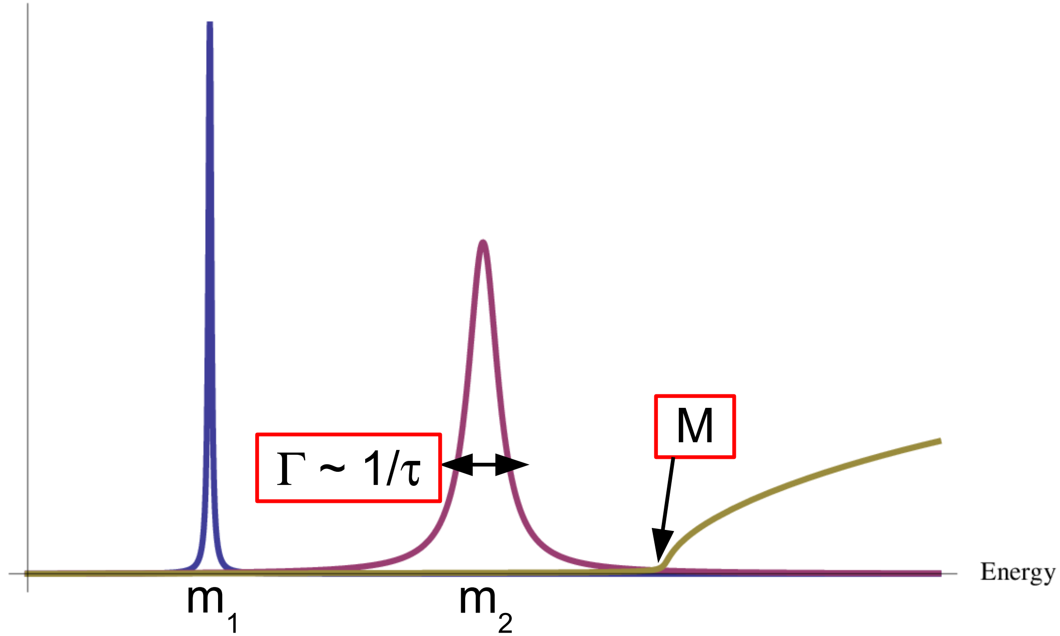
when perturbation theory breaks down (strong interaction)



- *scale dependence*
- *same symmetries*
- *PT works again*

Non-local quasi-particles

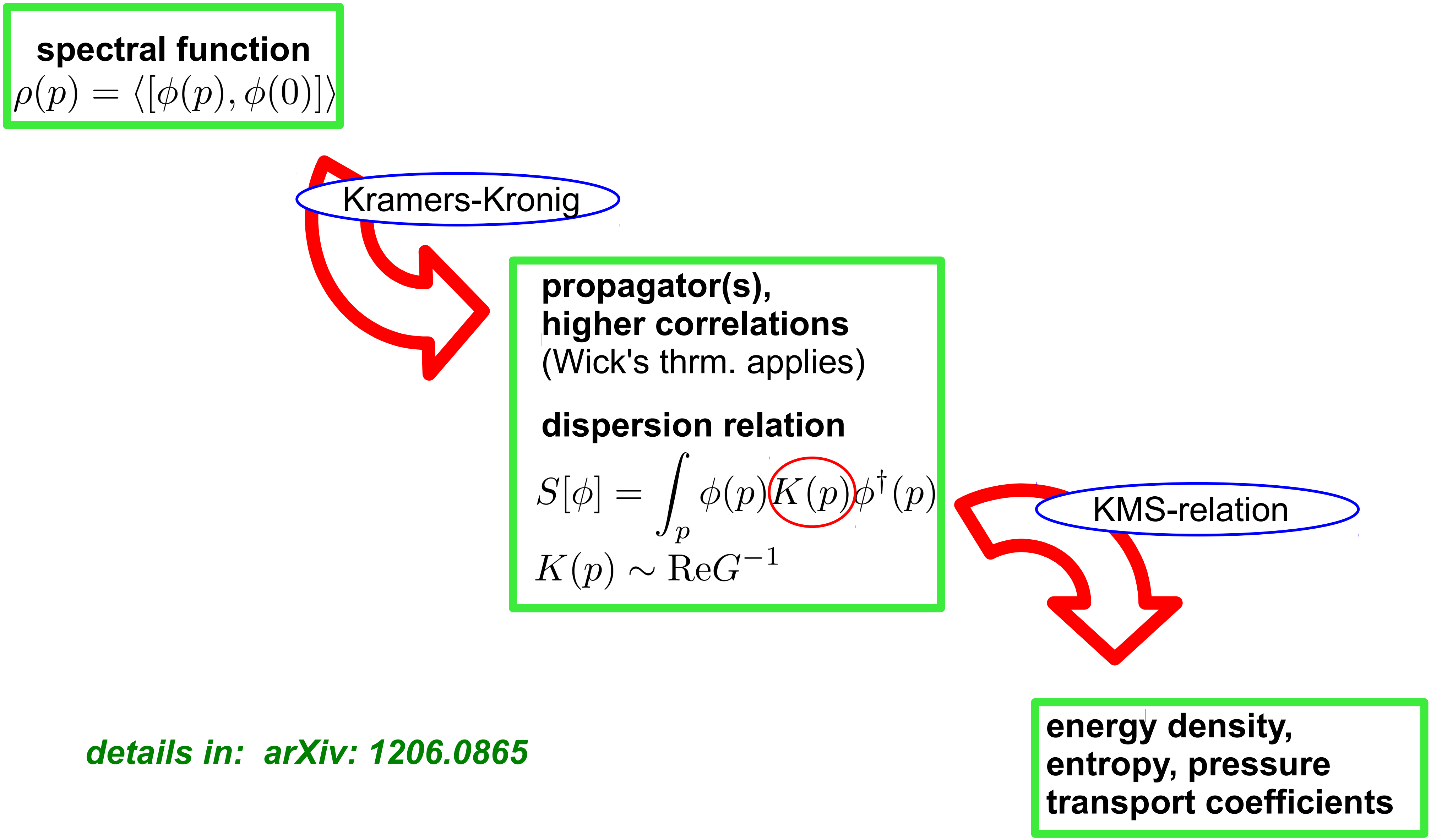
Parametrization: spectral function(s) = energy levels at fixed quantum numbers



- *all dynamical informations at 2-point level*
- *"resummed" correlations (~2PI at tree-level)*

$$\rho(x) = \langle [\phi(x), \phi(0)] \rangle$$

Non-local quasi-particles



details in: arXiv: 1206.0865

Thermodynamics

Energy density:

$$\varepsilon = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \left(\omega \frac{\partial K(\omega, \mathbf{p})}{\partial \omega} - K(\omega, \mathbf{p}) \right) \rho(\omega, \mathbf{p}) n(\omega)$$

Usual relations:

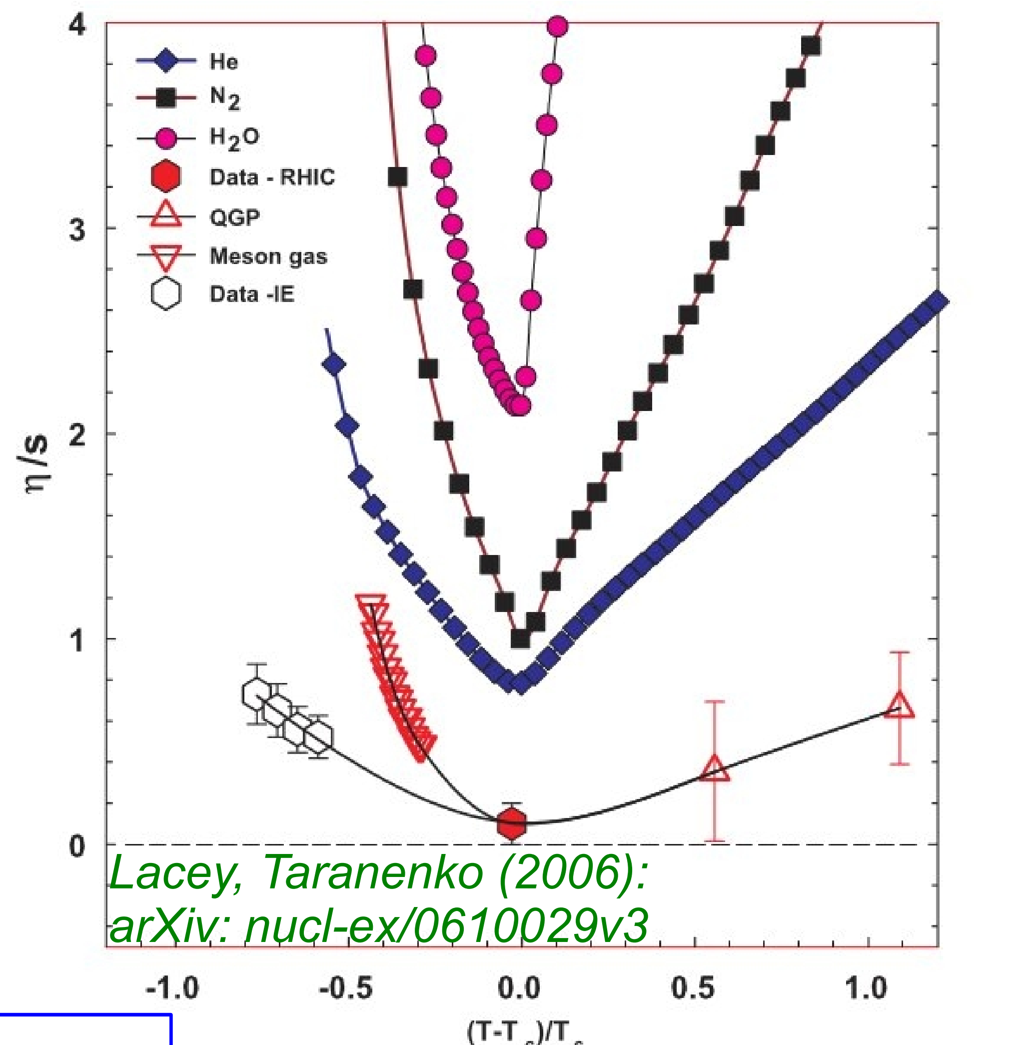
$$s = -\frac{\partial f}{\partial T} \qquad \varepsilon = -T^2 \frac{\partial (f/T)}{\partial T} \qquad P = Ts - \varepsilon$$

