Primordial radiation and vanishing of domain walls

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Outline

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Domain walls and other topological defects

- Domain walls are one of the simplest topological defects.
- They are responsible for breaking of discrete symmetries (ie. Z_2).
- Vacuum manifold is disconnected.
- Domain walls are interfaces between regions of different vacuum pieces.

Examples

- Z_2, $1d$ kinks in ϕ^4 are the simplest domain walls.
- Sine-Gordon kinks.
- Magnetic domains in ferromagnet.
- Universe could be partitioned by domain walls into cells - not confirmed.
- D-branes are also DW.

There are lots of other topological defects: strings, monopoles, skyrmions, textures. Some of them are believe to be responsible for certain observation facts in CMB (cosmic textures - cold spots)

Outline

Properties and creation

- Most defects are stable (Derek's theorem) and compact objects.
- Because of (conserved) topological charge they can be destroyed or created usually only in pairs
- Their interactions disappear exponentially
- They interact only when they meet
- They are created in Kibble-Zurek mechanism (vacuum expectation value in separated regions can be different)
- Mistery: why do we observe so little traces of topological defects in the Universe, if any?

The simple model

Let us consider relativistic real field theory with at least two degenerate vacua

$$\mathcal{L} = \frac{1}{2}\partial_\mu \partial^\mu \phi - U(\phi).$$

For potential $U(\phi)$ we assume that it has at least two equal minima ϕ_i but with different masses of small perturbations around them:

$$U'(\phi_i) = 0 \text{ and } U(\phi_i) = 0 \text{ and } U''(\phi_1) \neq U''(\phi_2)$$

The mechanism can be extended on more complicated theories with gauge fields Let us start with 1-dimensional kinks $\phi_s(-\infty) = \phi_1, \phi_s(\infty) = \phi_2$.
Adding a small perturbation $\phi = \phi_s + \xi$ to the static kink we obtain:

$$\ddot{\xi} + \left(-\partial_x^2 - U''(\phi_s)\right)\xi = 0.$$

The potential $V(x) = U''(\phi_s(x))$ yield different values as $|x| \to \pm\infty$

Scattering modes can form a traveling wave interacting with the defect. This solution must satisfy energy and momentum conservation law.

$$\partial_t \mathcal{E} = \partial_x \left(\phi' \dot{\phi} \right),$$
$$\partial_t \mathcal{P} = -\frac{1}{2}\partial_x \left(\dot{\phi}^2 + \phi'^2 - 2U(\phi) \right).$$

This can be done only if we add one more degree of freedom - the position of the defect. The scattered wave exerts a radiation pressure which accelerate the kink. After integration we can write the force acting on the initial motionless kink using only asimptotic values of the scattered wave: $\phi = \phi_s + \mathcal{A}\Re(e^{-i\omega t}\eta(x)))$:

$$\mathcal{A}^2\omega \left(\frac{k_2}{|A(k_1,k_2)|^2} + \frac{|A(-k_1,k_2)|^2}{|A(k_1,k_2)|^2} k_1 - (1-c^2)k_1 \right) = 0.$$

c is a higher order correction responsible for energy exchange.
$A(k_1, k_2)$ - amplitude of scattered wave (read of from asymptotic form of the solution)

The force can be expressed as:

$$F = \frac{1}{2}\frac{\mathcal{A}^2}{|A(k_1,k_2)|^2}\left(2|A(-k_1,k_2)|^2 k_1^2 + k_1 k_2 - k_2^2\right).$$

Special cases

- If $k_1 = k_2$ (symmetric case) than $F \geq 0$ - kink is pushed by the wave
- If $k_1 = k_2$ and (very rare $A(-k_1,k_2) = 0$ ie. sinus-Gordon, ϕ^4) in the first order the kink is transparent to the radiation. Higher order terms must be taken into account.
  - For integrable sG $F = 0$ in all orders
  - For nonintegrable ϕ^4 model $F = \mathcal{A}^4 F^{(2)} < 0$ Negative Radiation Pressure
- If $k_1 > k_2$ always $F > 0$ the potential has a threshold
- If $k_2 < k_1$ and reflection coefficient $|A(-k_1,k_2)|$ is small enough a negative radiation pressure can be observed (V-shaped potential, ϕ^6)

Outline

Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\phi^2\left(\phi^2 - 1\right)^2.$$

