Spinfoam gravity: progress and perspectives - Lecture 3

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Outline

Lecture 1:
Path integral and the Spinfoam amplitude

Lecture 2:
Quantum geometry in Spinfoams

Lecture 3:
- gravitons
- quantum cosmology
- black hole entropy

- The problem:

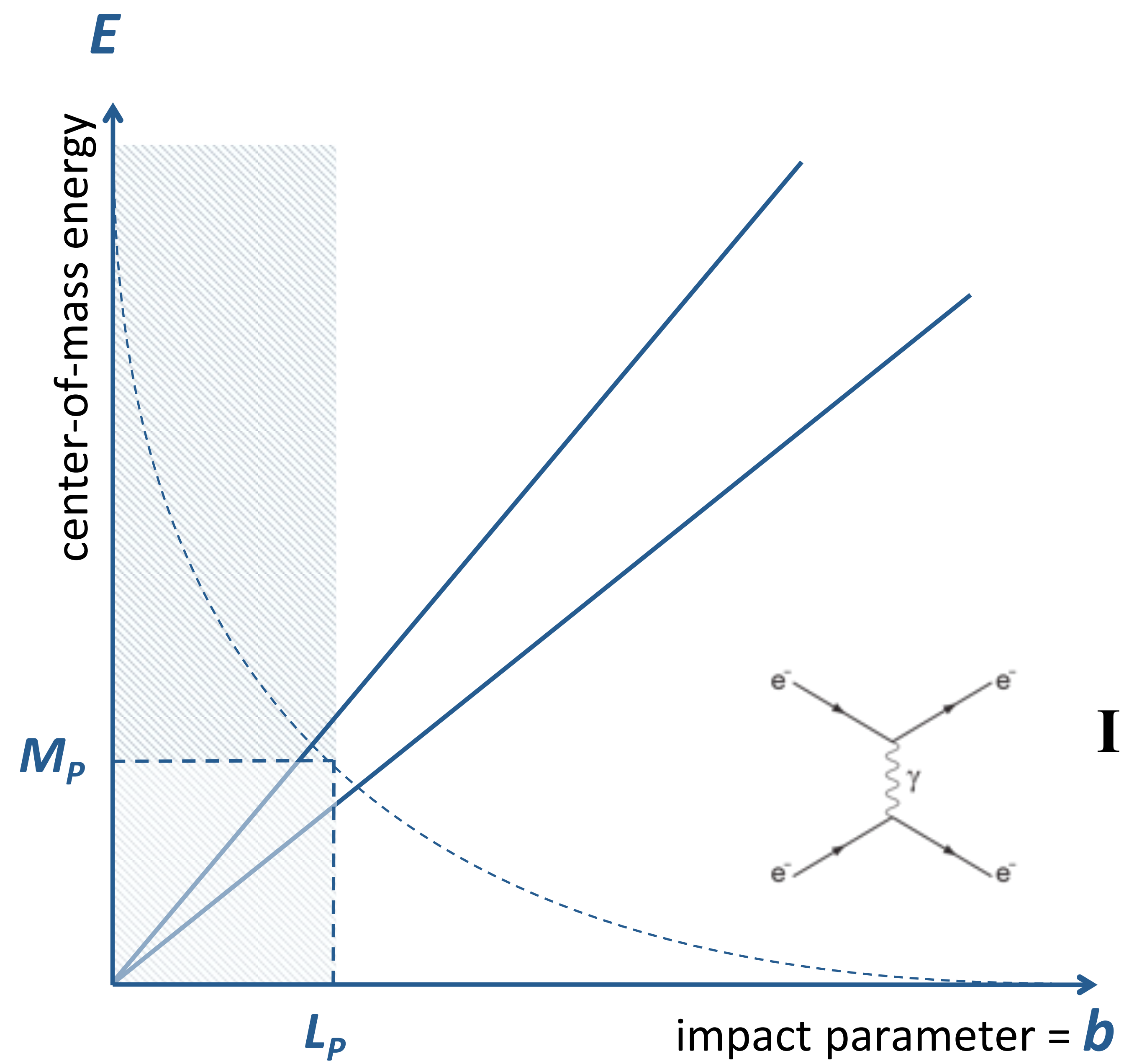
Loop Quantum Gravity (LQG) is a background-independent theory.
Does the theory admit a regime described by an Effective Field Theory of gravitons propagating on a flat background geometry?

Rovelli *PRL*'06, Bianchi-Modesto-Rovelli-Speziale *CQG*'06
Livine-Speziale *JHEP*'06, Bianchi-Modesto, Alesci-Rovelli *PRD*'07
Alesci-Bianchi-Magliaro-Perini *CQG*'08, Bianchi-Satz *NPB*'08
Bianchi-Magliaro-Perini *PRD*'09 Rovelli-Zang *PRD*'11, Bianchi-Ding *PRD*'12

- The main idea:

- the background flat geometry is to be coded in a coherent state
- gravitons from correlations of the *Penrose* metric operator on such state

- **I**. *Born scattering regime*, $G_N E << b$
  graviton exchange, perturbative QFT
  Does LQG admit an EFT description in I. ?



cf. [Amati-Ciafaloni-Veneziano 1987-], [Giddings-Porto 2009]

Scattering in Spinfoams? A phase diagram

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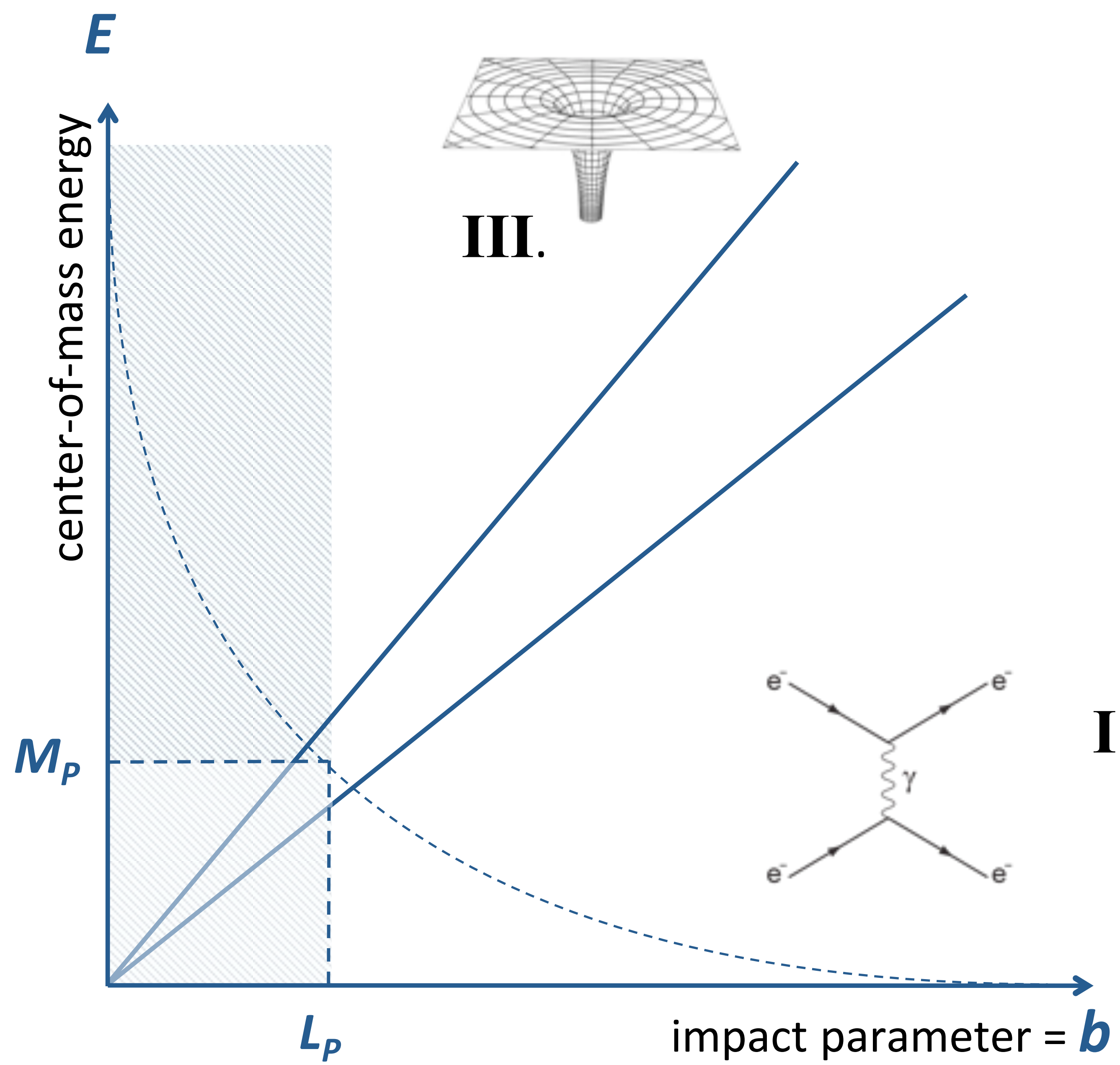
- **III**. *Strong-gravity regime*, $G_N E >> b$
  Scattering with BH production described by *classical* GR in III. [Eardley-Giddings 2002]
  Does LQG reproduce classical GR ?