Shear viscosity

~momentum-diffusion coefficient

$$\rho\frac{\delta v}{\tau} \sim \eta\frac{\delta v}{l^2} \quad \xrightarrow{\text{QP-appr.}} \quad \frac{\eta}{\rho} \sim \frac{\eta}{s} \sim \langle v\rangle\, l$$

$l \to \infty$	**ideal gas**	weakly interacting
$l \to 0$	**ideal fluid**	strongly interacting



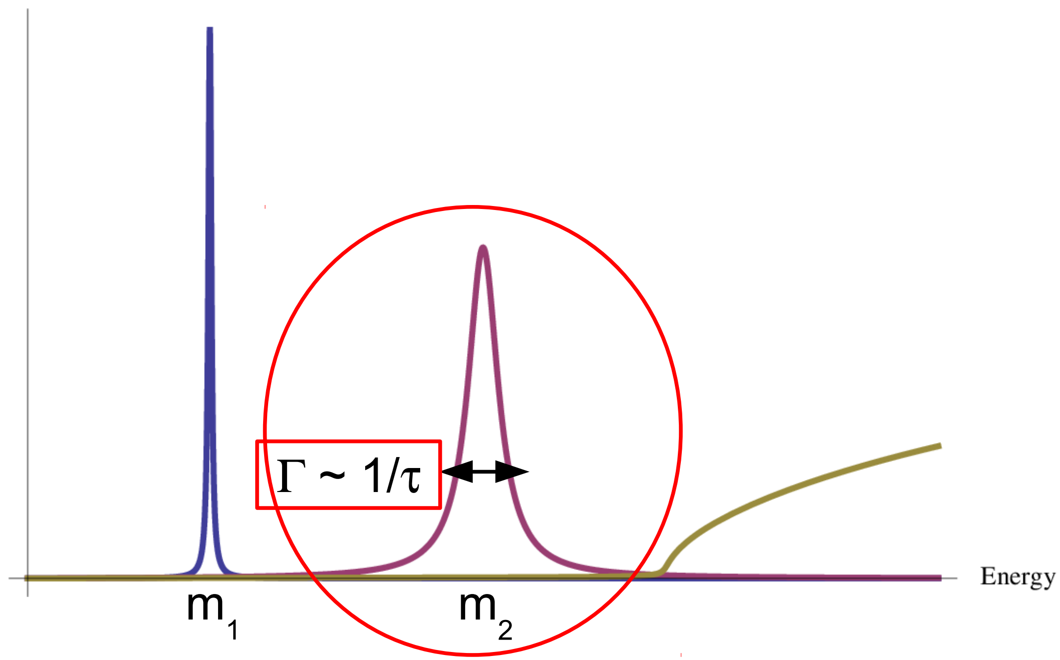
in linear response:

$$\eta = \frac{1}{6}\sum_{i\neq j}\frac{\langle[T_{ij}(\omega,\mathbf{p}=0);T_{ij}(0)]\rangle}{\omega}\Bigg|_{\omega\to 0}$$

$$\eta = \frac{1}{6T}\int_p \frac{\sum_{i\neq j}(p_i p_j)^2}{p^2}(K'(p)\rho(p))^2 n(p)(1+n(p))$$

