Exploring Quantum Universe. A transfer matrix model of volume fluctuations in CDT.

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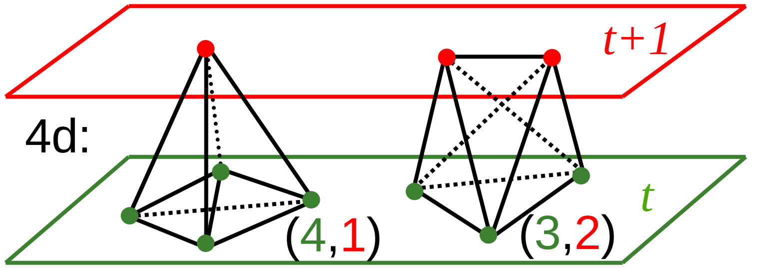
- **Causal Dynamical Triangulations (CDT)** is a non-perturbative approach to Quantum Gravity based on the path integral

$$Z = \int_{Geom} D[g]\exp(iS_{HE}[g])$$

- Regularization of Z is done by summing over all **causal triangulations** T constructed from 4-d simplices

$$Z = \sum_T \frac{1}{C_T}\exp(i\tilde{S}_R[T])$$

- We assume a **global time foliation** S^1 and **fixed spatial topology** $S^3 \Rightarrow$ resulting space-time $S^1 \times S^3$ can be built from two types of simplices



- The Einstein-Hilbert action:

$$S_{HE} = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} \left(R - 2\Lambda \right)$$

  - G – Newton's constant
  - g – metric determinant
  - R – curvature scalar
  - Λ – cosmological constant

- is regularized by the Regge action where curvature R is determined by the deficit angle „around" 2-d triangles

$$\tilde{S}_R = i\left[-k_0 N_0 + K_4 N_4 + \Delta \left(N_4^{(4,1)} - 6N_0 \right) \right] = iS_R$$

$k_0 \leftrightarrow 1/G$, $K_4 \leftrightarrow \Lambda$, $\Delta \leftrightarrow \alpha$ $(l_t^2 = -\alpha l_s^2)$

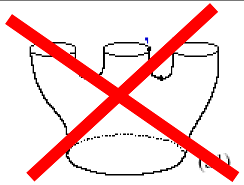
  - N_0 – # of vertices
  - N_4 – # of 4-simplices
  - $N_4^{(4,1)}$ – # of (4,1) & (1,4) simplices

- After Wick rotation: $\alpha \rightarrow -\alpha$ ($|\alpha| > 7/12$) S_R is purely real:

$$Z = \sum_T \frac{1}{C_T} \exp(i\tilde{S}_R[T]) = \sum_T \frac{1}{C_T} \exp(-S_R[T]) \quad \rightarrow \quad \text{probability distribution}$$

Triangulations are updated dynamically by **Monte Carlo** methods

- **Alexander moves …**
  - preserve global & local topology ⇔ **causality**



  - any triangulation is achievable by a sequence of moves ⇔ **ergodicity**
- …are performed with probability determined by a **detailed balance condition**:

$$P(T_1)P(T_1 \xrightarrow{M} T_2) = P(T_2)P(T_2 \xrightarrow{M^*} T_1)$$

- Monte Carlo simulations allow to compute probability distributions (histograms) of **observables**:

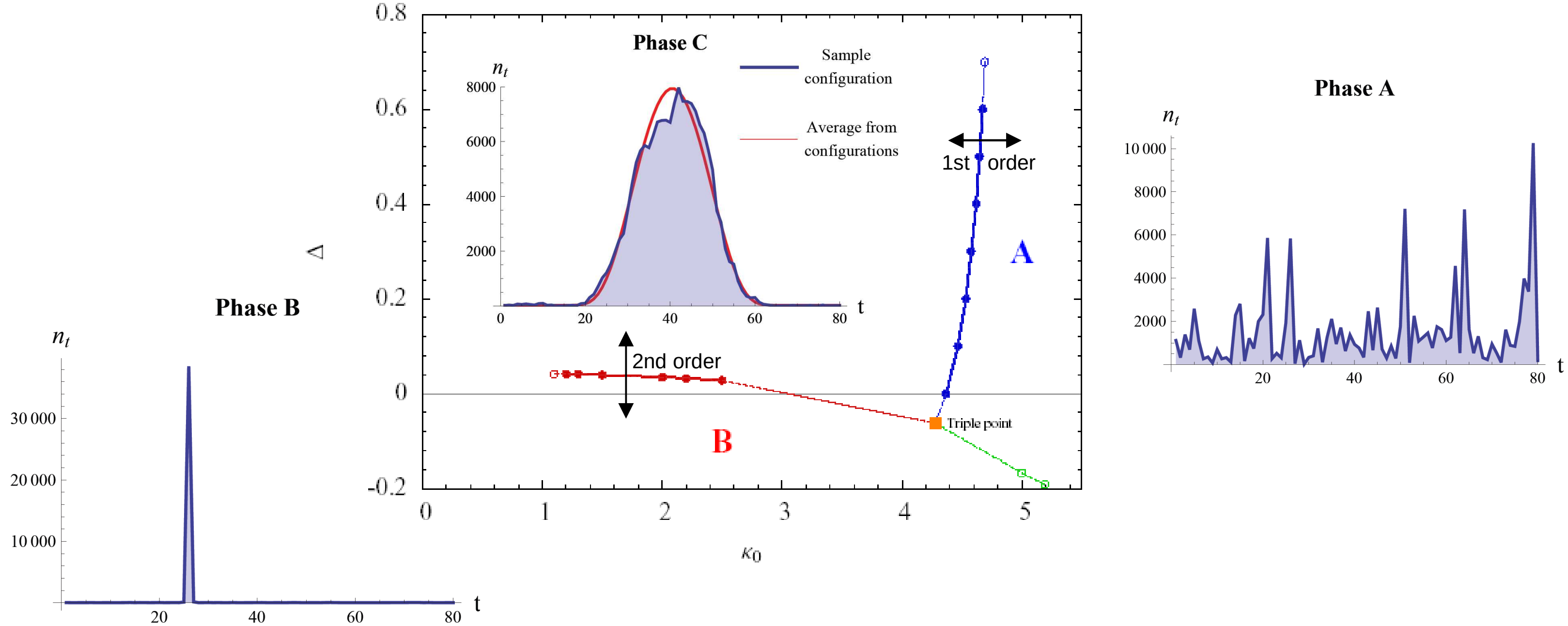
$$P(O) = \frac{1}{Z}\sum_{T(O)} \frac{1}{C_T} \exp(-S_R[T])$$

- We focus on the distribution of the **spatial volumes of the universe** in different time slices:

$$O \equiv n_t = N_4^{(4,1)}(t) \propto V_3(t)$$

$$S_R = -k_0 N_0 + K_4 N_4 + \Delta\left(N_4^{(4,1)} - 6N_0\right)$$

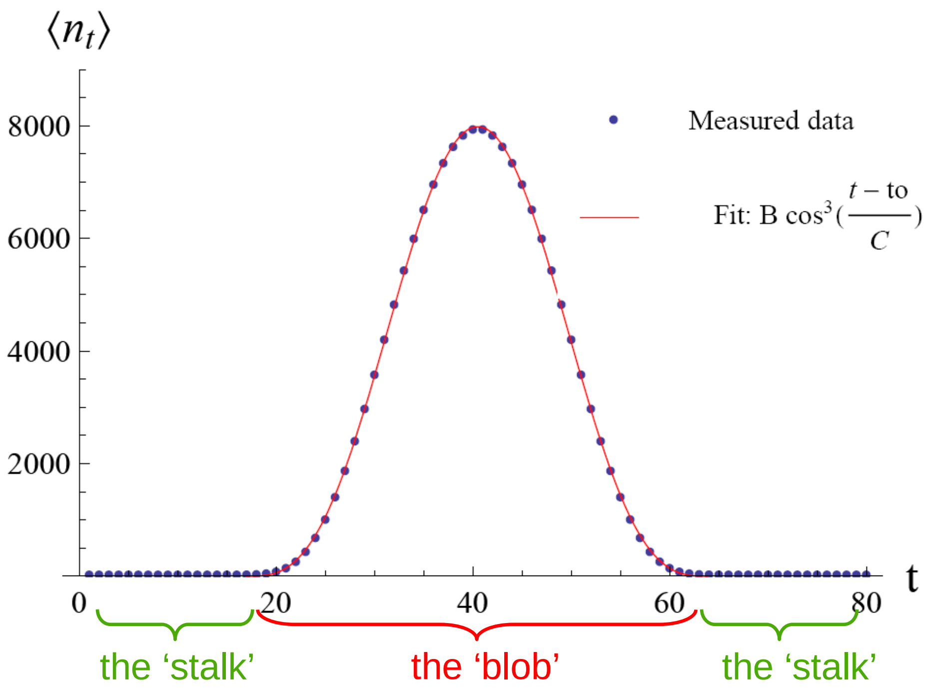
- Depending on the values of bare couplings k_0 and Δ ($K_4 \approx K_4^{crit}$) ***three phases*** emerge