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  Angle quantization effects? [Major 2010]
  Single intertwiner may be relevant
  cf. Stern-Gerlach experiment

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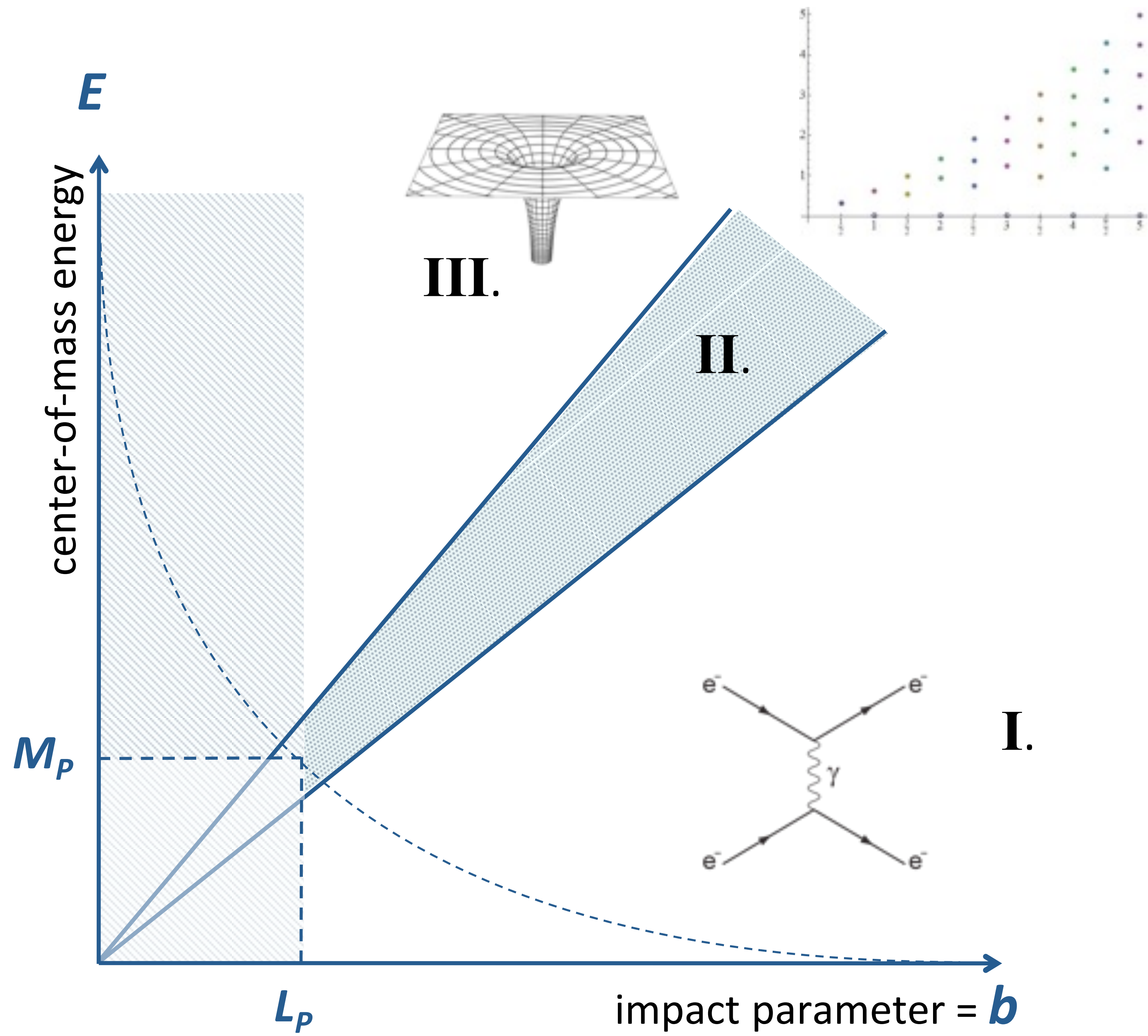

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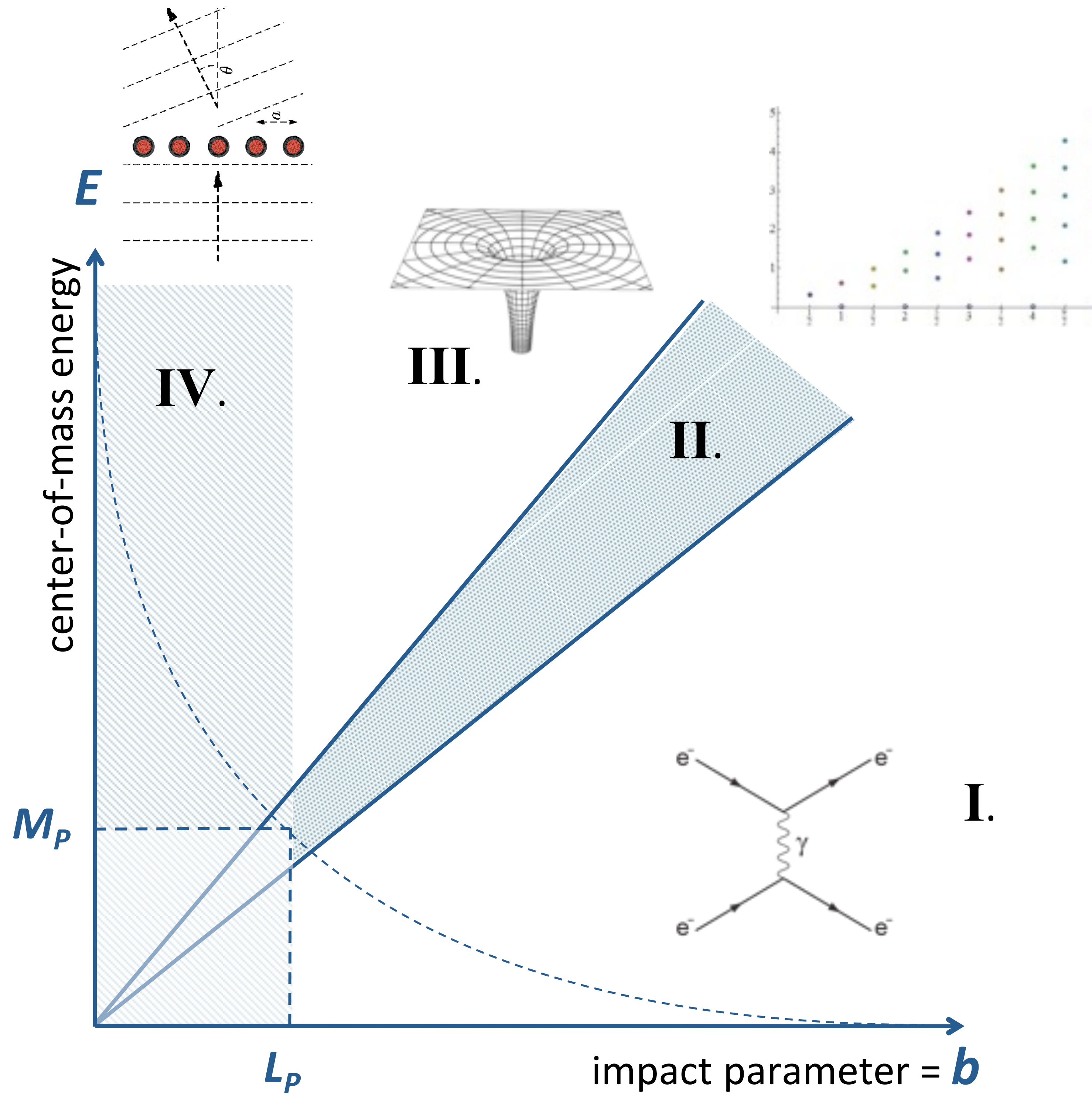
- **IV**. *Trans-Planckian regime*, $b < L_P$
  No Trans-Planckian d.o.f. in LQG.
  Planck-scale gaps