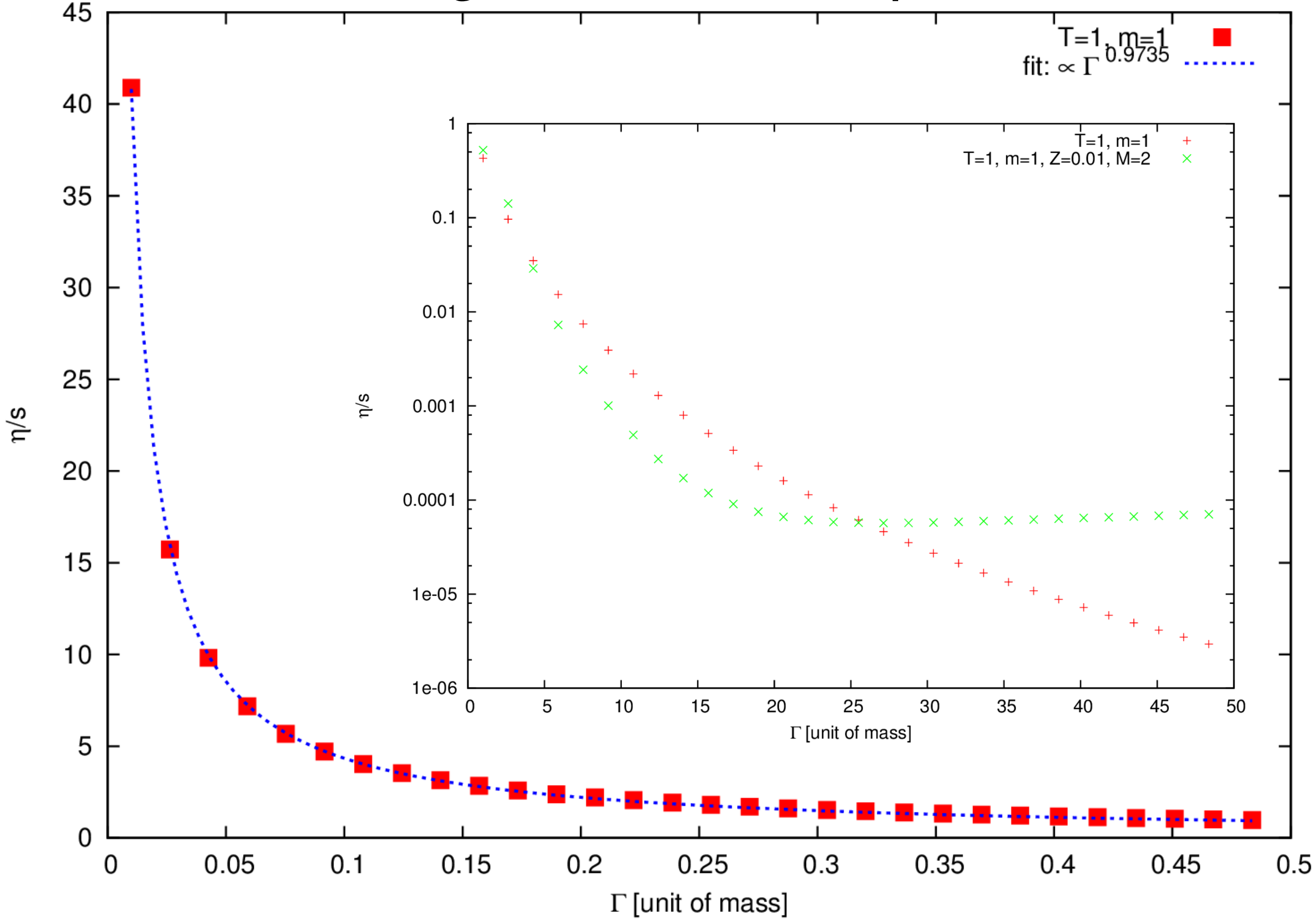
Non-local quasi-particles

Parametrization: spectral function(s) = energy levels at fixed quantum numbers

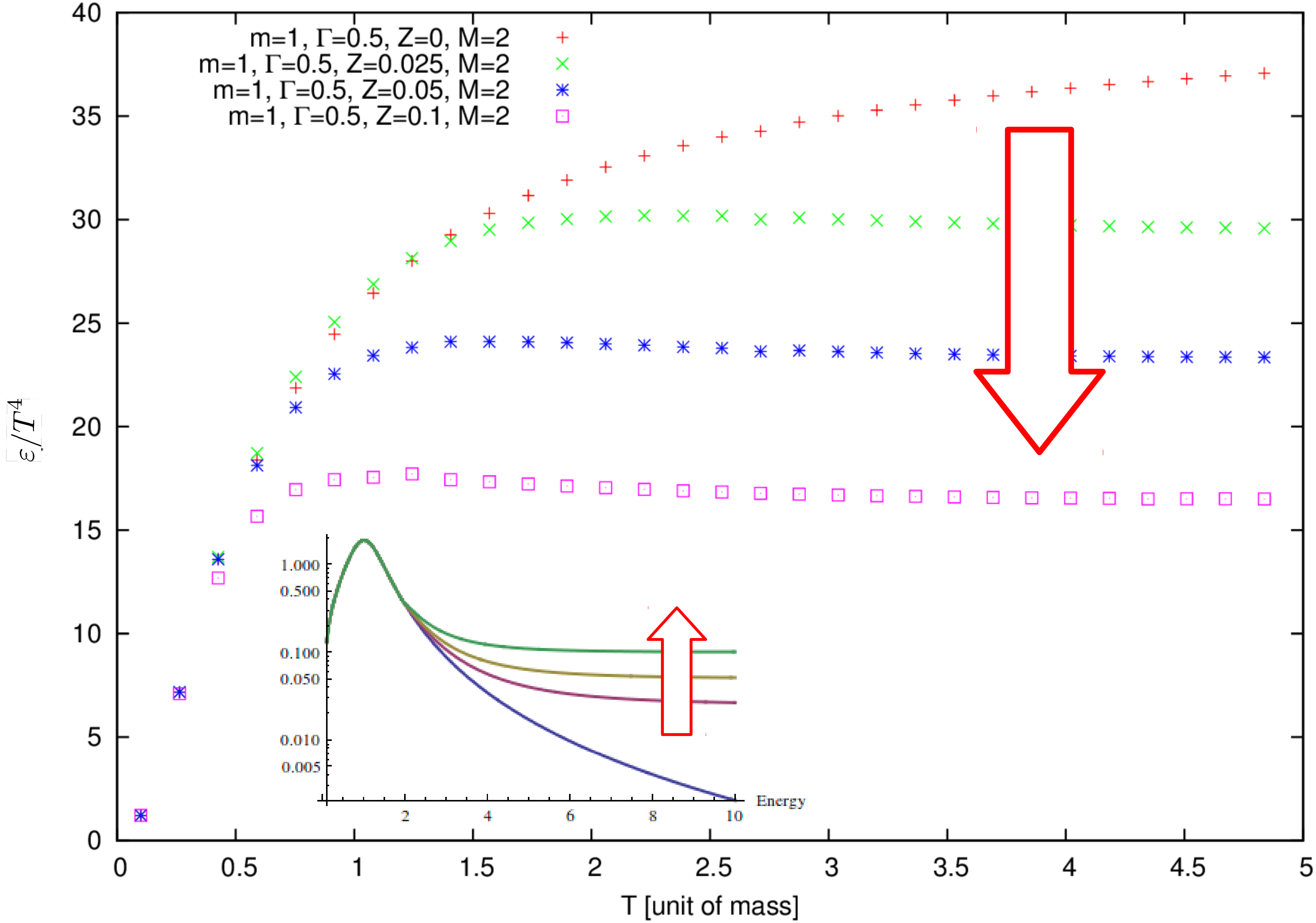


$$\rho(x) = \langle[\phi(x), \phi(0)]\rangle$$

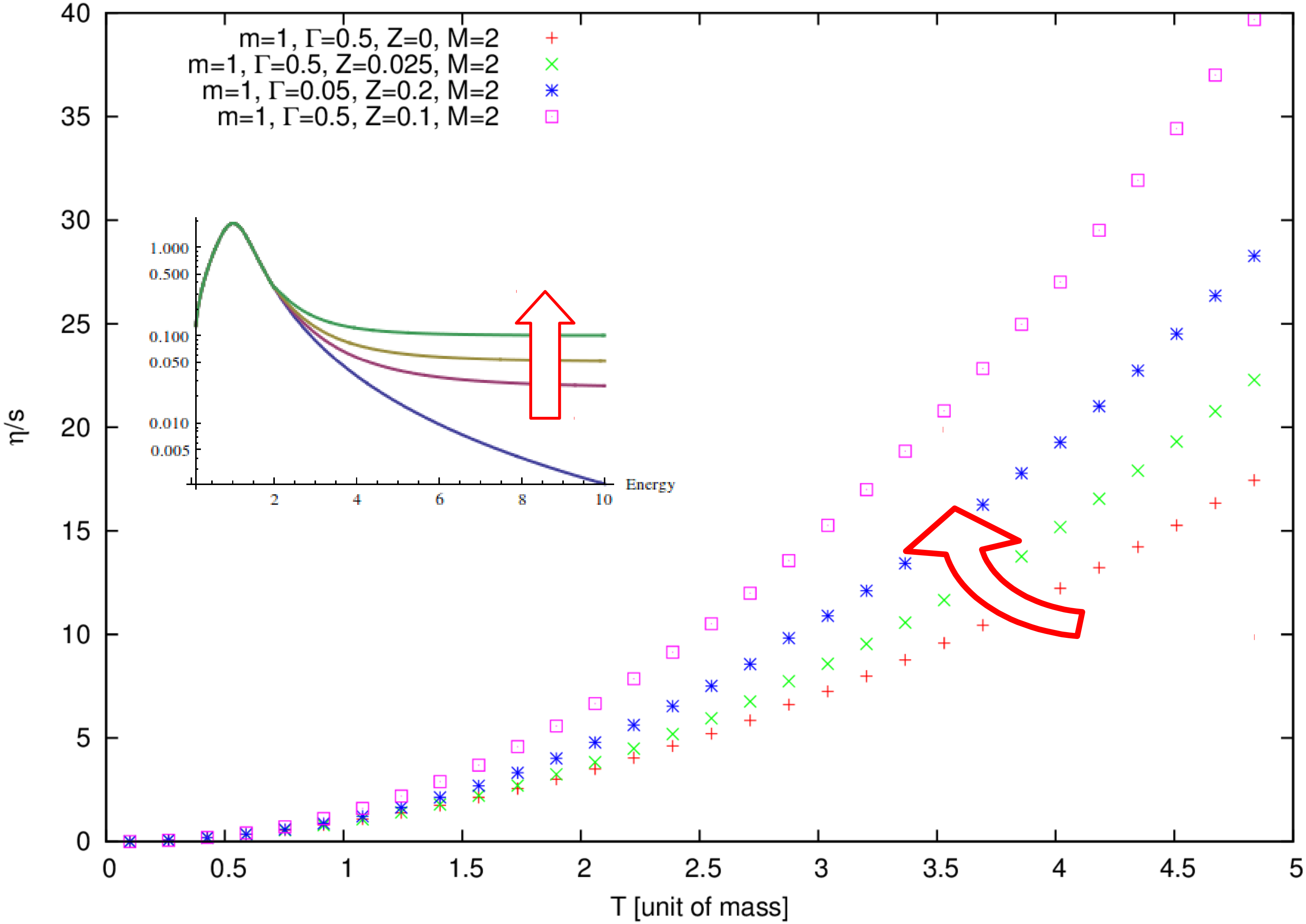
Shear viscosity of a wide peak



Peaks and continuum



Peaks and continuum



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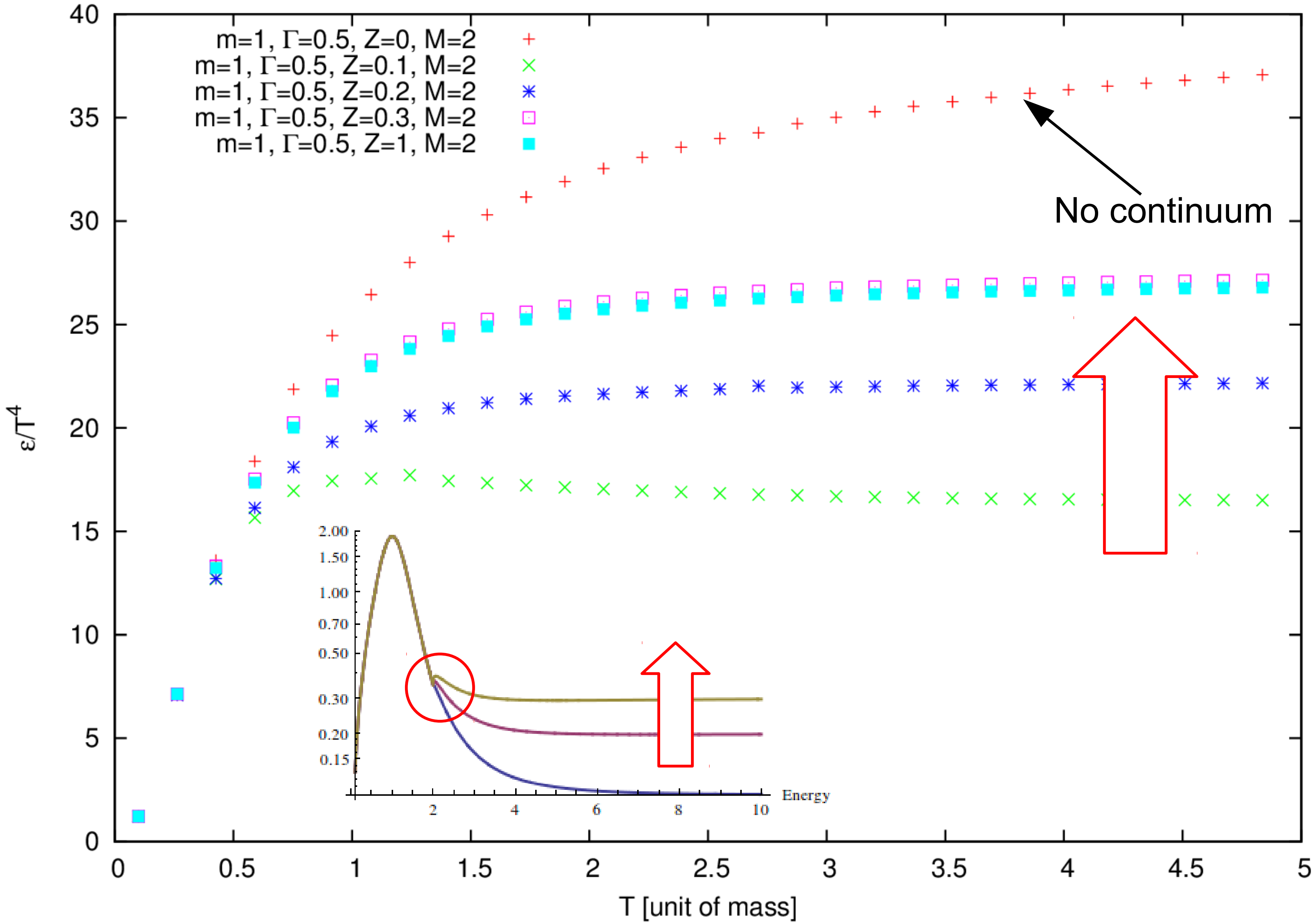

