Spinfoam gravity: progress and perspectives - Lecture 1

Eugenio Bianchi

Perimeter Institute - Canada

53. Cracow School of Theoretical Physics

Outline

Lecture 1:

Path integral and the Spinfoam amplitude

Lecture 2:

Quantum geometry in Spinfoams

Lecture 3:

- gravitons
- quantum cosmology
- black hole entropy

Spin-foam path integral

- Aim: provide a realization of the path-integral over geometries for 4d Lorentzian gravity

$$Z = \int \mathcal{D}g_{\mu\nu} \;\; e^{\frac{i}{\hbar} S[g_{\mu\nu}]}$$

* Action? Measure? Boundary conditions?
  How to compute it beyond perturbation theory?

Spin-foam path integral

- Aim: provide a realization of the path-integral over geometries for 4d Lorentzian gravity

$$Z = \int \mathcal{D}g_{\mu\nu} \;\; e^{\frac{i}{\hbar} S[g_{\mu\nu}]}$$

* Action? Measure? Boundary conditions?
How to compute it beyond perturbation theory?

- Spinfoams: covariant formulation of loop quantum gravity

$$\boxed{Z_{\Delta_2} = \sum_{j_f\;, i_e} \prod_{f\in\Delta_2^*} (2j_f+1) \prod_{v\in\Delta_2^*} \Big\{ \bigotimes_{e\in v} \mathcal{I}_\gamma(i_e) \Big\}}$$

spin

foam

invariant of the
Lorentz group SO(1,3)

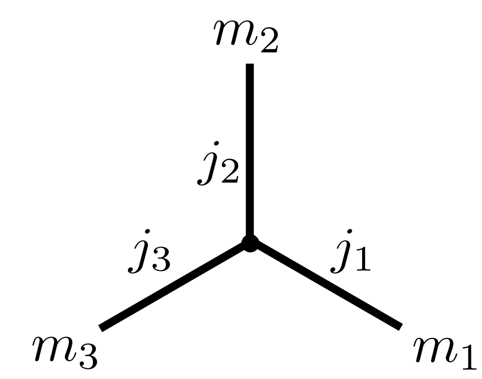
cf. Wigner {*6j*}*-symbol*

The simplest spinfoam: 3d Quantum Gravity and the Wigner {6*j*}-symbol

Spin: irreps of SU(2) $\quad |j, m\rangle \in \mathcal{H}_j$

Intertwiner: invariant tensor $\quad |i\rangle \in \mathrm{Inv}_{SU(2)}\left(\mathcal{H}_{j_1} \otimes \mathcal{H}_{j_2} \otimes \mathcal{H}_{j_3}\right)$

$$|i\rangle = \sum_{m_1 m_2 m_3} i_{m_1 m_2 m_3} \, |j_1, m_1\rangle |j_2, m_2\rangle |j_3, m_3\rangle \quad =$$

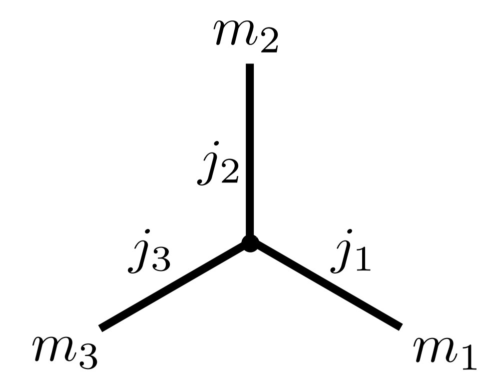


The simplest spinfoam: 3d Quantum Gravity and the Wigner {*6j*}-symbol

Spin: irreps of SU(2) $\quad |j,m\rangle \in \mathcal{H}_j$

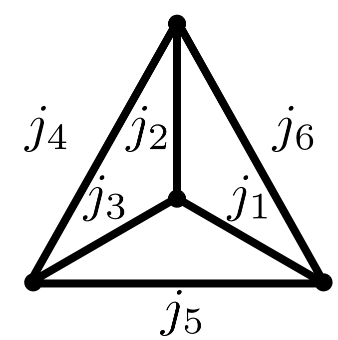
Intertwiner: invariant tensor $\quad |i\rangle \in \mathrm{Inv}_{SU(2)}\left(\mathcal{H}_{j_1} \otimes \mathcal{H}_{j_2} \otimes \mathcal{H}_{j_3}\right)$

$$|i\rangle = \sum_{m_1 m_2 m_3} i_{m_1 m_2 m_3}\, |j_1, m_1\rangle |j_2, m_2\rangle |j_3, m_3\rangle \qquad =$$



{*6j*}-symbol: invariant scalar

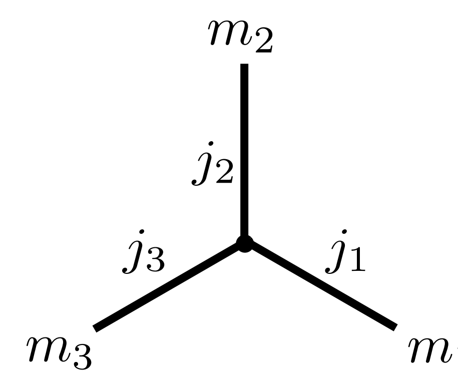
$$\{6j\} \equiv \left\{ \begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array} \right\} = \left\{ \bigotimes_{n=1}^{4} i_n \right\} =$$



Spin: irreps of SU(2) $|j,m\rangle \in \mathcal{H}_j$

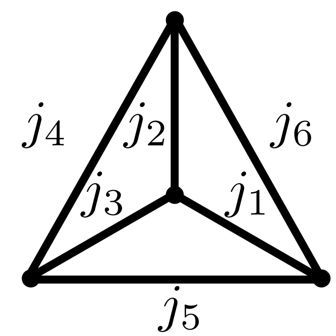
Intertwiner: invariant tensor $|i\rangle \in \mathrm{Inv}_{SU(2)}\,(\mathcal{H}_{j_1}\otimes\mathcal{H}_{j_2}\otimes\mathcal{H}_{j_3})$

$$|i\rangle = \sum_{m_1 m_2 m_3} i_{m_1 m_2 m_3}\,|j_1,m_1\rangle|j_2,m_2\rangle|j_3,m_3\rangle \quad =$$



{6j}-symbol: invariant scalar

$$\{6j\} \equiv \left\{ \begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array} \right\} = \left\{ \bigotimes_{n=1}^{4} i_n \right\} =$$



Semiclassical limit (Ponzano-Regge '68)

$$\boxed{\{6j\} \approx e^{+\frac{i}{\hbar}S(L_e)} + \text{c.c.}}$$

edge length $L_e = 8\pi G\hbar\,(j_e + \frac{1}{2})$

The Regge action ('61)

$$S(L_e) = \frac{1}{16\pi G}\int_{\triangle} \sqrt{g}R\,d^3x$$

$$= \frac{1}{8\pi G}\sum_e L_e\,\theta_e$$

Outline

Lecture 1:

Path integral and the Spinfoam amplitude

Background material:

- representations of the Lorentz group
- γ-simple representation
- Lorentz interwiners $\mathcal{I}_\gamma(i)$

The Lorentz group $SO^{\uparrow}(1,3)$ and its double cover $SL(2,\mathbb{C})$

- Finite-dim reps, e.g.:

  - Spinor rep (Weyl) $\qquad L_i = \mathrm{i}\,\frac{\sigma_i}{2} \qquad K_i = \frac{\sigma_i}{2}$

  - Vector rep $\qquad x'^{\mu} = \Lambda^{\mu}{}_{\nu}\, x^{\nu}$

  - ...

<u>Non-Unitary</u> $\qquad$ Non-Hermitian Generator of Boosts

The Lorentz group $SO^{\uparrow}(1,3)$ and its double cover $SL(2,\mathbb{C})$

- Finite-dim reps, e.g.:

  - Spinor rep (Weyl) $\quad L_i = \mathrm{i}\,\dfrac{\sigma_i}{2} \qquad K_i = \dfrac{\sigma_i}{2}$

  - Vector rep $\quad x'^{\mu} = \Lambda^{\mu}_{\ \nu}\, x^{\nu}$

  - ...

  Non-Unitary $\qquad$ Non-Hermitian Generator of Boosts

- Infinite-dim reps, e.g.:

  - Field Theoretical rep $\quad U(\Lambda)\phi(x^{\mu}) = \phi(\Lambda^{-1\ \mu}_{\ \ \nu}\, x^{\nu})$

  Unitary but reducible

$$U(\Lambda) = e^{\,\mathrm{i}\,\omega_{\mu\nu} J^{\mu\nu}} \qquad J^{\mu\nu} = -\mathrm{i}\,(x^{\mu}\partial^{\nu} - x^{\nu}\partial^{\mu})$$

$\longrightarrow$ decompose in irreducible blocks

$V^{(p,k)}$ = Unitary Irreducible Representations of $SL(2,\mathbb{C})$

ref: Ruhl '70

$\boxed{V^{(p,k)}}$ $\quad p \in \mathbb{R} \quad k \in \mathbb{N}/2 = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots \quad \dim V^{(p,k)} = \infty$

Casimirs

$$C_1 \equiv \frac{1}{2} J_{IJ} J^{IJ} = \vec{K}^2 - \vec{L}^2$$

$$C_2 \equiv \frac{1}{8} \epsilon_{IJKL} J^{IJ} J^{KL} = \vec{K} \cdot \vec{L}$$

on $V^{(p,k)}$

$$= p^2 - k^2 + 1$$

$$= p\, k$$

Hermitian Generators $\quad J^{IJ} = -J^{JI}$

$$[J^{IJ}, J^{KL}] = -\mathrm{i}\,(\eta^{IK} J^{JL} - \eta^{IL} J^{JK} + \eta^{JL} J^{IK} - \eta^{JK} J^{IL})$$