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- gravitons
- ⇨ quantum cosmology
- black hole entropy

- The problem:

How to identify the cosmological regime of Spinfoams?
Homogeneity and Isotropy cannot be imposed at the quantum level
because of Heisenberg uncertainty relations for quantum geometry

Bianchi-Rovelli-Vidotto *PRD* '10, Rovelli-Vidotto *CQG* '08
Bianchi-Krajewski-Rovelli-Vidotto *PRD* '11, Magliaro-Marciano-Perini '11,
Hellmann *PRD* '11, Vidotto'11, Livine-MartinBenito '11
Kisielowski-Lewandowski-Puchta '12, Alesci-Cianfrani '13, Rennert-Sloan '13

- The main idea:

- Coherent state peaked on a homogeneous and isotropic geometry of space
- Evolution with the spinfoam path integral:
  effective dynamics for the large scale degrees of freedom
  Friedmann eqs + corrections $\Rightarrow$ signatures?
- cf. Loop Quantum Cosmology [Bojowald, Ashtekar]

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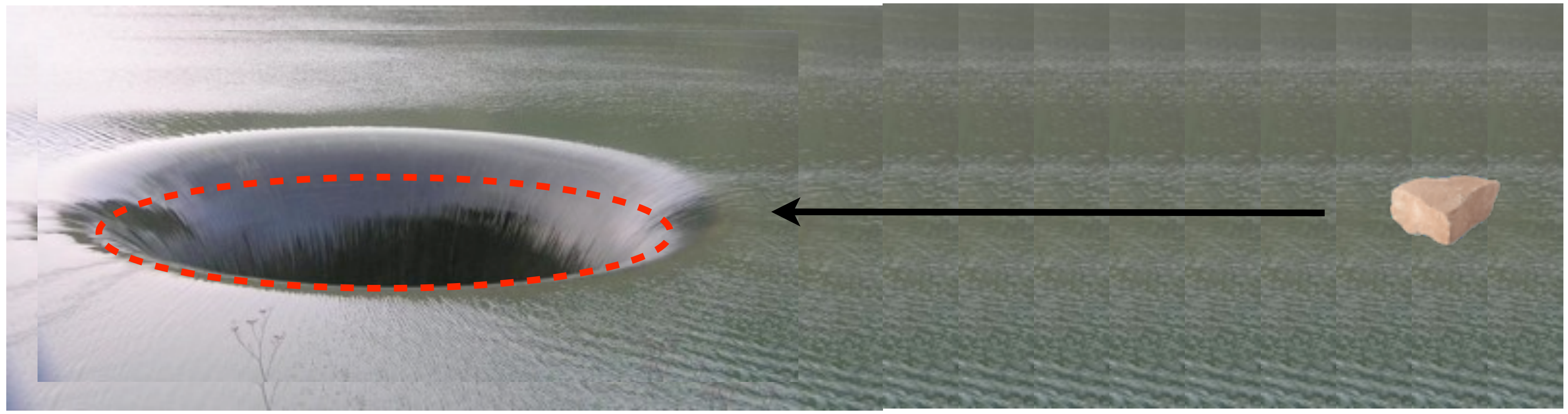
Process:

matter falling in a black hole,



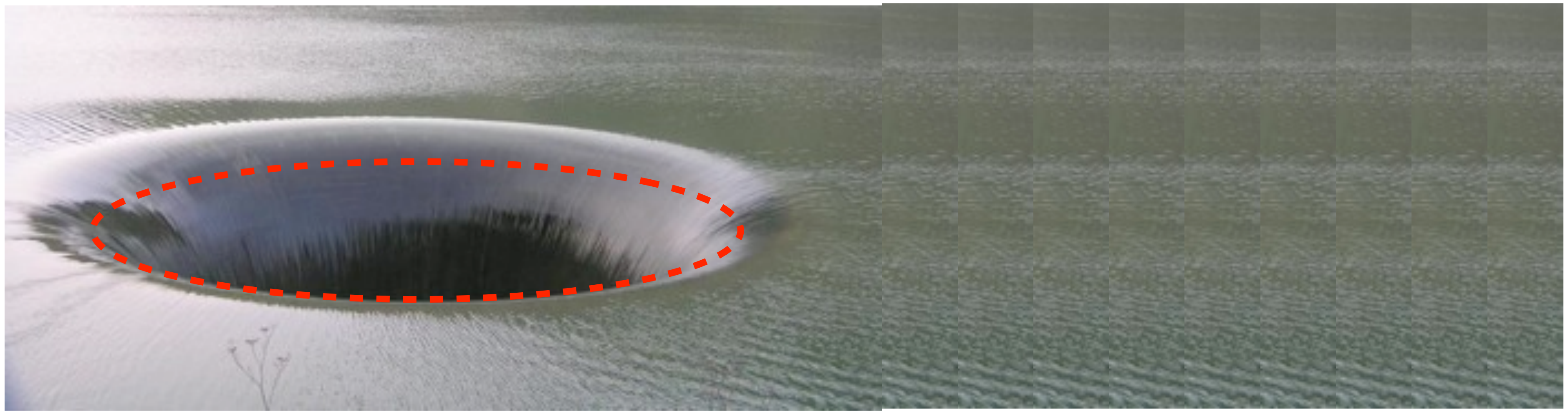
M A δE

Entropy change δS_{BH}

Process:

matter falling in
a black hole,

area of the horizon
increases



$M + \delta E$ $A + \delta A$

Entropy change δS_{BH}

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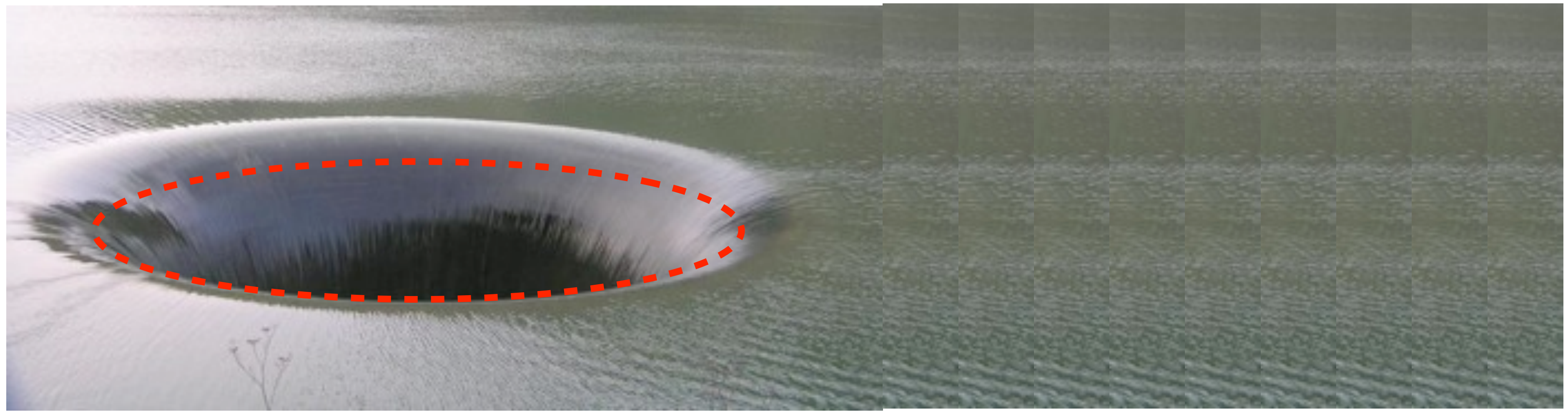

