Quantum gravitational fluctuations of a time delay observable in the Minkowski vacuum of linearized gravity

[arXiv:1307.0256 ; follow-up on arXiv:1111.7127]

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Motivation

If I had a theory of Quantum Gravity (QG), what would I do with it?

- Our answer: Compute qualitative and quantitative QG corrections to experiments and observations.
- Idea: Work backwards! Start with a potential experiment (even if only in principle possible), described operationally.
  Construct a mathematical model of it and obtain an observable quantity with an unambiguous interpretation.

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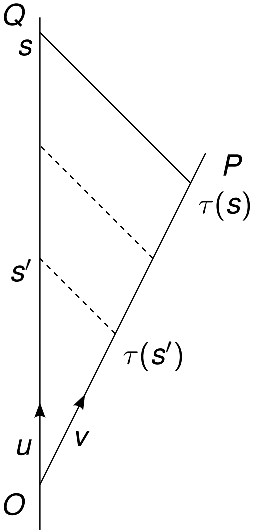
Outline

Thought experiment set-up I



- Two inertially moving, localized systems: the lab and the probe.
- Probe is launched from the lab at event O.
- Both carry a proper-time clock, which is synchronized at O.
- Probe sends signals that are time stamped with the emission time τ at P.
- The lab records the signals at reception time, s at Q, together with the time stamp $\tau(s)$.

Observable

time delay $= \delta\tau(s) = s - \tau(s)$

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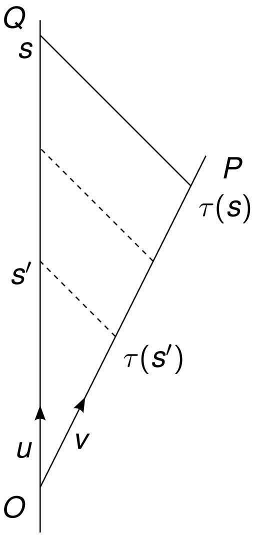

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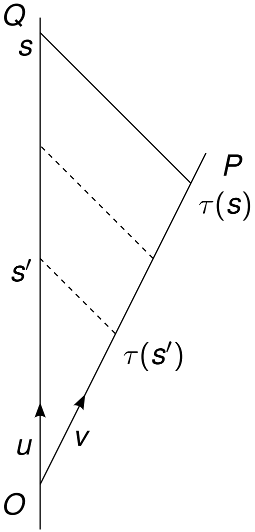

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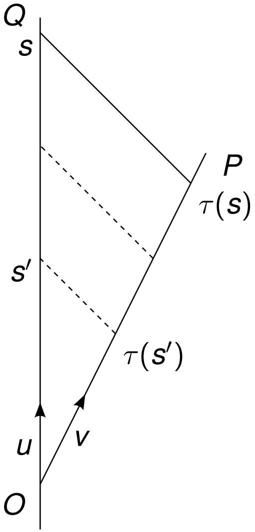

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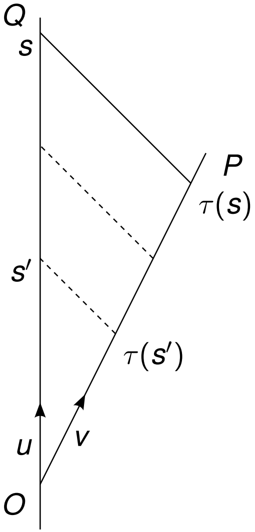

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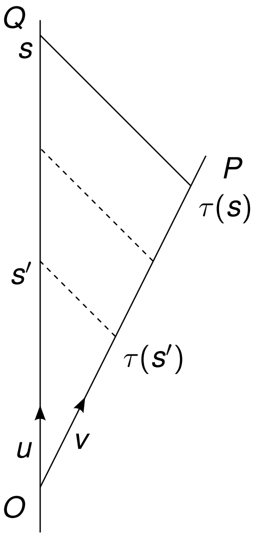

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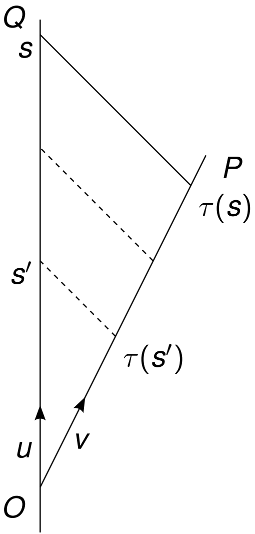

Introduce quantum gravitational effects

- $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- $h_{\mu\nu} \longrightarrow \hat{h}_{\mu\nu}$

Why is this reasonable?