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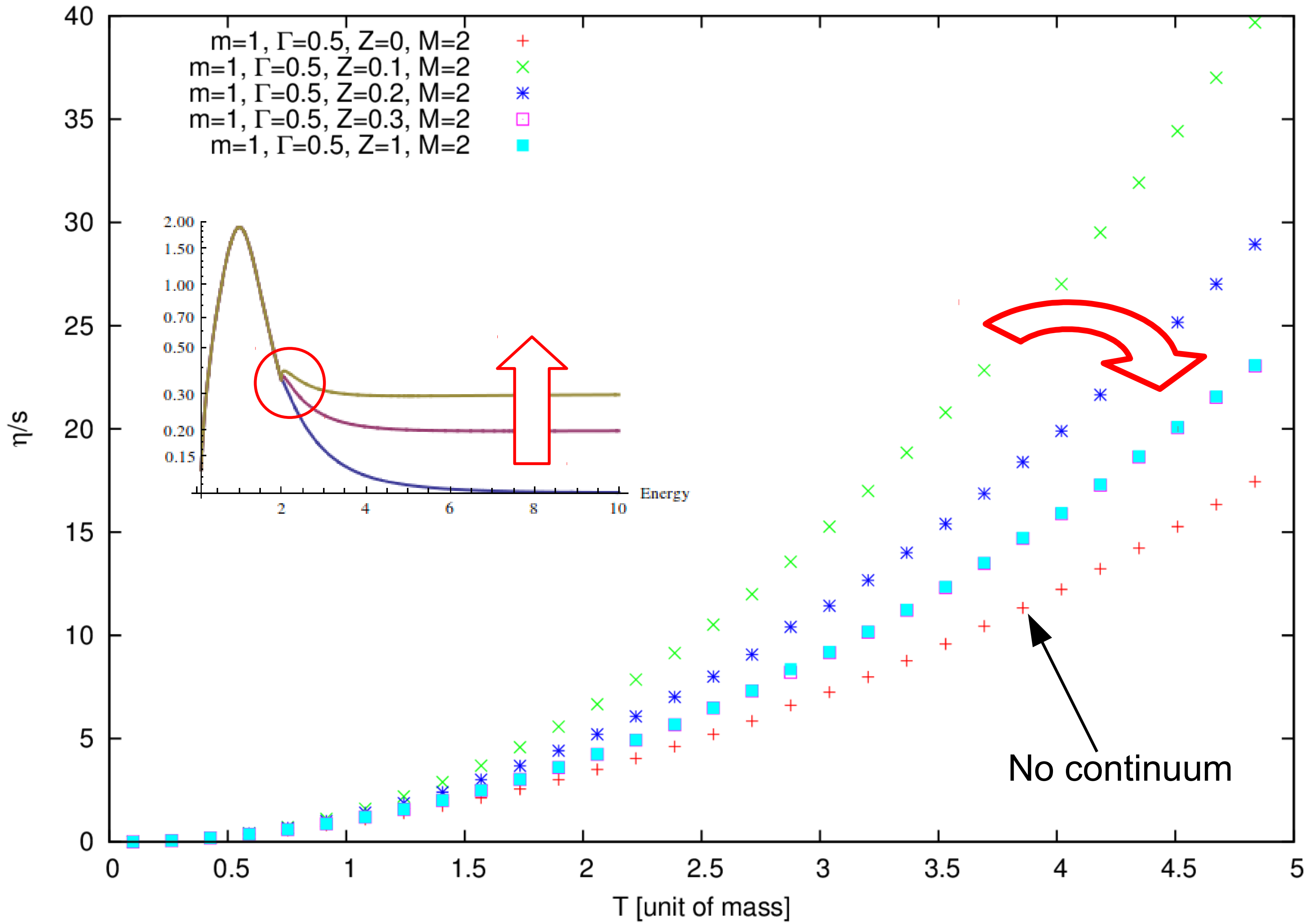
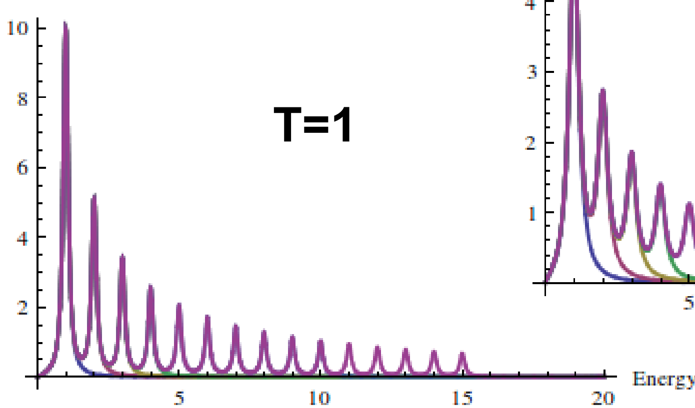
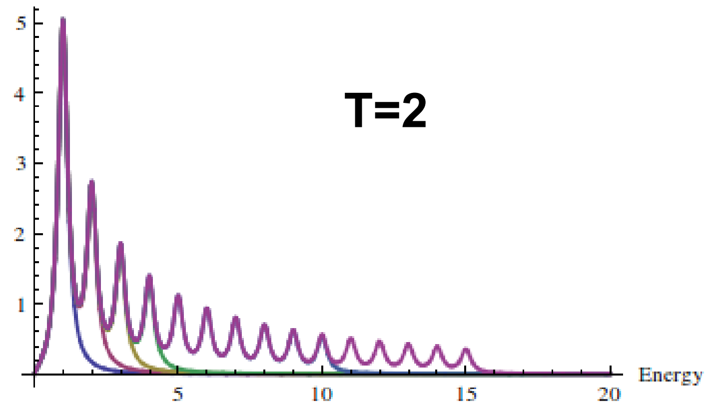
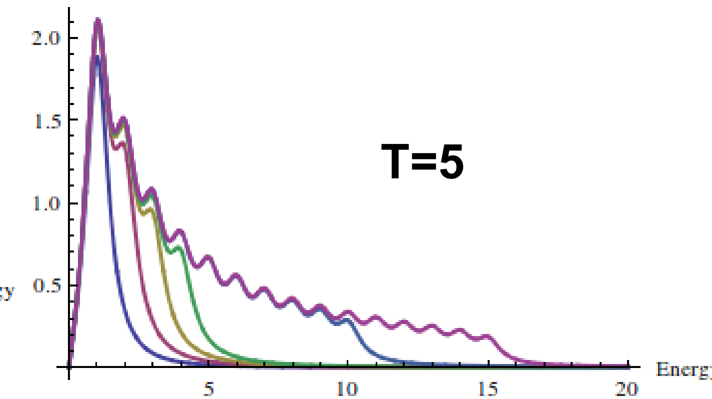