$M + \delta E$ $A + \delta A$

Entropy change δS_{BH}

$$\delta S_{BH} = \frac{\kappa\, c^3}{\hbar} \frac{\delta A}{4\, G}$$

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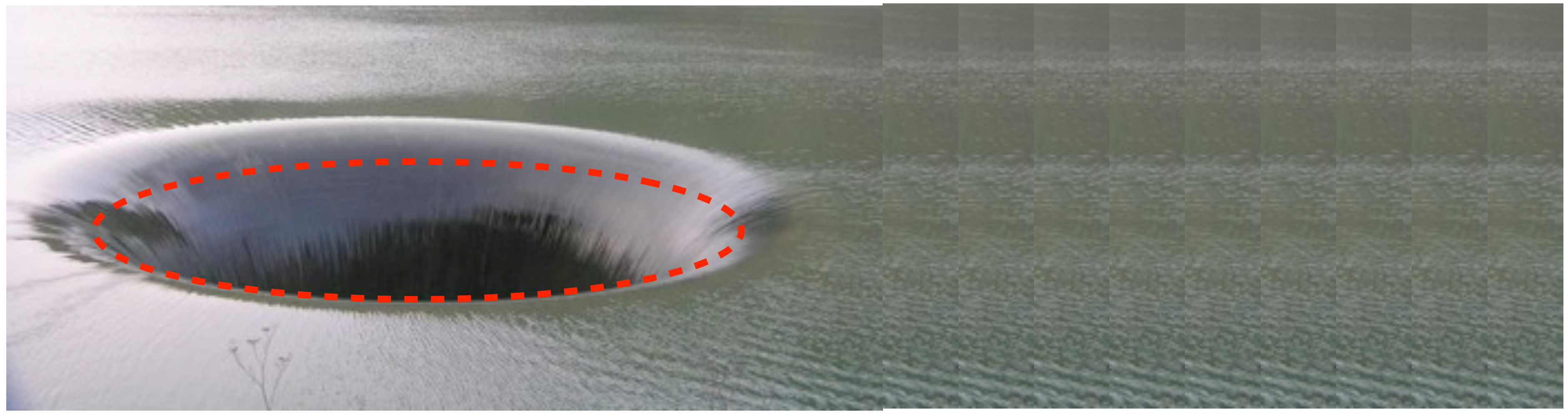


$M + \delta E$ $A + \delta A$

Entropy change δS_{BH}

thermodynamics

geometry

$$\delta S_{BH} = \frac{\kappa\, c^3}{\hbar} \frac{\delta A}{4\, G}$$

statistical mechanics

quantum mechanics

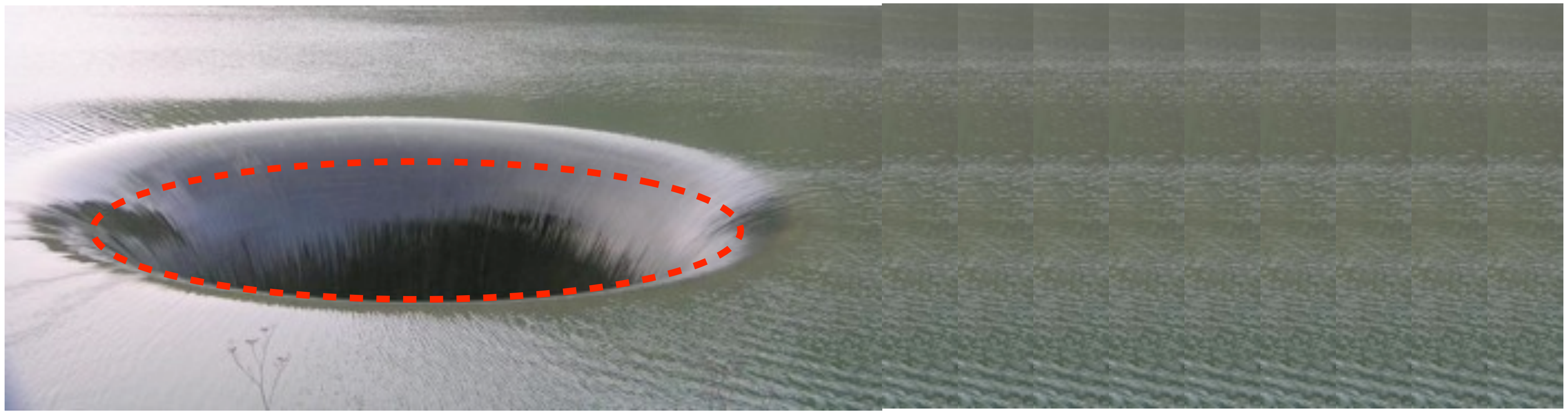
relativity

gravity

Process:

matter falling in a black hole,

area of the horizon increases



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Entropy change δS_{BH}

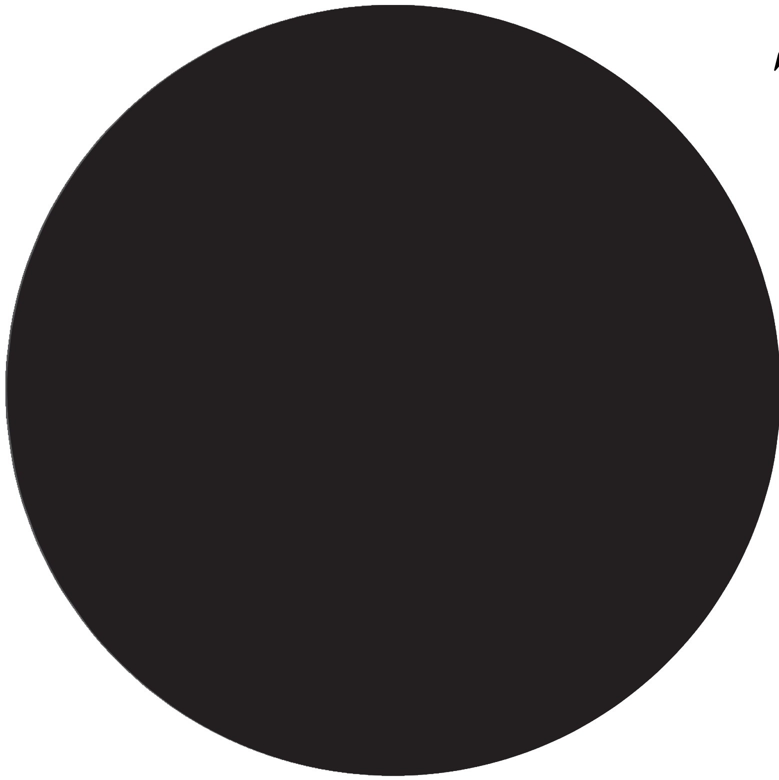
thermodynamics geometry

$$\delta S_{BH} = \frac{\kappa\, c^3}{\hbar}\, \frac{\delta A}{4\, G}$$

statistical mechanics quantum mechanics relativity gravity

Puzzle: the entropy of what?

Black hole entropy from Loop Quantum Gravity and Spinfoams



$$S_{BH} = \frac{A}{4G\hbar}$$

Rovelli, Smolin, Krasnov '95
Ashtekar, Baez, Corichi, Krasnov '98
Engle, Perez, Noui '10
Ghosh, Perez '11

Loop quantum gravity:

- microstates, quantum geometries
- proportionality to the area
- finiteness of the entropy
- Barbero-Immirzi γ puzzle
- ensemble?

