Microcanonical Entropy of Quantum Isolated Horizon, ‘quantum hair N’ and fixation of γ

Abhishek Majhi

Astro-particle Physics and Cosmology Division;
Saha Institute of Nuclear Physics

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An Outline

- *Isolated Horizon(IH)* : A brief introduction, some relevant properties
- *Spin Network* : A few words on Quantum Geometry
- *Punctures on IH* : Visualization of *Quantum* IH (QIH)
- *Macrostates of QIH* : Spin Configuration
- *Macroscopic* Parameters of the theory according to a recent proposal
- *Microcanonical Entropy* : Calculation
- Restriction on the Barbero-Immirzi parameter(γ)

Isolated Horizon

Introduction and some Properties

IH : An inner boundary of spacetime

- 3D *Null* hypersurface
- Topologically $R \times S^2$
- Non-expanding $\Rightarrow$ *Constant* Area (A_{cl})
- Thermodynamical variables and parameters can be defined *locally* on IH
- IH allows matter and radiation arbitrarily close to it
- IH boundary conditions $\rightarrow$ Laws of IH Mechanics
- Surface gravity (κ_{IH}) constant on IH (Zeroth Law on IH)
- $E_{radiation} = E_{ADM} - E_{IH}$; E_{IH} satisfies 1st law of IH Mechanics: $\delta E_{IH} = \frac{\kappa_{IH}}{8\pi}\delta A_{IH} + \cdots$

Isolated Horizon

Event Horizon : a *special* case

Event Horizon(EH) is a *special* case of IH :

- IH + *stationarity* $\Rightarrow$ Event Horizon(EH)
- Surface gravity (κ_{EH}) constant on EH (Zeroth Law on EH)
- For EH, $E_{radiation} = 0 \Rightarrow E_{ADM} = E_{EH}$ =mass of black hole
- E_{ADM} satisfies 1st law of black hole Mechanics

Quantum Geometry

Spin Networks

Bulk quantum geometry(spatial) described by *Spin Networks* :

- Non-trivial, flexible lattice – much like a 'fish-net'
- Collection of *edges* and *vertices*
- *Edges* assigned with *spins*
- Two distinct *edges* meet ONLY at a *vertex*

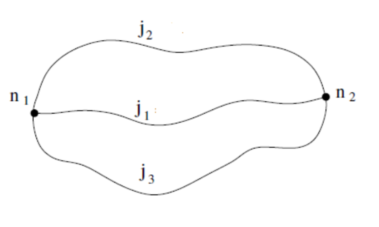

Figure: A *Spin Network* with 3 *edges* and 2 *vertices*

Punctures : Intersection points of *Spin Network* and IH

- Defects on smooth topology of IH
- *Edges* deposit corresponding spins at the *punctures*
- The distribution of *punctures* on IH gives the quantum picture : *Quantum Isolated Horizon* (QIH)
- Effect of a *puncture* with spin j, manifested by its area contribution : $A_j = 8\pi\gamma\ell_p^2\sqrt{j(j+1)}$ where γ =Barbero-Immirzi parameter, ℓ_p =Planck length
- Contribution from N no. of *punctures*, n th puncture having spin j_n : $A = 8\pi\gamma\ell_p^2\sum_{n=1}^{N}\sqrt{j_n(j_n+1)}$

Quantum Geometry

An Intuitive Picture of QIH

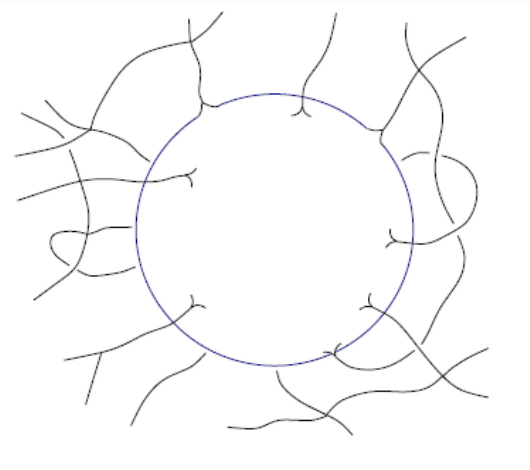

Figure: Bulk spin network intersects with IH at the punctures : QIH

Quantum Geometry

Degrees of Freedom on QIH

Action for spacetime admitting IH $\Rightarrow$ Chern-Simons(CS) action as boundary term $\Rightarrow$ EoM (Einstein equation)

