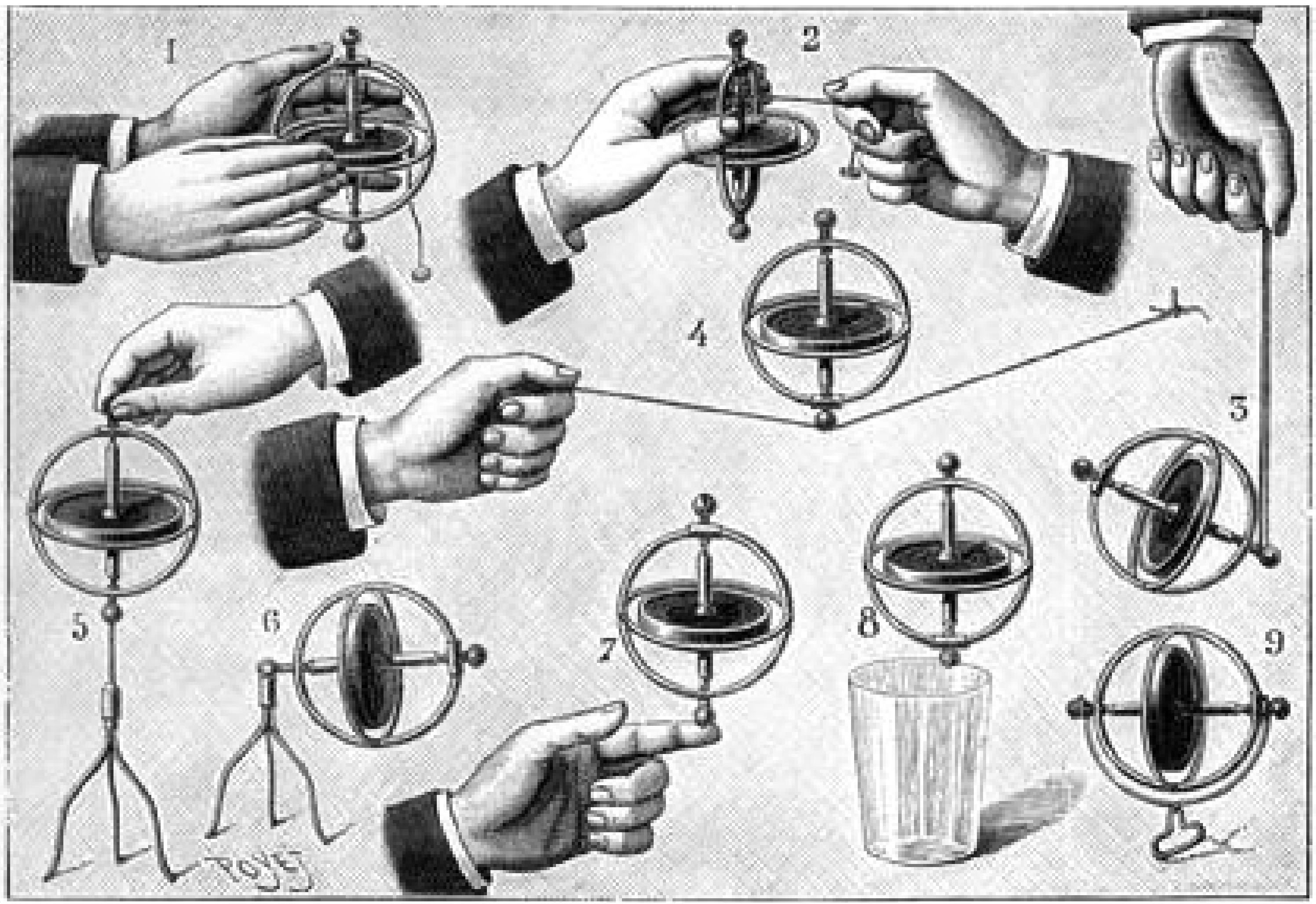
Collective Neutrino Flavor Oscillations



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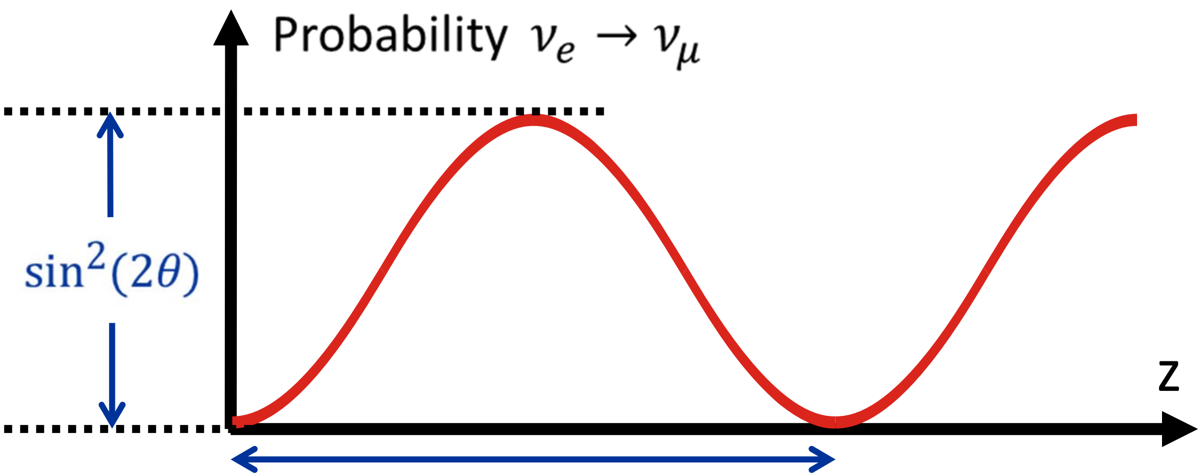
Neutrino Flavor Oscillations

