Gravitational Waves from Preheating in Matrix Inflation

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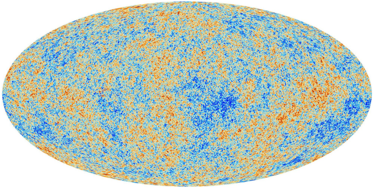
Motivation: Problems with Scalar Field Inflation

Inflation is an add-on to the Big Bang cosmology at early times to explain:

- Flatness/isotropy of the Universe.
- Primordial density perturbations.



Definition

$$\text{Inflation} \quad \Leftrightarrow \quad \ddot{a} > 0.$$

Scalar Field Inflation: The simplest inflationary models are obtained by minimally coupling Einstein gravity to a scalar field ϕ, the *inflaton*, with some choice of potential, e.g. $V(\phi) = m^2\phi^2/2$.

Problems:

- Does not explain the flatness of the potential, i.e. why $m \ll m_{\mathrm{P}}$.
- Requires super-Planckian field excursions, i.e. $\Delta\phi \simeq 14 m_{\mathrm{P}} > m_{\mathrm{P}}$.
- And what is this ϕ field anyway?

However: A plethora of other inflationary scenarios are possible!

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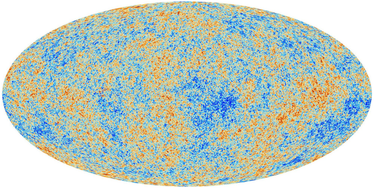

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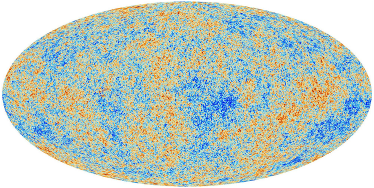

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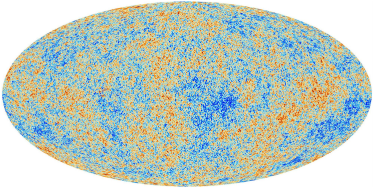

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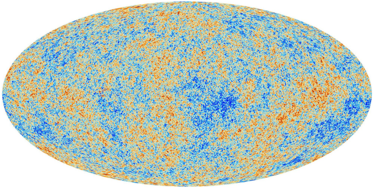

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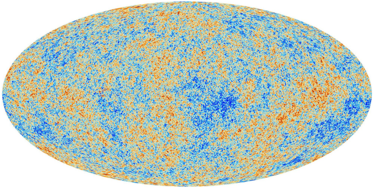

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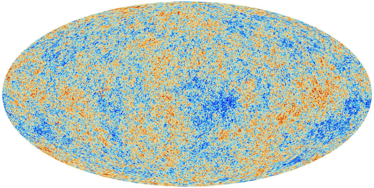

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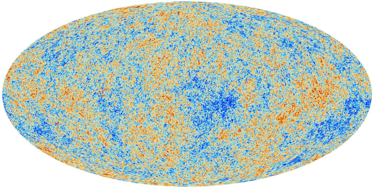

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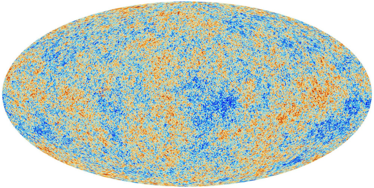

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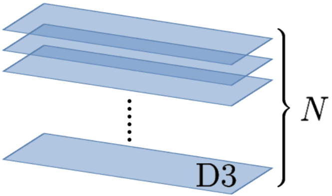
Matrix Inflation from String Theory

Matrix inflation (or *M-flation*):

- The inflaton: three $N \times N$ Hermitian matrices $\mathbf{\Phi}_i$ $(\forall 1 \leq i \leq 3)$.
- The potential: from the dynamics of N D3-branes in a specific $d = 10$ IIB SUGRA background, so that $\mathbf{\Phi}_i \propto X_i$ transverse to the D3-branes.



