Lifshitz solutions and flows in Type IIB Supergravity

Philipp Burda

Centre for Particle Theory, Durham University
Durham, UK

(Work in progress with Prof. Ruth Gregory and Prof. Simon Ross)

2013 Cracow School of Theoretical Physics
Zakopane, June 29, 2013

Strings and Branes

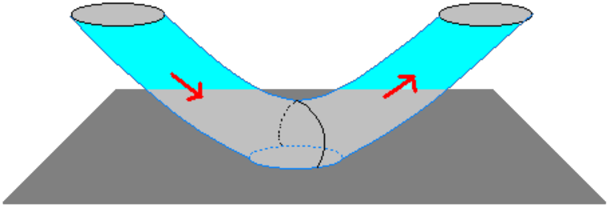


Figure : Any process in string theory could be seen from both, open and closed strings perspective, this suggests, co-called String-duality or Open/Closed string duality

D branes: Dp brane is a $p+1$-dimensional hypersurface on which open strings can end.

AdS/CFT Correspondence

$$\begin{array}{ccccc} \text{String-duality} & \rightarrow & \text{Open string pic.} & \longleftrightarrow & \text{Closed string pic.} \\ & & \downarrow & & \downarrow \\ \text{D3 branes} & \rightarrow & \mathcal{N}=4 \text{ SYM} & \longleftrightarrow & \text{SUGRA on } AdS_5 \times S_5 \end{array}$$

D3 branes have two equivalent descriptions in *low-energy limit*:

- Action for massless modes

$$S_1 = S_{DBI} + S_{bulk} + S_{int}(\sim \alpha'^2) \overset{\alpha' \to 0}{=} S_{SYM} + S_{freeSUGRA}$$

- Polyakov action for σ-model of strings on D3 background

$$S_2 = \frac{1}{\alpha'} \int d^2\sigma \sqrt{h} h^{ab} G_{MN} \partial_a x^M \partial_b x^N \overset{\alpha' \to 0}{=} S_{String} + S_{freeSUGRA}$$

$$S_1 = S_2$$

$$\Downarrow$$

$$\boxed{4d\, \mathcal{N} = 4 \text{ SYM in flat space } \leftrightarrow \text{ Type IIB string theory on } AdS_5 \times S_5}$$

Matching of Symmetries

- $\mathcal{N} = 4$ SYM – superconformal relativistic field theory in $4d$, hence the symmetry of the theory is present by superconformal group SU(2,2|4), its subgroup in bosonic sector is

$$SU(2,2|4) \supset SO(2,4) \times SU(4)_R$$

- Strings on $AdS_5 \times S_5$ - isometries of spacetime came from AdS_5 piece - SO(2,4), and from sphere S_5 - SO(6) ($\simeq$ SU(4)). So here group of isometries is

$$SO(2,4) \times SU(4)$$

Symmetry of boundary gauge theory coincides with isometries of supergravity geometry in the bulk. Hence, in order to consider more realistic field theories (break conformal and/or relativistic invariance) one needs to proceed towards more involved geometries in the bulk!

Lifshitz spacetime

Lifhitz scalling

$$t \to \lambda^z t, \quad x \to \lambda x \quad (z \neq 1)$$

In context of holography, metric of Lishitz spacetime reads

$$ds^2 = r^{2z} dt^2 - \frac{dr^2}{r^2} - r^2 d\vec{x}^2$$

and $z \to 1$ being AdS limit

- Important for many physical applications of holography to the real physical systems (AdS/CMT). Some quantum critical points, itinerant fermion systems etc.
- It has recently been shown, that one can find solutions in 6D and 5D Romans gauge supergravity theories, which respect Lifshtiz scaling. They can be uplifted to the Type IIA and Type IIB higher dimensional SUGRA theories, and after that be interpreted in terms of the intersecting D3 branes.

