Basic phenomenology for heavy-ion collisions: Lecture III

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Outline

LECTURE I

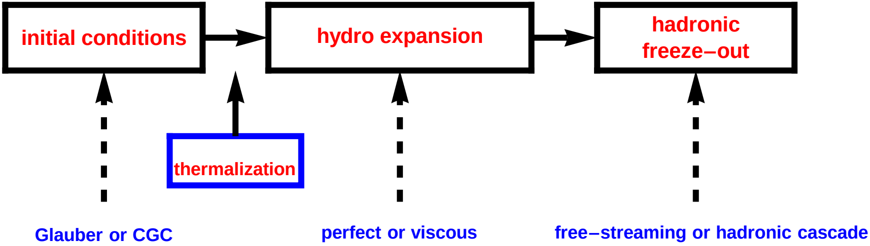
Outline

LECTURE II

Outline

LECTURE III

STANDARD MODEL (MODULES) of HEAVY–ION COLLISIONS



NEW: FLUCTUATIONS IN THE INITIAL STATE / EVENT–BY–EVENT HYDRO / FINAL–STATE FLUCTUATIONS

EQUATION OF STATE?

VISCOSITY?

5. HYDRODYNAMIC DESCRIPTION OF NUCLEAR COLLISIONS

The use of relativistic hydrodynamics to describe particle production in hadronic collisions has a long history which starts with the famous work of Landau in the early 1950s (Khalatnikov 1954, Belenkij 1956).

Landau's considerations were preceded, however, by a few approaches that used pure statistical and thermodynamic concepts in the analysis of the hadronic collisions (Koppe 1948,Koppe 1949,Fermi 1950). Such approaches may be regarded as **pre-hydrodynamic models** and it is important to present them before we turn to the discussion of the genuine hydrodynamic models.

5.1 Historical Perspective: Fermi Statistical Model

Fermi assumed that when two relativistic nucleons collide, **the energy available in their center-of-mass frame is released in a very small volume** V, whose magnitude corresponds to the Lorentz contracted characteristic pion field volume V_0, i.e., $V = 2m_{\mathrm{N}}\, V_0/\sqrt{s}$, where $V_0 = (4/3)\,\pi R_\pi^3$ with $R_\pi = 1/m_\pi$, and $\sqrt{s}$ is the center-of-mass energy.

Subsequently, **such a dense system decays into one of many accessible multiparticle states**. The decay probability is calculated in the framework of the standard statistical physics.

5.1 Historical Perspective: Fermi Statistical Model

The reason for the introduction of the statistical approach was the **breakdown of the perturbation theory in the attempts to describe strongly interacting systems**. Clearly, the large values of the coupling constant prohibit the application of the perturbation theory. On the other hand, the large value of the coupling is responsible for the phenomenon of multiple production of particles, which is a characteristic feature of strong interactions.

Generally speaking, the probability of the transition into a given state is proportional to the square of the corresponding matrix element and to the density of states. **In the statistical description the matrix elements are treated as constants and the main effect comes from the phase space**. Thus, the statistical approach represents a simple theoretical modeling of collisions which may be regarded as the complementary approach to the perturbation schemes which typically break down at a certain scale. The main heuristic argument for the justification of the use of the statistical approach is that the role of the phase space naturally grows with the increasing energy of the collisions.

5.1 Historical Perspective: Fermi Statistical Model

The probability for the formation of the state with n particles has the form

$$S(n) = \left[\frac{V}{(2\pi)^3}\right]^{(n-1)} \frac{dQ(W)}{dW}. \quad (1)$$

Here W is the total energy of the colliding system, dQ/dW is the number of states per unit energy, and V is the interaction volume. The power $n-1$ arises from the fact that in the center-of-mass frame the momenta of only $n-1$ particles are independent. We note that the probabilities $S(n)$ given by (1) are not normalized and the appropriate normalization should be done in the end of the calculations.