Time-like vector $\quad N^I = (1,0,0,0)$

Rotations $\quad L^I = \frac{1}{2} \epsilon^I{}_{JKL} J^{JK} N^L = (0, L^i)$

Boosts $\quad K^I = J^{IJ} N_J = (0, K^i)$

Algebra

$$[L^i, L^j] = \mathrm{i}\,\epsilon^{ij}{}_k L^k$$

$$[L^i, K^j] = \mathrm{i}\,\epsilon^{ij}{}_k K^k$$

$$[K^i, K^j] = -\mathrm{i}\,\epsilon^{ij}{}_k L^k$$

Decomposition of $V^{(p,k)}$ in *SU(2)*-irreducible blocks

Little group that preserves the time-like vector N^I

$$SU(2) \subseteq SL(2,\mathbb{C})$$

$V^{(p,k)}$ is also a rep of *SU(2)*, but reducible

$$V^{(p,k)} = \mathcal{H}_k \oplus \mathcal{H}_{k+1} \oplus \cdots = \bigoplus_{j=k}^{\infty} \mathcal{H}_j$$

e.g.:

$$D^{(p,\frac{1}{2})}(U) = \begin{pmatrix} \underset{D^{(\frac{1}{2})}}{\boxed{\ddots}} & & \\ & \boxed{\ddots} & \\ & & \ddots \end{pmatrix}$$

O.N. basis of $V^{(p,k)}$

$\longrightarrow$ diagonalize simultaneously $C_1\,,\ C_2\,,\ \vec{L}^2\,,\ L_z$

$$|(p,k);\, j, m\rangle \qquad j \geq k$$

Outline

Lecture 1:

Path integral and the Spinfoam amplitude

Background material:

- representations of the Lorentz group
- γ-simple representation and the map $Y_\gamma : \mathrm{Irrep}_{SU(2)} \to \mathrm{Irrep}_{SL(2,\mathbb{C})}$
- Lorentz interwiners $\mathcal{I}_\gamma(i)$

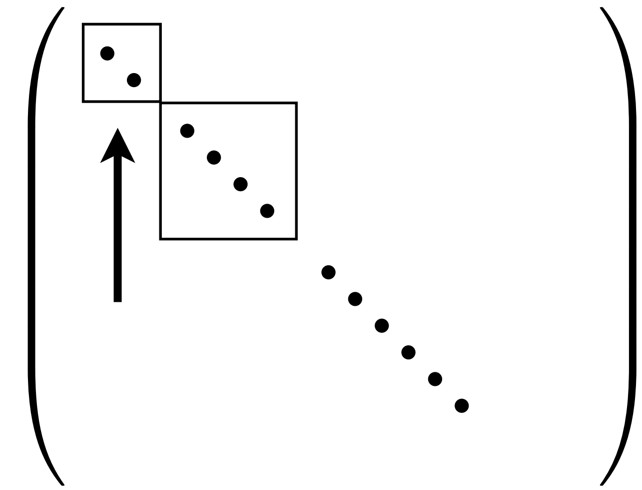
γ -simple representations: $V^{(p,k)}$ with $p=\gamma\,(k+1)$ $\qquad \gamma \in \mathbb{R}$

Vogan minimal K-type subspace $V_{\min}^{(p,k)} \subset V^{(p,k)}$

= minimal SU(2)-invariant block

O.N. basis $|(p,k);\; k,m\rangle, \quad m=-k,\ldots,+k$

$$\dim V_{\min}^{(p,k)} \;=\; 2k+1$$

$D^{(p,\frac{1}{2})}(U) \;=$ 

γ-simple representations: $V^{(p,k)}$ with $p = \gamma\,(k+1)$ $\quad \gamma \in \mathbb{R}$

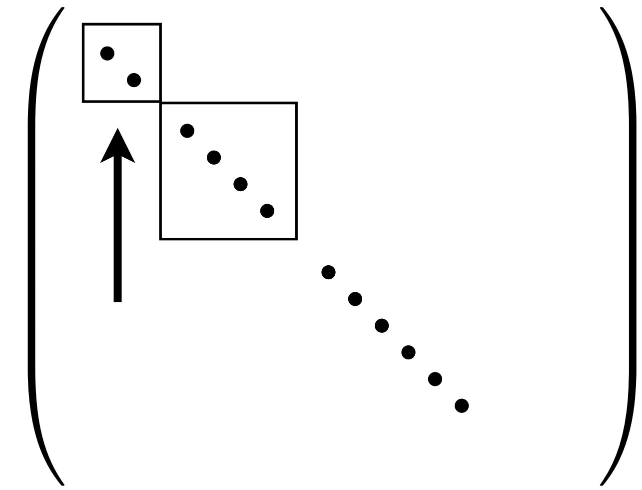
Vogan minimal K-type subspace $V^{(p,k)}_{\min} \subset V^{(p,k)}$

= minimal SU(2)-invariant block

O.N. basis $|(p,k);\; k, m\rangle, \quad m = -k, \ldots, +k$

$$\dim V^{(p,k)}_{\min} = 2k+1$$

$$D^{(p,\frac{1}{2})}(U) =$$



Projection to $V^{(p,k)}_{\min}$ of the generators L^i , K^i :

$$L^3_{\min} = \sum_{m=-k}^{+k} m\; |(p,k);k,m\rangle\langle(p,k);k,m| \quad , \qquad K^3_{\min} = \sum_{m=-k}^{+k} \frac{p\,m}{k+1}\; |(p,k);k,m\rangle\langle(p,k);k,m|$$

γ -simple representations: $\quad V^{(p,k)}$ with $\quad p = \gamma\,(k+1) \qquad \gamma \in \mathbb{R}$

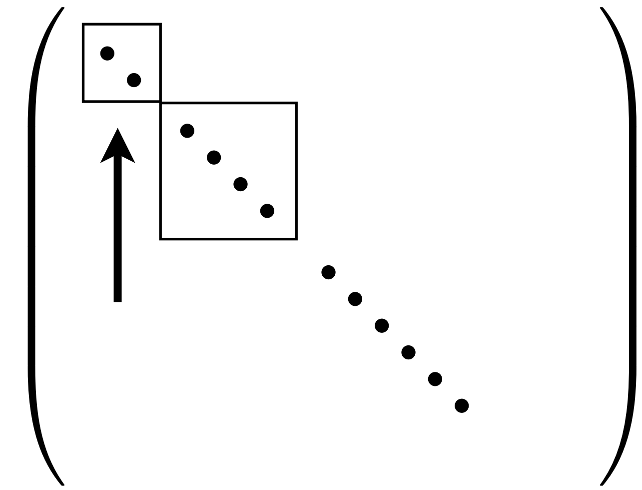
Vogan minimal K-type subspace $\;V^{(p,k)}_{\mathrm{min}} \subset V^{(p,k)}$

= minimal SU(2)-invariant block

O.N. basis $\quad |(p,k);\,k,m\rangle, \qquad m = -k,\ldots,+k$

$$\mathrm{dim} V^{(p,k)}_{\mathrm{min}} \;=\; 2k+1$$

$$D^{(p,\frac{1}{2})}(U) \;=$$



Projection to $V^{(p,k)}_{\mathrm{min}}$ of the generators $\;L^i\,,\;K^i$:

$$L^3_{\mathrm{min}} = \sum_{m=-k}^{+k} m\;|(p,k);k,m\rangle\langle(p,k);k,m| \qquad , \qquad K^3_{\mathrm{min}} = \sum_{m=-k}^{+k} \frac{p\,m}{k+1}\;|(p,k);k,m\rangle\langle(p,k);k,m|$$

<u>γ -simple representations:</u> $\qquad \boxed{K^i_{\mathrm{min}} = \gamma\,L^i_{\mathrm{min}}} \qquad$ for some $\;\gamma \in \mathbb{R}$

$$\Longleftrightarrow \qquad V^{(p,k)} \quad \text{with} \quad \boxed{p = \gamma\,(k+1)}$$

Outline

Lecture 1:

Path integral and the Spinfoam amplitude

Background material:

- representations of the Lorentz group
- γ-simple representation and the map $Y_\gamma : \mathrm{Irrep}_{SU(2)} \to \mathrm{Irrep}_{SL(2,\mathbb{C})}$
- Lorentz interwiners $\mathcal{I}_\gamma(i)$

Lorentz intertwiners from *SU(2)* intertwiners

Map $\quad \mathrm{Irrep}_{\mathrm{SU}(2)} \;\rightarrow\; \gamma\text{-simple } \mathrm{Irrep}_{SL(2,\mathbb{C})}$

$$Y_\gamma : \; \mathcal{H}_j \;\rightarrow\; V^{(p,k)} \qquad \text{with} \qquad p = \gamma\,(j+1)\;, \quad k = j$$

$$|j,m\rangle \mapsto |(\gamma(j+1),j);j,m\rangle$$

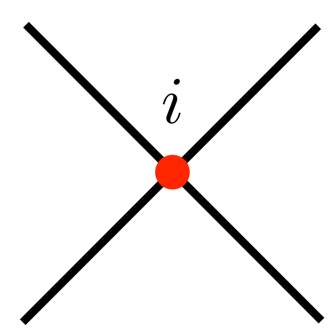
SU(2) intertwiners

$$i \;\in\; \mathrm{Inv}_{SU(2)}(\mathcal{H}_{j_1} \otimes \cdots \otimes \mathcal{H}_{j_N})$$

<u>EPRL</u>: *SL(2,C)* intertwiners from *SU(2) intetwiners* (Engle-Pereira-Rovelli-Livine *NPB* '08)

$$\boxed{\mathcal{I}_\gamma(i) \;=\; \mathrm{Inv}_{SL(2,\mathbb{C})}(Y_\gamma\, i)}$$

