Causal Structure for Noncommutative Geometry

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Joint project with Nicolas Franco(CC, Kraków)
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INNOVATIVE ECONOMY
NATIONAL COHESION STRATEGY

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DEVELOPMENT FUND

Introduction & motivation

- Why (non)commutative geometry?
  - Dual description of geometry - forget about points!
  - New noncommutative horizons
- Drawbacks of the standard spectral approach
  - Relativistic physics is Lorentzian not Riemannian
  - Applications - need for a Wick rotation ($t \to it$)
  - We loose the causal structure
- Lorentzian spectral triples - a remedy?
  - Wick rotation implemented in a controllable way
  - Can encompass the causal structure

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Outline

The axioms of noncommutative geometry

$(\mathcal{A}, \mathcal{H}, \mathcal{D})$ - spectral triple

- $\mathcal{A}$ - pre-C^*-algebra (unital)
- $\mathcal{H}$ - Hilbert space
  $\exists$ a faithful representation $\pi(\mathcal{A}) \subset \mathcal{B}(\mathcal{H})$
- $\mathcal{D}$ - the Dirac operator - selfadjoint, unbounded
  - $(\mathcal{D} - \lambda)^{-1}$ for any $\lambda \notin \mathbb{R}$- compact resolvent
  - $[\mathcal{D}, \pi(a)] \in \mathcal{B}(\mathcal{H})$ for all $a \in \mathcal{A}$
- $\ldots$
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Lorentzian spectral triples

$(\mathcal{A}, \widetilde{\mathcal{A}}, \mathcal{H}, \mathcal{D}, \mathfrak{J})$ - Lorentzian spectral triple

- A Hilbert space $\mathcal{H}$.
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- An unbounded operator $\mathcal{D}$ densely defined on $\mathcal{H}$ such that, $\forall a \in \widetilde{\mathcal{A}}$:
  - $[\mathcal{D}, a]$ extends to a bounded operator on $\mathcal{H}$,
  - $a\Delta_{\mathfrak{J}}^{-1}$ is compact, with $\Delta_{\mathfrak{J}} := \left(\frac{1}{2}(\mathcal{D}\mathcal{D}^* + \mathcal{D}^*\mathcal{D}) + 1\right)^{1/2}$.
- A bounded operator $\mathfrak{J}$ on $\mathcal{H}$ - fundamental symmetry - such that:
  - $\mathfrak{J}^2 = 1$,
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Globally hyperbolic manifold

A commutative Lorentzian spectral triple on a Lorentzian manifold M

- $\mathcal{A} \subset C_0^\infty(M)$ - smooth functions vanishing at ∞
- $\widetilde{\mathcal{A}} \subset C_b^\infty(M)$ - smooth bounded functions with bounded derivatives
- $\mathcal{H} = L^2(M, S)$ - Hilbert space of square integrable spinor sections over M .
- $\mathcal{D} = -i(c \circ \nabla^S) = -i\gamma^\mu \nabla_\mu^S$ is the Dirac operator.
- spacelike reflection $r \in \mathrm{Aut}(TM)$, $r^2 = 1$, $g(r\cdot, r\cdot) = g(\cdot, \cdot)$
  $g^r(\cdot, \cdot) := g(\cdot, r\cdot)$ - positive definite metric on $TM = F^- \oplus F^+$
- $\mathfrak{J}_r$ - fundamental symmetry associated with r

$$\mathfrak{J}_r c(e_0) \mathfrak{J}_r = -c(re_0), \qquad \mathfrak{J}_r = ic(e_0) = i\gamma^0$$

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Causality - a reminder

- Two points p, q are **causally related** $p \preceq q$ iff $p = q$ or $\exists$ a future directed causal curve linking p and q.
- $\preceq$ induces a partial order relation on the set of points of M.
- Causal futures and pasts
  - $J^+(p) = \{q \in M \ : \ p \preceq q\}$ - causal future of p.
  - $J^-(p) = \{q \in M \ : \ q \preceq p\}$ - causal past of p.
- **global hyperbolicity** $\Longrightarrow$ no closed causal curves

Theorem [Geroch (1967)]

Compact Lorentzian manifold always contain closed causal curves.

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Gelfand - Naimark theorem [1943]

commutative C^*-algbras $\overset{1:1}{\longleftrightarrow}$ (locally) compact Hausdorff topological spaces

- States $S(\mathcal{A}) = \{\varphi\}$ on $\mathcal{A}$:
  - positive linear functionals with $\|\varphi\| = 1$
  - $S(\mathcal{A})$ is a closed convex set
  - $P(\mathcal{A})$ - extremal points - pure states
- Points of $X \overset{1:1}{\longleftrightarrow} P(C(X)) \quad \forall_{x \in X} \quad \chi_x : \mathcal{A} \to \mathbb{C}, \quad \chi_x(f) := f(x)$

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$\mathcal{C}(M) = \{f \in C^\infty(M, \mathbb{R}) \ : \ f -$ non-decreasing along future dir. causal curves$\}$

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A causal cone $\mathcal{C}$ is a subset of elements in $\widetilde{\mathcal{A}}$ such that:

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Theorem (N. Franco, M.E. [2013])

Let $(\mathcal{A}, \widetilde{\mathcal{A}}, \mathcal{H}, \mathcal{D}, \mathfrak{J})$ be a commutative Lorentzian spectral triple constructed from a globally hyperbolic Lorentzian manifold M. Then,

$$P(\mathcal{A}) \cong \mathrm{Spec}(\mathcal{A}) \cong M,$$

and the partial order relation $\preceq$ on $S(\widetilde{\mathcal{A}})$ restricted to $P(\mathcal{A})$ corresponds to the usual causal relation on M.

Causal future and past of states

Let $X = S(\widetilde{\mathcal{A}})$, $P(\widetilde{\mathcal{A}})$, $P(\mathcal{A})$, for every $\chi \in X$ define

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Almost commutative causality

Noncommutative "extra" dimensions

Consider $\mathcal{A} = M_2(\mathbb{C})$, then $\dim_g(\mathcal{A}) = 0$, but $P(\mathcal{A}) \cong S^2$.

- Take $\mathcal{A} = C_0^\infty(M) \otimes M_2(\mathbb{C})$
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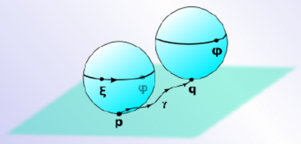

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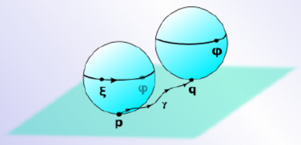

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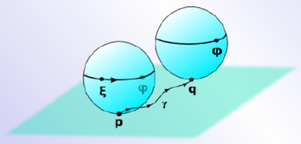

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Outline

Take-home messages

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- Is there a hidden causal structure in gauge theories?
- Is the Universe commutative? - Check it twice!

Thank you for your attention!

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