5.1.1 Fermi Statistical Model: pion production in low-energy nucleon collisions

Now assume that V is equal to V_0 and W is the kinetic energy of the two colliding nucleons – $T_{\rm kin}$. The phase space volume is the two-nucleon phase space corresponding to $T_{\rm kin}$. At low energy, i.e., for $T_{\rm kin}$ slightly exceeding the pion mass m_π, $Q_2(T_{\rm kin})$ may be calculated in the non-relativistic limit

$$\begin{aligned} Q_2(T_{\rm kin}) &= \int d^3q_1 d^3q_2\, \theta\left(T_{\rm kin} - \frac{q_1^2}{2m_{\rm N}} - \frac{q_2^2}{2m_{\rm N}}\right) \delta^{(3)}\left(\mathbf{q}_1 + \mathbf{q}_2\right) \\ &= \int d^3q_1 \theta\left(T_{\rm kin} - \frac{q_1^2}{m_{\rm N}}\right) = 4\pi \int_0^{\sqrt{T_{\rm kin}\, m_{\rm N}}} q_1^2 dq_1 = \frac{4\pi}{3}\left(T_{\rm kin}\, m_{\rm N}\right)^{3/2}. \end{aligned} \tag{2}$$

Here $\mathbf{q}_1$ and $\mathbf{q}_2$ are the final nucleon momenta and θ denotes the step function. According to Fermi's theory, the probability for the formation of such a state is given by the expression

$$S(2) = \frac{V}{(2\pi)^3} \frac{dQ_2}{dT_{\rm kin}} = \frac{V m_{\rm N}^{3/2}}{4\pi^2} T_{\rm kin}^{1/2}. \tag{3}$$

5.1.1 Fermi Statistical Model: pion production in low-energy nucleon collisions

Similarly, the non-relativistic volume of the three-particle phase space of two nucleons and a pion is obtained.

$$S(3) = \frac{V^2 m_{\rm N}^{3/2} m_\pi^{3/2} (\Delta T_{\rm kin})^2}{32\sqrt{2}\pi^3}. \qquad (4)$$

Here $\Delta T_{\rm kin} = T_{\rm kin} - m_\pi$ is the kinetic energy left over after a pion was created.

We may consider the ratio $S(3)/(S(2)+S(3)) \approx S(3)/S(2)$ as the probability of the pion formation,

$$\frac{S(3)}{S(2)} = \frac{V m_\pi^{3/2}}{8\sqrt{2}\pi} \frac{(T_{\rm kin} - m_\pi)^2}{T_{\rm kin}^{1/2}} \approx \frac{V m_\pi}{8\sqrt{2}\pi} (T_{\rm kin} - m_\pi)^2 . \qquad (5)$$

5.1.2 Fermi Statistical Model: from statistical to thermodynamic production

In the last chapter of his seminal paper from 1950, Fermi considers the collisions at extremely high energies. He argues that in this case a detailed statistical considerations may be replaced by the simple thermodynamic arguments. Assuming that the matter is thermalized one can calculate the temperature of the produced hadronic system from the thermodynamic relations valid for massless particles.

Fermi took into account only the production of pions, nucleons and antinucleons,

$$\left(\varepsilon_\pi + \varepsilon_{N+\bar{N}}\right) V = \frac{\pi^2 V T^4}{3} = W. \tag{6}$$

Using the expression for the Lorentz contracted volume we rewrite (6) in the following form

$$T^4 = \frac{3}{2\pi^2} \frac{W^2}{V_0 m_{\rm N}} = \frac{9}{8\pi^3} \frac{W^2 m_\pi^3}{m_{\rm N}}. \tag{7}$$

This equation may be used to calculate the abundances of the produced pions, nucleons and antinucleons from the thermodynamic relations giving the particle densities in terms of the temperature.

5.2 Landau Model

Fermi statistical model – the system reaches equilibrium at the stage of maximum compression and then instantaneously breaks up into multiparticle hadronic states.

Landau hydrodynamic model – instead of assuming instantaneous break-up, expansion of matter before the hadron decoupling.