Type IIB SUGRA via 5d

Lagrangian of bosonic part of 5D Romans SUGRA theory reads

$$\mathcal{L} = -\frac{R}{4} + \frac{1}{2}D_\mu\phi D^\mu\phi - \frac{1}{4}\xi^{-4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{4}\xi^2\Big(F^{(i)}_{\mu\nu}F^{\mu\nu(i)} + \\ + B^{\mu\nu\alpha}B^a_{\mu\nu}\Big) + \frac{1}{4}\epsilon^{\mu\nu\rho\sigma\lambda}\left(\frac{1}{g_1}\epsilon_{\alpha\beta}B^\alpha_{\mu\nu}D_\rho B^\beta_{\sigma\lambda} - F^{(i)}_{\mu\nu}F^{(i)}_{\rho\sigma}\mathcal{A}_\lambda\right) + P(\phi)$$

here $\xi = e^{\sqrt{\frac{2}{3}}\phi}$, scalar field potential is

$$P(\phi) = \frac{g_2}{8}\left(g_2\xi^{-2} + 2\sqrt{2}g_1\xi\right)$$

and field strengths are

$$\mathcal{F}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu$$
$$F^{(i)}_{\mu\nu} = \partial_\mu A^{(i)}_\nu - \partial_\nu A^{(i)}_\mu + g_2\epsilon^{ijk}A^{(j)}_\mu A^{(k)}_\nu$$

Theory has two independent parameters g_2 and g_1. This gives rise to the three physically distinct theories, when $g_1 g_2 > 0$, when $g_2 = 0$ and when $g_1 g_2 < 0$. We restrict ourself to the first case, i.e. impose $g_1 g_2 > 0$.

e.o.m. of Type IIB SUGRA via 5d

$$
\begin{aligned}
R_{\mu\nu} &= 2\partial_\mu\phi\partial_\nu\phi + \frac{4}{3}g_{\mu\nu}P(\phi) - \xi^{-4}\Big(2\mathcal{F}_{\mu\rho}\mathcal{F}^{\rho}_{\nu} - \\
&\quad -\frac{1}{3}g_{\mu\nu}\mathcal{F}_{\rho\sigma}\mathcal{F}^{\rho\sigma}\Big) - \xi^{2}\left(2F^{(i)}_{\mu\rho}F^{\rho(i)}_{\nu} - \frac{1}{3}g_{\mu\nu}F^{(i)}_{\rho\sigma}F^{\rho\sigma(i)}\right) \\
\Box\phi &= \frac{\partial P}{\partial\phi} + \sqrt{\frac{2}{3}}\xi^{-4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \sqrt{\frac{1}{6}}\xi^{2}F^{(i)}_{\rho\sigma}F^{\rho\sigma(i)} \\
D_\nu\left(\xi^{-4}\mathcal{F}^{\nu\mu}\right) &= \frac{1}{4}\epsilon^{\mu\nu\rho\sigma\tau}F^{(i)}_{\nu\rho}F^{(i)}_{\sigma\tau} \\
D_\nu\left(\xi^{2}F^{\nu\mu(i)}\right) &= \frac{1}{2}\epsilon^{\mu\nu\rho\sigma\tau}F^{(i)}_{\nu\rho}\mathcal{F}_{\sigma\tau} \\
\epsilon^{\mu\nu\rho\sigma\lambda}\epsilon_{\alpha\beta}D_\rho B^{\beta}_{\sigma\lambda} &= g_1 B^{\mu\nu}_{\alpha}
\end{aligned}
$$

Ansatz for the metric:

$$
ds^2 = r^{2z}dt^2 - r^2dx^2 - e^{2d_0}\cdot dr^2/r^2 - e^{2h_0}/y_2^2\cdot\left(dy_1^2 + dy_2^2\right)
$$

Looking for solutions in the form of $AdS_3 \times H_2$ and $Li_3 \times H_2$

AdS and Li Solutions

Ansatz for the fields:

$$\begin{aligned} \mathcal{F}_{rt} &= \xi_0^2 A e^{d_0} & \mathcal{F}_{rx} &= \xi_0^2 \tilde{B} e^{d_0} \\ F^{(3)}_{rt} &= \xi_0^{-1} \tilde{A} e^{d_0} & F^{(3)}_{rx} &= \xi_0^{-1} B e^{d_0} \end{aligned}$$

$$\xi = \xi_0 = e^{\sqrt{\frac{2}{3}}\phi_0} \qquad F^{(3)}_{y_1 y_2} = q/y_2^2$$

charge q in the flux through hyperbolic space is conserved quantity

- AdS solution: $A = B = \tilde{A} = \tilde{B} = 0$, $e^{-2d_0} = \sqrt{2}\xi/4, e^{-2h_0} = \xi^{-1}/2$, and charge $q^2 = \xi^3\sqrt{2} - 1$
- Li solutions:
  1. tilded Lifshitz $(z \geq 1)$, $A = B = 0$,