Two-flavor mixing $\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$

Each mass eigenstate propagates as e^{ipz}

with $p = \sqrt{E^2 - m^2} \approx E - m^2/2E$

Phase difference $\frac{\delta m^2}{2E} z$ implies flavor oscillations

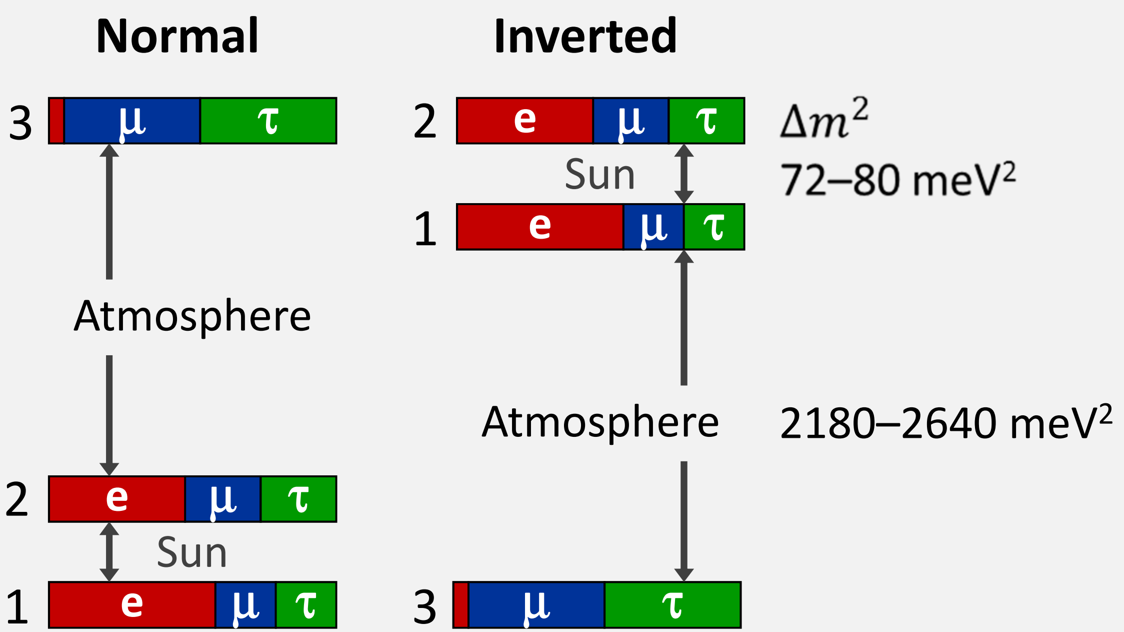


Oscillation Length $\frac{4\pi E}{\delta m^2} = 2.5\ \mathrm{m}\ \frac{E}{\mathrm{MeV}} \left(\frac{\mathrm{eV}}{\delta m}\right)^2$

Three-Flavor Neutrino Parameters

Three mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$ (Euler angles for 3D rotation), $c_{ij} = \cos\theta_{ij}$, a CP-violating "Dirac phase" δ, and two "Majorana phases" α_2 and α_3

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \underbrace{\begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}}_{\substack{39^\circ < \theta_{23} < 53^\circ\\ \text{Atmospheric/LBL-Beams}}} \underbrace{\begin{pmatrix}c_{13} & 0 & e^{-i\delta}s_{13}\\ 0 & 1 & 0\\ -e^{i\delta}s_{13} & 0 & c_{13}\end{pmatrix}}_{\substack{7^\circ < \theta_{13} <\\ \text{Reactor}}} \underbrace{\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}}_{\substack{33^\circ < \theta_{12} < 37^\circ\\ \text{Solar/KamLAND}}} \underbrace{\begin{pmatrix}1 & 0 & 0\\ 0 & e^{i\frac{\alpha_2}{2}} & 0\\ 0 & 0 & e^{i\frac{\alpha_3}{2}}\end{pmatrix}}_{\substack{\text{Relevant for}\\ 0\nu2\beta\text{ decay}}} \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$



Tasks and Open Questions

- Precision for all angles
- CP-violating phase δ?
- Mass ordering ? (normal vs inverted)
- Absolute masses ? (hierarchical vs degenerate)
- Dirac or Majorana ?

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3400 citations

Neutrino oscillations in matter

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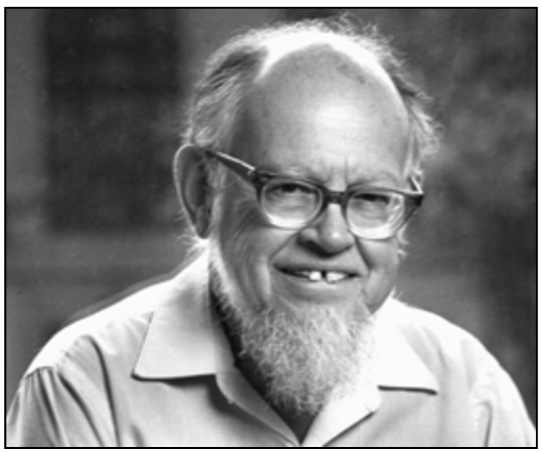
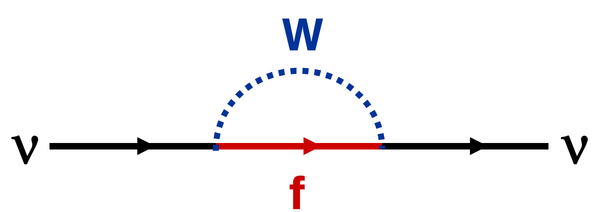
(Received 6 October 1977; revised manuscript received 5 December 1977)

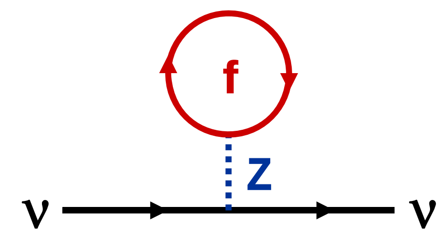
The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.



