Onset of Hawking radiation in 1+1 dimensions

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1 July 2013

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Outline

Motivation

What do we want to do?

We'd like to examine thermality in time-dependent situations. In particular, what does one feel as one falls into a BH? Under which circumstance can we talk about thermality?

Why 1+1 spacetimes?

- Great simplification: exact quantisation in many interesting cases.
- Qualitatively similar.

What is a particle detector?

(3+1) Unruh-DeWitt detector

Two-level system coupled to a scalar field. Heuristically, can think of an atom interacting with a scalar field by absorbing or emitting field quanta. Mathematically:

$H_{\text{int}} = \mu(x(\tau))\chi(\tau)\phi(x(\tau))$,

- χ: switching function,
- μ: monopole moment of the detector,
- ϕ: scalar field

For uniformly accelerated trajectory $\Rightarrow$ Unruh effect, i.e.,

$$\dot{\mathcal{F}}(\omega) = \omega \frac{1}{e^{2\pi\omega/a} - 1}$$

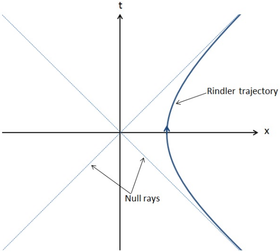

Fig 1. A Rindler detector feels bath of particles at temperature $T = \frac{\hbar a}{2\pi ck}$.

The Unruh-DeWitt detector in 1+1

- Transition probability = (Detector internal properties) × (Response function), i.e. $P(\omega) \propto \mathcal{F}(\omega)$.
- Transition rate, $d\mathcal{F}/d\tau$ depends on the Wightman function:

$$\dot{\mathcal{F}}(\omega,\tau) = 2\int_0^{\Delta\tau} ds\, \mathrm{Re}\left[\mathrm{e}^{-\mathrm{i}\omega s}\mathcal{W}(\tau,\tau-s) + \frac{1}{2\pi s^2}\right] + \frac{1}{2\pi^2\Delta\tau} - \frac{\omega}{4\pi}.$$

where $\mathcal{W}(\tau,\tau') \equiv \langle 0|\phi(\tau)\phi(\tau')|0\rangle$.

- But Hadamard $\mathcal{W}(\tau,\tau')$ only exists for massive fields in 1+1 dimensions!

The derivative coupling detector

- Infrared divergence for massless fields $\sim \log(m)$!
- Is the Unruh-DeWitt detector natural in 1+1 dimensions?
- Introduce a derivative coupling detector.
- Suggested by Don Marolf and first mentioned in the literature at least in (Davies and Ottewill, 2002).

The derivative coupling detector

Consider

$$H_{\text{int}} = c\chi(\tau)\mu(\tau)\dot{x}^a\partial_a\phi(x(\tau)).$$

where $\chi(\tau)$ is a smooth switching function of compact support controlling the interaction time. Then

$$\mathcal{F}(\omega) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' \, \chi(\tau)\chi(\tau')\mathrm{e}^{\mathrm{i}\omega(\tau-\tau')}\mathcal{A}(\tau,\tau'),$$

where $\mathcal{A}(\tau,\tau') \equiv \frac{d}{d\tau}\frac{d}{d\tau'}\mathcal{W}(\tau,\tau')$ is finite for a massless 1+1 scalar field.

Smooth switching and sharp switching limit

Choosing a sensible switching function, a general formula, *independent* of the details of the switching, may be obtained for the transition rate:

$$\dot{\mathcal{F}}(\omega,\tau) = 2\int_0^{\Delta\tau} ds\,\mathrm{Re}\left[\mathrm{e}^{-\mathrm{i}\omega s}\mathcal{A}(\tau,\tau-s) + \frac{1}{2\pi s^2}\right] + \frac{1}{\pi\Delta\tau} - \frac{\omega}{2}.$$

Star collapse model: receding mirror

A spacetime in which the boundary moves following the trajectory (lightcone coods.) $v(u) = -\frac{1}{\kappa}\log(1+e^{-\kappa u})$ emulates the spacetime of a collapsing star.

- Exact field quantisation.
- We consider (asymptotic) early and late time response of inertial detector.

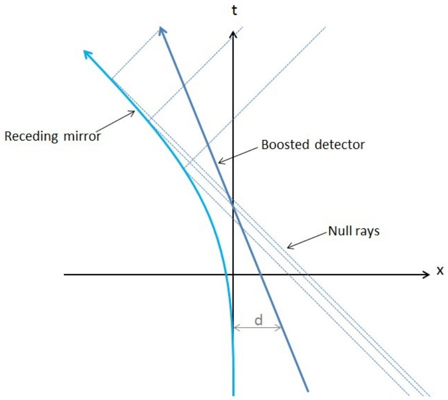

Fig 3. Inertial detector in a Receding mirror spacetime.

Receding mirror: Asymptotics

A boosted detector follows the trajectory $x^a = \gamma(\tau, -\beta\tau)$, where $\gamma = 1/\sqrt{1-\beta^2}$.
In the asymptotic past the transition rate is dominantly

$$\dot{\mathcal{F}}(\omega)_{\tau\to-\infty} = -\omega\left[1 - \frac{1+\beta}{1-\beta}\cos\left(\frac{2\beta\tau}{1-\beta}\omega)\right)\right]\Theta(-\omega). \quad (1)$$

In the asymptotic future the leading contribution is

$$\dot{\mathcal{F}}(\omega)_{\tau\to+\infty} = -\frac{\omega}{2}\Theta(-\omega) + \frac{\omega}{2}\frac{1}{\exp\left[\frac{2\pi\omega}{\kappa}\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}\right]-1}. \quad (2)$$