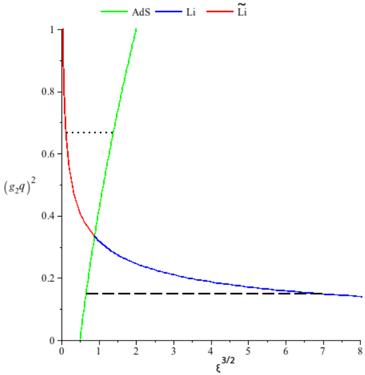
$$\begin{aligned} \tilde{A}^2 &= z(z-1)/2e^{-2d_0} & \tilde{B}^2 &= (z-1)/2e^{-2d_0} \\ q^2 &= (2z^2 + 3z - 2)/9z & \xi^3 &= \sqrt{2}(z+1)/(2z^2 + 3z - 2) \end{aligned}$$

  2. untilded Lifshitz $(1 \leq z \geq 2)$, $\tilde{A} = \tilde{B} = 0$,

$$\begin{aligned} A^2 &= z(z-1)/2e^{-2d_0} & B^2 &= (z-1)/2e^{-2d_0} \\ q^2 &= (-2z^2 + 3z + 2)/9z & \xi^3 &= \sqrt{2}z(z+1)/(-2z^2 + 3z + 2) \end{aligned}$$

Flows

We can plot our solutions:



We will look for solutions of our SUGRA theory with non-zero and non-constant fluxes with different asymptotic behavior in different regions of spacetime (near the boundary, where $r \to \infty$, and near horizon when $r \to 0$)

Ansatz for flows

for the metric:

$$ds^2 = e^{2F(r)}dt^2 - r^2dx^2 - e^{2d(r)}\frac{dr^2}{r^2} - e^{2h(r)}d\Omega^2_{2,-1}$$

for the fields:

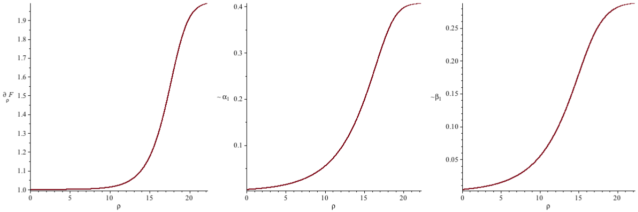
$$\begin{array}{rclcrcl}
\mathcal{F}_{rt} & = & \xi^2 A(r)e^{F(r)+d(r)} & \qquad & \mathcal{F}_{rx} & = & \xi^2\tilde{B}(r)e^{d(r)} \\
F^{(3)}_{rt} & = & \xi^{-1}\tilde{A}(r)e^{F(r)+d(r)} & \qquad & F^{(3)}_{rx} & = & \xi^{-1}B(r)e^{d(r)} \\
 & & & & & & \\
\xi & = & e^{\sqrt{\frac{2}{3}}\phi(r)} & \qquad & F^{(3)}_{y_1y_2} & = & q/y_2^2
\end{array}$$

Substituting all this into the e.o.m. we get 8-dimensional system of nonlinear differential equations with several fixed points – AdS and Lifshitz, and also AdS_5 case as an attractor.
We have solved this system numerically by linearising equations near each fixed point, identifying unstable directions and performing 'shooting' procedure.

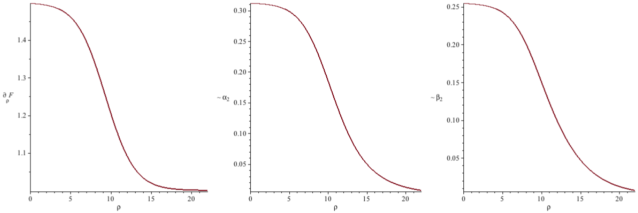
Numerical results

We use $\rho = \ln r$ and recalling $ds^2 = e^{2F(r)}dt^2 + \ldots$, we should get $\partial_\rho F \to 1$ for AdS, and $\partial_\rho F \to z$ for Lifshitz

- Flow from AdS_3 to tilded Lifshitz with $z = 2$



- Flow from untilded Lifshitz with $z = 3/2$ to AdS_3



Discussion

- Constracted $AdS \to Li$ flows dual to RG-flows in boundary field theory from conformal fixed points towards more complicated states.
- Lifshitz solutions brakes SUSY $\Rightarrow$ Possible instabilities in boundary field theory
- Maldacena & Nunez: partial SUSY breaking via wrapping branes on curved surfaces + twisting. Instabilities cured by supersymmetry and exactly right amount of twisting

What about our case with no Supersymmetry at all?
We have plenty of "experimental results" (numerical flows) in order to address this and all the related questions.

Thank you very much!
Stay tuned!