(A. Ashoorioon, H. Firouzjahi and M. M. Sheikh-Jabbari, JCAP 0906:018, 2009, arXiv:0903.1481)

Gauged M-flation Action

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left\{ -\frac{m_{\mathrm{P}}^2}{2} R - \frac{1}{4} \mathrm{Tr} \left(\mathbf{F}_{\mu\nu} \mathbf{F}^{\mu\nu} \right) - \frac{1}{2} \mathrm{Tr} \left(D_\mu \mathbf{\Phi}_i D^\mu \mathbf{\Phi}_i \right) - V \right\},$$

$$V = \mathrm{Tr} \left(-\frac{\lambda}{4} [\mathbf{\Phi}_i, \mathbf{\Phi}_j][\mathbf{\Phi}_i, \mathbf{\Phi}_j] + \frac{\mathrm{i}\kappa}{3} \epsilon_{jkl} [\mathbf{\Phi}_k, \mathbf{\Phi}_l] \mathbf{\Phi}_j + \frac{m^2}{2} \mathbf{\Phi}_i^2 \right)$$

where $\mathbf{F}_{\mu\nu} = 2\partial_{[\mu} \mathbf{A}_{\nu]} + \mathrm{i} g_{\mathrm{YM}} [\mathbf{A}_\mu, \mathbf{A}_\nu]$, $D_\mu \mathbf{\Phi}_i = \partial_\mu \mathbf{\Phi}_i + \mathrm{i} g_{\mathrm{YM}} [\mathbf{A}_\mu, \mathbf{\Phi}_i]$.

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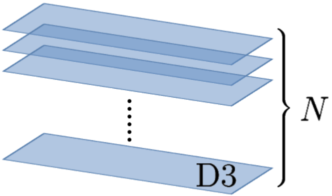

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Equations of Motion and Truncation to the $\mathbf{SU}(2)$ Sector

The EOMs for the scalar and gauge fields are:

$$D_\mu D^\mu \mathbf{\Phi}_i + \lambda[\mathbf{\Phi}_j, [\mathbf{\Phi}_i, \mathbf{\Phi}_j]] - \mathrm{i}\kappa\epsilon_{ijk}[\mathbf{\Phi}_j, \mathbf{\Phi}_k] - m^2\mathbf{\Phi}_i = 0,$$

$$D_\mu \mathbf{F}^{\mu\nu} - \mathrm{i}g_{\mathrm{YM}}[\mathbf{\Phi}_i, D^\nu\mathbf{\Phi}_i] = 0.$$

If $\{\mathbf{J}_i\}_{i=1}^3$ are the $N \times N$ generators of $\mathbf{SU}(2)$: $[\mathbf{J}_i, \mathbf{J}_j] = \mathrm{i}\epsilon_{ijk}\mathbf{J}_k$, let

$$\mathbf{\Phi}_i = \hat{\phi}\mathbf{J}_i + \mathbf{\Psi}_i.$$

Remark

If initially $\mathbf{\Psi}_i = \dot{\mathbf{\Psi}}_i = 0$ and $\hat{\phi} \neq 0$, the EOMs imply $\mathbf{\Psi}_i = 0$ for all time. Moreover, if $\mathbf{A}_\mu = 0$ as well, $\mathbf{\Phi}_i$ will not source it.

Hence, $\mathbf{\Psi}_i$ and $\mathbf{A}_\mu$ are *spectators* – they can be turned off classically:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left\{-\frac{m_{\mathrm{P}}^2}{2}R + \mathrm{Tr}\mathbf{J}^2\left(-\frac{1}{2}\partial_\mu\hat{\phi}\partial^\mu\hat{\phi} - \frac{\lambda}{2}\hat{\phi}^4 + \frac{2\kappa}{3}\hat{\phi}^3 - \frac{m^2}{2}\hat{\phi}^2\right)\right\}$$

where $\mathrm{Tr}\mathbf{J}^2 = \mathrm{Tr}(\mathbf{J}_i\mathbf{J}_i) = N(N^2-1)/4$. Setting $\phi = \sqrt{\mathrm{Tr}\mathbf{J}^2}\hat{\phi}$, this just becomes the usual scalar field inflationary action with a quartic potential.

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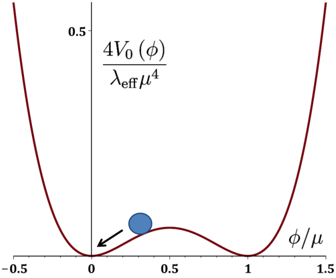
The Effective Potential

With $\lambda_{\text{eff}} = \frac{8\lambda}{N(N^2-1)}$, $\kappa_{\text{eff}} = \frac{2\kappa}{[N(N^2-1)]^{1/2}}$,

$$V_0(\phi) = \frac{\lambda_{\text{eff}}}{4}\phi^4 - \frac{2\kappa_{\text{eff}}}{3}\phi^3 + \frac{m^2}{2}\phi^2$$
$$= \frac{\lambda_{\text{eff}}}{4}\phi^2(\phi-\mu)^2, \quad \mu := \frac{\sqrt{2}m}{\sqrt{\lambda_{\text{eff}}}}$$

where $\lambda m^2 = (2\kappa/3)^2$ is fixed, to have a constant dilaton in the SUGRA theory.