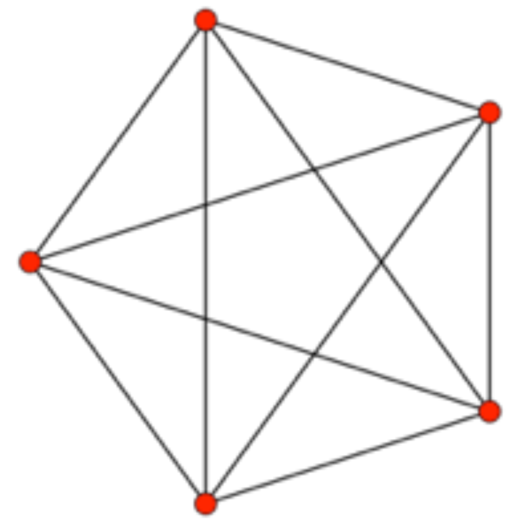
e.g.: N=4, $\quad \mathcal{I}_\gamma(i) \;=$



The spinfoam vertex amplitude

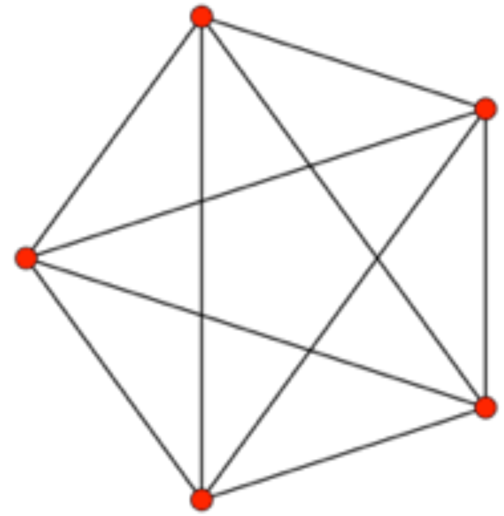
Barrett-Crane *JMP* '98
Engle-Pereira-Rovelli *PRL* '07
Freidel-Krasnov *CQG*'07
Livine-Speziale *PRD*'07
Engle-Pereira-Rovelli-Livine *NPB* '08

$$\mathcal{A}_v = \Big\{ \bigotimes_{e \in v} \mathcal{I}_\gamma(i_e) \Big\} =$$



The spinfoam vertex amplitude

$$\mathcal{A}_v \;=\; \Big\{ \bigotimes_{e\in v} \mathcal{I}_\gamma(i_e) \Big\} \;=$$



Barrett-Crane *JMP* '98
Engle-Pereira-Rovelli *PRL* '07
Freidel-Krasnov *CQG*'07
Livine-Speziale *PRD*'07
Engle-Pereira-Rovelli-Livine *NPB* '08

Semiclassical limit: $\hbar \to 0$

$$\boxed{\mathcal{A}_v \;\approx\; e^{+\frac{i}{\hbar} S_{GR}(\;)} + c.c.}$$

Barrett-Dowdall-Fairbairn-Hellmann-
-Gomes-Pereira *JMP* '09

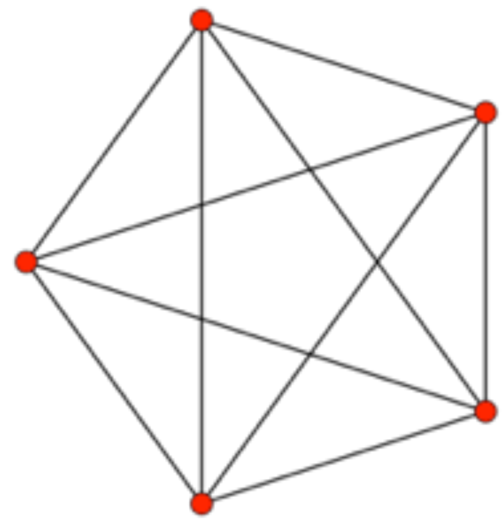
Area of a face $\quad A_f \;=\; 8\pi G \hbar \gamma \sqrt{j_f(j_f+1)}$

Regge action in 4d ('61)

$$S_{GR}(\;) \;=\; \frac{1}{16\pi G}\int \sqrt{-g}\,R\,d^4x \qquad = \frac{1}{8\pi G}\sum_f A_f \theta_f$$

The spinfoam vertex amplitude

$$\mathcal{A}_v \;=\; \Big\{ \bigotimes_{e\in v} \mathcal{I}_\gamma(i_e) \Big\} \;=$$



Barrett-Crane *JMP* '98
Engle-Pereira-Rovelli *PRL* '07
Freidel-Krasnov *CQG*'07
Livine-Speziale *PRD*'07
Engle-Pereira-Rovelli-Livine *NPB* '08

Semiclassical limit: $\hbar \to 0$

$$\boxed{\mathcal{A}_v \;\approx\; e^{+\frac{\mathrm{i}}{\hbar} S_{GR}(\;)} \;+\; c.c.}$$

Barrett-Dowdall-Fairbairn-Hellmann-
-Gomes-Pereira *JMP* '09

Area of a face $\quad A_f \;=\; 8\pi G\hbar\gamma\,\sqrt{j_f(j_f+1)}$

Geometry and action from invariants of the Lorentz group

Regge action in 4d ('61)

$$S_{GR}(\;) \;=\; \frac{1}{16\pi G}\int \sqrt{-g}R\,d^4x \qquad = \; \frac{1}{8\pi G}\sum_f A_f\theta_f$$

Spin-foam path integral

- Aim: provide a realization of the path-integral over geometries for 4d Lorentzian gravity

$$Z = \int \mathcal{D}g_{\mu\nu} \;\; e^{\frac{i}{\hbar} S[g_{\mu\nu}]}$$

* Action? Measure? Boundary conditions?
How to compute it beyond perturbation theory?

- Spinfoams: covariant formulation of loop quantum gravity

$$\boxed{Z_{\Delta_2} = \sum_{j_f\,,\,i_e} \prod_{f\in\Delta_2^*} (2j_f+1) \prod_{v\in\Delta_2^*} \{\bigotimes_{e\in v} \mathcal{I}_\gamma(i_e)\}}$$

spin

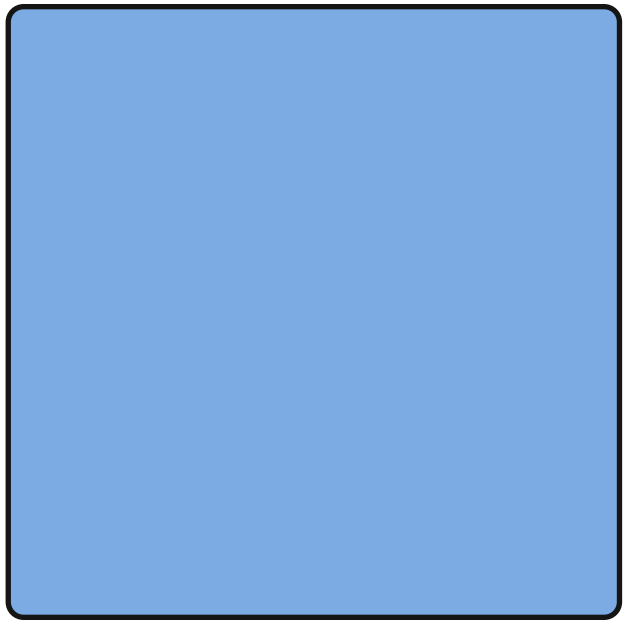
foam

invariant of the
Lorentz group SO(1,3)

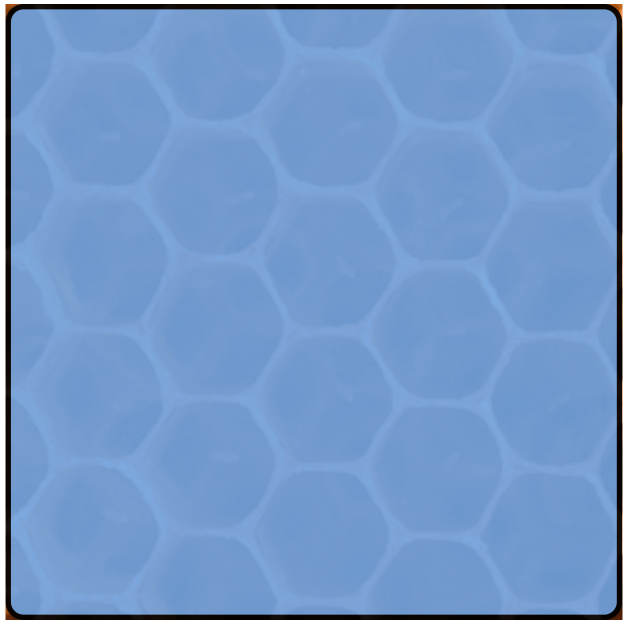
cf. Wigner *{6j}-symbol*

Spacetime manifold and the notion of 2d-foam

- M = Spacetime = 4d Manifold of trivial topology



Spacetime manifold and the notion of 2d-foam



- M = Spacetime = 4d Manifold of trivial topology
- Δ = Topological decomposition of M in cells