- Model has three vacua $\{-1, 0, 1\}$,
- small perturbations around those vacua have masses $m_{\pm 1} = 2, m_0 = 1$;
- in 1d there are two types of kinks $(-1, 0), (0, 1)$

$$\phi_{(0,\pm 1)}(x) = \pm\sqrt{\frac{1 + \tanh x}{2}},$$

other (anti-)kinks can be obtained using symmetries $x \to -x$ i $\phi \to -\phi$:

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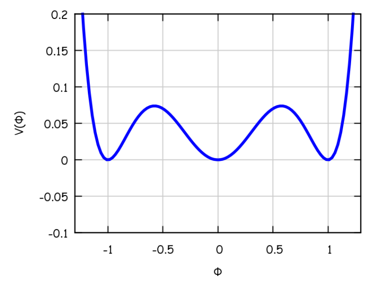

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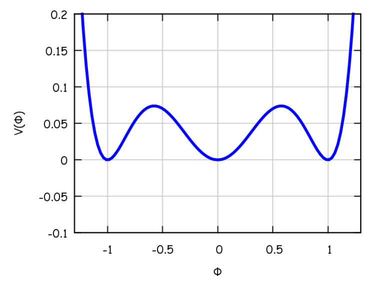

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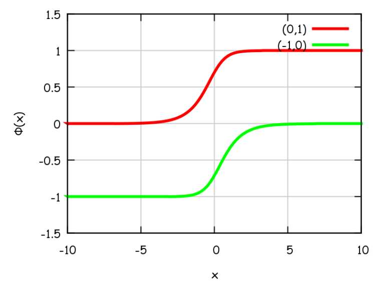

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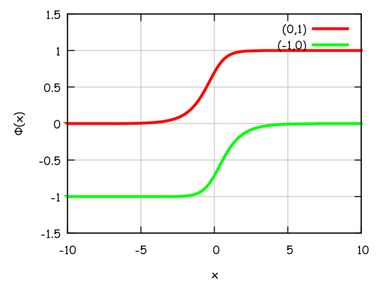
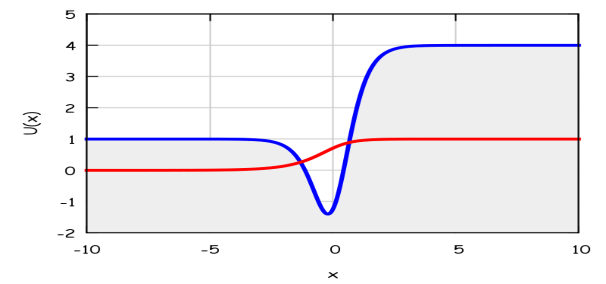


Spectral structure of linear perturbations around kink: $\phi = \phi_s + e^{i\omega t}\eta(x)$:

$$-\eta'' + V(x)\eta = \omega^2\eta, \text{ gdzie } V(x) = U''(\phi_s(x)) = 15\phi^4 - 12\phi^2 + 1.$$

There is no oscillational discrete mode in this model.



Note the difference in the mass threshold on both sides of the kink.

In ϕ^6 model one can find the analytic expression for the force.

$$k_1 = \sqrt{\omega^2 - 1},\ k_2 = \sqrt{\omega^2 - 4},$$

$$A(k_1, k_2) = \frac{\Gamma(1 - ik_2)\Gamma(-ik_1)}{\Gamma(-\frac{1}{2}ik_2 - \frac{1}{2}ik_1 + \frac{5}{2})\Gamma(-\frac{1}{2}ik_2 - \frac{1}{2}ik_1 - \frac{3}{2})}.$$

The force acting on the $(0, 1)$ kink exerted by the wave coming from the lhs of the kink:

$$F_{+\infty}(k_1, k_2) = \frac{1}{2}\frac{\mathcal{A}^2}{|A(k_1, k_2)|^2}\left(2|A(-k_1, k_2)|^2 k_1^2 + k_1 k_2 - k_2^2\right).$$

and from the rhs of the kink:

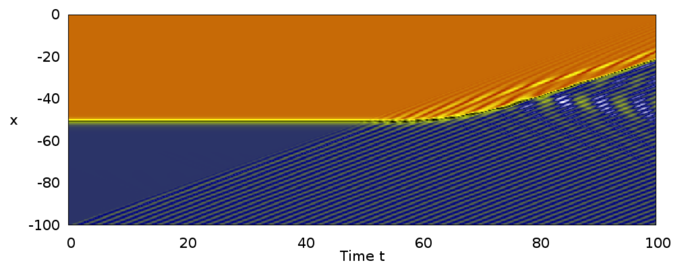
$$F_{-\infty}(k_1, k_2) = -F_{+\infty}(k_2, k_1) \equiv \mathcal{A}^2(\omega)f(\omega)$$