- The necessary values for successful inflation can be determined by imposing 60 e-foldings and some CMB observations (δ_H and n_s).
- Solves all three problems raised earlier: the couplings are naturally small; the field displacement is less than the UV cutoff (1101.0048); and the inflaton has a known (stringy) origin.
- The next question is: How can we probe this with observations? Look to preheating.

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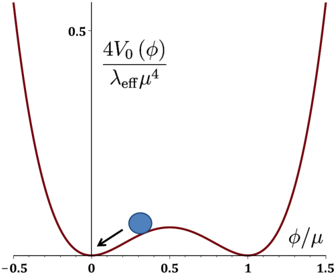

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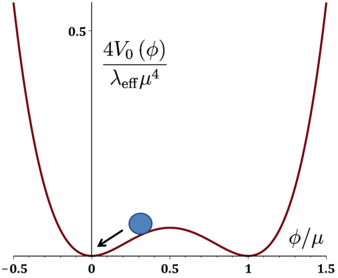

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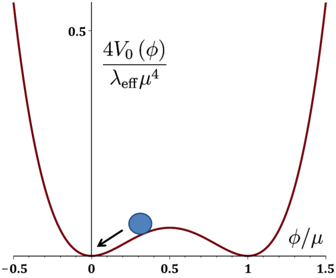

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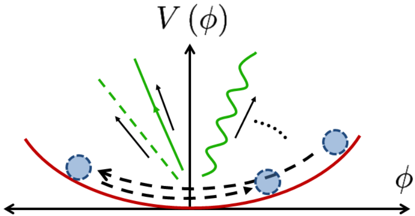
- Solves all three problems raised earlier: the couplings are naturally small; the field displacement is less than the UV cutoff (1101.0048); and the inflaton has a known (stringy) origin.
- The next question is: How can we probe this with observations? Look to preheating.

Preheating and Parametric Resonance: Basic Idea

Definition

Preheating is the epoch just after inflation, during which the inflaton decays into SM particles via damped oscillations about its minimum.



The idea is to treat ϕ classically, but the matter field χ quantumly:

$$\hat{\chi}(t,\mathbf{x}) = \int \frac{\mathrm{d}^3 k}{(2\pi)^{3/2}} \left(\chi_k^*(t)\, \hat{a}_k e^{\mathbf{i}\mathbf{k}\cdot\mathbf{x}} + \chi_k(t)\, \hat{a}_k^\dagger e^{-\mathbf{i}\mathbf{k}\cdot\mathbf{x}} \right).$$

Then, the simplest choice for an interaction is:

$$S_{\mathrm{int}} \propto \int \mathrm{d}^4 x \sqrt{-g}\phi^2\chi^2 \;\Rightarrow\; \ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + m_{\mathrm{eff}}^2(\phi) \right)\chi_k = 0,$$

where $m_{\mathrm{eff}}^2(\phi)$ is oscillatory and hence the EOM for χ has the form of a Mathieu equation: well-known instabilities for certain ranges of k, leading to exponential growth, i.e. *parametric resonance*.

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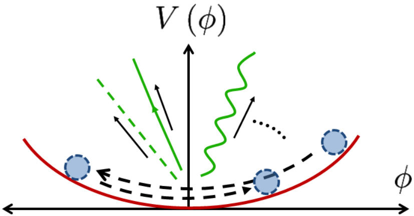

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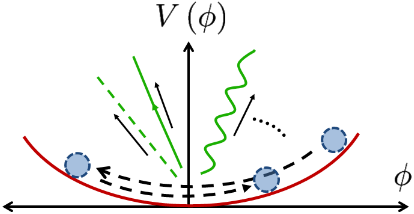

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M-flation Spectators as Preheat Fields

Although the spectators $\mathbf{\Psi}_i$ and $\mathbf{A}_\mu$ are turned off classically during M-flation, they can be excited quantumly and thus serve as preheat fields!