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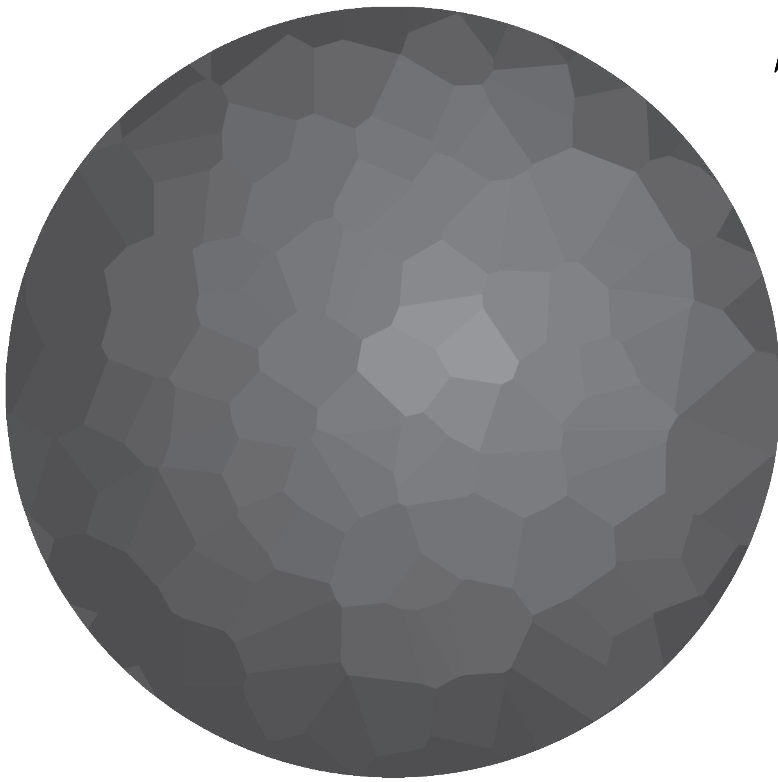

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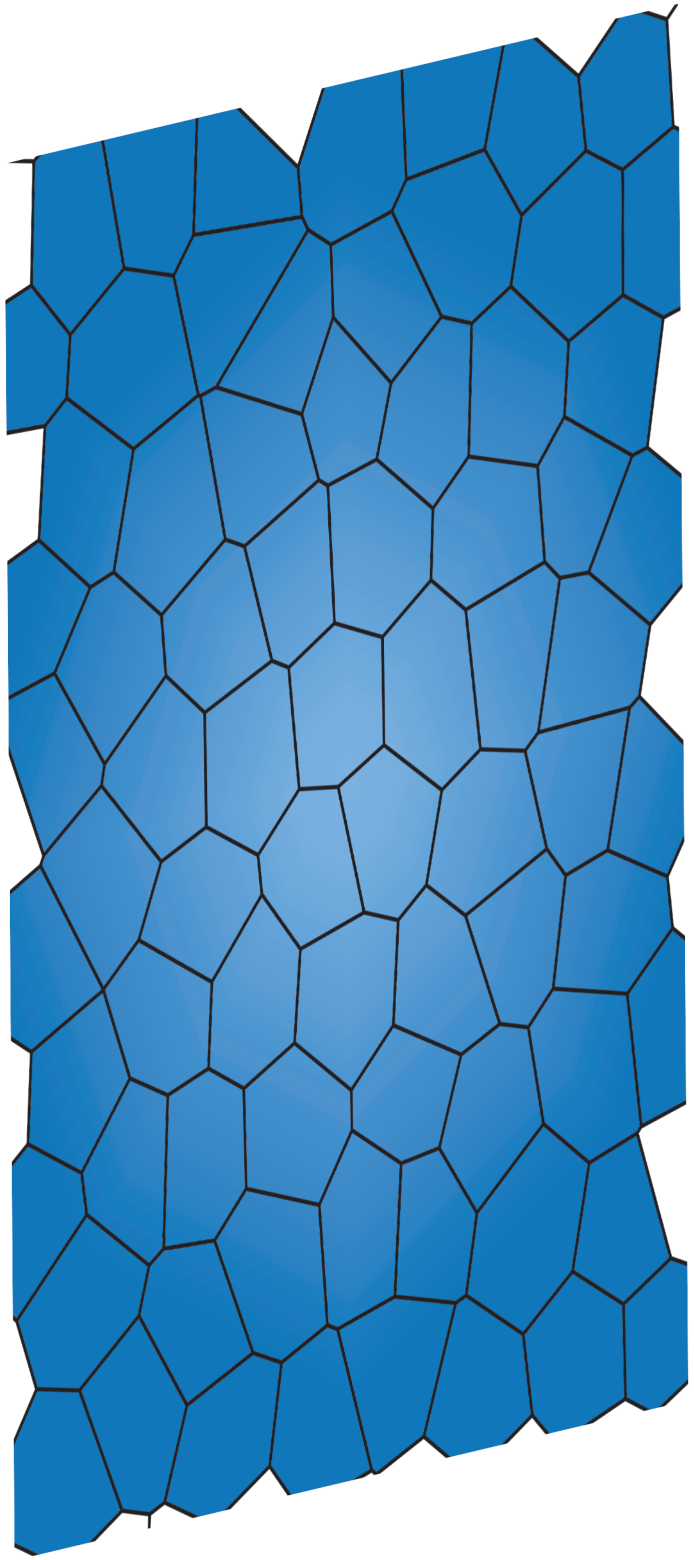
New results, 2012-2013:
- dynamics of the quantum horizon d.o.f.
- derivation of the $1/4$ prefactor in S_{BH}
- role played by horizon entanglement

Bianchi '12, Frodden-Ghosh-Perez '12
Pranzetti '12, Bianchi-Wieland '12
Bianchi '13, Bianchi-Satz '13
Bianchi-Haggard-Rovelli '13

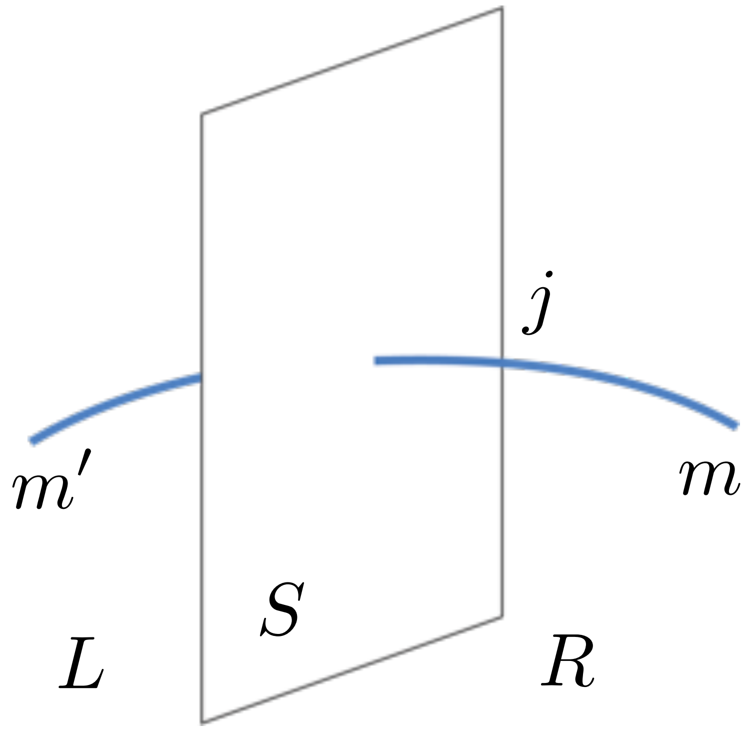


- a link piercing the horizon
- equilibrium state, perturbation δj
- area variation $\delta A = 8\pi G\hbar\gamma\,\delta j$
- entropy variation $\delta S = 2\pi\,\gamma\,\delta j$

Entropy/Area: γ neatly cancels in the ratio

$$\delta S = 2\pi\,\frac{\delta A}{8\pi G\hbar} = \frac{\delta A}{4G\hbar}$$

Reproduces the Bekenstein-Hawking entropy formula



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Bipartite quantum system

- Hilbert space $$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

- Observables on *B* $$\mathcal{O} = I_A \otimes \mathcal{O}_B$$

$$\longrightarrow \quad \langle\psi|\mathcal{O}|\psi\rangle = \mathrm{Tr}_B(\mathcal{O}_B\, \rho_B)$$

- Reduced density matrix $$\rho_B = \mathrm{Tr}_A(|\psi\rangle\langle\psi|)$$

- Entanglement entropy $$S_{\mathrm{ent}}(|\psi\rangle) = -\mathrm{Tr}_B(\rho_B \log \rho_B)$$