The idea of modification of the Fermi approach was put forward by **Pomeranchuk** already in 1951 – the strong interaction between produced particles cannot cease immediately. The particles in the system should interact until the average distance between them becomes larger than the typical interaction distance, that is of the order of the inverse pion mass, m_π^{-1}.

Landau proposed his hydrodynamic approach to describe proton-proton collisions. Following Fermi, he assumed that the two colliding protons released their energy in the volume corresponding to the Lorentz-contracted size of a proton.

Under the influence of the longitudinal gradient the system starts expanding. The transverse gradient is also present but initially the gradient in the longitudinal direction is much larger and **the early expansion may be regarded as one-dimensional**.

5.2 Landau Model

One-dimensional expansion of matter along the collision axis z, the equations of **perfect-fluid relativistic hydrodynamics with zero baryon chemical potential**,

$$\begin{aligned} u^0 \partial_0 (T u^0) + u^3 \partial_3 (T u^0) &= \partial^0 T, \\ u^0 \partial_0 (T u^3) + u^3 \partial_3 (T u^3) &= \partial^3 T. \end{aligned} \tag{8}$$

Equations (8) together with the normalization condition for four-velocity give

$$\frac{\partial}{\partial x^0} \left(T u_3 \right) = \frac{\partial}{\partial x^3} \left(T u_0 \right), \tag{9}$$

which means that $T u_0$ and $T u_3$ may be written as the derivatives of a potential field $\Phi_{\rm L}$,

$$T u_0 = -\partial_0 \Phi_{\rm L}, \quad T u_3 = -\partial_3 \Phi_{\rm L}. \tag{10}$$

The total differential of the potential $\Phi_{\rm L}$ is then

$$d\Phi_{\rm L} = \partial_0 \Phi_{\rm L}\, dx^0 + \partial_3 \Phi_{\rm L}\, dx^3 = -T u^0\, dt + T u^3\, dz. \tag{11}$$

5.2.1 Landau Model: Khalatnikov equation

The next convenient step is to perform the **Legendre transformation** and switch from the potential $\Phi_{\rm L}$ to the potential χ defined as

$$\chi = \Phi_{\rm L} + Tu^0 t - Tu^3 z. \tag{12}$$

The total differential of χ is (with $u^0 = \cosh\vartheta$, $u^3 = \sinh\vartheta$)

$$\begin{aligned} d\chi &= \left(u^0 t - u^3 z\right) dT + tT\, du_0 - Tz\, du^3 \\ &= (t\cosh\vartheta - z\sinh\vartheta)\, dT + (t\sinh\vartheta - z\cosh\vartheta)\, T d\vartheta. \end{aligned} \tag{13}$$

From the entropy conservation

$$\frac{\partial}{\partial t}(\sigma\cosh\vartheta) + \frac{\partial}{\partial z}(\sigma\sinh\vartheta) = 0. \tag{14}$$

we come to the single, partial differential equation for χ,

$$\frac{1}{\sigma}\frac{d\sigma}{dT}\left(\frac{\partial\chi}{\partial T} - \frac{1}{T}\frac{\partial^2\chi}{\partial\vartheta^2}\right) + \frac{\partial^2\chi}{\partial T^2} = 0. \tag{15}$$

5.2.1 Landau Model: Khalatnikov equation

Further simplifications may be achieved if we introduce

$$Y = \ln\left(\frac{T}{T_{\rm i}}\right). \tag{16}$$

In this way we obtain the **Khalatnikov equation**

$$\frac{\partial^2\chi}{\partial\vartheta^2} - c_{\rm s}^2\frac{\partial^2\chi}{\partial Y^2} + (c_{\rm s}^2 - 1)\frac{\partial\chi}{\partial Y} = 0. \tag{17}$$

For constant sound velocity it becomes a partial differential equation with constant coefficients. For example, in the case $c_{\rm s}^2 = 1/3$,