Treating both in turn as perturbations, at quadratic order (1101.0048):

Spectator Masses (with degeneracy $2j+1$ for each mode)

scalar $\begin{cases} M^2_{\alpha_j} = \frac{1}{2}\lambda_{\rm eff}\phi^2(j+2)(j+3) - 2\kappa_{\rm eff}\phi(j+2) + m^2, & 0\leq j\leq N-2, \\ M^2_{\beta_j} = \frac{1}{2}\lambda_{\rm eff}\phi^2(j-1)(j-2) + 2\kappa_{\rm eff}\phi(j-1) + m^2, & 1\leq j\leq N, \end{cases}$

gauge $\begin{cases} M^2_{A_j} = \frac{1}{4}\lambda_{\rm eff}\phi^2 j(j+1), & 0\leq j\leq N-1. \end{cases}$

Remark

The potential for each scalar mode must also receive a (ϕ-independent) quartic correction to ameliorate the possibility of a tachyonic mass.

Parametric resonance during preheating can be a good source of GW. We numerically computed their spectra in M-flation using Zhiqi Huang's HLattice 2.0 (Z. Huang, Phys. Rev. D83:123509, 2011, arXiv:1102.0227).

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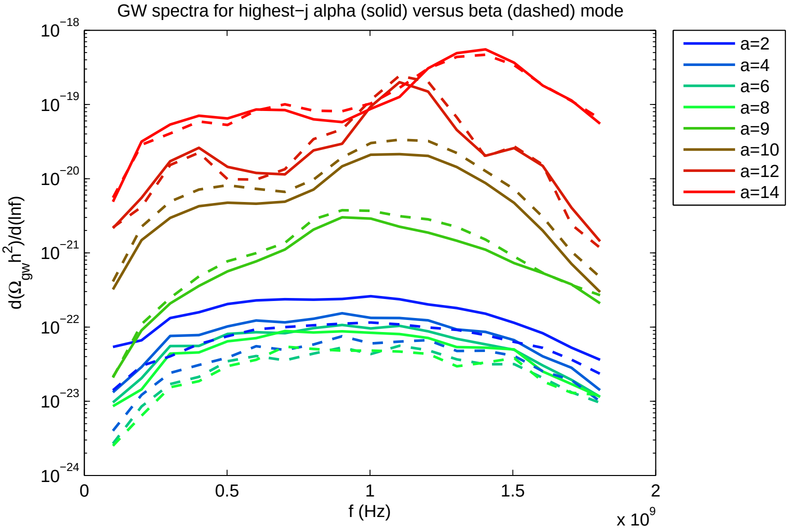
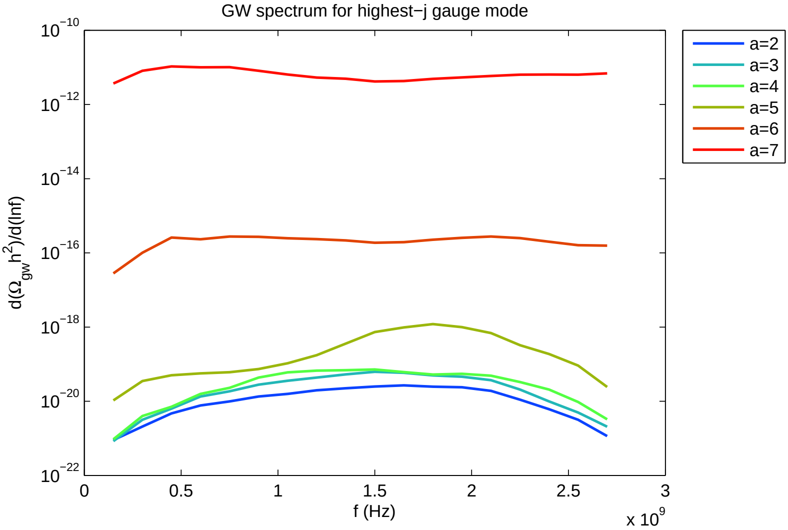
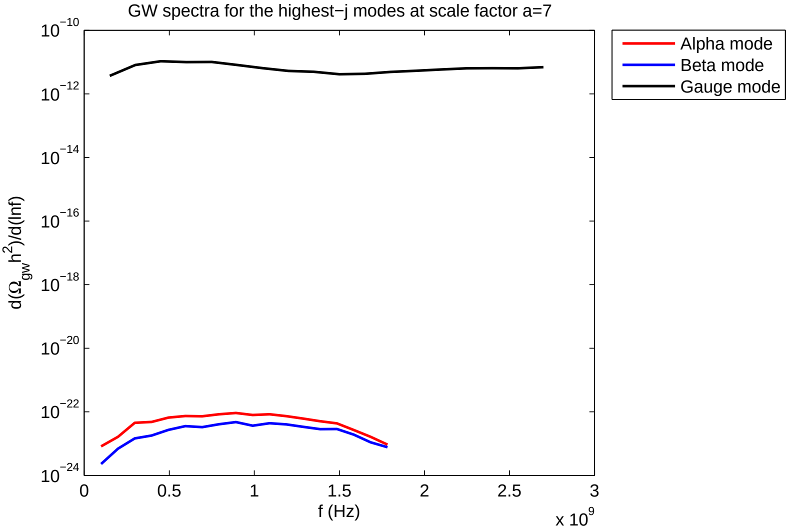
Gravity Waves from M-flation Preheating: Scalar Modes