Vacuum Entanglement in QFT

[Sorkin 1983]

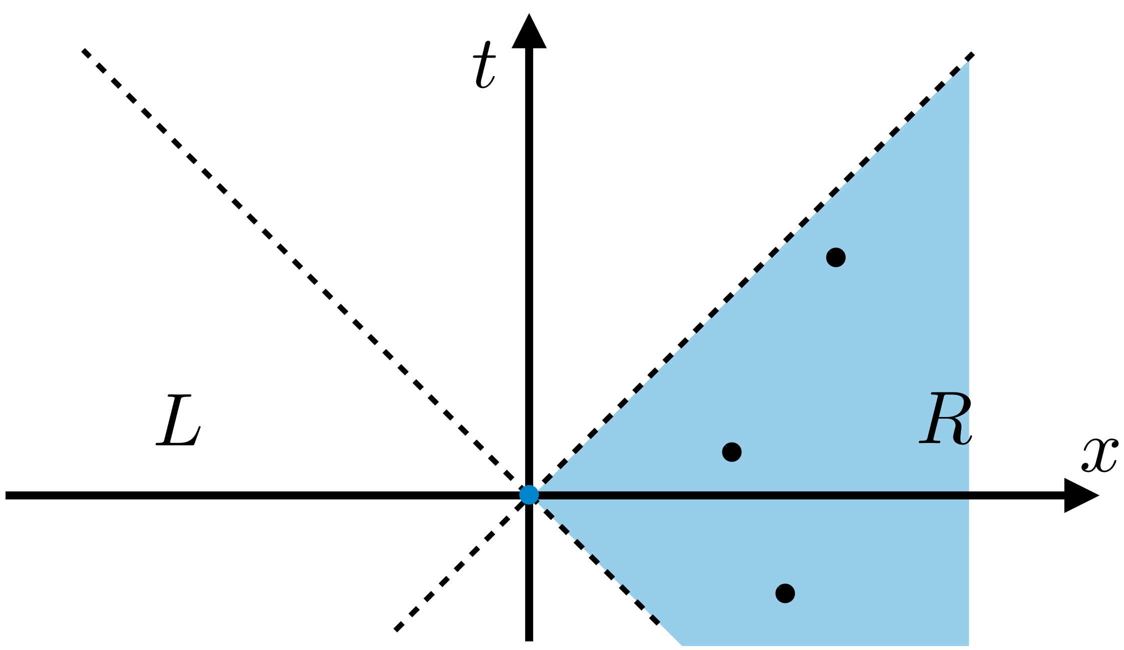
- 4d Minkowski space, $x^\mu = (t, x, y_1, y_2)$

- Regions: $L = \{x < |t|\}$, $R = \{x > |t|\}$

- QFT $\mathcal{H} = \mathcal{H}_L \otimes \mathcal{H}_R$

- Observable $\mathcal{O} = I_L \otimes \mathcal{O}_R$

$$= \int_{x_1,\ldots,x_N \in R} f(x_1,\ldots,x_N)\, \varphi(x_1)\cdots\varphi(x_N)$$



- Vacuum $|0\rangle$, reduced density matrix $\rho_0 = \mathrm{Tr}_L(|0\rangle\langle 0|) = \dfrac{e^{-2\pi K_R}}{Z}$

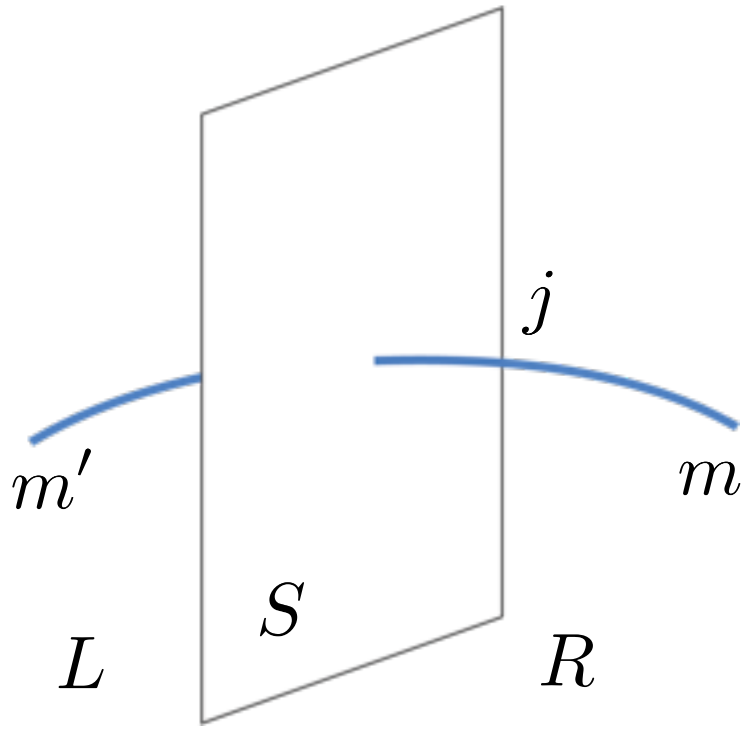
- Vacuum Entanglement Entropy

$$S_{ent}(|0\rangle) = -\mathrm{Tr}_R(\rho_0 \log \rho_0)$$

$$= c_0\, A\, \Lambda^2 + \ldots$$

- UV cutoff $\Lambda \sim 1/\sqrt{G}$?

The state of the Quantum Horizon



The Hartle-Hawking state:

$$|HH\rangle = \frac{1}{\sqrt{Z}} \sum_{m=-j}^{+j} \langle j,m|e^{-\pi K}|j,m\rangle \;\; |j,m\rangle_L \otimes |j,m\rangle_R$$

Entanglement across the Horizon

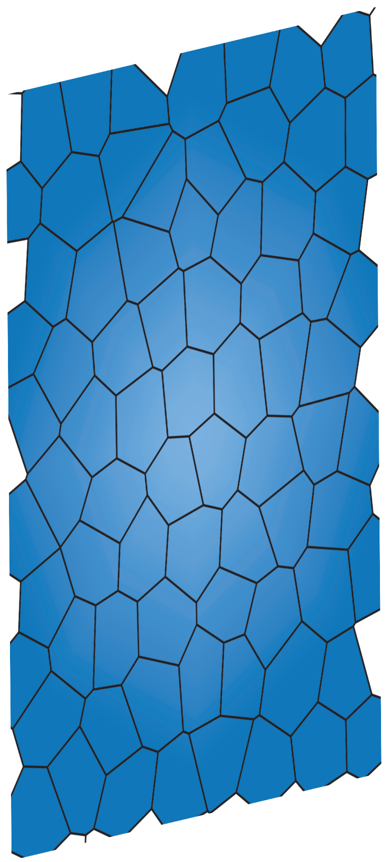
Expectation value of operators acting on the *Right* Hilbert space
= Trace over the *Left*

e.g. Area $$A = \langle HH|\, 8\pi G\hbar\gamma\, \vec{L}\cdot\vec{n}\, |HH\rangle = \mathrm{Tr}_R(\,\hat{A}\,\sigma) = 8\pi G\hbar\gamma m_*$$

reduced density matrix: $$\sigma = \mathrm{Tr}_L |HH\rangle\langle HH| = \frac{1}{Z} P e^{-\pi K} P e^{-\pi K} P$$

Many facets: density matrix $$\rho_0 = \bigotimes_{i=1}^{N} \sigma_i$$

Entanglement entropy variation = *thermodynamic entropy*



$$\sigma = \mathrm{Tr}_L |HH\rangle\langle HH|$$
$$= \frac{1}{Z} P e^{-\pi K} P e^{-\pi K} P$$

use: $\mathrm{Tr}(\delta\rho) = 0$

$P \log P = 0$

Process: $\rho_0 = \bigotimes_{i=1}^{N} \sigma$

$\Downarrow$

$$\rho_1 = \frac{1}{N} \sum_{i=1}^{N} \sigma \otimes \cdots \otimes \underset{i}{\alpha} \otimes \cdots \otimes \sigma \qquad \alpha = |j,j\rangle\langle j,j|$$