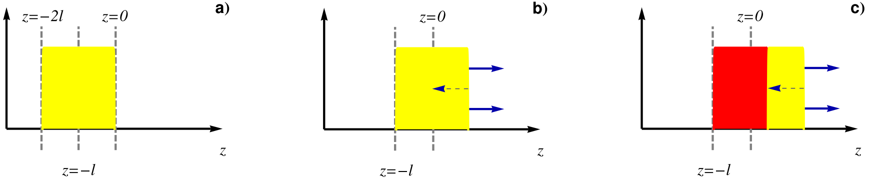
$$3\frac{\partial^2\chi}{\partial\vartheta^2} - \frac{\partial^2\chi}{\partial Y^2} - 2\frac{\partial\chi}{\partial Y} = 0. \tag{18}$$

5.2.1 Landau Model: Khalatnikov equation

(a) at the beginning, matter forms a highly compressed disk of the width $\Delta = 2l$, because of the reflection symmetry with respect to the plane $z = -l$ we may consider $z \geq -l$,

(b) next, the evolution of matter consists of the expansion into vacuum (indicated by the two solid arrows) and the rarefaction wave entering the system (indicated by the dashed arrow),

(c) after $t_0 = l/c_s$, when the rarefaction wave hits the plane $z = -l$, the evolution of the central region becomes quite complicated, it consists of the incident rarefaction wave and the reflected waves, in the outer region the simple Riemann solution always holds, and should be matched to the non-trivial solution found by Khalatnikov.



5.2.2 Landau Model: approximate solution

Landau finds that in the leading order of magnitude the function χ is given by

$$\chi = -l T_{\rm i} \exp\left[-Y + \sqrt{Y^2 - c_{\rm s}^2 \vartheta^2}\right] \qquad (19)$$

where

$$l = \frac{\Delta}{2} = R\,\frac{2m_{\rm N}}{\sqrt{s_{\rm NN}}}. \qquad (20)$$

Landau further argues that the end of one-dimensional motion takes place when

$$t^2 - z^2 = R_{\rm f}^2 \quad \text{(connection between } t \text{ and } z,\ R_{\rm f} \approx 2R). \qquad (21)$$

New variable L is introduced

$$L = -2Y + \sqrt{Y^2 - \vartheta^2/3} \quad \text{(connection between } T \text{ and } \theta). \qquad (22)$$

$$L = \ln\left(\frac{R_{\rm f}}{\Delta}\right) = \ln\left(\frac{\sqrt{s_{\rm NN}}}{2m_{\rm N}}\right). \qquad (23)$$

5.2.2 Landau Model: approximate solution

Rapidity profile of temperature,

$$T = T_{\rm i} \exp\left[-\frac{1}{3}\left(2L - \sqrt{L^2 - \vartheta^2}\right)\right]. \tag{24}$$

Similarly, we find the **rapidity profile of the entropy density**,

$$\sigma = \sigma_{\rm i} \exp\left[-2L + \sqrt{L^2 - \vartheta^2}\right]. \tag{25}$$

In order to calculate the rapidity distribution we first consider a thin slice of the fluid. Since the fluid element moves in the center-of-mass frame with the velocity $v = \tanh\vartheta$, may write

$$dS = \pi R^2\, R_{\rm f}\, \sigma_{\rm i} \exp\left[-2L + \sqrt{L^2 - \vartheta^2}\right]\, d\vartheta. \tag{26}$$

For $\vartheta \ll L$ we find

$$\frac{dS}{d\vartheta} = \pi R^2\, R_{\rm f}\, e^{-L}\, \sigma_{\rm i} \exp\left(-\frac{\vartheta^2}{2L}\right). \tag{27}$$

If we identify the entropy density with the particle density and the fluid rapidity ϑ with the particle rapidity y we may write

$$\frac{dN}{d\mathrm{y}} = \frac{N}{(2\pi L)^{1/2}} \exp\left(-\frac{\mathrm{y}^2}{2L}\right). \tag{28}$$

GAUSSIAN RAPIDITY DISTRIBUTION!

5.3 Bjorken Model

Landau model – initial conditions are specified for a given laboratory time, considered in the center-of-mass frame, when the matter is highly compressed and at rest.

Landau's description looses one aspect of high-energy hadronic collisions – fast particles are produced later and further away from the collision center than the slow ones. It is possible to account for this effect in the hydrodynamic description by imposing special initial conditions. This idea was proposed and studied by Bjorken.