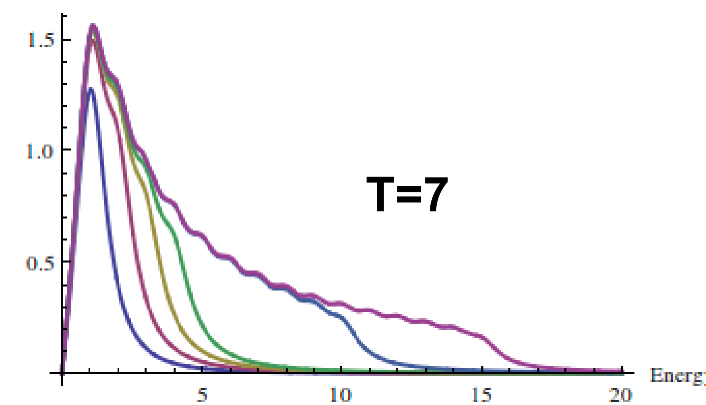
Comb of melting peaks

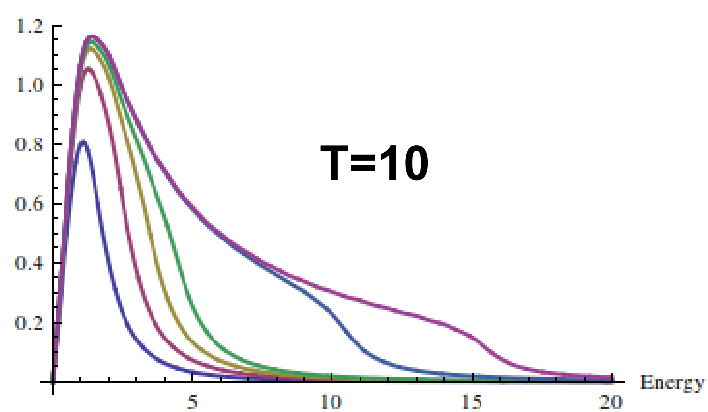






*widths grow with **T** as* $\Gamma_i = \sqrt{\gamma_0^2 + \gamma_1^2 T^2}$





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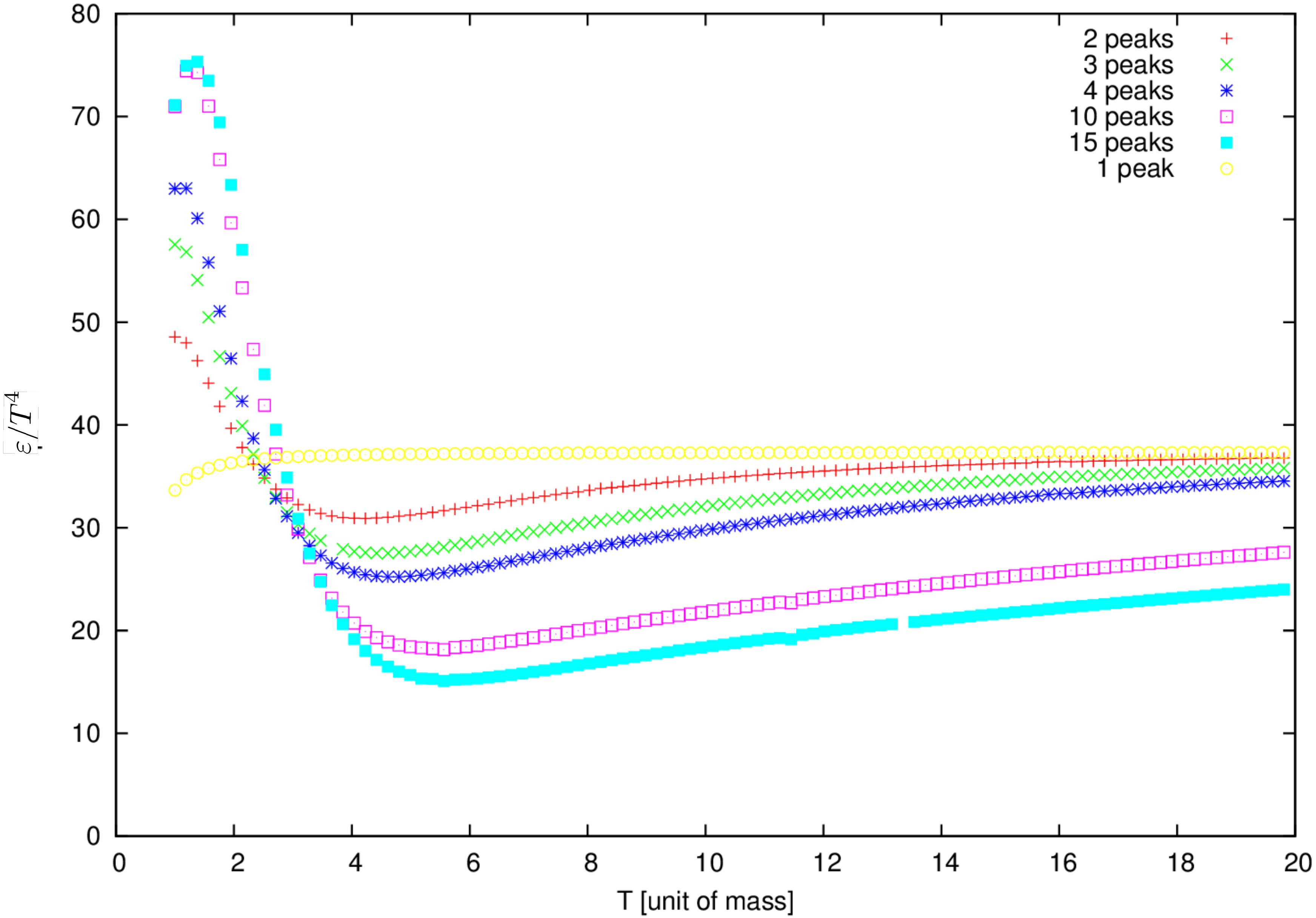

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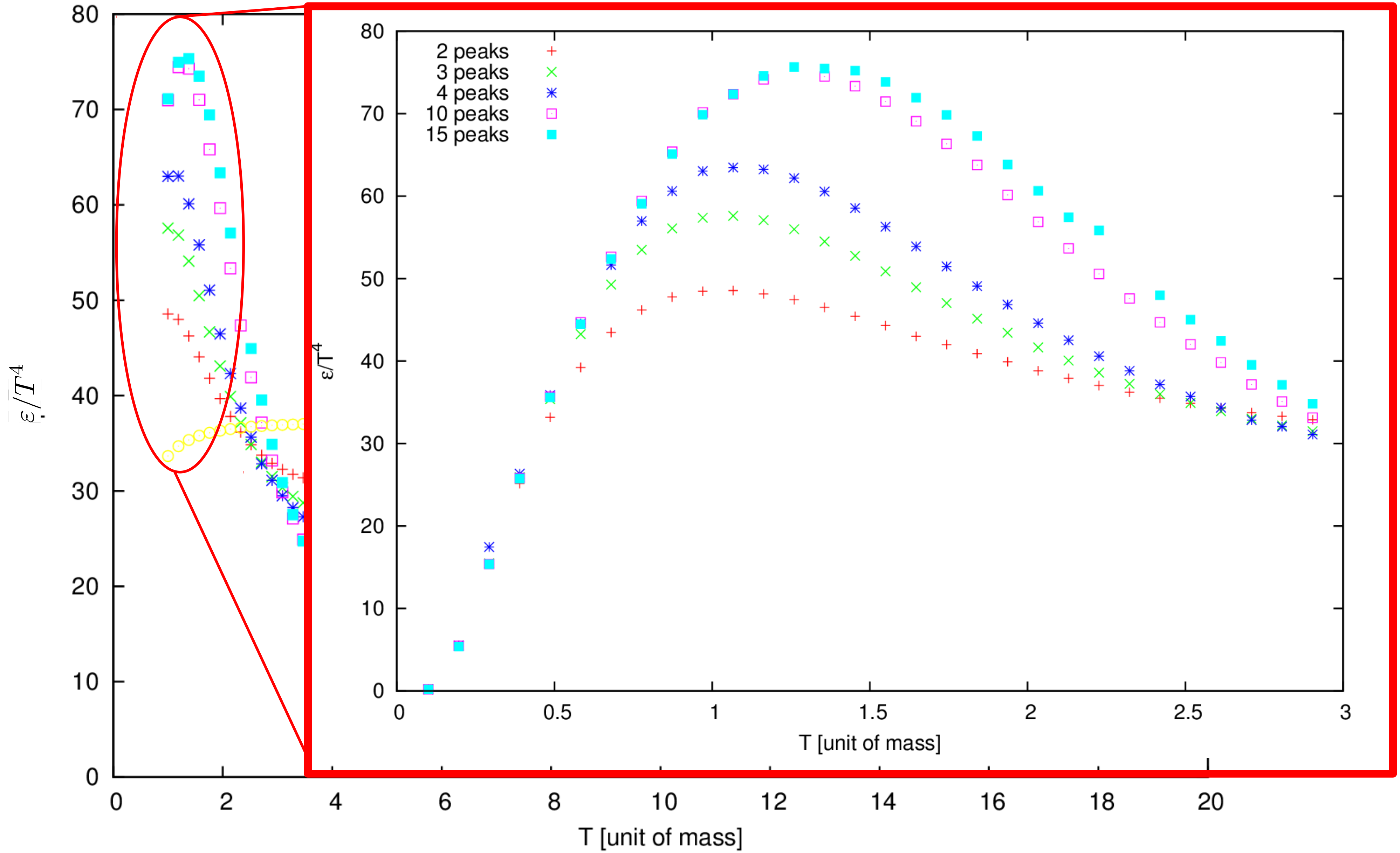

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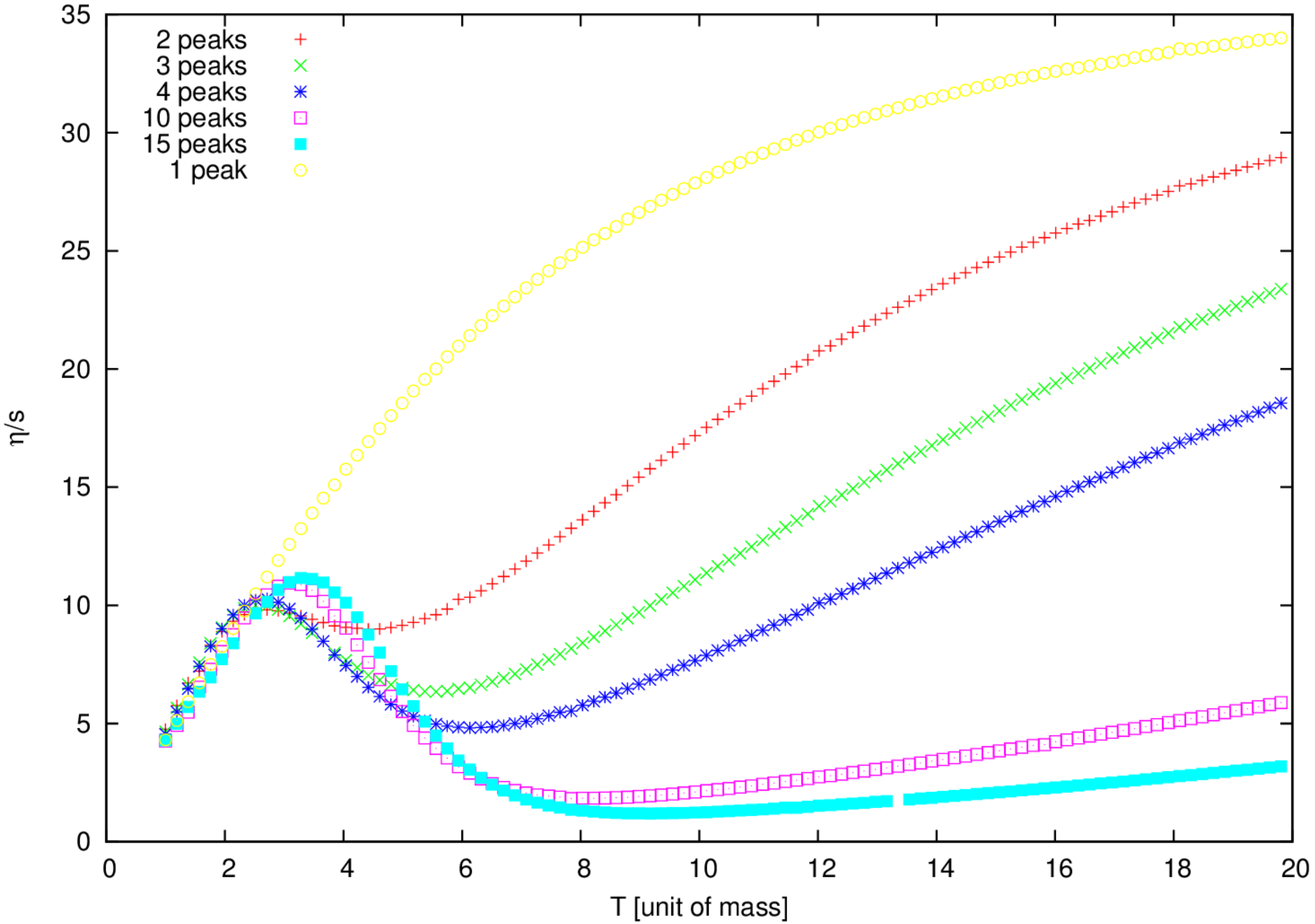