Lincoln Wolfenstein

Neutrinos in a medium suffer flavor-dependent refraction





$$V_{\rm weak} = \sqrt{2} G_{\rm F} \times \begin{cases} N_{\rm e} - N_{\rm n}/2 & \text{for } \nu_{\rm e} \\ -N_{\rm n}/2 & \text{for } \nu_\mu \end{cases}$$

Typical density of Earth: 5 g/cm^3

$$\Delta V_{\rm weak} \approx 2 \times 10^{-13}\ {\rm eV} = 0.2\ {\rm peV}$$

Neutrino Oscillations in Matter

2-flavor neutrino evolution as an effective 2-level problem

$$\mathrm{i}\frac{\partial}{\partial z}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = H\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

With a 2×2 Hamiltonian matrix

Negative for $\overline{\nu}$

$$H = \frac{1}{2E}\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}m_1^2 & 0\\ 0 & m_2^2\end{pmatrix}\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix} \pm \sqrt{2}G_\mathrm{F}\begin{pmatrix}N_e - \frac{N_n}{2} & 0\\ 0 & -\frac{N_n}{2}\end{pmatrix}$$

Mass-squared matrix, rotated by mixing angle θ relative to interaction basis, drives oscillations

$$\frac{\Delta m^2}{2E} \sim \begin{cases} 4\ \mathrm{peV} & \text{for 12 mass splitting}\\ 120\ \mathrm{peV} & \text{for 13 mass splitting}\end{cases}$$

Solar, reactor and supernova neutrinos:
E ~ 10 MeV

Weak potential difference

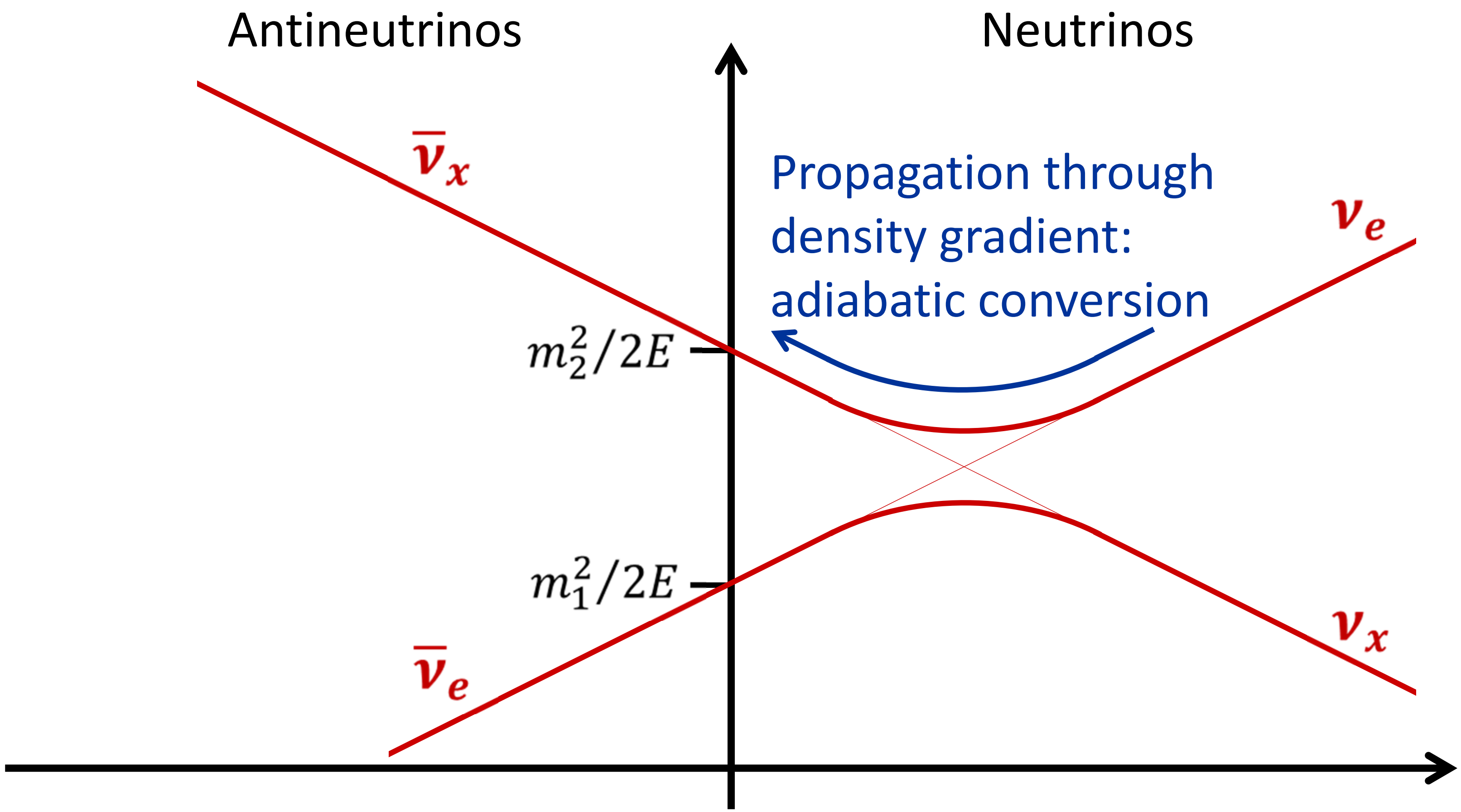
$$\Delta V_\mathrm{weak} = \sqrt{2}G_\mathrm{F}N_e \sim 0.2\ \mathrm{peV}$$