The Bjorken hydrodynamic model (Bjorken, 1983) was based on the assumption that the rapidity distribution of the charged particles, $dN_{\rm ch}/d{\rm y}$, is constant in the mid-rapidity region. This fact means that the central region is invariant under Lorentz boosts along the beam axis. This in turn implies that the longitudinal flow has the form $v_z = z/t$ and all thermodynamic quantities characterizing the central region depend only on the longitudinal proper time $\tau = \sqrt{t^2 - z^2}$ and the transverse coordinates x and y.

5.3.1 Pure longitudinal expansion

One-dimensional hydrodynamic model where thermodynamic variables are functions of the longitudinal proper time only,

$$\varepsilon = \varepsilon(\tau), \qquad P = P(\tau), \qquad T = T(\tau), \qquad \text{etc.} \tag{29}$$

The initial conditions for the hydrodynamic expansion are imposed along the hyperbola of the constant proper time

$$\sqrt{t^2 - z^2} = \tau_{\rm i}. \tag{30}$$

In this way one accounts for the time dilation effects characterizing the particle production. The fluid four-velocity field has the form

$$u^\mu = \frac{1}{\tau}(t,0,0,z) = \gamma\left(1,0,0,\frac{z}{t}\right). \tag{31}$$

This form implies

$$\partial_\mu u^\mu = \frac{1}{\tau}. \tag{32}$$

5.3.1 Pure longitudinal expansion

We identify all kinds of rapidities

$$\mathrm{y} = \text{arctanh}\left(\mathrm{v}_{\parallel}\right) = \text{arctanh}\left(\frac{z}{t}\right), \tag{33}$$

$$t = \tau \cosh \mathrm{y}, \qquad z = \tau \sinh \mathrm{y}. \tag{34}$$

At high energies, the baryon number density in the central region is negligible. The entropy conservation gives

$$\partial_\mu \left(\sigma u^\mu\right) = \frac{d\sigma\left(\tau\right)}{d\tau} + \frac{\sigma\left(\tau\right)}{\tau} = 0. \tag{35}$$

The solution of this equation is

$$\sigma\left(\tau\right) = \sigma\left(\tau_{\mathrm{i}}\right) \frac{\tau_{\mathrm{i}}}{\tau}. \tag{36}$$

On the other hand, for the energy density we find

$$\frac{d\varepsilon}{\varepsilon + P} = -\frac{d\tau}{\tau}. \tag{37}$$

This equation can be solved if the equation of state is known. For ultra-relativistic particles $P = \lambda\,\varepsilon$ (with $\lambda = c_s^2 = 1/3$) and we find

$$\varepsilon\left(\tau\right) = \varepsilon\left(\tau_{\mathrm{i}}\right) \left(\frac{\tau_{\mathrm{i}}}{\tau}\right)^{1+\lambda}. \tag{38}$$

5.3.2 Simple estimates

In the reference frame where the fluid element is at rest we have

$$d^3x = d^2x_\perp \tau \, d\mathrm{y}. \tag{39}$$

Thus the entropy contained in the interval $d\mathrm{y}$ around $\mathrm{y} = 0$ is

$$dS = \tau \, \sigma(\tau) \int d^2x_\perp \, d\mathrm{y} \tag{40}$$

and

$$\frac{d}{d\tau}\left[\frac{dS}{d\mathrm{y}}\right] = \int d^2x_\perp \frac{d}{d\tau}\left[\tau \, \sigma(\tau)\right] = 0. \tag{41}$$

This result allows us to make the simple estimate of the energy density achieved in the central region of the ultra-relativistic heavy-ion collisions. We first calculate the entropy density $\sigma(\tau_\mathrm{i})$ at the time τ_i when the hydrodynamic description starts

$$\sigma(\tau_\mathrm{i}) = \frac{1}{\tau_\mathrm{i}\,\mathcal{A}} \frac{dS}{d\mathrm{y}}(\mathrm{y}=0) \approx \frac{3.6}{\tau_\mathrm{i}\,\mathcal{A}} \frac{dN}{d\mathrm{y}}(\mathrm{y}=0). \tag{42}$$