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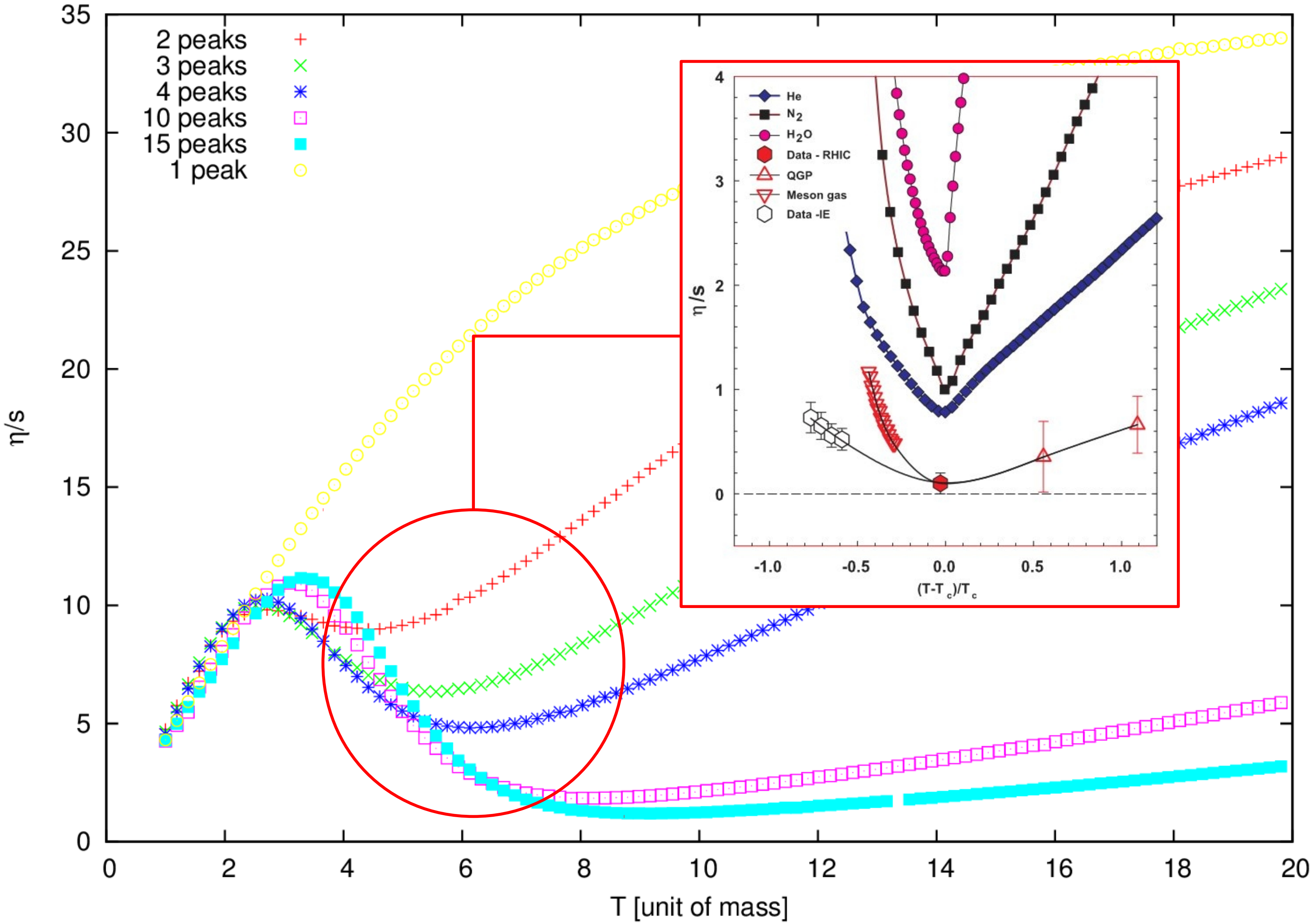
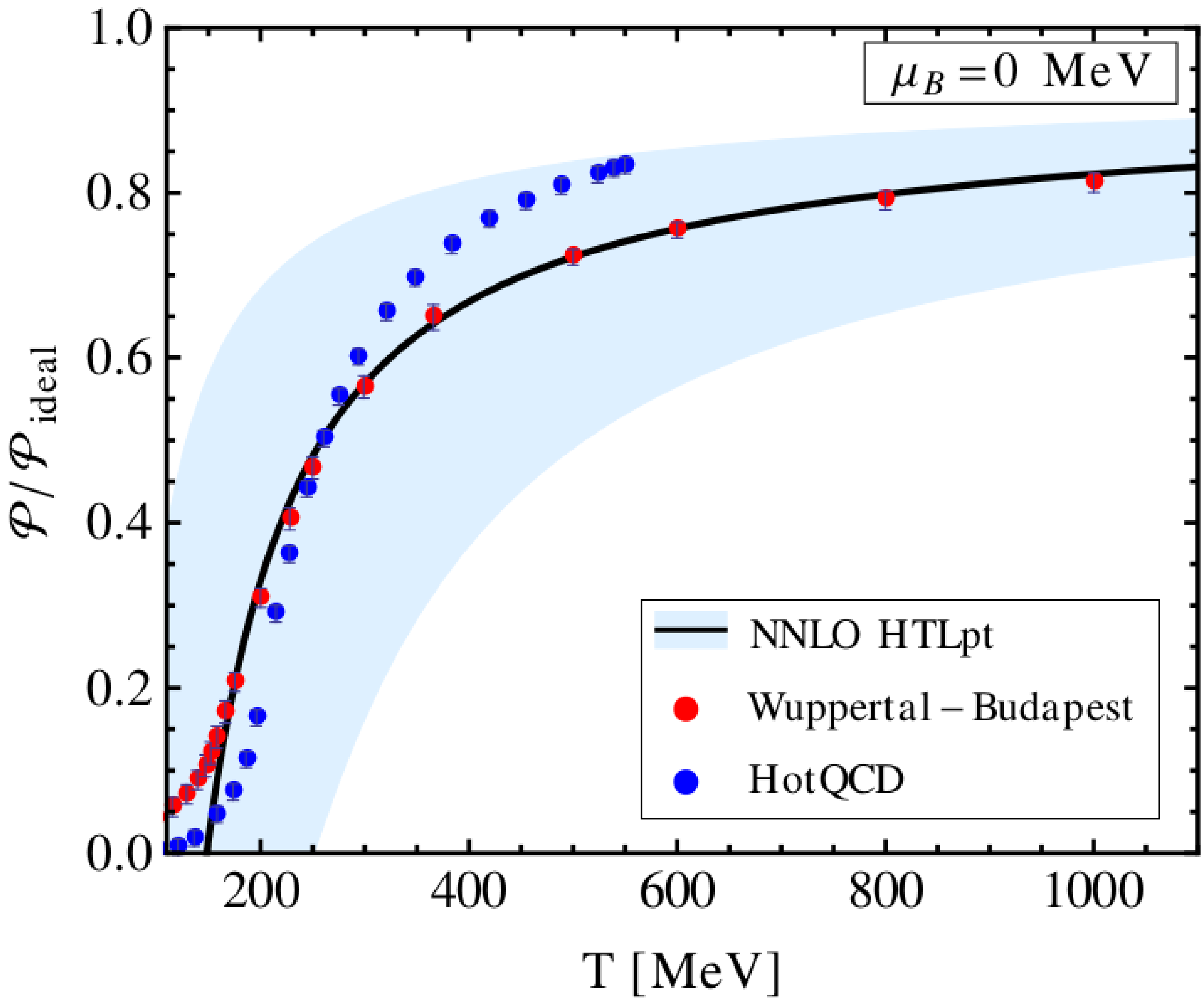