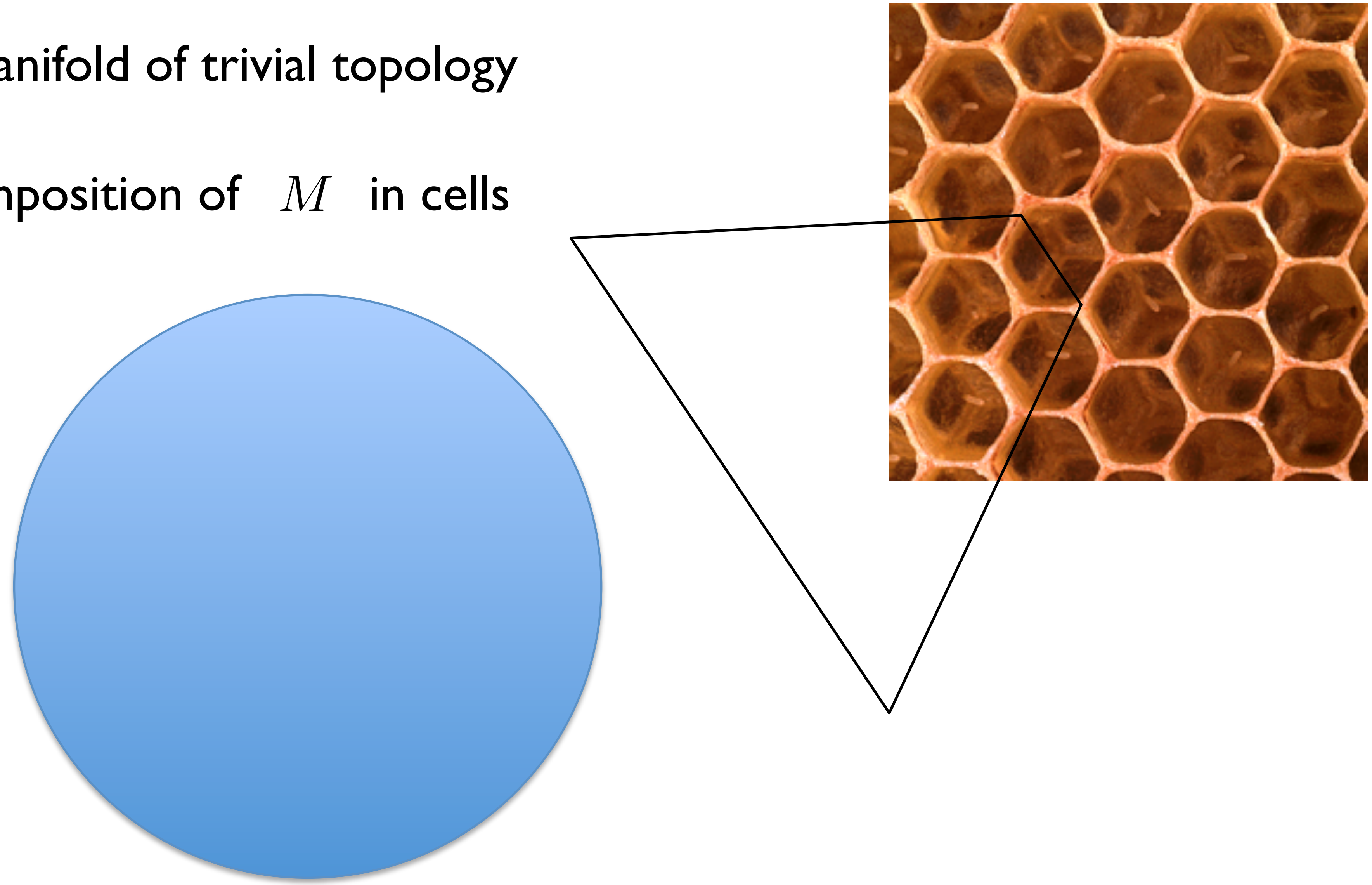
Spacetime manifold and the notion of 2d-foam

- M = Spacetime = 4d Manifold of trivial topology
- Δ = Topological decomposition of M in cells

4-cells Δ_4 = 4-ball

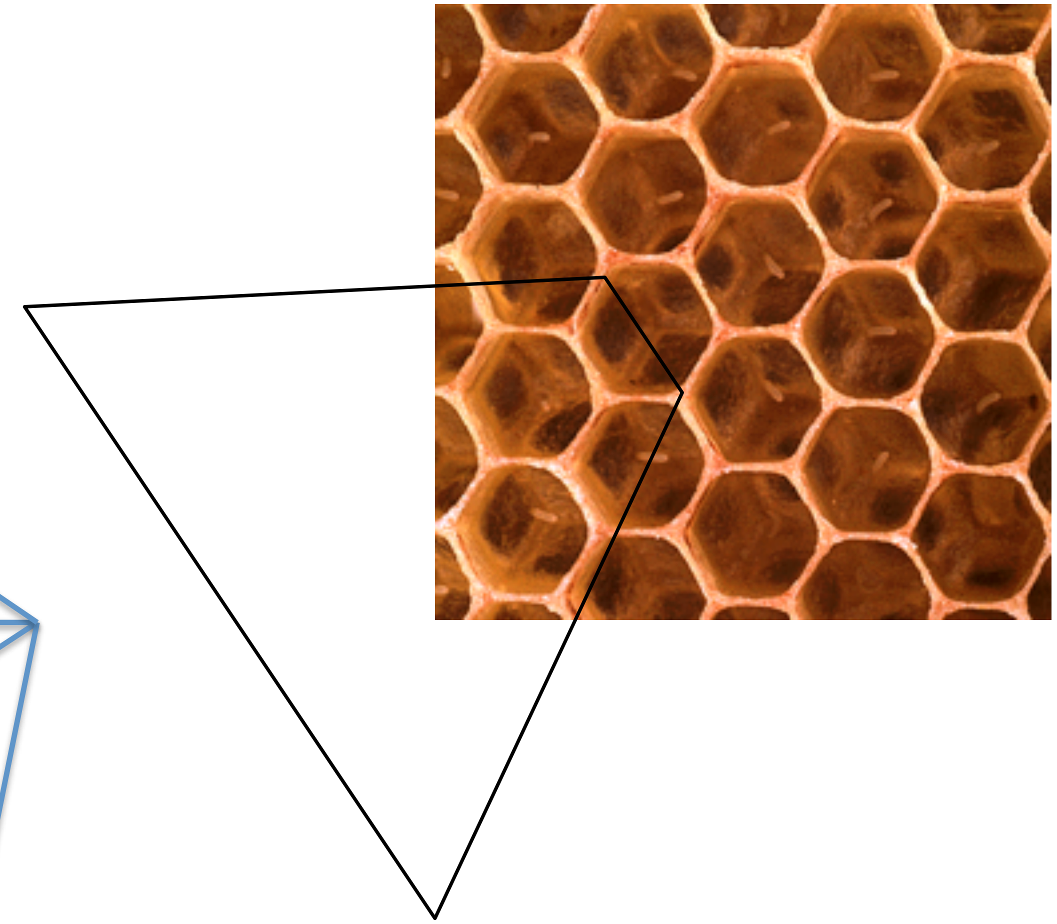
$\partial\Delta_4$ = 3-cells Δ_3

$\partial\Delta_3$ = 2-cells Δ_2

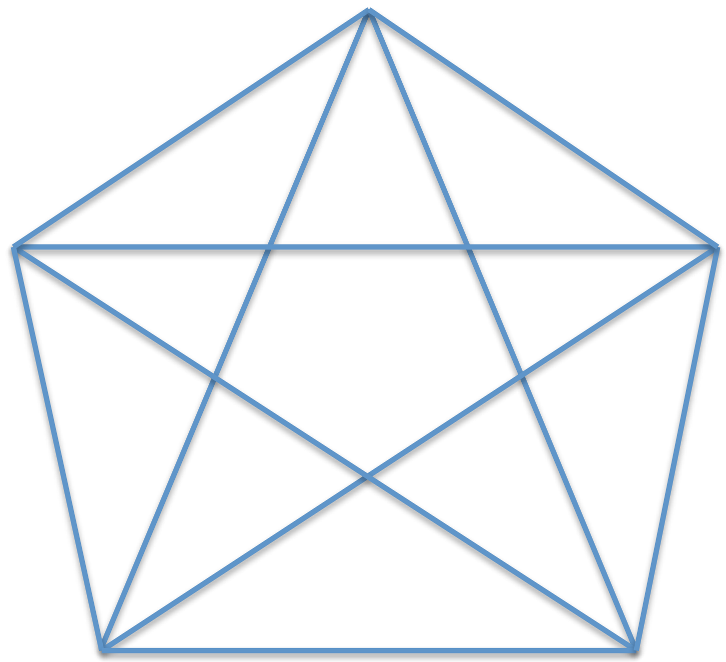


Spacetime manifold and the notion of 2d-foam

- M = Spacetime = 4d Manifold of trivial topology
- Δ = Topological decomposition of M in cells