Here we used the result $\sigma/n = 2\pi^4/(45\zeta(3)) \approx 3.6$, the transverse overlap area of the two colliding nuclei $\mathcal{A} = \pi\left(3A/(4\pi\rho_0)\right)^{2/3}$, where A is the mass number of the nuclei and ρ_0 is the nuclear saturation density,

$$\varepsilon(\tau_\mathrm{i}) = \left[\frac{1215\,\sigma^4(\tau_\mathrm{i})}{128\,g\,\pi^2}\right]^{\frac{1}{3}} \approx \frac{5.4}{g^{\frac{1}{3}}}\left[\frac{1}{\tau_\mathrm{i}\,\mathcal{A}}\frac{dN}{d\mathrm{y}}\right]^{\frac{4}{3}}. \tag{43}$$

5.3.2 Simple estimates

For the most energetic ($\sqrt{s_{\rm NN}} = 200$ GeV) central Au + Au collisions studied at RHIC, the multiplicity of charged particles per unit rapidity at $\mathrm{y} = 0$ is about 600, hence

$$\frac{dN}{d\mathrm{y}} \approx \frac{3}{2} 600 \approx 900. \tag{44}$$

Taking $g_\pi = 3$ and the standard value $\tau_{\rm i} = 1$ fm, one gets

$$\varepsilon_{\rm pions}(\tau_{\rm i}) = 9.6 \, \frac{\mathrm{GeV}}{\mathrm{fm}^3}. \tag{45}$$

This suggests that the quark-gluon plasma might have been formed at the initial stages of the collisions. However, since the concept of the pion gas at such a high energy density breaks down, we should make another estimate, with the degeneracy factor reflecting the number of the internal degrees of freedom in the plasma phase.

5.3.2 Simple estimates

Assuming that the system at $\tau = \tau_{\rm i}$ is the weakly-interacting quark-gluon plasma that may be effectively regarded as an ideal gas of massless bosons with $g_{\rm qgp}$ degrees of freedom we may again use (43). For $g_{\rm qgp} = 37$, one finds

$$\varepsilon_{\rm qgp}\left(\tau_{\rm i}\right) \approx 4.2 \, \frac{\rm GeV}{\rm fm^3}. \tag{46}$$

With the inclusion of strangeness, the number of the internal degrees of freedom in the plasma is larger, $g_{\rm qgp} = 47.5$, and

$$\varepsilon_{\rm qgp}\left(\tau_{\rm i}\right) \approx 3.8 \, \frac{\rm GeV}{\rm fm^3}. \tag{47}$$

We conclude that the two values, (46) and (47), are again consistent with the hypothesis of the plasma formation.

5.4 Hydrodynamic modeling of the data

Modern hydrodynamic calculations follow general concepts introduced in the Landau and Bjorken models. However, they differ from the Landau original description in the way how they treat the initial conditions. They also **go beyond the simple Bjorken approach by including transverse expansion and, eventually, by breaking the boost-invariance**. Additionally, the recent hydrodynamic codes use modern equations of state inspired by the lattice simulations of QCD and advanced hadron-gas calculations.

In the Landau and Bjorken models, the thermalized matter was a gas of ultra-relativistic particles, mainly pions, satisfying the extreme relativistic equation of state $P = \varepsilon/3$. At present, more accurate equations of state for hot and dense matter are known. **Expecting the phase transition to the quark-gluon plasma, one can use the plasma equation of state including the phase transition back to ordinary hadronic matter**.

The fact that the phase transition may be easily implemented in the hydrodynamic description is one of the great advantages of using hydrodynamics as a tool to describe the evolution of matter formed in heavy-ion collisions.