- **In phase 'C' the dynamically generated semi-classical background …**

$$\langle n_t \rangle = \frac{3}{4}\tilde{V}_4 \frac{1}{\tilde{A}\tilde{V}_4^{1/4}} \cos^3\left(\frac{t-t_0}{\tilde{A}\tilde{V}_4^{1/4}}\right)$$

$$\sqrt{g_{tt}}V_3(t) = \frac{3}{4}V_4 \frac{1}{AV_4^{1/4}} \cos^3\left(\frac{t-t_0}{AV_4^{1/4}}\right)$$



- The solution (in the '**blob**') is consistent with the Euclidean (Wick's rotation) **de Sitter Universe** (no matter, positive cosmological constant)

- **… and quantum fluctuations are governed by the action:**

$$S = \frac{1}{24\pi G}\int dt\sqrt{g_{tt}}\left(\frac{g^{tt}\dot{V}_3(t)^2}{V_3(t)} + \mu V_3(t)^{1/3} - \lambda V_3(t)\right)$$

  - $\mu = 9\left(\frac{3}{4}\right)^{2/3} A^{-8/3}$
  - $\lambda = 9V_4^{-1/2}A^{-2}$

- This is the (Euclidean) ***minisupespace*** (MS) action obtained from S_{HE} for the maximally symmetric metric: $ds^2 = g_{tt}dt^2 + a^2(t)d\Omega_3^{\,2}$

- **CDT conjecture:** the effective action in the phase 'C' is a discretization of the minisuperspace action:

$$S_{dis} = \frac{1}{\Gamma}\sum_t\left(\frac{(n_{t+1}-n_t)^2}{n_{t+1}+n_t} + \tilde{\mu}n_t^{\,1/3} - \tilde{\lambda}n_t + \mathrm{O}\left(n_t^{\,-1/3}\right)\right)$$

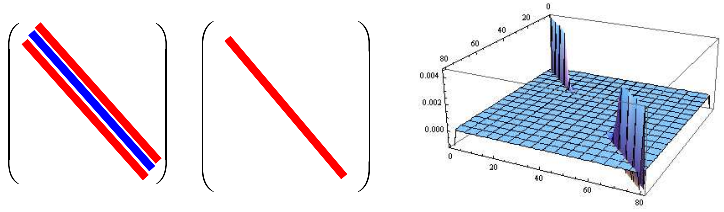
$$S_{dis} = \sum_t \frac{1}{\Gamma}\left(\underbrace{\frac{\left(n_{t+1}-n_t\right)^2}{n_{t+1}+n_t}}_{\text{kinetic part}} + \underbrace{\tilde{\mu} n_t^{1/3} - \tilde{\lambda} n_t}_{\text{potential part}} \right)$$

- Second order expansion of S_{dis} around semi-classical solution $\bar{n}$

$$S_{dis}[\bar{n}+\delta n] = S_{dis}[\bar{n}] + \frac{1}{2}\sum_{t,t'} \delta n_t P_{tt'} \delta n_{t'} + O\left(\delta n^3\right)$$

can be compared with the covariance of volume fluctuations: $C(\delta n_t\,, \delta n_{t'})$

$$P = P_{kin} + P_{pot} = C^{-1}$$



Motivation:

- The ‘blob’ range in phase ‘C’ is described by the MS effective action:
  - How good is this description ?
  - Can we measure the effective action directly (not only its 2nd derivative around $\bar{n}$) ?
- How to describe the behaviour of the ‘stalk’ range ?
  - Average spatial volume in the ‘stalk’ does not scale with total 4-volume
  - Small volume discretization effects
  - Relatively large quantum fluctuations
- ‘Cleaning’ of discretization artifacts might lead to discovery of some non-trivial physical effects in the ‘tail’ (small volume) region
  - $P_{kin} \sim n^{-1}$
  - $P_{pot} \sim n^{-5/3}$

$$S_{blob} = \sum_t \frac{1}{\Gamma}\left(\frac{(n_{t+1}-n_t)^2}{n_{t+1}+n_t} + \tilde{\mu} n_t^{1/3} - \tilde{\lambda} n_t\right) = \sum_t L_{blob}(n_t, n_{t+1})$$