Quasi-stationary, $\delta\rho = \rho_1 - \rho_0$ is small for large N

- Area variation $\delta A = \mathrm{Tr}(A\,\delta\rho) = 8\pi G\hbar\,\gamma(j - m_*)$

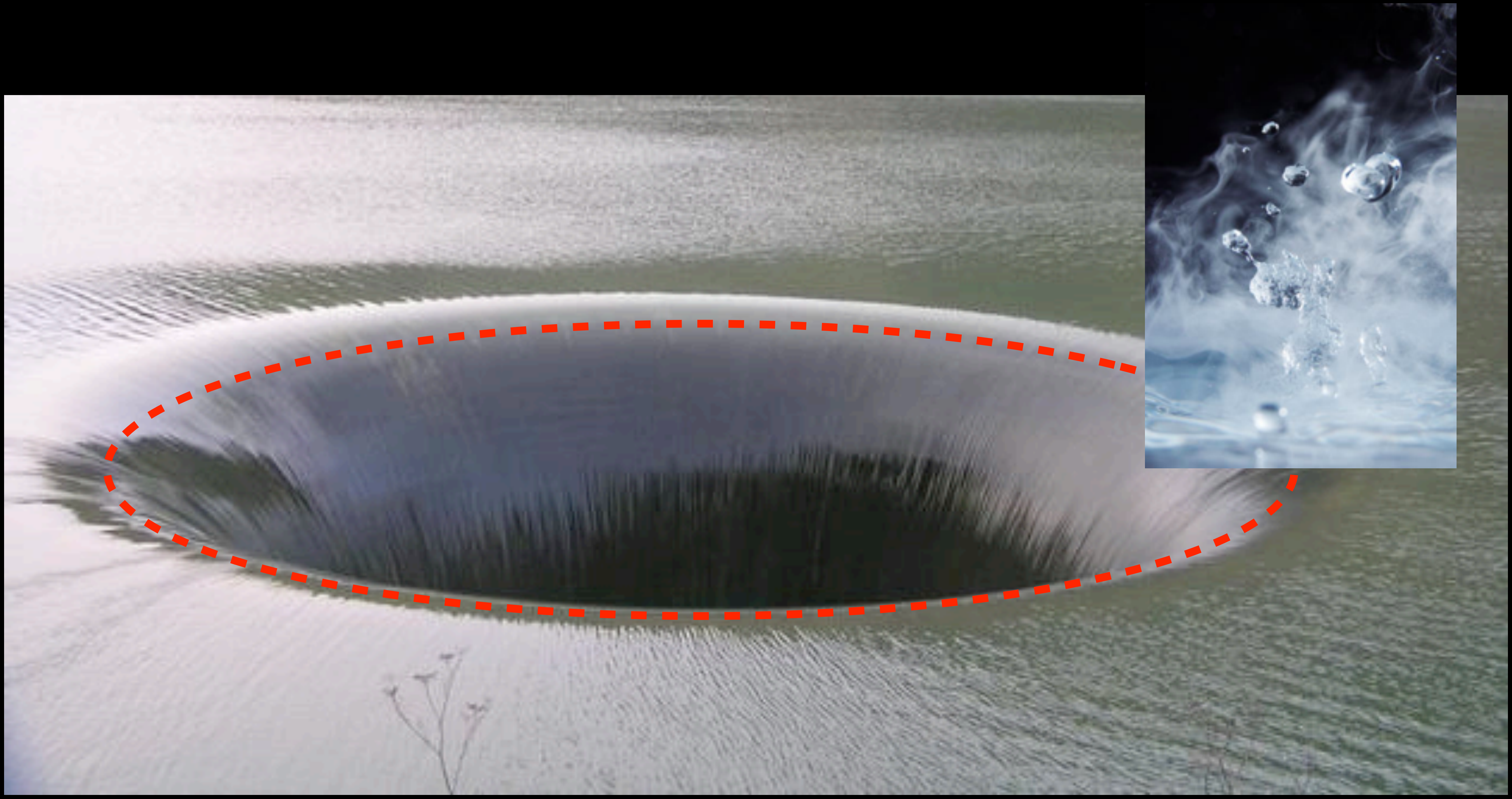
- Energy variation $\delta E = \mathrm{Tr}(K\,\delta\rho) = \gamma\,(j - m_*)$

- Variantion of the entanglement entropy

$$\delta S = -\delta\,\mathrm{Tr}(\rho \log \rho)$$
$$= -\mathrm{Tr}(\delta\rho \log \rho_0) \quad -\mathrm{Tr}(\rho_0 \frac{1}{\rho_0} \delta\rho)$$
$$= 2\pi\,\mathrm{Tr}(K\,\delta\rho)$$
$$= 2\pi\,\mathrm{Tr}(\frac{A}{8\pi G\hbar}\,\delta\rho)$$
$$= 2\pi \frac{\delta A}{8\pi G\hbar} = \frac{\delta A}{4G\hbar}$$

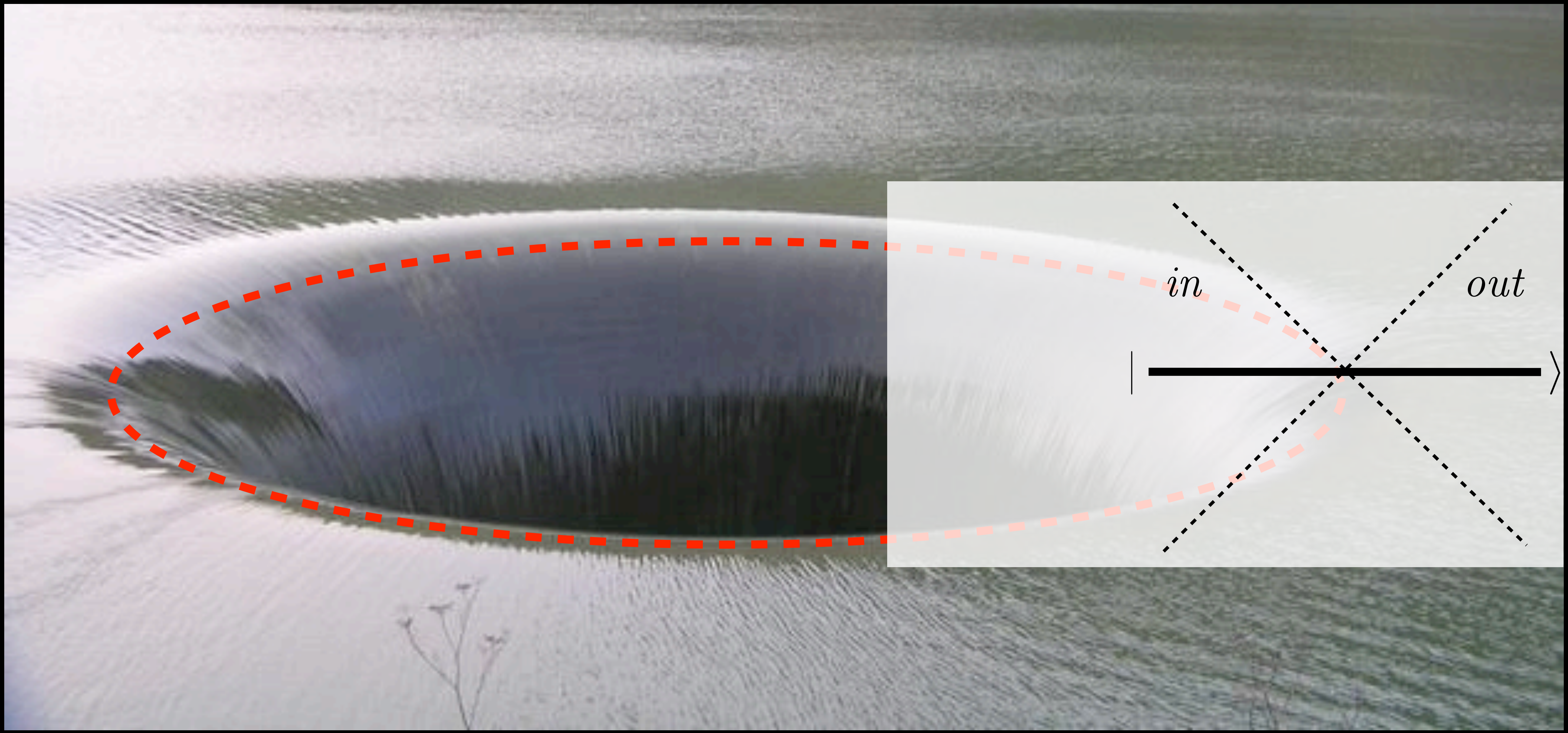
■

Why are black holes hot ?