for normal Earth matter, but large effect in SN core (nuclear density $3\times10^{14}\ \mathrm{g/cm^3}$)

$$\Delta V_\mathrm{weak} \sim 10\ \mathrm{eV}$$

Mikheev-Smirnov-Wolfenstein (MSW) effect

Eigenvalue diagram of 2×2 Hamiltonian matrix for 2-flavor oscillations

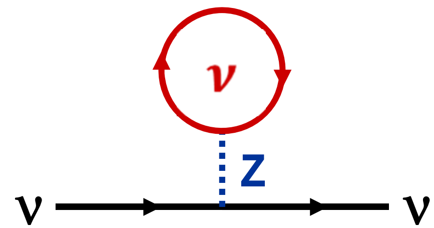


"Negative density" represents antineutrinos in the same diagram

Flavor-Off-Diagonal Refractive Index

2-flavor neutrino evolution as an effective 2-level problem

$$\mathrm{i}\frac{\partial}{\partial t}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = H\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$



Effective mixing Hamiltonian

$$H = \frac{M^2}{2E} + \sqrt{2}G_\mathrm{F}\begin{pmatrix} N_e - \frac{N_n}{2} & 0 \\ 0 & -\frac{N_n}{2}\end{pmatrix} + \sqrt{2}G_\mathrm{F}\begin{pmatrix} N_{\nu_e} & N_{\langle\nu_e|\nu_\mu\rangle} \\ N_{\langle\nu_\mu|\nu_e\rangle} & N_{\nu_\mu}\end{pmatrix}$$

Mass term in flavor basis: causes vacuum oscillations

Wolfenstein's weak potential, causes MSW "resonant" conversion together with vacuum term