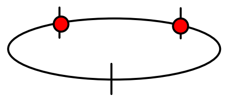
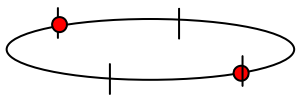
$$\langle n_t | M_{blob} | n_{t+1} \rangle = \exp\left[-L_{blob}(n_t, n_{t+1})\right]$$

$$Z_{blob} = \sum_{n_1 \dots n_t \dots n_T} \exp(-S_{blob}) = Tr\left(M_{blob}{}^T\right)$$

- **Assumptions:**
  - we consider only effective aggregate ‘states’ $|n_t\rangle$
  - description by the transfer matrix is also viable in the ‘stalk’ range
  - identical probability distributions for all slices in ‘stalk’ range $\Rightarrow M$ ~~(t)~~
  - time reflection symmetry $\Rightarrow \langle n_t | M | n_{t+1} \rangle = \langle n_{t+1} | M | n_t \rangle$

$$Z = \sum_{n_1 \dots n_t \dots n_T} \prod_{t=1}^{T} \langle n_t | M | n_{t+1} \rangle = Tr\left(M^T\right)$$

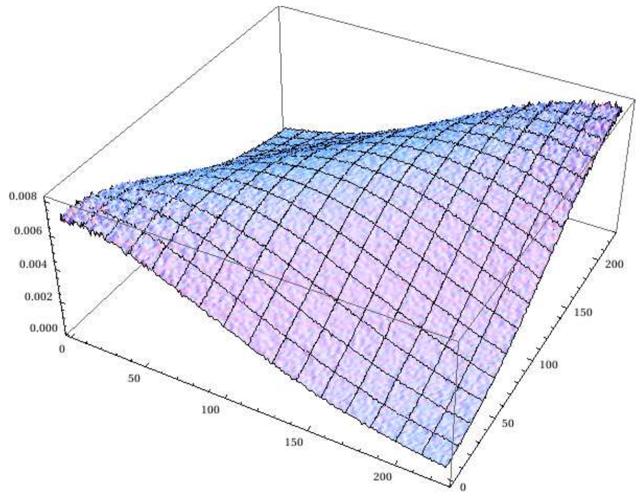
- 1-point probability distribution: $$P_1^{(T)}(n) = \frac{1}{Z}\langle n | M^T | n \rangle$$
- 2-point probability distribution: $$P_2^{(T,\Delta t)}(n,m) = \frac{1}{Z}\langle n | M^{\Delta t} | m \rangle \langle m | M^{T-\Delta t} | n \rangle$$

- Measured probabilities:
  - $T = 3,\ \Delta t = 1$  $$P_2^{(3,1)}(n,m) \propto \langle n | M | m \rangle \langle m | M^2 | n \rangle$$
  - $T = 4,\ \Delta t = 2$  $$P_2^{(4,2)}(n,m) \propto \langle n | M^2 | m \rangle^2$$
- can be used to determine M : $$\langle n | M | m \rangle = N \frac{P_2^{(3,1)}(n,m)}{\sqrt{P_2^{(4,2)}(n,m)}}$$

- The effective action for large volumes (in the 'blob') is measured:

$$\langle n | M | m \rangle = N e^{-L_{eff}(n,m)}$$

$$L_{eff}(n,m) = \frac{1}{\Gamma}\left[\frac{(n-m)^2}{n+m-2n_o} + \mu\left(\frac{n+m}{2}\right)^{1/3} - \lambda\left(\frac{n+m}{2}\right)\right]$$