- In classical limit, QG $\longrightarrow$ GR
- Generalize this idea – knowing that the Fock quantization of a linear theory is often a good approximation of the quantum theory of any field whose dynamics is approximated by a linear theory – in the appropriate limit, QG $\longrightarrow$ quantum linearized gravity

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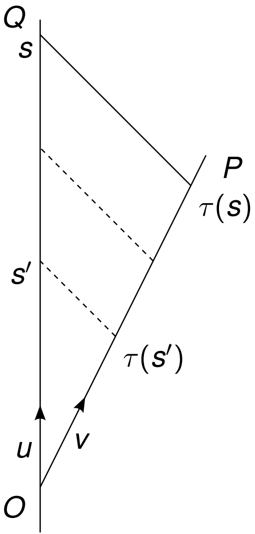

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Observable expectation values I

- Choose $|\psi\rangle$ to be the Poincaré invariant Fock vacuum
  - Natural choice
  - Simplifies calculations
- Physical quantities $\langle 0|\hat{\tau}^n(s)|0\rangle = \int \mathrm{d}\tau\, \tau^n P(\tau) \sim \int \mathrm{d}\tau\, \tau^n e^{-\frac{\tau-\mu}{2\sigma^2}}$
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$$\tau(s) = \frac{s}{\gamma} \longrightarrow \tilde{\tau}(s) = e^{r(h)}\tau_\eta(s) = \frac{s}{\gamma}(1 + r(\hat{h}) + \mathcal{O}(h^2))$$

where γ = time dilation factor

$$\mu = \langle 0|\tilde{\tau}(s)|0\rangle = \frac{s}{\gamma}\langle 0|1|0\rangle + \langle 0|r(\hat{h})|0\rangle + \mathcal{O}(h^2) = \frac{s}{\gamma} + \mathcal{O}(h^2)$$

$$\sigma^2 = \langle 0|\tilde{\tau}(s)^2|0\rangle - \langle 0|\tilde{\tau}(s)|0\rangle^2$$

$$= \frac{s^2}{\gamma^2}(1 + 0 + \langle 0|r(\hat{h})^2|0\rangle + \langle 0|r_2(\hat{h})|0\rangle - 1 + \mathcal{O}(h^3)) \sim \frac{s^2}{\gamma^2}\langle 0|r(\hat{h})^2|0\rangle$$

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- All terms contain $\langle \hat{h}(x)\hat{h}(y)\rangle \sim \frac{1}{(x-y)^2}$
- Need regularization for $x \longrightarrow y$
- Compare with QED, $\langle E(x)^2\rangle$ diverges, but $\langle \tilde{E}(x)^2\rangle$ is finite and represents the vacuum noise in a detector with sensitivity profile $\tilde{g}(x)$.
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$$\hat{h}(x) \longrightarrow \tilde{h}(x) = \int \mathrm{d}z\, \hat{h}(x-z)\, g(z)$$

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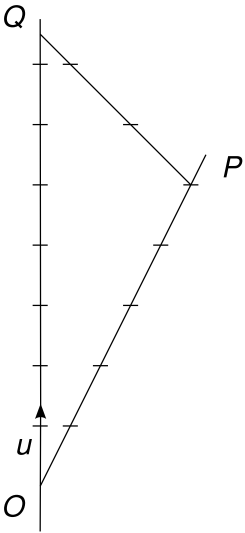

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$$\langle \tilde{h}(x)\tilde{h}(y)\rangle \sim \int \mathrm{d}z_1 \int \mathrm{d}z_2 \frac{\tilde{g}_u(z_1)\tilde{g}_u(z_2)}{(x - z_1 - y + z_2)^2}$$
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where $\mu =$ spatio-temporal resolution of $g_u(z)$