4-cells Δ_4 = 4-ball

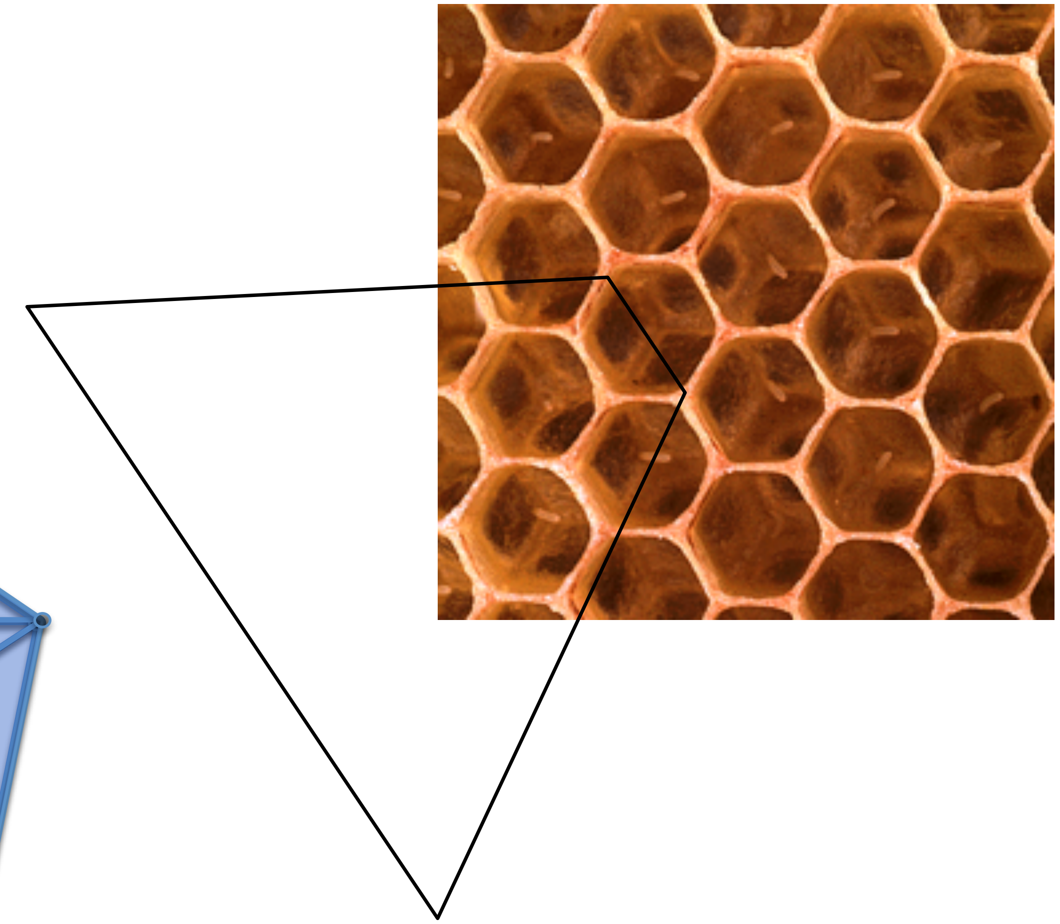


$\partial\Delta_4$ = 3-cells Δ_3

$\partial\Delta_3$ = 2-cells Δ_2

Spacetime manifold and the notion of 2d-foam

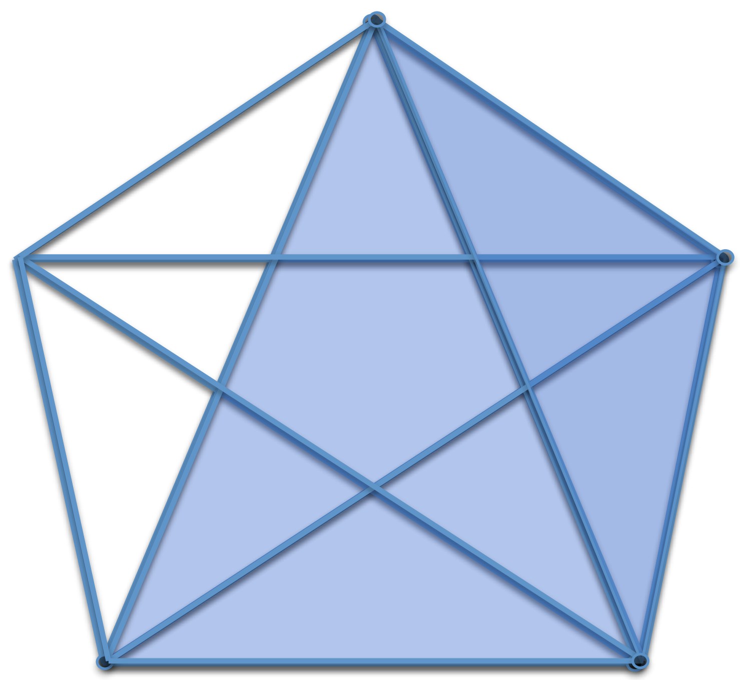
- M = Spacetime = 4d Manifold of trivial topology
- Δ = Topological decomposition of M in cells



4-cells Δ_4 = 4-ball

$\partial\Delta_4$ = 3-cells Δ_3

$\partial\Delta_3$ = 2-cells Δ_2



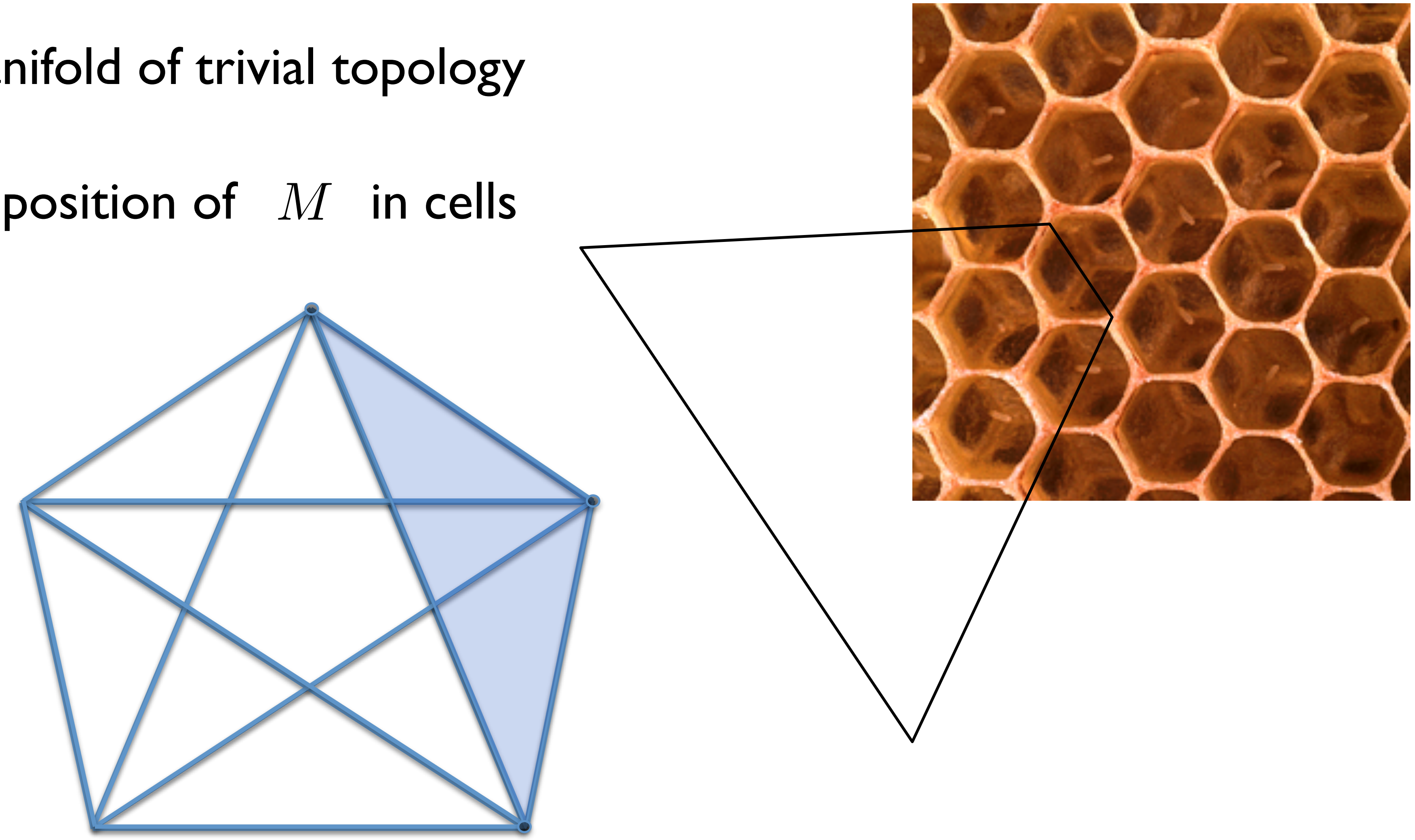
Spacetime manifold and the notion of 2d-foam

- M = Spacetime = 4d Manifold of trivial topology
- Δ = Topological decomposition of M in cells

4-cells Δ_4 = 4-ball

$\partial\Delta_4$ = 3-cells Δ_3

$\partial\Delta_3$ = 2-cells Δ_2



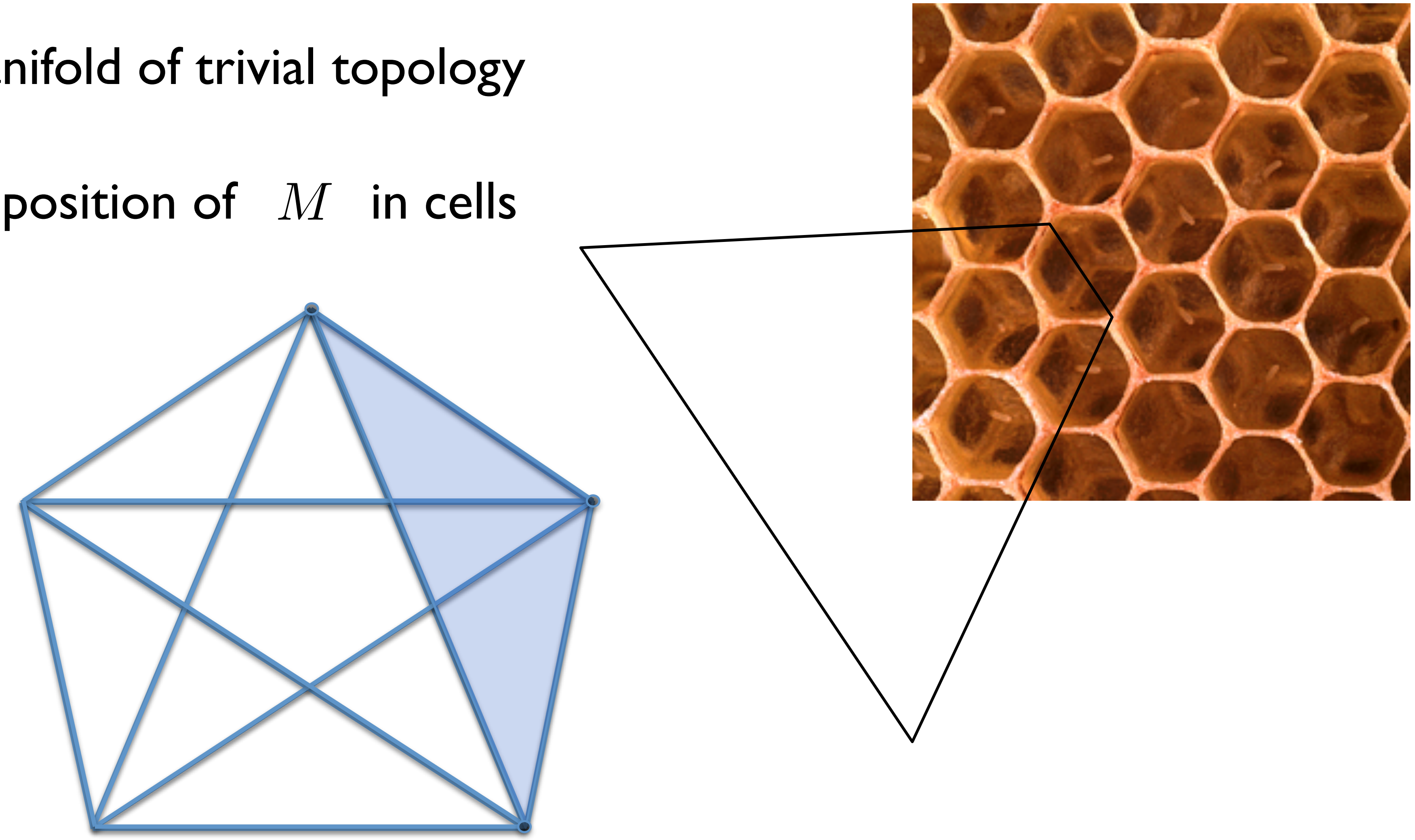
Spacetime manifold and the notion of 2d-foam