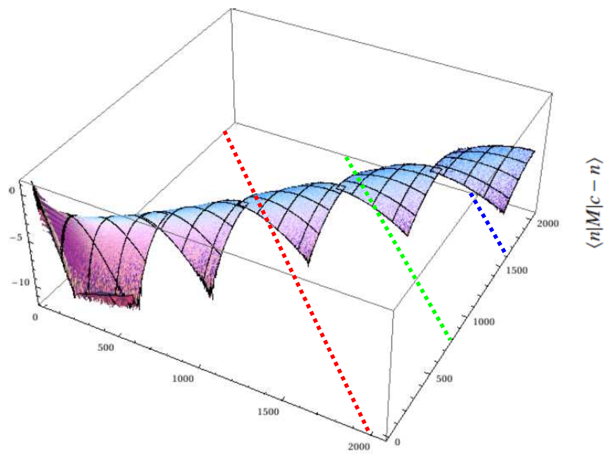


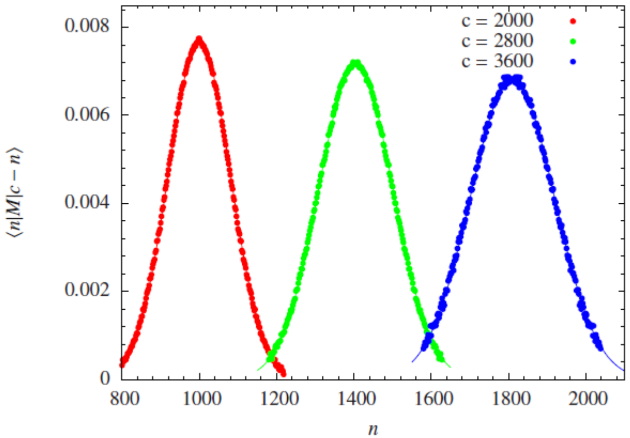
- The kinetic term …

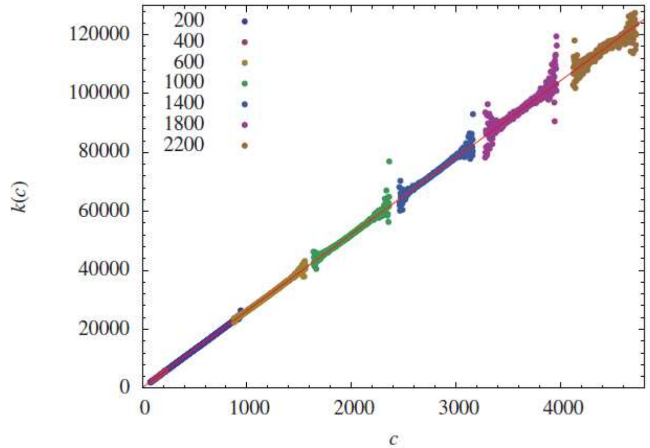
$$\langle n | M | m \rangle = N e^{-\frac{1}{\Gamma}\left[\frac{(n-m)^2}{n+m-2n_0} + \mu\left(\frac{n+m}{2}\right)^{1/3} - \lambda\left(\frac{n+m}{2}\right)\right]}$$

$k(n,m) = \Gamma\,(n+m-2n0)$

  - Gaussian bahaviour for $n+m=c$: $$\langle n | M | c-n \rangle = \tilde{N}(c)\exp\left[-\frac{(2n-c)^2}{k(c)}\right]$$