SU(2) Singlet States of CS theory coupled to Punctures (sources) $\equiv$ DoF of QIH

- Physical Hilbert space of a quantum black hole *for a particular set of punctures* $\{\mathcal{P}\}$: $\mathcal{H}_{Phys} = \mathcal{H}_V \otimes \mathcal{H}_S \;\; mod \; \mathcal{G}$
  $V \equiv$ Bulk, $S \equiv$ Boundary(QIH), $\mathcal{G} \equiv$ Gauge Group
- $\mathcal{G} \supset$ internal $SU(2)$ rotations (results from local Lorentz invariance after fixing the local boost d.o.f [R.Kaul, P.Majumdar; PRD(2011)])
- Rotational Invariant states $\implies$ Physical DoF of QIH

Quantum Geometry

Microstates of QIH

- Given a set of N punctures $(j_1, \cdots, j_N)$ on a QIH, no. of microstates :

  $$\Omega(j_1, \cdots, j_N) = \frac{2}{k+2} \sum_{a=1}^{k+1} \frac{\sin \frac{a\pi(2j_1+1)}{k+2} \cdots \sin \frac{a\pi(2j_N+1)}{k+2}}{\left(\sin \frac{a\pi}{k+2}\right)^{N-2}}$$

  (Kaul-Majumdar, PLB 98), and also $j \leq k/2$
- $k \equiv A_{cl}/4\pi\gamma\ell_p^2$ is the level of the CS theory

Statistical Mechanics of QIH

Macrostates

- Macrostates of QIH : Spin configurations $\{s_j\}$
- Degeneracy of a particular configuration :

$$\Omega[\{s_j\}] = \frac{2}{k+2} \sum_{a=1}^{k+1} \sin^2 \frac{a\pi}{k+2} \prod_j \frac{N!}{s_j!} \left\{ \frac{\sin \frac{a\pi(2j+1)}{k+2}}{\sin \frac{a\pi}{k+2}} \right\}^{s_j}$$

- $\{s_j\} \Rightarrow \quad s_{1/2}$ punctures with spin $1/2$, $\quad s_1$ punctures with spin 1, $\cdots$, $\quad s_{k/2}$ punctures with spin $k/2$

Statistical Mechanics of QIH

Macroscopic parameters

According to a recent proposal by Ghosh-Perez (PRL, 2011), total number of punctures N is a macroscopic parameter of QIH.

- Two *macroscopic* parameters of the theory : k and N
- *Macrostates* designated by N, k
- Configurations obey two constraints :

$$\sum_j s_j \sqrt{j(j+1)} \; = \; k/2 \quad , \quad \sum_j s_j = N$$

Statistical Mechanics of QIH

Most probable distribution

- Microcanonical Entropy : $S_{MC} = \log \Omega(N, k)$
- Most probable configuration $\{s_j^\star\}$ for given k, N maximizes entropy
- Hence, $S_{MC} \simeq \log \Omega[\{s_j^\star\}]$
- Distribution $s_j^\star$ obtained from

$$\delta \log \Omega[\{s_j\}] - \lambda \sum_j \delta s_j \sqrt{j(j+1)} - \sigma \sum_j \delta s_j = 0$$

Statistical Mechanics of QIH

Most probable distribution(contd. ...)

- Most probable distribution

$$s_j^\star = N \exp\left[-\lambda\sqrt{j(j+1)} - \sigma - \frac{\delta}{\delta s_j}\log g[\{s_j\}]\right]$$

where

$$g[\{s_j\}] = \frac{2}{k+2}\sum_{a=1}^{k+1}\sin^2\frac{a\pi}{k+2}\left\{\frac{\sin\frac{a\pi(2j+1)}{k+2}}{\sin\frac{a\pi}{k+2}}\right\}^{s_j}$$

- A rigorous calculation approximately yields

$$\Omega[\{s_j\}] \sim \frac{N!}{\prod_j s_j!}(2j+1)^{s_j}N^{-3/2}$$

Statistical Mechanics of QIH

Microcanonical Entropy in terms of k and N

- Microcanonical entropy of QIH

$$S = \log \Omega[\{s_j^\star\}] = \frac{\lambda k}{2} + N\sigma - \frac{3}{2}\log N + \cdots$$