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- This breaks Lorentz symmetry. *Up to this point entire calculation was Lorentz invariant!*

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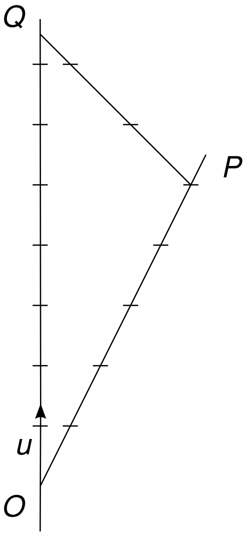

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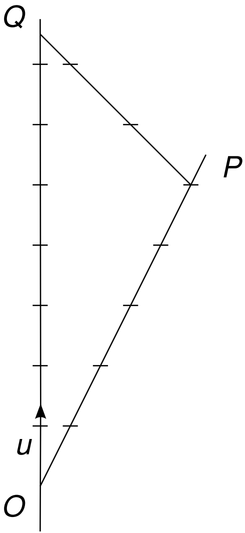

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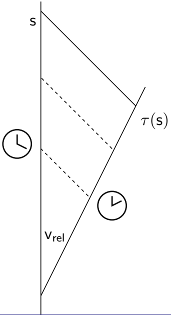
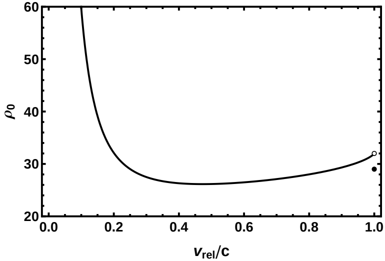
Result I

$$\langle \tilde{r}^2 \rangle = \frac{\ell_p^2}{\mu^2}\left(\rho_0 + \rho_1 \frac{\mu}{s} + \rho_2 \frac{\mu^2}{s^2} + \left(\frac{\mu^3}{s^3}\right)\right) + \left(\frac{\ell_p^2}{\mu^2}\right)$$

$v = v_{rel}/c < 1$	$v = 1$
$\rho_0 = \frac{1}{v^2}\left(\frac{51}{8} + 8v + \frac{141}{8}v^2\right) - \frac{1}{v^2}(3+4v)\,\frac{1-v^2}{v}\log\left(\frac{1+v}{1-v}\right)$	$\rho_0 = 29$
$\rho_1 = -2\pi^2$	$\rho_1 = -\frac{1}{4}\pi^2$
$\rho_2 = 0$	$\rho_2 = 0$





Result II

- Low v approximation

$$\langle \triangle\tau(s)\rangle \sim \sqrt{\frac{3}{8}}\left(\frac{1}{v}\frac{s}{\mu}\right)\ell_p$$

with enhancement factors

1. s/μ = set-up length scale/detector resolution
2. $1/v = c/v_{rel}$

s	1 m/s	10^5 m/s (Hubble recession velocity at 1 Mpc)	10^8 m/s (relativis velocity)
lab scale (1 m/s)	10^{17}	10^{12}	10^9
cosmological (1 Mpc)	10^{39}	10^{34}	10^{31}

The (dimensionless) enhancement factor (on top of the Planck scale $\ell_p \sim 10^{-44}$ s) for $\Delta\tau$ for a detector resolution scale $\mu = 10^{-9}$ m (X-ray wavelength).

- Currently this is not observable, however, it is not out of the question that alternative lab scenarios may be closer to the current state of the art.
- Lorentz invariance is broken, which is likely due to our choice of smearing.
- The divergence for $v \to 0$ is puzzling.
  1. our approximation scheme requires $\mu/s \ll 1$, but as $v \to 0$ this is violated (this is also true for the limit $v \to 1$);
  2. the quadratic correction that we neglected for pragmatic reasons may cancel the low velocity divergence.

Conclusion

- Aim was to construct an observable within realm of QG and calculate its effect.
- Nice features
  - + Easily extendable to other planar geometries.
  - + Generalizable to curved (background) spacetimes
  - + Can be used to contrast predictions of the conservative linearized gravity with more 'exotic quantum gravity' models.
- Future application: Which observable can tell us the 'size' of a black hole? And what can it say about BH evaporation?

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