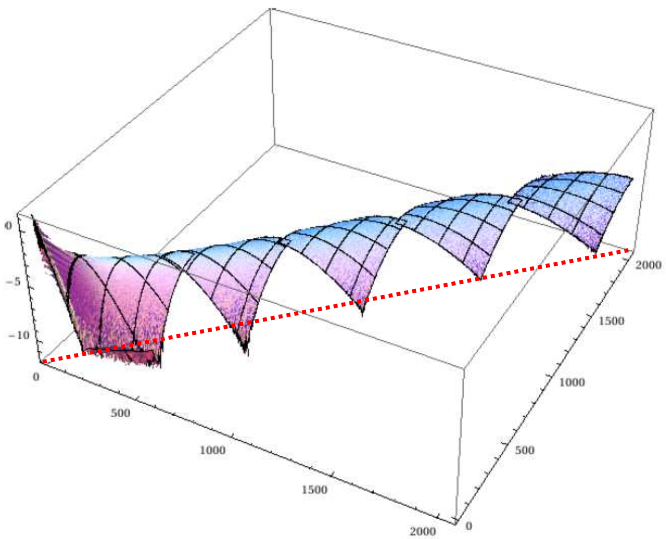


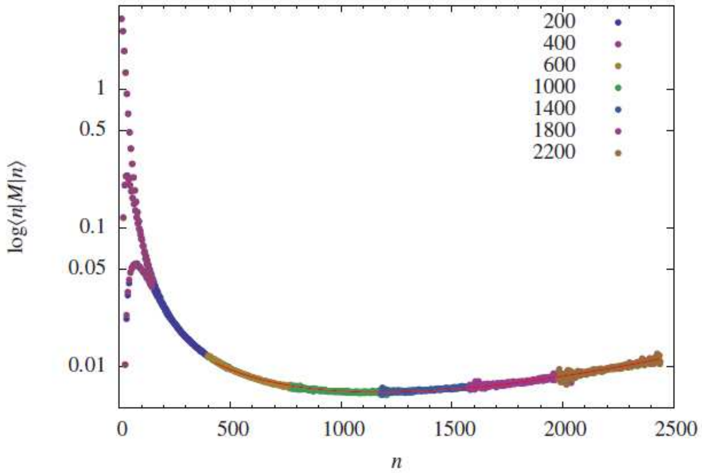
- … and the potential term can be analyzed in detail

$$\langle n | M | m \rangle = N e^{-\frac{1}{\Gamma}\left[\frac{(n-m)^2}{n+m-2n_0} + \mu\left(\frac{n+m}{2}\right)^{1/3} - \lambda\left(\frac{n+m}{2}\right)\right]}$$

- For $n=m$:

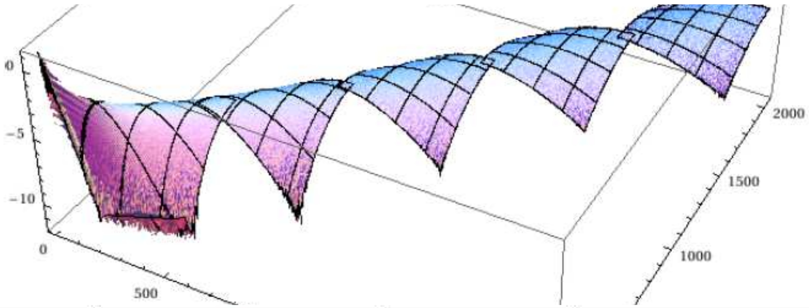
$$\log\langle n | M | n \rangle = -\frac{1}{\Gamma}\left(\mu n^{1/3} - \lambda n\right) + \alpha$$





- The fits agree with the previous method based on the covariance matrix

$$\langle n | M | m \rangle = N e^{-\frac{1}{\Gamma}\left[\frac{(n-m)^2}{n+m-2n_0} + \mu\left(\frac{n+m}{2}\right)^{1/3} - \lambda\left(\frac{n+m}{2}\right)\right]}$$

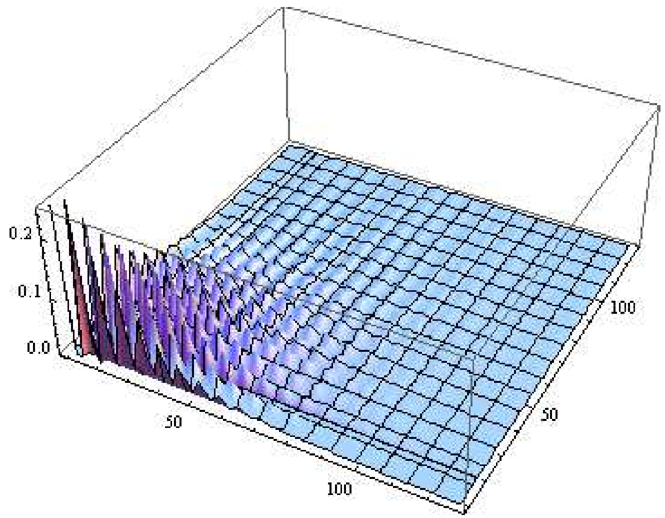


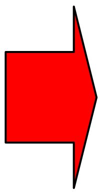
Method	Γ	n_0	μ	λ
Cross-diagonals	26.07 ± 0.02	-3 ± 1	–	–
Diagonal	(26.07)	–	16.5 ± 0.2	0.049 ± 0.001
Full fit	26.17 ± 0.01	7 ± 1	15.0 ± 0.1	0.046 ± 0.001
Previous method*	23 ± 1	–	13.9 ± 0.7	0.027 ± 0.003