- M = Spacetime = 4d Manifold of trivial topology
- Δ = Topological decomposition of M in cells

4-cells Δ_4 = 4-ball

$\partial\Delta_4$ = 3-cells Δ_3

$\partial\Delta_3$ = 2-cells Δ_2



- Set $\{\Delta_2\}$ = 2-skeleton of (M, Δ) = *2d-foam*

* The manifold $M' = M - \{\Delta_2\}$ is non simply-connected, non-trivial π_1

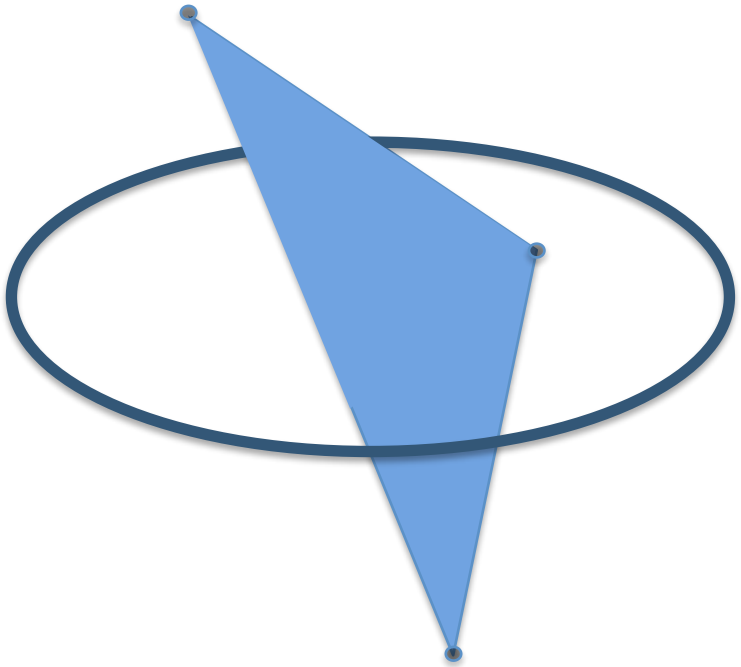
Spacetime manifold and the notion of 2d-foam

- M = Spacetime = 4d Manifold of trivial topology
- Δ = Topological decomposition of M in cells

4-cells Δ_4 = 4-ball

$\partial\Delta_4$ = 3-cells Δ_3

$\partial\Delta_3$ = 2-cells Δ_2



- Set $\{\Delta_2\}$ = 2-skeleton of (M, Δ) = *2d-foam*

* The manifold $M' = M - \{\Delta_2\}$ is non simply-connected, non-trivial π_1

non-contractible loops around Δ_2

- Gravity: Einstein-Cartan action

$$S[e,\omega]=\frac{1}{16\pi G}\int\ \frac{1}{2}\epsilon_{IJKL}e^I\wedge e^J\wedge F^{KL}(\omega)$$

- Gravity: Einstein-Cartan action + Holst term

$\gamma \in \mathbb{R}$ = Barbero-Immirzi parameter

$$S[e,\omega] = \frac{1}{16\pi G} \int \frac{1}{2} \epsilon_{IJKL} e^I \wedge e^J \wedge F^{KL}(\omega) \;\; + \;\; \frac{1}{\gamma} e_I \wedge e_J \wedge F^{IJ}(\omega)$$

Gravity and Topological field theory

- Gravity: Einstein-Cartan action + Holst term

$\gamma \in \mathbb{R}$ = Barbero-Immirzi parameter

$$S[e,\omega] = \frac{1}{16\pi G}\int \frac{1}{2}\epsilon_{IJKL}e^I \wedge e^J \wedge F^{KL}(\omega) \;+\; \frac{1}{\gamma} e_I \wedge e_J \wedge F^{IJ}(\omega)$$

- Topological Field Theory: BF action

B^{IJ} = two-form field

$$S[B,\omega] = \frac{1}{2}\int_{M_4} (\frac{1}{2}\epsilon_{IJKL}B^{KL} + \frac{1}{\gamma}B_{IJ}) \wedge F^{IJ}(\omega) \qquad \Rightarrow \qquad F^{IJ}(\omega) = 0$$

Gravity and Topological field theory

- Gravity: Einstein-Cartan action + Holst term

$\gamma \in \mathbb{R}$ = Barbero-Immirzi parameter

$$S[e,\omega] = \frac{1}{16\pi G}\int \frac{1}{2}\epsilon_{IJKL}e^I \wedge e^J \wedge F^{KL}(\omega) \; + \; \frac{1}{\gamma} e_I \wedge e_J \wedge F^{IJ}(\omega)$$

- Topological Field Theory: BF action

B^{IJ} = two-form field

$$S[B,\omega] = \frac{1}{2}\int_{M_4} \left(\frac{1}{2}\epsilon_{IJKL}B^{KL} + \frac{1}{\gamma}B_{IJ}\right) \wedge F^{IJ}(\omega) \qquad \Rightarrow \qquad F^{IJ}(\omega) = 0$$

* Gravity as a Topological Theory with constrained B-field:

Constraint $B^{IJ} = \dfrac{1}{8\pi G}\, e^I \wedge e^J$ unfreezes $F^{IJ}(\omega)$

The Spin Foam action: main idea

- Topological Field Theory: BF action

B^{IJ} = two-form field

$$S[B,\omega] = \int \frac{1}{2}\epsilon_{IJKL}B^{IJ} \wedge F^{KL}(\omega) \;+\; \frac{1}{\gamma}B_{IJ} \wedge F^{IJ}(\omega) \quad \Rightarrow \quad F^{IJ}(\omega) = 0$$

Constraint imposed on

$$\boxed{B^{IJ} = \frac{1}{8\pi G}\, e^I \wedge e^J}$$

- everywhere on *M* $\Rightarrow$ General Relativity

The Spin Foam action: main idea

- Topological Field Theory: BF action

B^{IJ} = two-form field

$$S[B,\omega] = \int \frac{1}{2}\epsilon_{IJKL}B^{IJ} \wedge F^{KL}(\omega) \;+\; \frac{1}{\gamma}B_{IJ} \wedge F^{IJ}(\omega) \quad \Rightarrow \quad F^{IJ}(\omega) = 0$$

Constraint imposed on

$$\boxed{B^{IJ} = \frac{1}{8\pi G}\, e^I \wedge e^J}$$

- everywhere on M ➡ General Relativity
- on a 2d-foam in M ➡ Spin Foam action

2d-foam allows to unfreeze a finite number of gravitational degrees of freedom:

- quantization straightforward

- perspective: General Relativity as Effective field theory description

The spinfoam action: BF theory + space-like simplicity of B on Δ_2

Spinfoam action

$$S_{SF}[B,\omega,\lambda] = \frac{1}{2}\int_M \big(\frac{1}{2}\epsilon_{IJKL}B^{KL} + \frac{1}{\gamma}B_{IJ}\big)\wedge F^{IJ}(\omega) \;\; + \int_{\Delta_2} B_{IJ}N^I\,\lambda^J$$

λ^I = Lagrange multiplier $\Rightarrow$ constraint $B_{IJ}N^I = 0$

The spinfoam action: BF theory + space-like simplicity of B on Δ_2

Spinfoam action

$$S_{SF}[B,\omega,\lambda] = \frac{1}{2}\int_M \big(\frac{1}{2}\epsilon_{IJKL}B^{KL} + \frac{1}{\gamma}B_{IJ}\big)\wedge F^{IJ}(\omega) \; + \int_{\Delta_2} B_{IJ}N^I\,\lambda^J$$

λ^I = Lagrange multiplier $\Rightarrow$ constraint $B_{IJ}N^I = 0$

Momentum conjugated to ω^{IJ}, generator of Lorentz transformations

$$\Pi_{IJ} = \underbrace{\frac{1}{2}\epsilon_{IJKL}B^{KL} + \frac{1}{\gamma}B_{IJ}}$$

boosts $K^I = t_J \Pi^{JI}$

rotations $L^I = \frac{1}{2}\epsilon_{KIJL}t^K\,\Pi^{JL}$

Spinfoam action

$$S_{SF}[B,\omega,\lambda] = \frac{1}{2}\int_M \left(\frac{1}{2}\epsilon_{IJKL}B^{KL} + \frac{1}{\gamma}B_{IJ}\right)\wedge F^{IJ}(\omega) \;+\; \int_{\Delta_2} B_{IJ}N^I\,\lambda^J$$