5.4 Hydrodynamic modeling of the data

The hydrodynamic codes may be divided into three classes: The first class consists of the boost-invariant models, whose validity is restricted to the central rapidity region (Huovinen, Kolb, Heinz, Ruuskanen, Voloshin, Teaney, Shuryak, Rapp). Such models are usually called the 2+1 hydrodynamic models. The number 2 refers in this case to the two non-trivial space dimensions and the number 1 refers to one time dimension. The evolution in the third space dimension is trivial due to the assumption of boost-invariance.

The second class consists of fully three-dimensional models, which are known as 3+1 codes (Nonaka, Hirano, Morita). Finally, the third class includes the hydrodynamic models which take into account the effects of viscosity and other transport phenomena (Heinz, Chaudhuri, Baier, Romatschke, Song, Luzum, Denicol, Bozek, Schenke).

The inclusion of the viscous effects and fluctuations represents the forefront of the present investigations in the field of ultra-relativistic heavy-ion collisions.

there is a lower bound on the shear viscosity $\rightarrow$ one should do the viscous hydro
Danielewicz, Gyulassy, PRD31, 53 (1985)
Kovtun, Son, Starinets, PRL94, 111601 (2005)

5.5 HBT Puzzle

Long time evolution is good for v_2 but bad for HBT!

- W. Broniowski, M. Chojnacki, WF, A. Kisiel, PRL **101** (2008) 022301
- S. Pratt, PRL **102** (2009) 232301 (considerations without v_2)

However, with several improvements done in the hydrodynamic models, the HBT puzzle is practically eliminated (discrepancies smaller than 10%)

- realistic equation of state (++)
- early start of hydrodynamics (++)
- modified initial conditions (+-)
- shear viscosity included (-+)
- fluctuations of the initial eccentricity (+-)
- two-particle method for the correlation functions important (++)
- Coulomb corrections not important (++)
- fast freeze-out (+-)

5.6 Early Thermalization Puzzle

Good description of the data requires early start of hydrodynamics!

The most accepted solution: plasma is strongly interacting!

QGP $\rightarrow$ sQGP

Even if the plasma is strongly interacting, as described by the AdS/CFT correspondence, Janik, the system produced initially in the collisions is highly anisotropic.

This suggests reorganization of the hydrodynamic expansion as discussed in the lectures by Strickland.

The data can be described also with a delayed thermalization (delayed approach towards local thermodynamic equilibrium, 1–2 fm/c), Ryblewski.

6. QCD PHASE TRANSITION IN THE EARLY UNIVERSE

6.1 Friedmann equation

for isentropic expansion one may use the Friedmann equation

$$\frac{d\varepsilon_R}{dt} = -3\sqrt{\frac{8\pi G\varepsilon_R}{3}}(\varepsilon_R + P_R)$$

ε_R and P_R are the total energy density and pressure
combined strong and electro-weak sector

$$\varepsilon_R = \varepsilon + \varepsilon_{\rm ew}, \quad P_R = P + P_{\rm ew}$$

$$\varepsilon_{\rm ew} = g_{\rm ew}\frac{\pi^2}{30}T^4, \quad P_{\rm ew} = g_{\rm ew}\frac{\pi^2}{90}T^4, \quad \sigma_{\rm ew} = \frac{4\varepsilon_{\rm ew}}{3T}$$

6.1 Friedmann equation

temperature may be treated as an independent variable

$$\left[c_s^{-2}\sigma + 3\sigma_{\rm ew}\right]\frac{dT_R}{dt} = -3\sqrt{\frac{8\pi G(\varepsilon+\varepsilon_{\rm ew})}{3}}(\varepsilon+\varepsilon_{\rm ew}+P+P_{\rm ew})$$