Radiation pressure

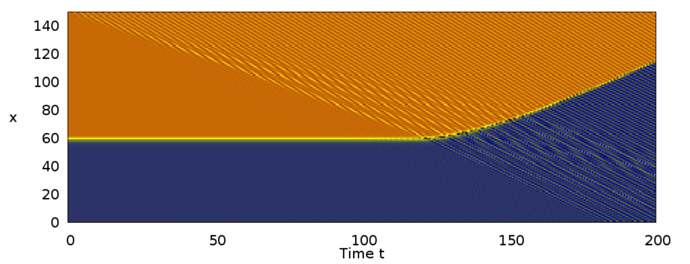
Both forces push the kink in the same direction $F > 0$

- for wave coming from the $\phi = 0$ $(m = 1)$ vacuum - positive radiation pressure
- for wave coming from the $\phi = \pm 1$ $(m = 2)$ vacuum - negative radiation pressure

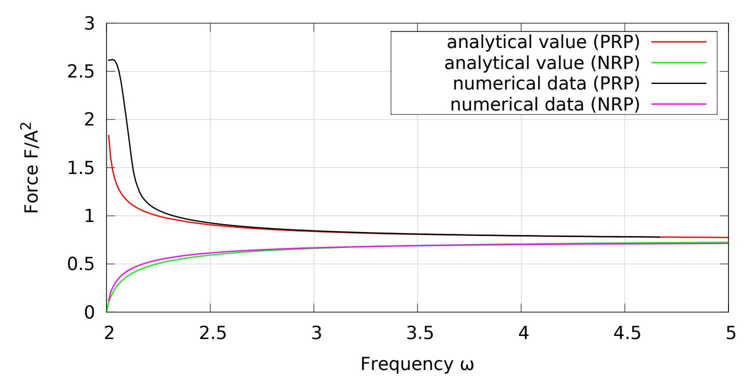
- Positive radiation pressure



- Negative radiation pressure



Comparison between the theory and numerical calculations:



The difference is large only near the mass threshold

Consequences

- Each monochromatic wave pushes the kink towards one of the vacua $\phi = \pm 1$ no matter where the wave comes from
- The waves obey superposition rule and so the force:

$$\delta\phi = \int_{-\infty}^{\infty} dk\, \mathcal{A}(\omega(k))\eta_k(x)e^{-i\omega(k)t}$$

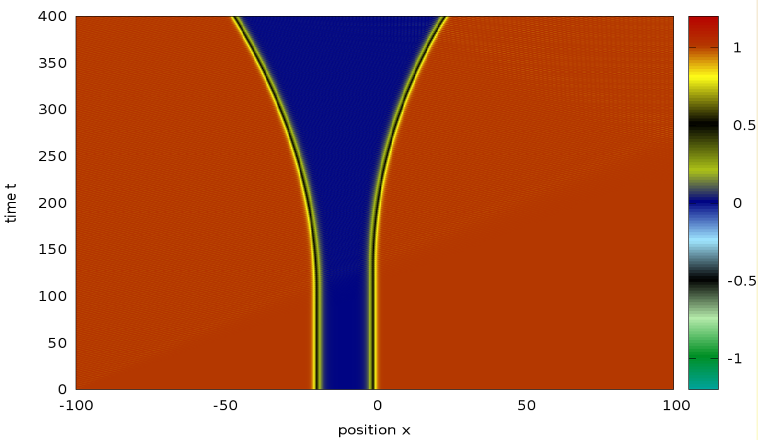
total force would be equal to

$$F_{tot} = \int_{-\infty}^{\infty} d\omega\, \mathcal{A}(\omega)^2 f(\omega) = \alpha\langle\mathcal{A}\rangle^2.$$

α depends on the particular distribution, but in most cases a good approximation is $\alpha \approx 0.75$