λ^I = Lagrange multiplier ⇨ constraint $B_{IJ}N^I = 0$

Momentum conjugated to ω^{IJ}, generator of Lorentz transformations

$$\Pi_{IJ} = \underbrace{\frac{1}{2}\epsilon_{IJKL}B^{KL} + \frac{1}{\gamma}B_{IJ}}$$

boosts $K^I = t_J\Pi^{JI}$

rotations $L^I = \frac{1}{2}\epsilon_{KIJL}t^K\,\Pi^{JL}$

$B_{IJ}N^I = 0$ on Δ_2 ⇨ $\boxed{K^I = \gamma\,L^I}$

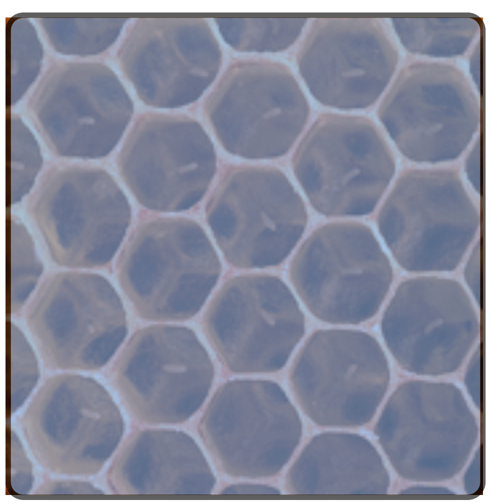
γ -simple representations of the Lorentz group

The Spin-Foam partition function for 4d Lorentzian Quantum Gravity

$$Z(\Delta_2) = \int \mathcal{D}B_{IJ}\mathcal{D}\omega_{IJ}\mathcal{D}\lambda_I \quad e^{\,\mathrm{i}\ \frac{1}{2}\int_M \left(\frac{1}{2}\epsilon_{IJKL}B^{KL} + \frac{1}{\gamma}B_{IJ}\right)\wedge F^{IJ}(\omega)\ +\int_{\Delta_2} B_{IJ}N^I\,\lambda^J}$$

$$= \sum_{j_f\ ,\, i_e} \prod_f (2j_f+1) \prod_v \Big\{ \bigotimes_{e\in v} \mathcal{I}_\gamma(i_e) \Big\}$$

Cellular decomposition of M



4-cells Δ_4

3-cells Δ_3

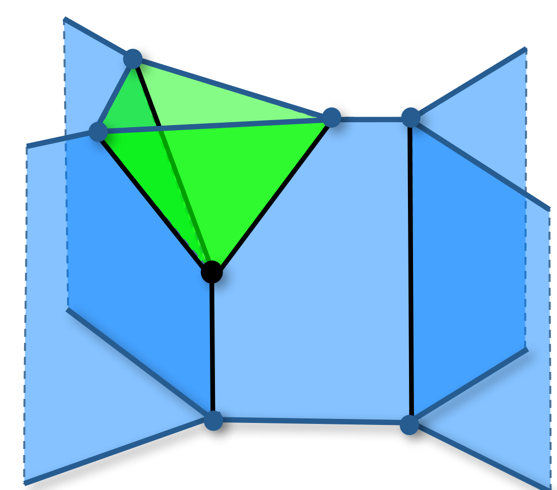
2-cells Δ_2

Dual 2-complex

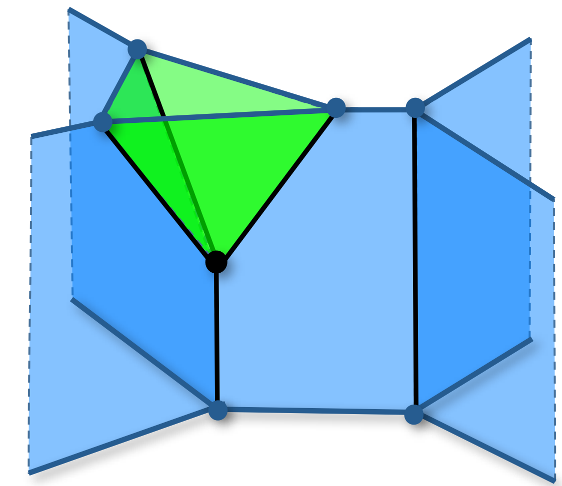
vertex v

edge e

face f



$$\boxed{Z(\Delta_2) = \sum_{j_f\,,\,i_e} \prod_f (2j_f+1) \prod_v \{ \bigotimes_{e\in v} \mathcal{I}_\gamma(i_e) \}}$$



Semiclassical limit:

- vertex amplitude (Theorem)

$$\{ \bigotimes_{e\in v} \mathcal{I}_\gamma(i_e) \} \approx e^{\frac{i}{\hbar} S_{\text{Gravity}}(\;)} + c.c.$$

cf. Ponzano-Regge '68 {6*j*} and 3d Quantum Gravity

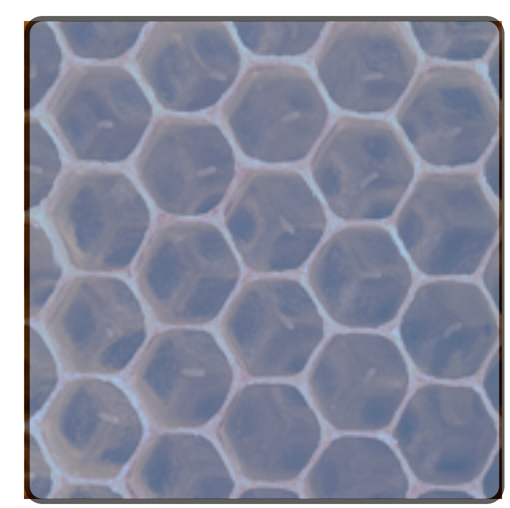
Barrett-Dowdall-Fairbairn-Hellmann-Gomes-Pereira *JMP*'09

- 2-complex (evidence)

$$Z(\Delta) \approx \int \mathcal{D}g_\Delta \, e^{\frac{i}{\hbar} S[g_\Delta]} + c.c.$$

g_Δ piecewise-flat metric on Δ

Conrady-Freidel *PRD*'09
EB-Magliaro-Perini *PRD*'10
Magliaro-Perini *CQG*'11
Bahr-Dittrich-Hellmann-Kaminsky '12

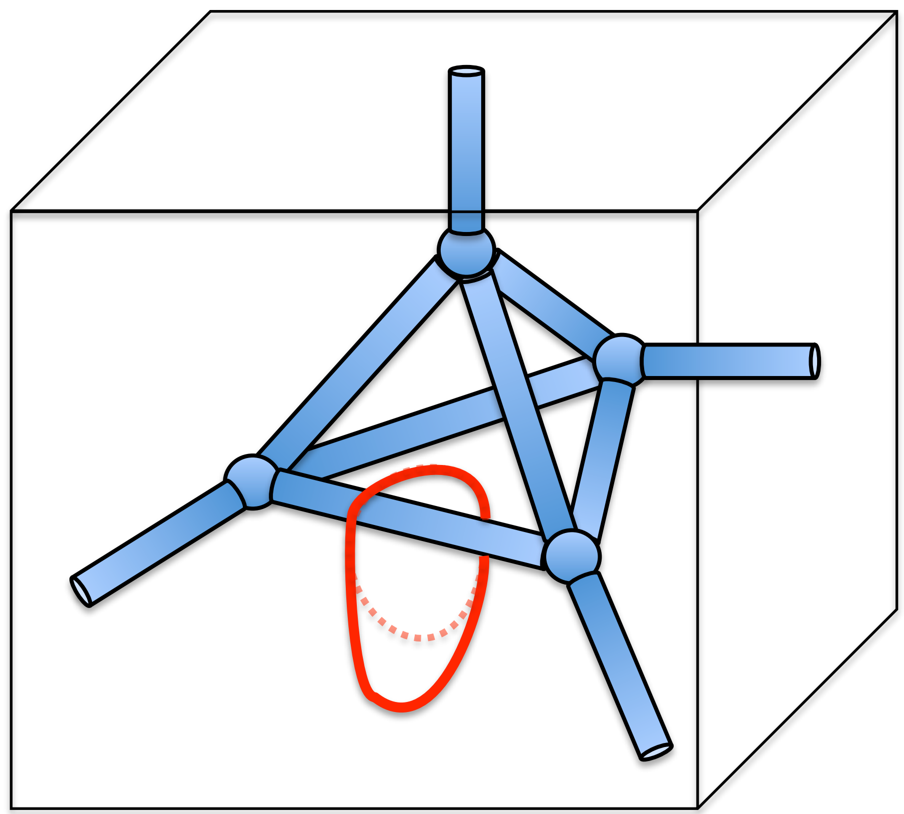


The 2d-foam unfreezes a finite number of local gravitational degrees of freedom:

$F^{IJ}(\omega) = 0$ everywhere, except on the 2d-foam

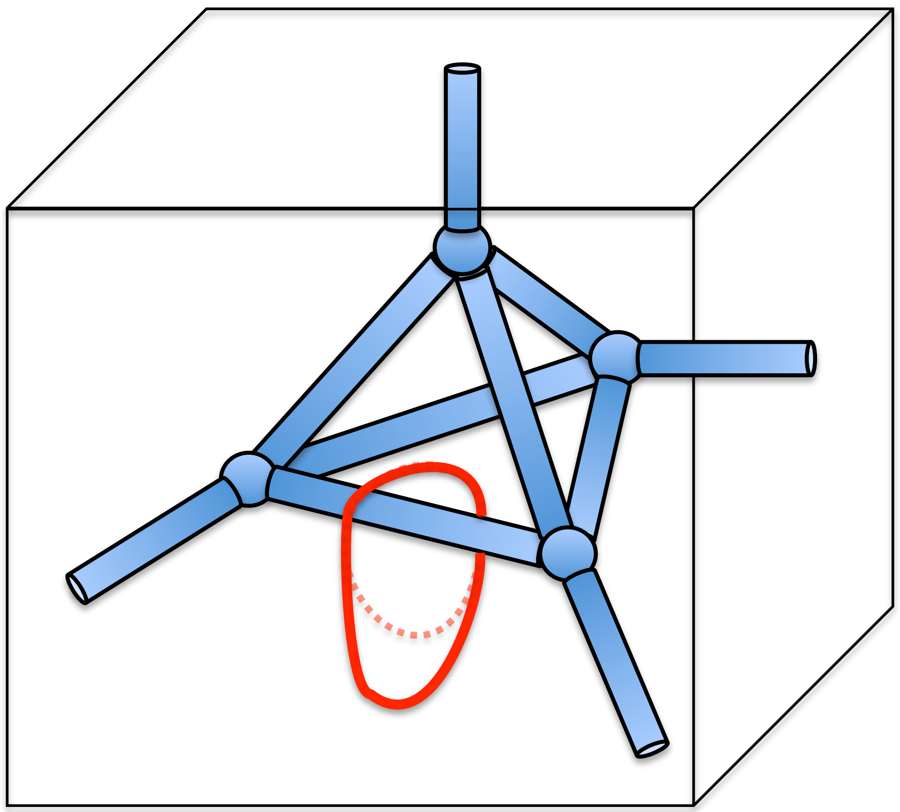
Finite number of gravitational d.o.f., completely captured by *Wilson loops*

Diff-invariant truncation of General Relativity. Full quantum theory via completion.