QCD thermodynamics

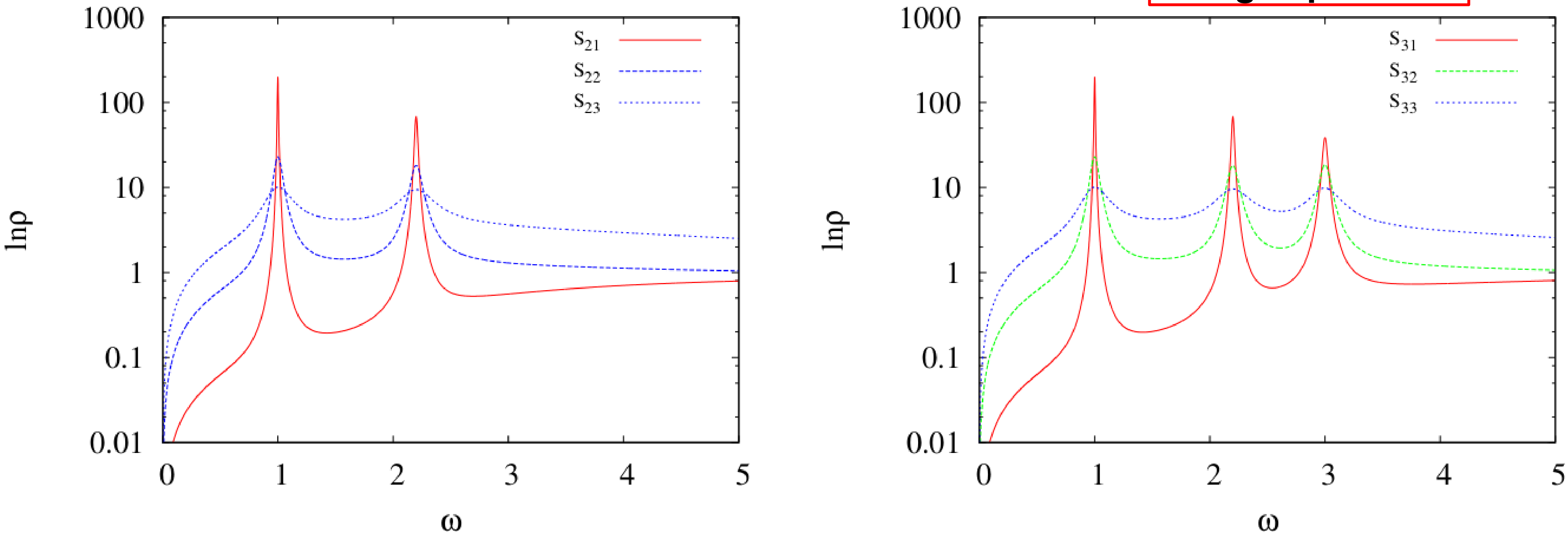


arXiv:1309.3968

QCD thermodynamics

$$N_{\mathrm{eff}}(\,\mathrm{params.}\,) = \frac{P(T,\,\mathrm{params.}\,)}{P_0(T)}$$

free gas pressure



A. Jakovác & T. S. Biró, arXiv:1405.5471

QCD thermodynamics

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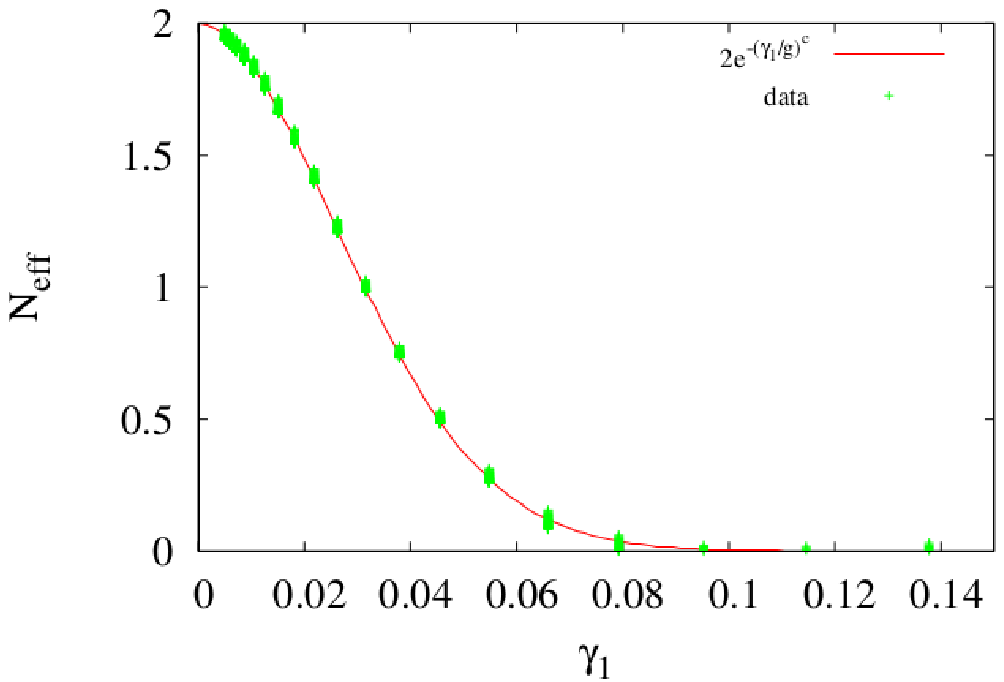
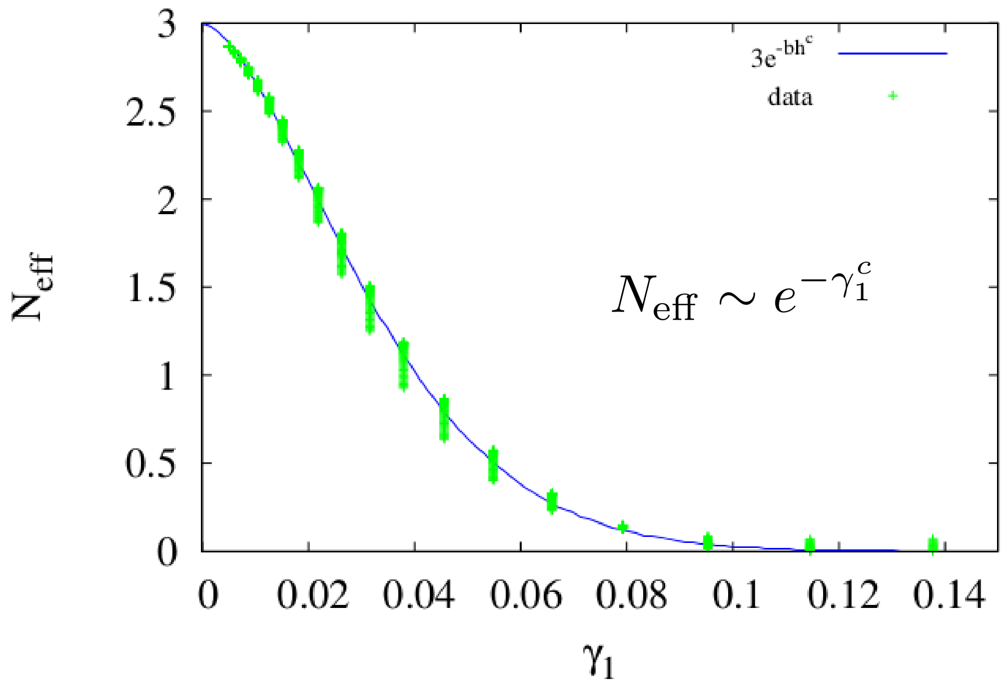
free gas pressure





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QCD thermodynamics

$$P^{(\mathrm{hadr.})}(T) = N_{\mathrm{eff}}^{(\mathrm{hadr.})} \sum_{n \in \mathrm{hadrons}} P_0(T, m_n) \qquad \ln N_{\mathrm{eff}}^{(\mathrm{hadr.})} = (T/T_0)^b$$

$$m_n = m_\pi + T_H \ln n$$

- *the effective number of DoF is dominated by the continuum effects*

$$\ln N_{\mathrm{eff}} \sim \gamma$$

- *the height of the continuum grows with T*

$$\gamma \sim T$$

$$P^{(\mathrm{part.})}(T) = N_{\mathrm{eff}}^{(\mathrm{part.})} \sum_{n \in \mathrm{partons}} P_0(T, m_n) \qquad \ln N_{\mathrm{eff}}^{(\mathrm{part.})} = G_0 + c(N_{\mathrm{eff}}^{(\mathrm{hadr.})})^d$$

- *partonic cross section grows when hadronic excitations are present*
- *partons dominate the large T region*

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QCD thermodynamics

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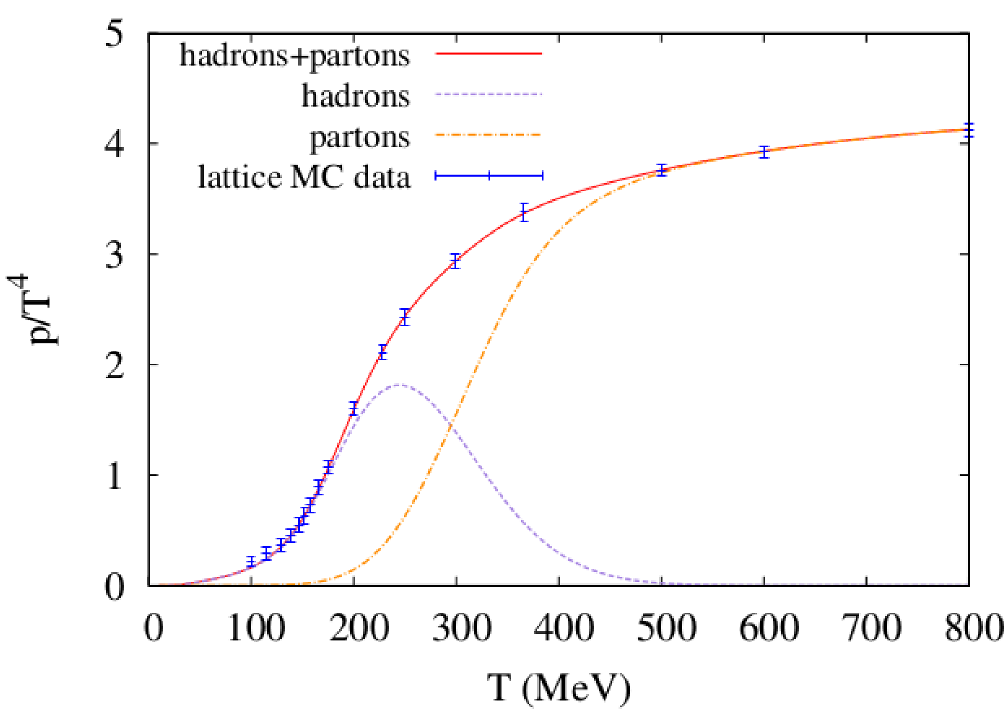