- λ and σ are the solutions of the following equations

$$\begin{aligned}\exp[\sigma] &= \sum_j (2j+1)\exp[-\lambda\sqrt{j(j+1)}] \\ k/2 &= N\sum_j \sqrt{j(j+1)}(2j+1)\exp[\lambda\sqrt{j(j+1)} - \sigma]\end{aligned}$$

Statistical Mechanics of QIH

Approximations involving λ and σ

In the limit $k \to \infty$, the two equations can approximated as follows :

$$
\begin{aligned}
e^{\sigma} &\simeq \int_{1/2}^{\infty} (2j+1) e^{-\lambda\sqrt{j(j+1)}} dj \\
&= \frac{2}{\lambda^2}\left(1 + \frac{\sqrt{3}}{2}\lambda\right) e^{-\frac{\sqrt{3}}{2}\lambda} \\
\frac{k}{2N} &\simeq \frac{\int_{1/2}^{\infty} (2j+1)\sqrt{j(j+1)}\, e^{-\lambda\sqrt{j(j+1)}} dj}{\int_{1/2}^{\infty} (2j+1) e^{-\lambda\sqrt{j(j+1)}} dj} \\
&= 1 + \frac{2}{\lambda} + \frac{4}{\lambda(\sqrt{3}\lambda + 2)}
\end{aligned}
$$

Plot of e^{σ} as a function of λ

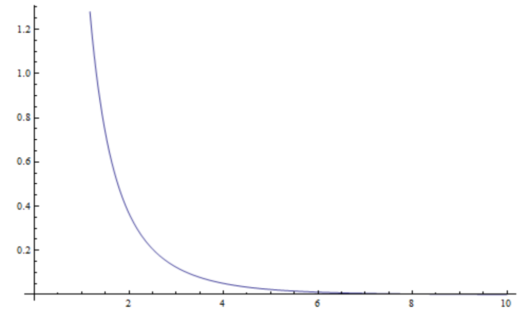

Figure: e^{σ} is plotted as a function of λ

$\sigma < 0$ i.e. $e^{\sigma} < 1$ happens for $\lambda > 1.313$

Fixing γ to obtain $S_{MC} = A_{cl}/4\ell_p^2 + N\sigma(\gamma)$

- Define the *microcanonical* ensemble by choosing $k = k_1$ and $N = N_1$; then $\lambda = \lambda(k_1/N_1) \equiv \lambda_1$.
- We can claim $\gamma = \lambda_1/2\pi = \gamma_1$(say) to obtain $S_{MC} = A_{cl1}/4\ell_p^2 + N\sigma(\gamma_1)$, where $A_{cl1} = 4\pi\gamma_1 k_1 \ell_p^2$.
- Similarly, one can make another choice $k = k_2$ and $N = N_2$, such that $(k_1/N_1) \neq (k_2/N_2)$, for which there exists a corresponding γ_2 and A_{cl2} so that we obtain $S_{MC} = A_{cl2}/4\ell_p^2 + N\sigma(\gamma_2)$.
- For every such choice of k/N there exists an unique value of γ, given by $\lambda(k/N)/2\pi$, which results in the *microcanonical* entropy given by $S_{MC} = A_{cl}/4\ell_p^2 + N\sigma(\gamma)$.

Whose entropy is this ?

Hilbert space of a QIH

- The *full* Hilbert space of a QIH is given by

$$\mathcal{H}^k_{QIH} = \bigoplus_{\{\mathcal{P}\}} \mathrm{Inv}\left(\bigotimes_{l=1}^{N} \mathcal{H}_{j_l}\right)$$

  where there is a SUM over all possible sets of punctures
- Fluctuations of N purely quantum mechanical, devoid of any thermodynamic phenomenon
- Quantum fluctuations underlying a classical equilibrium are the sole origin of microcanonical entropy of a system
- Fixing N is tantamount to look at a subspace of the full Hilbert space

Nature of the term $N\sigma(\gamma)$

- Entropy is the measure of unavailability of information
- Fixing N restricts to a subspace of the full Hilbert space, which in other words is equivalent to providing more information about the system
- Entropy should be reduced
- Hence, we can conclude that

$$N\sigma(\gamma) < 0$$

Bound on γ

- We have $\gamma = \lambda/2\pi$
- $\sigma(\gamma) < 0$ imposes a bound on γ given by

$$0.209 < \gamma < \infty$$

THANK YOU FOR YOUR KIND ATTENTION !