Gravity Waves from M-flation Preheating: Gauge Mode



Gravity Waves from M-flation Preheating: Scalar vs Gauge



Conclusions

- M-flation, which resolves many of the theoretical difficulties associated with standard chaotic inflation, can also make concrete predictions from its built-in preheating mechanism.
- In particular, M-flation preheating produces a large-amplitude GW spectrum in the GHz band, chiefly thanks to its gauge spectators.
- Such a spectrum could be observed by ultra-high frequency GW detectors that probe the GHz band, e.g. the Birmingham HFGW detector (below) or the INFN Genoa HFGW resonant antenna.

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Thank you for your attention.

Appendix I: Spectator Perturbations to Quadratic Order

Scalar spectators: Inserting $\mathbf{\Phi}_i = \hat{\phi}\mathbf{J}_i + \mathbf{\Psi}_i$, $\mathbf{A}_\mu = 0$ into the action,

$$\mathcal{L}^{(2)}_{\mathbf{\Psi}} = -\frac{1}{2}\mathrm{Tr}(\partial_\mu \mathbf{\Psi}_i)^2 \overbrace{+\mathrm{Tr}\left[\frac{\lambda}{2}\hat{\phi}^2\left(\epsilon_{ijk}[\mathbf{J}_j,\mathbf{\Psi}_k]\right)^2 + \mathrm{i}\left(\frac{\lambda}{2}\hat{\phi}^2 - \kappa\hat{\phi}\right)\epsilon_{ijk}[\mathbf{J}_i,\mathbf{\Psi}_j]\mathbf{\Psi}_k - \frac{m^2}{2}\mathbf{\Psi}_i^2\right]}^{-\left(M^2/2\right)\mathbf{\Psi}_i^2,\ \text{if}\ \mathrm{i}\epsilon_{ijk}[\mathbf{J}_j,\mathbf{\Psi}_k] = \omega\mathbf{\Psi}_i}$$

Gauge spectators: Inserting $\mathbf{\Psi}_i = \mathbf{\Phi}_i - \hat{\phi}\mathbf{J}_i = 0$ into the action and expanding to second order in $\mathbf{A}_\mu$, we get

$$\mathcal{L}^{(2)}_{\mathbf{A}_\mu} = -\frac{1}{4}\mathrm{Tr}\left(\partial_{[\mu}\mathbf{A}_{\nu]}\right)^2 \underbrace{+\frac{1}{2}g^2_{\mathrm{YM}}\hat{\phi}^2\mathrm{Tr}\left([\mathbf{J}_i,\mathbf{A}_\mu][\mathbf{J}_i,\mathbf{A}_\mu]\right)}_{-\left(M_A^2/2\right)\mathrm{Tr}\left(\mathbf{A}_\mu^2\right),\ \text{if}\ [\mathbf{J}_i,[\mathbf{J}_i,\mathbf{A}_\mu]] = \omega\mathbf{A}_\mu}$$

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Appendix II: Scalar Spectator Quartic Couplings

For the potential of any scalar mode,

$$V = V_0(\phi) + \frac{M^2(\phi)}{2}\chi^2 + \Lambda\chi^4,$$

we calculate the quartic coupling to be:

$$\left.\begin{array}{c} \Lambda_{\alpha_{j-2}} \\ \Lambda_{\beta_{j+2}} \end{array}\right\} = \left.\begin{array}{c} (j+1)^2 \\ j(j+1) \end{array}\right\} \times \frac{\lambda_{\text{eff}}}{4} N\left(N^2-1\right)$$
$$\times \sum_{c=0}^{2j} (2c+1) \begin{pmatrix} j & j & c \\ 1 & -1 & 0 \end{pmatrix}^2 \begin{Bmatrix} j & j & c \\ \frac{N-1}{2} & \frac{N-1}{2} & \frac{N-1}{2} \end{Bmatrix}^2$$

where $(:::)$ and $\{:::\}$ are Wigner $3j$ and $6j$ symbols, respectively (expressible in terms of Clebsch–Gordan coefficients).