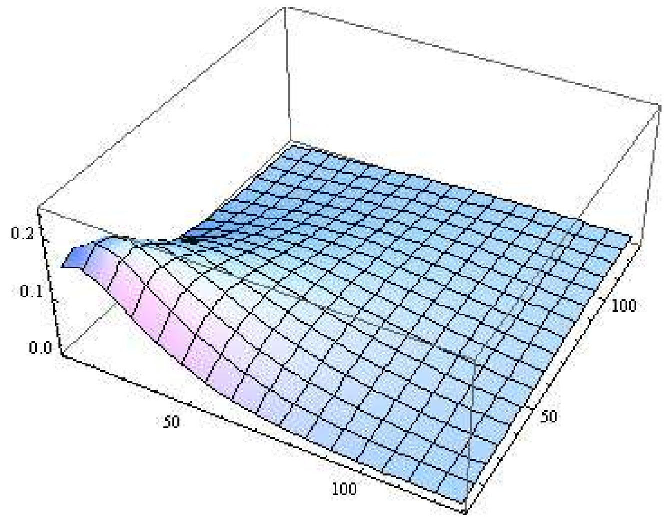
- The transfer matrix gives access to the effective action in the 'stalk' range

$$\langle n | M | m \rangle = N e^{-L_{eff}(n,m)}$$

- Discretization effects (split into three families of states) makes analytical modeling difficult
- If we average over these three families M becomes smooth



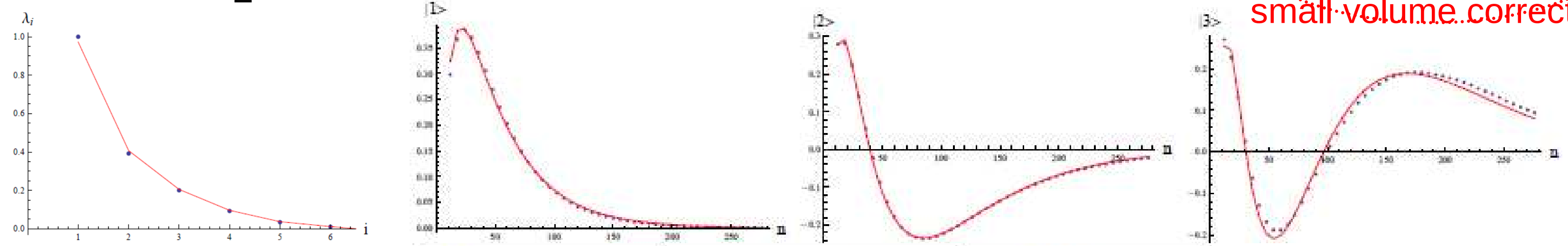




- The effective action for the stalk is basically the same as for the 'blob'

$$S_{eff}^{stalk} = \sum_t \frac{1}{\Gamma}\left[\frac{(n_t - n_{t+1})^2}{n_t + n_{t+1} - 2n_0} + \mu\left(\frac{n_t + n_{t+1}}{2}\right)^{1/3} - \lambda\left(\frac{n_t + n_{t+1}}{2}\right)^{1/3} + \delta\left(\frac{n_t + n_{t+1}}{2}\right)^{-\rho}\right]$$

small volume correction



Parameter	Stalk	Blob
Γ	27.2 ± 0.1	$25.7 - 26.2$
n_0	5 ± 1	$-3 - +7$
μ	34 ± 2	$13 - 30$
λ	0.12 ± 0.02	$0.04 - 0.07$
δ	$(4 \pm 7) \times 10^4$	–
ρ	3 ± 1	–

Conclusions

- The transfer matrix model allows to measure the effective action directly
- The effective action in phase ‘C’ of CDT is very well described by the minisuperspace model
- Despite the nature of the stalk seems quite different from the blob on the first sight it is well explained by the same effective action when discretization effects are „cleared”
- We observe some correction of the potential term in the small volume range, however it is small and cannot be determined with high precision (it may be as well the effect of discretization artifacts)
- Our new method of measurement based on the transfer matrix model has a lot of advantages:
  - the effective action is measured directly $\Rightarrow$ higher precision of the fits
  - the method is much faster (small systems $\Rightarrow$ fast termalization) and allows high statistics of the measurements

Thank You for attention !!!

The results presented here were obtained in collaboration with:

- ***Jan Ambjørn***, *The Niels Bohr Institute, Copenhagen University*
- ***Andrzej Görlich***, *The Niels Bohr Institute, Copenhagen University*
  Institute of Physics, Jagellonian University
- ***Jerzy Jurkiewicz***, *Institute of Physics, Jagellonian University*

The transfer matrix in four-dimensional CDT, **arXiv:1205.3791v1 [hep-th]**