initial condition

$$T_R(t_0) = 500\,\mathrm{MeV}$$

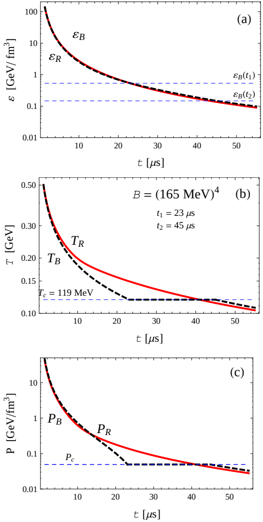
corresponding energy density

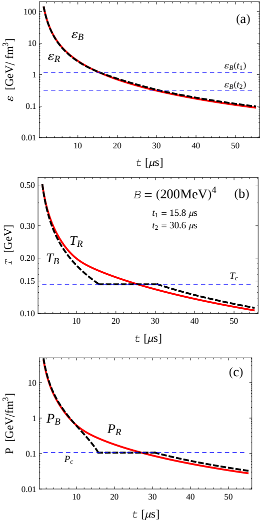
$$\varepsilon_{R,0} = \varepsilon(T_R(t_0)) + \varepsilon_{\rm ew}(T_R(t_0)) = 138\,\mathrm{GeV/fm}^3$$

assuming that for $t < t_0$ the evolution is dominated by radiation, we get

$$t_0 = \sqrt{\frac{3}{32\pi G\varepsilon_{R,0}}} = 1.35\,\mu s$$

6.2 Solutions of Friedmann equation





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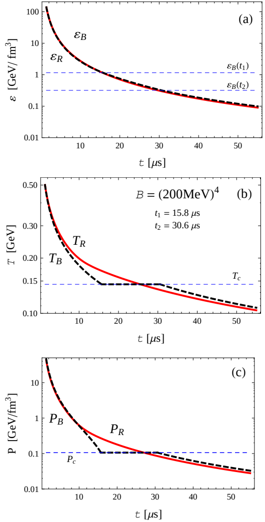
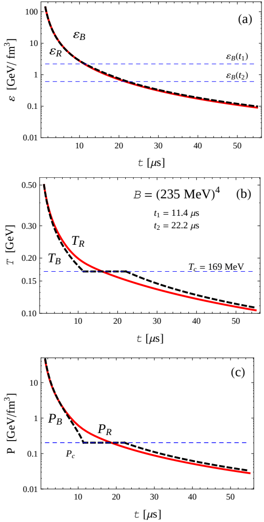
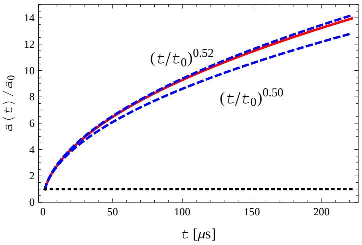




6.3 Hubble's constant and scale factor

time changes of the scale factor $a(t)$

$$a(t) = a(t_0) \exp\left[\int_{t_0}^{t} \sqrt{\frac{8\pi G \varepsilon_R(t')}{3}} dt' \right]$$

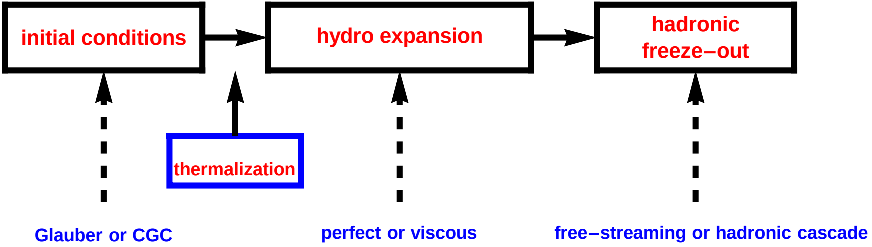


$$\frac{a(t)}{a(t_0)} = (t/t_0)^p, \quad p = 0.517, \quad H = \frac{da}{a\,dt} = \frac{p}{t}, \quad \frac{d\eta}{dt} = \frac{1}{a}, \quad \mathcal{H} = aH = \frac{da}{a\,d\eta}$$

H – Hubble's constant, η – conformal time

7. CONCLUSIONS

STANDARD MODEL (MODULES) of HEAVY–ION COLLISIONS



NEW: FLUCTUATIONS IN THE INITIAL STATE / EVENT–BY–EVENT HYDRO / FINAL–STATE FLUCTUATIONS

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Shuryak: *Heavy ion collisions, described well by hydro/thermodynamics, are in fact much simpler than more "elementary" pp or e+ e- collisions.*