Receding mirror: Transient behaviour

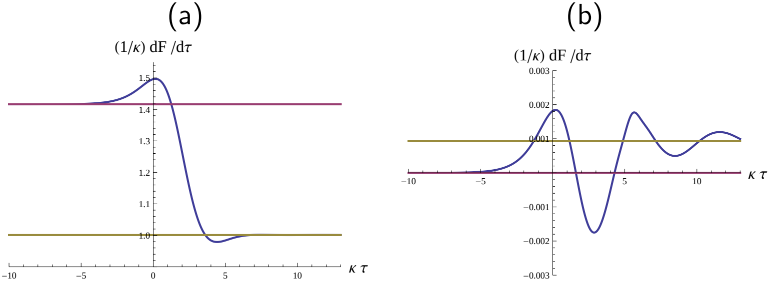

Fig 5. Transient behaviour of the response function in the emergence of thermality for $\kappa d = 1$ with (a) $\omega/\kappa = -1$ and (b) $\omega/\kappa = 1$.

The 1+1 Schwarzschild BH

$$ds^2 = -(1 - 2M/r)du\,dv = -[2M\exp(-r/2M)/r]d\bar{u}\,d\bar{v}$$

Consider an infalling detector following a timelike geodesic.

- Cannot be solved analytically in full generality. Can consider asymptotic limits.
- Early time measurements ($\tau \to -\infty$).
- Can be done for freely falling detector or boosted detector in the past timelike infinity.

The 1+1 Schwarzschild BH: Asymptotics

Consider a detector falling with initial velocity β at $\tau \to -\infty$. A measurement in the asymptotic past yields:

Vacuum	Transition rate, $\dot{\mathcal{F}}(\omega)_{\tau\to-\infty}$
$\mid 0_B\rangle$ (Boulware)	$-\omega\Theta(-\omega)$
$\mid 0_H\rangle$ (Hartle-Hawking)	$\frac{\omega/2}{\exp\left[2\pi\omega/\sqrt{\frac{1+\beta}{1-\beta}}\frac{1}{4M}\right]-1} + \frac{\omega/2}{\exp\left[2\pi\omega/\sqrt{\frac{1-\beta}{1+\beta}}\frac{1}{4M}\right]-1}$
$\mid 0_U\rangle$ (Unruh)	$-\frac{\omega}{2}\Theta(-\omega) + \frac{\omega}{2}\frac{1}{\exp\left[2\pi\omega/\sqrt{\frac{1+\beta}{1-\beta}}\frac{1}{4M}\right]-1}$

Thermality appears with a blue(red)shifted factor in the temperature, $T = \sqrt{\frac{1\pm\beta}{1\mp\beta}}\frac{1}{4M}/2\pi k_B$.

The 1+1 Schwarzschild BH: Loss of thermality

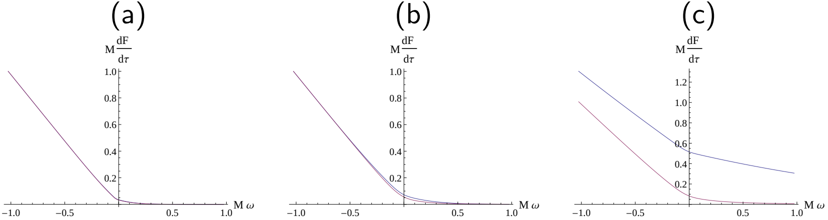

Fig. 6. Transition rate for a freely falling detector interacting with a field in the Unruh state. Schwarzschild radius crossing at $\tau = -4M/3$. The blue curve portrays a purely thermal spectrum while the red curve indicates the numerical value of the transition rate at proper time values (a) $\tau = -10M$, (b) $\tau = -3.5M$ and (c) $\tau = -1.5M$.

The 1+1 Schwarzschild BH: Approaching the spacetime singularity

- Numerical evidence that nothing special occurs at horizon crossing (HHI, Unruh states).
- The transition rate diverges as the BH singularity is approached.

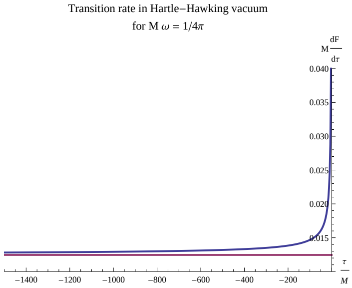

Fig 7. Transition rate along a geodesically freely falling detector coupled to a field in the HHI state.

The 1+1 Schwarzschild BH: Approaching the spacetime singularity

- For concreteness, consider the HHI state.
- Take a family of infalling observers characterised by the real parameter E coming from the eq. of motion $\dot{t} = E/(2M/r - 1)$.
- For example, $E = 1$ is a freely falling detector from infinity and $E = 0$ is a 'straight-up' detector from the white hole region in the Kruskal manifold. Then

$$\dot{\mathcal{F}} \to \frac{1}{4\pi}\left(\frac{(2M)^{1/2}}{r^{3/2}} + \frac{1+E^2}{2(2Mr)^{1/2}}\right) + \text{finite const.},$$

as $r \to 0$.

Conclusions

- Results are reliable despite the divergent Wightman function.
- Asymptotics and numerics under control in interesting examples.
- Detector coupled to field in the Unruh state also sensible to both left moving (from past null infinity) and right moving modes (from past the horizon).
- No divergences at the Schwarzschild radius in HHI and Unruh cases.
- Transition rate divergence at singularity calculated analytically.
- Detector will shed light on results found recently by Barbado et. al. (arXiv:1201.3820 [gr-qc]) and Smerlak and Singh (arXiv:1304.2858 [gr-qc]) for the value of the transition rate at horizon crossing.

Thanks for your attention!

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