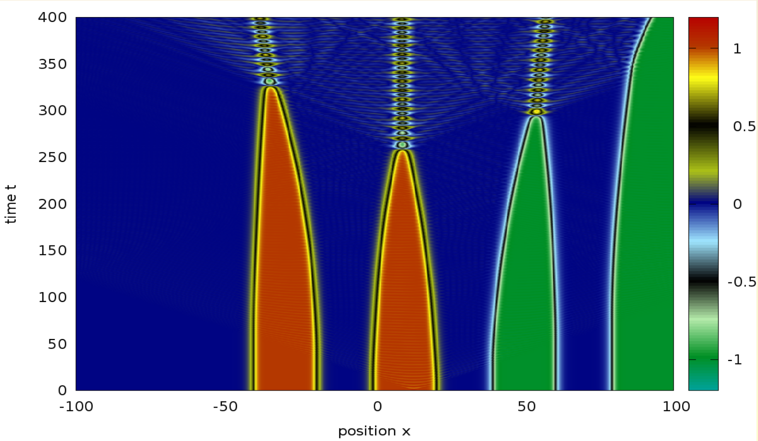
- All kinds of perturbation would push the kink towards ± 1
- Domains of $\phi = \pm 1$ will shrink (˜ false vacua)
- Domains of $\phi = 0$ would grow

Two kinks with oscillating background conditions $0.05\sin(3t)$

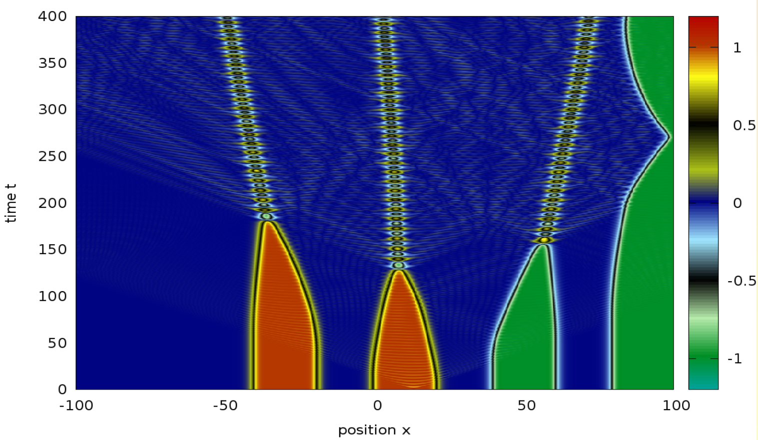


System of kinks with perturbation $-0.1\exp(-(x-13)^2)$

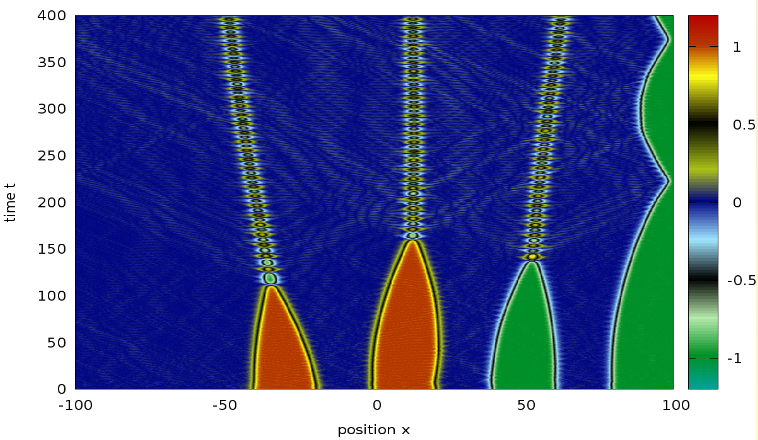
Time between collisions without perturbation $T \approx 2e^{L/2}/\sqrt{L} \approx 10^4$.
Time between collisions with perturbation $T \approx \sqrt{ML/F_{tot}} \approx 310$.



The system of kinks with gaussian perturbation $-0.2\exp(-(x-13)^2)$



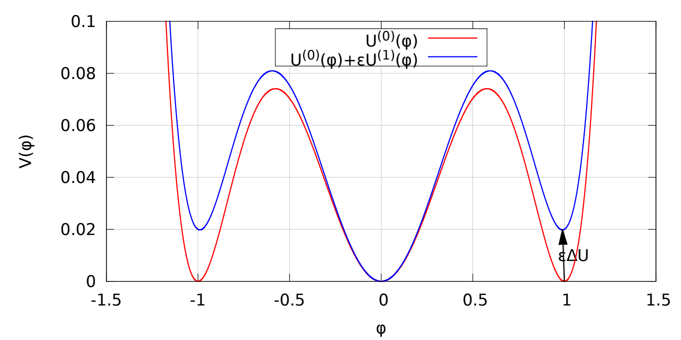
The system of kinks with random perturbation