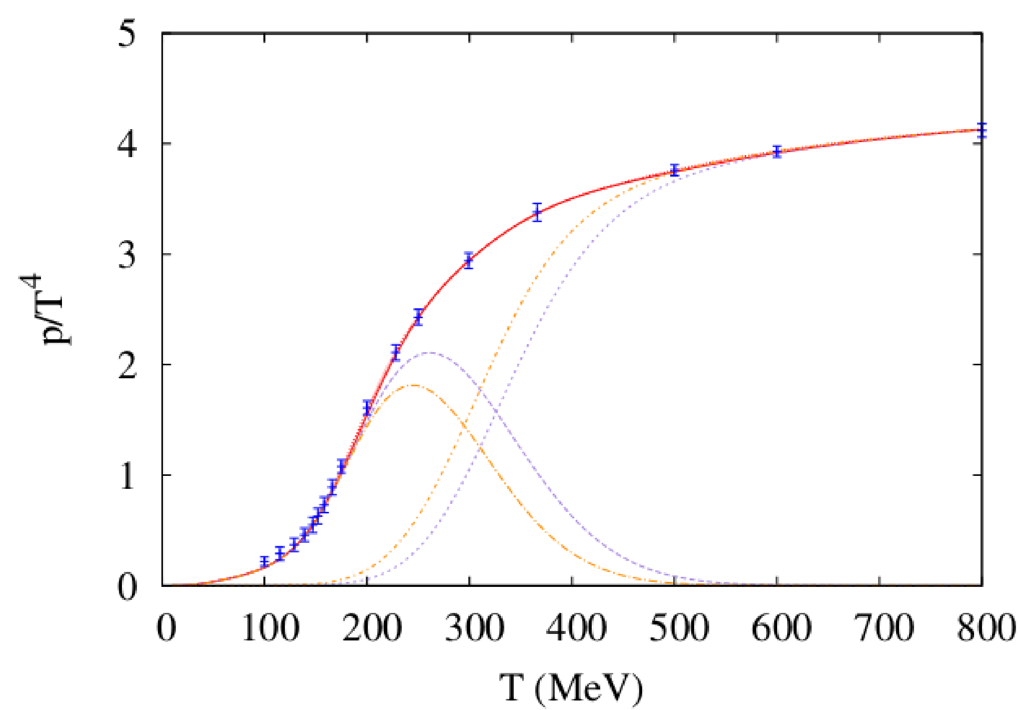

(data: Budapest-Wuppertal)

A. Jakovác & T. S. Biró, arXiv:1405.5471

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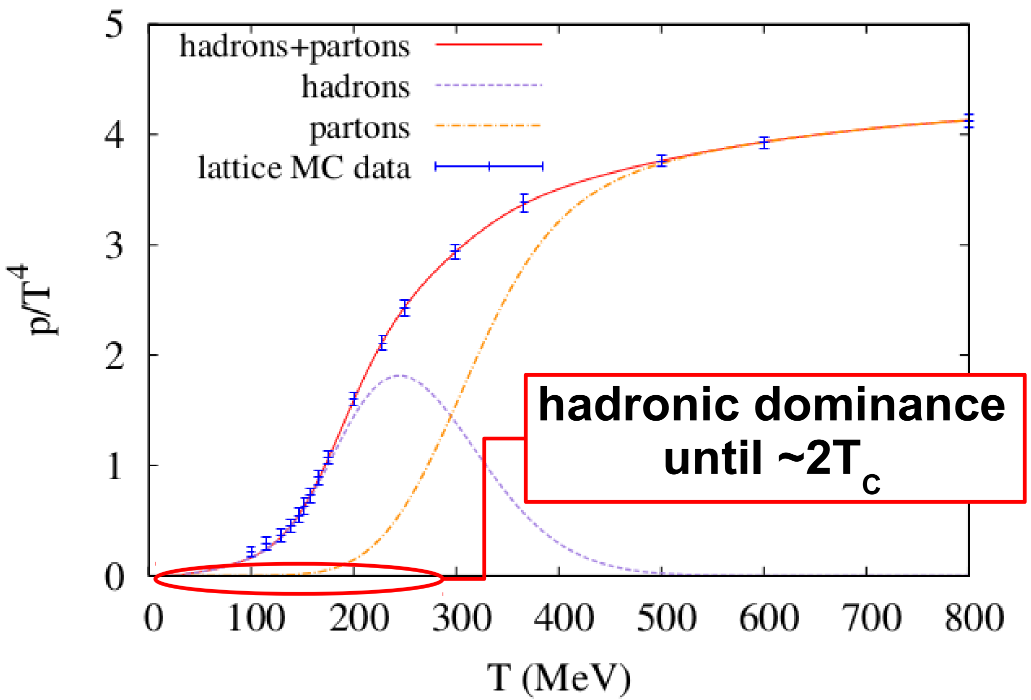

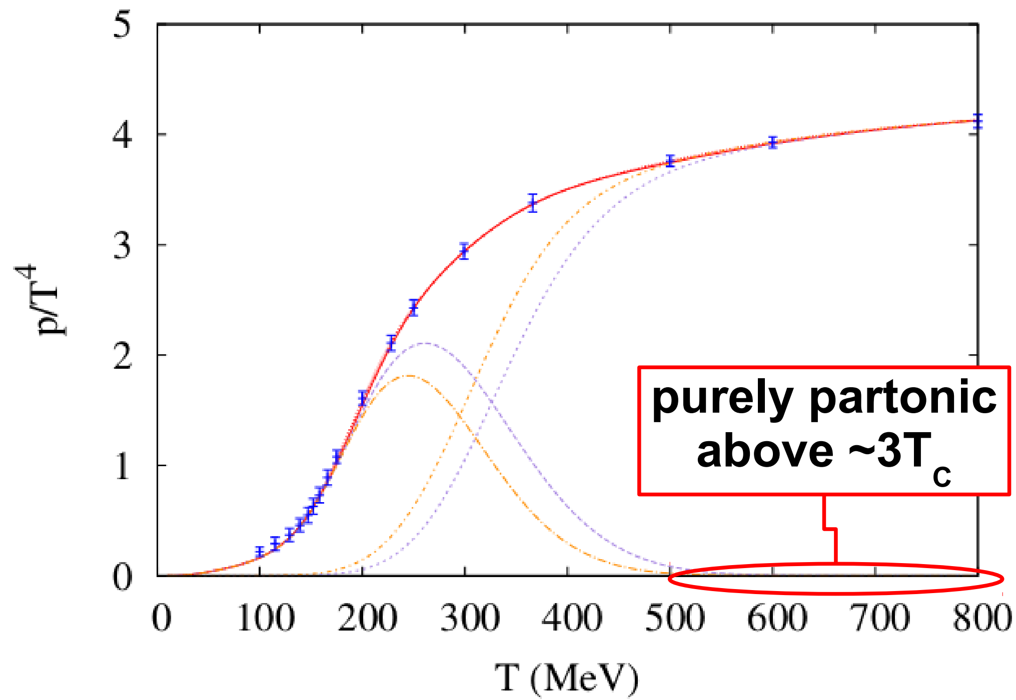

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What have we learned?

- how to test the thermal and transport properties of a quantum channel with known spectral density in an ~~lazy~~ easy way
- **presence of continuum: decreased thermal contribution**
- statistical hadrons (beyond HRG) mixed partons gas for describing QCD lattice data
- future prospects: finite chemical potential, conserved charge of finite lifetime objects ...

Thank you for the attention!

Questions? Comments?

arXiv:
1206.0865
1306.2657
1405.5471
1408.????
stay tuned!

Backup slides

Non-local quasi-particles – Backup

$$\langle \ldots \rangle = \mathrm{Tr}\left(e^{\beta \widehat{T}^{00}} (\ldots) \right)$$

real part of the propagator:

$$\mathrm{Re}G(p) = \mathrm{P.V.} \int_0^\infty \frac{\mathrm{d}y}{2\pi} \frac{y\rho(y)}{p^2 - y^2}$$

dispersion relation:

$$K(p) = \frac{\mathrm{Re}G(p)}{(\mathrm{Re}G(p))^2 + \frac{1}{4}\rho^2(p)}$$

energy density:

$$\varepsilon = \langle T^{00} \rangle = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \left(\frac{\omega^2}{p} \frac{\partial K(p)}{\partial p} - K(p) \right) \rho(p) n(\omega)$$

shear viscosity:

$$\eta = \frac{1}{3} \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \frac{\sum_{i \neq j} (p_i p_j)^2}{\omega^2 - \mathbf{p}^2} \left(\frac{\partial K(p)}{\partial p} \rho(p) \right)^2 \frac{1}{2T} \frac{1}{\cosh \frac{\omega}{T} - 1}$$

Non-local quasi-particles – Backup

energy-momentum tensor:

$$T^{\mu\nu}(k) = \int_p \int_q \left(\frac{p^\mu (q-p)^\nu}{q^2 - p^2} (K(p) - K(q)) - g^{\mu\nu} K(p) \right) \phi(p)^\dagger \phi(q) \delta(p+q-k)$$

U(1) Noether-current:

$$j^\mu(k) = \int_p \int_q \frac{(q-p)^\mu}{q^2-p^2} (K(p) - K(q)) \phi^\dagger(p) \phi(q) \delta(p+q-k)$$

$$\int_p \equiv \int \frac{\mathrm{d}^4 p}{(2\pi)^4}$$