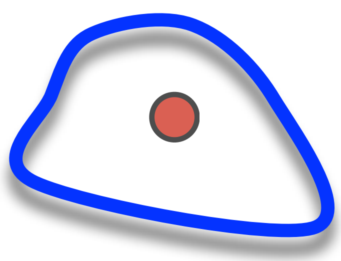
The microscopic degrees of freedom

d.o.f. = flux-tube excitation of a topologically invariant vacuum



The microscopic degrees of freedom

d.o.f. = flux-tube excitation of a topologically invariant vacuum

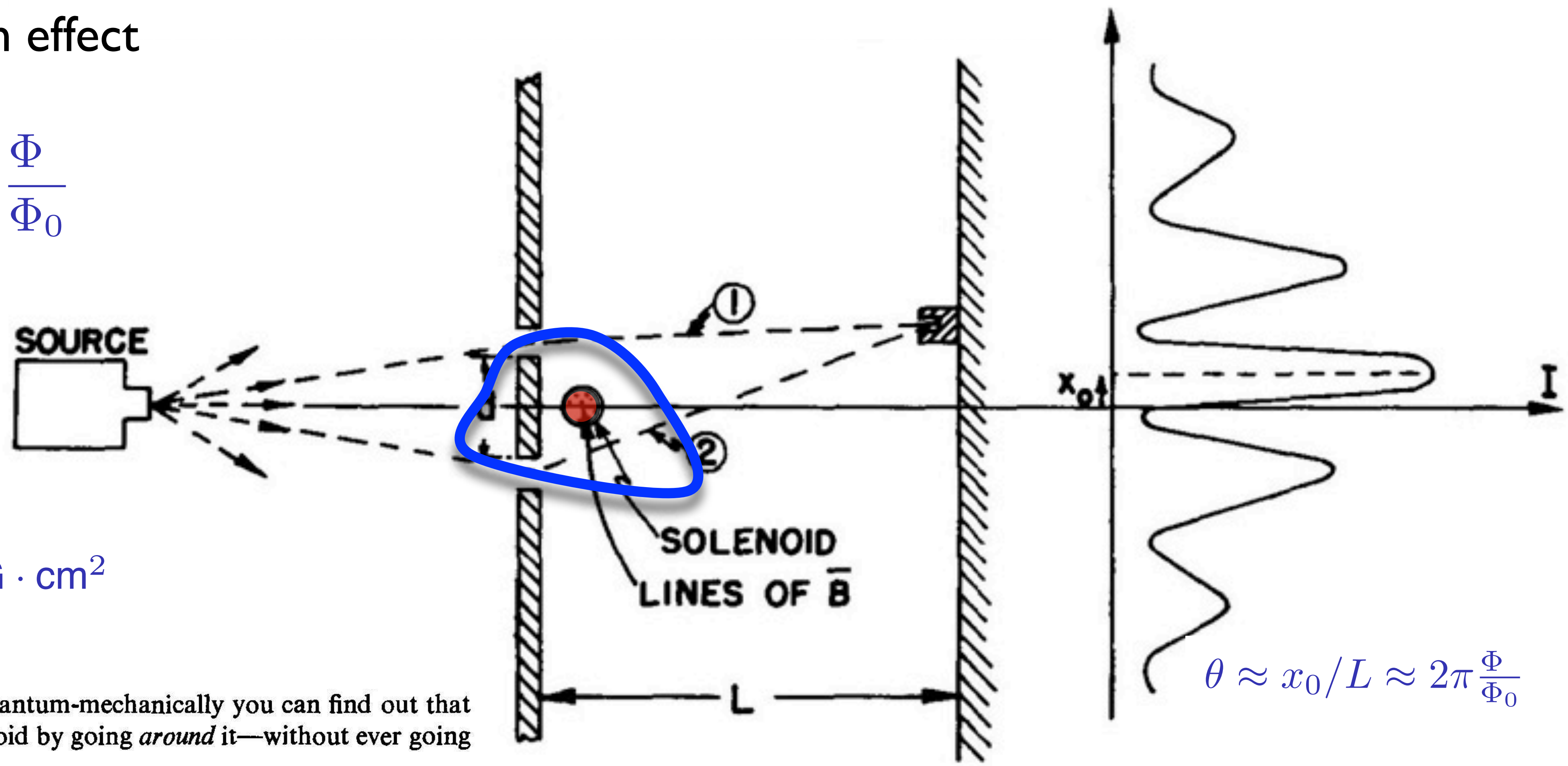


The microscopic degrees of freedom

d.o.f. = flux-tube excitation of a topologically invariant vacuum

cf. Aharonov-Bohm effect

$$e^{\frac{i}{\hbar} e \int_\gamma A dx} = e^{i\, 2\pi \frac{\Phi}{\Phi_0}}$$

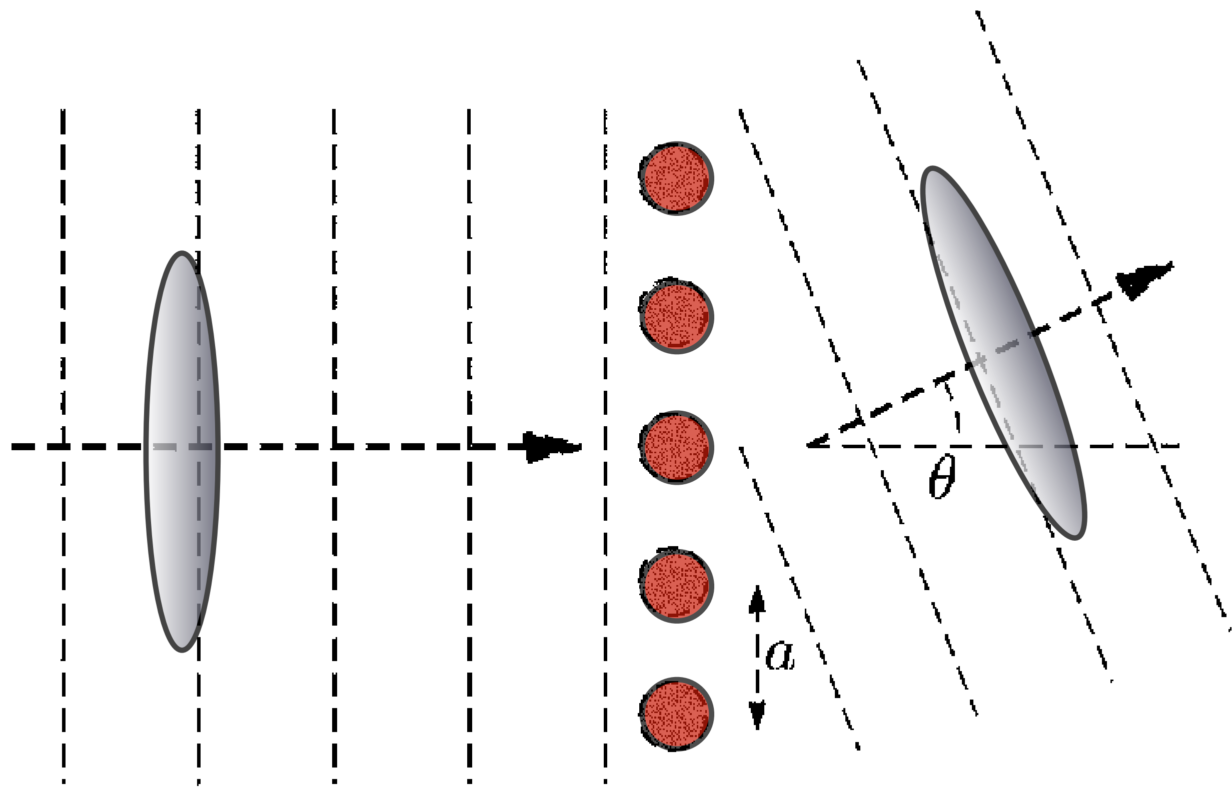


$\Phi =$ flux of B
$\Phi_0 = \frac{2\pi\hbar}{e} \approx 4 \times 10^{-7} \mathrm{G} \cdot \mathrm{cm}^2$

$\theta \approx x_0/L \approx 2\pi \frac{\Phi}{\Phi_0}$

From Feynman Lectures: But quantum-mechanically you can find out that there is a magnetic field inside the solenoid by going *around* it—without ever going close to it!

Recovering a classical geometry from singular configurations



wavepacket deflected by an array of solenoids

(Feynman Lectures vol 2. sec. 15-5)

Appealing scenario for Quantum Gravity:

No trans-Planckian d.o.f. because topological (and therefore finite) at small scales

At larger scales, finitely many d.o.f. which can be described effectively in terms of a local quantum field theory.

Outline

Lecture 1:

Path integral and the Spinfoam amplitude

Lecture 2:

Quantum geometry in Spinfoams

Lecture 3:

- gravitons
- quantum cosmology
- black hole entropy