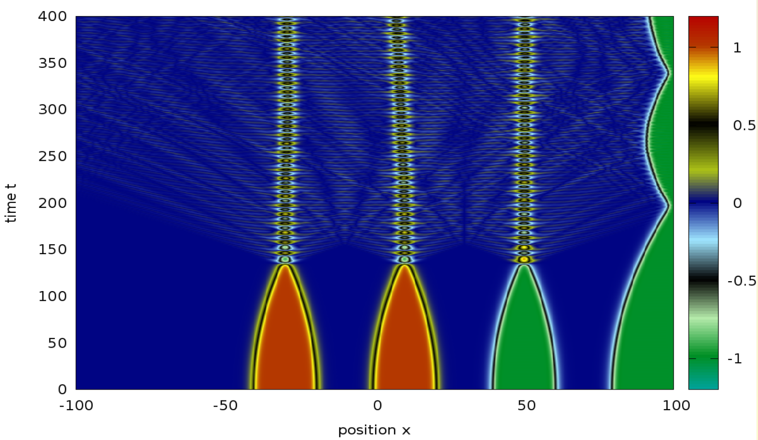
Remark

For uniform distribution the motion of kinks collectively can be described by raising the potential

$$F = \epsilon \Delta U \tag{1}$$



Motion of the kinks for modified potential $U(\phi) = U^{(0)}(\phi) + 0.00025\phi^2$

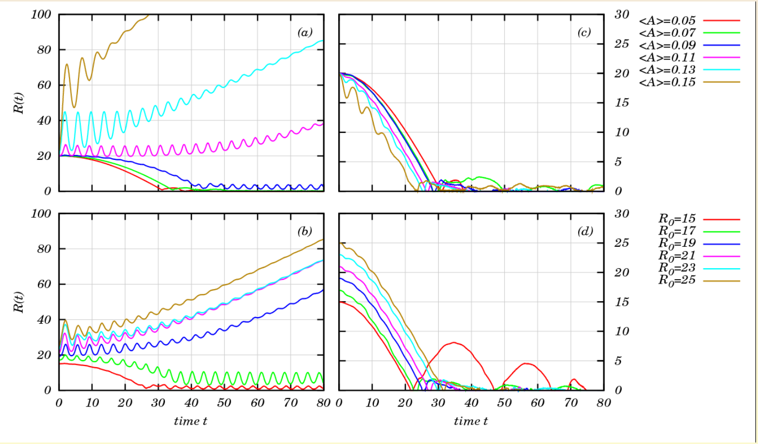


2-d bulks

- The energy of a circular domain wall with large enough radius is equal to $E(R) = 2\pi MR$.
- Tension trying to close the domain is equal to $F = -M/R$.
- If $\phi_{in} = \pm 1$ and $\phi_{out} = 0$ the forces of radiation pressure and tension act in the same direction.
- If $\phi_{in} = 0$ and $\phi_{out} = \pm 1$ the forces of radiation pressure and tension act in opposite directions.
  Critical radius: $R_{crit} = \frac{M}{\alpha\langle\mathcal{A}\rangle^2}$.
- Similarly in 3D: $R_{crit}^{(3D)} = \frac{2M}{\alpha\langle\mathcal{A}\rangle^2}$.

Simulations

Radius of the bulk



(a) $R_0 = 20$ and variable $\langle \mathcal{A} \rangle$, $\phi_{in} = 0$.
(b) Variable radius and constant $\langle \mathcal{A} \rangle = 0.1$, $\phi_{in} = 0$.
(c) $R_0 = 20$ and variable $\langle \mathcal{A} \rangle$, $\phi_{in} = 1$.
(d) Variable radius and constant $\langle \mathcal{A} \rangle = 0.1$, $\phi_{in} = 1$.

Outline

Conclusions

- In ϕ^6 kinks are always pushed in one direction, antikinks in opposite direction.
- No matter in wich direction the wave travells it poushes the kink towards $\phi = \pm 1$ vacuum.
- Result of different masses of small perturbation.
- Domains $\phi = \pm 1$ disappear and domains of $\phi = 0$ grow.
- In higher dimensions the tension (proportional to the curvature of the wall) wants to squeeze domains.
- For enclosed $\phi = 0$ vacuum the radiation can slow down or even invert the proces of shrinking domain (for domain walls with larger than critical radius).
- The mechanism is much more effective $T \sim \sqrt{L}$ than the static interaction between the kinks $T \sim e^L/\sqrt{L}$.
- The radiation can indeed speed up the process of domain wall vanishing, leaving only the domain with the smallest mass.