Why are black holes hot ?

because of entanglement across the horizon



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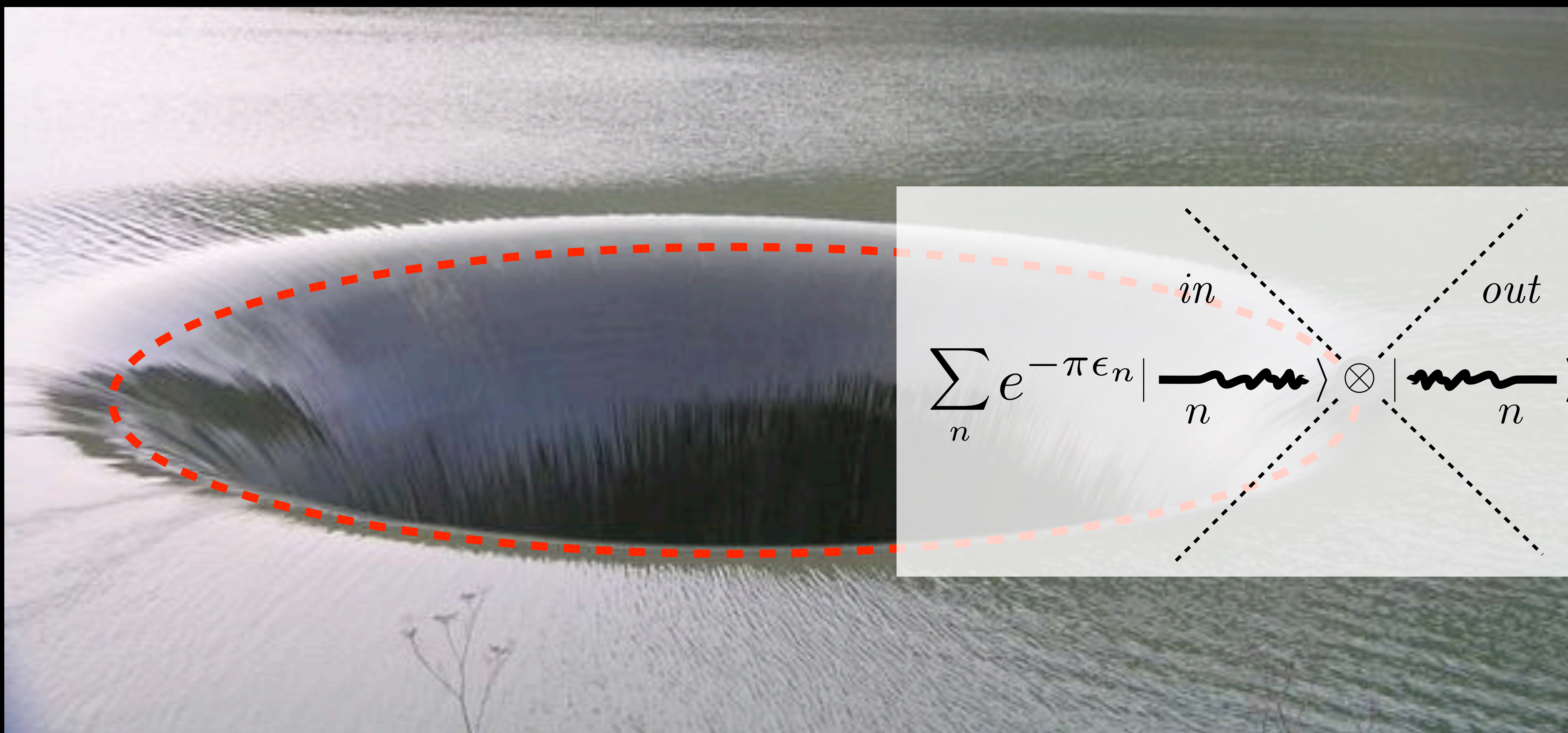

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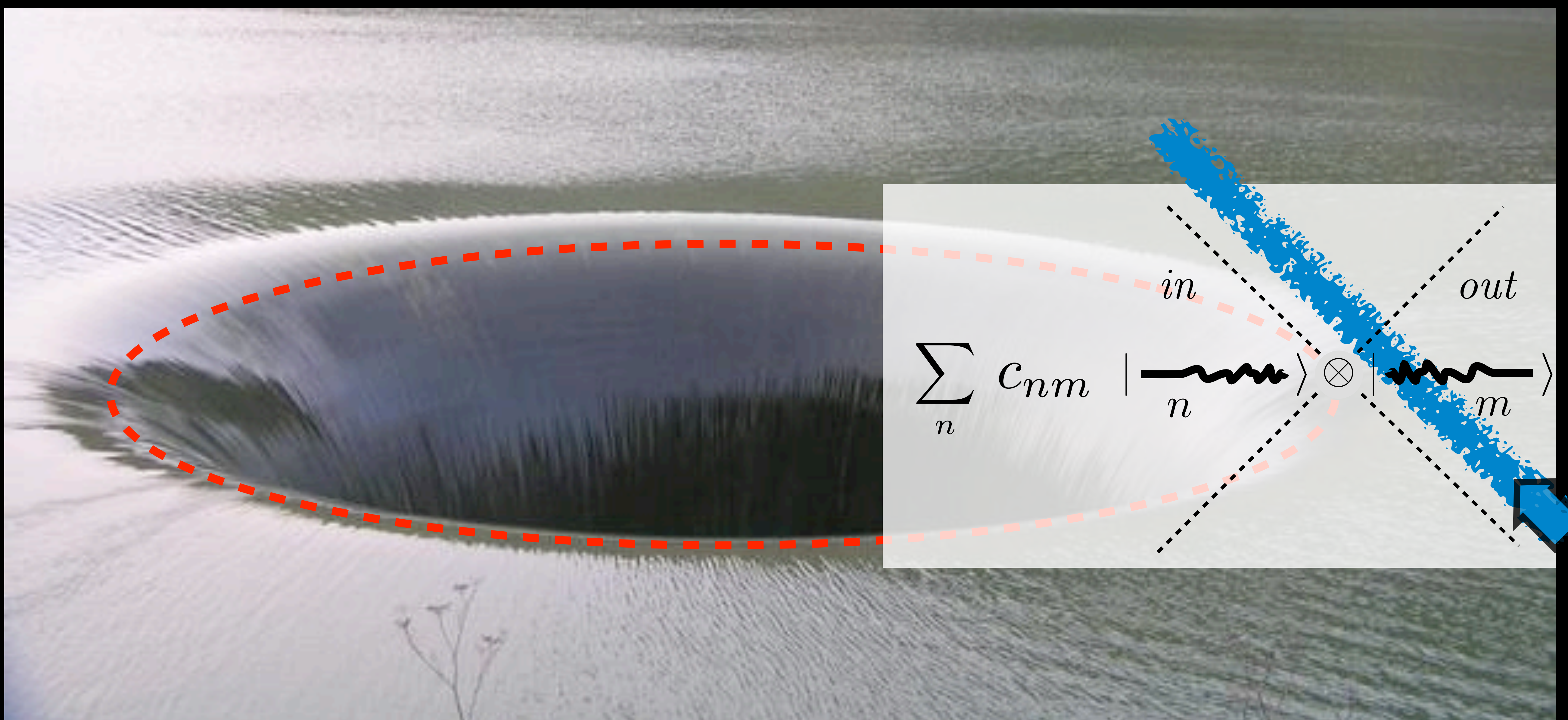
$$\delta S_{\text{ent}} = S_{\text{ent}}(|\mathcal{E}\rangle) - S_{\text{ent}}(|\Omega\rangle) = \frac{\delta A}{4G}$$



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