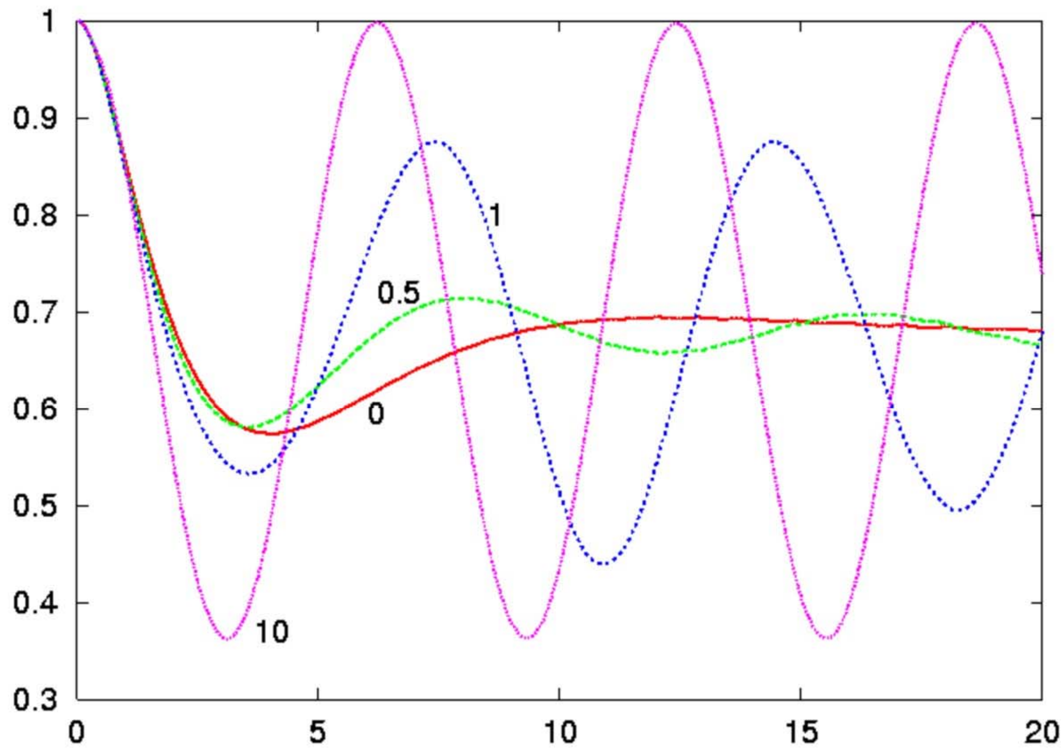
Flavor-off-diagonal potential, caused by flavor oscillations. (J.Pantaleone, PLB 287:128,1992)

Flavor oscillations feed back on the Hamiltonian: Nonlinear effects!

Synchronizing Oscillations by Neutrino Interactions

- Vacuum oscillation frequency depends on energy $\omega = \Delta m^2/2E$
- Ensemble with broad spectrum quickly decoheres kinematically
- ν-ν interactions “synchronize” the oscillations: $\omega_{\rm sync} = \langle \Delta m^2/2E \rangle$

Average e-flavor component of polarization vector



Time

Pastor, Raffelt & Semikoz, hep-ph/0109035

Slide ca. 2004

…rature on Synchronized Oscillations

…nization … first …overed and …udied numerically	Samuel: PRD 48 (1993) 1462; PRD 53 (1996) 5382. Kostelecký & Samuel: PRD 49 (1994) 1740; PLB 318 (1993) 127; PRD 52 (1995) 621; PRD 52 (1995) 3184; PLB 385 (1996) 159. Kostelecký, Pantaleone & Samuel: PLB 315 (1993) 46. Pantaleone: PRD 58 (1998) 073002.
Application to early-universe flavor oscillations and limits to lepton asymmetry	Lunardini & Smirnov: PRD 64 (2001) 073006. Dolgov, Hansen, Pastor, Petcov, Raffelt & Semikoz: NPB 632 (2002) 363. Wong: PRD 66 (2002) 025015. Abazajian, Beacom & Bell: PRD 66 (2002) 013008
Simple physical interpretation	Pastor, Raffelt & Semikoz: PRD 65 (2002) 053011
Application to SN hot bubble region	Pastor & Raffelt: astro-ph/0207281

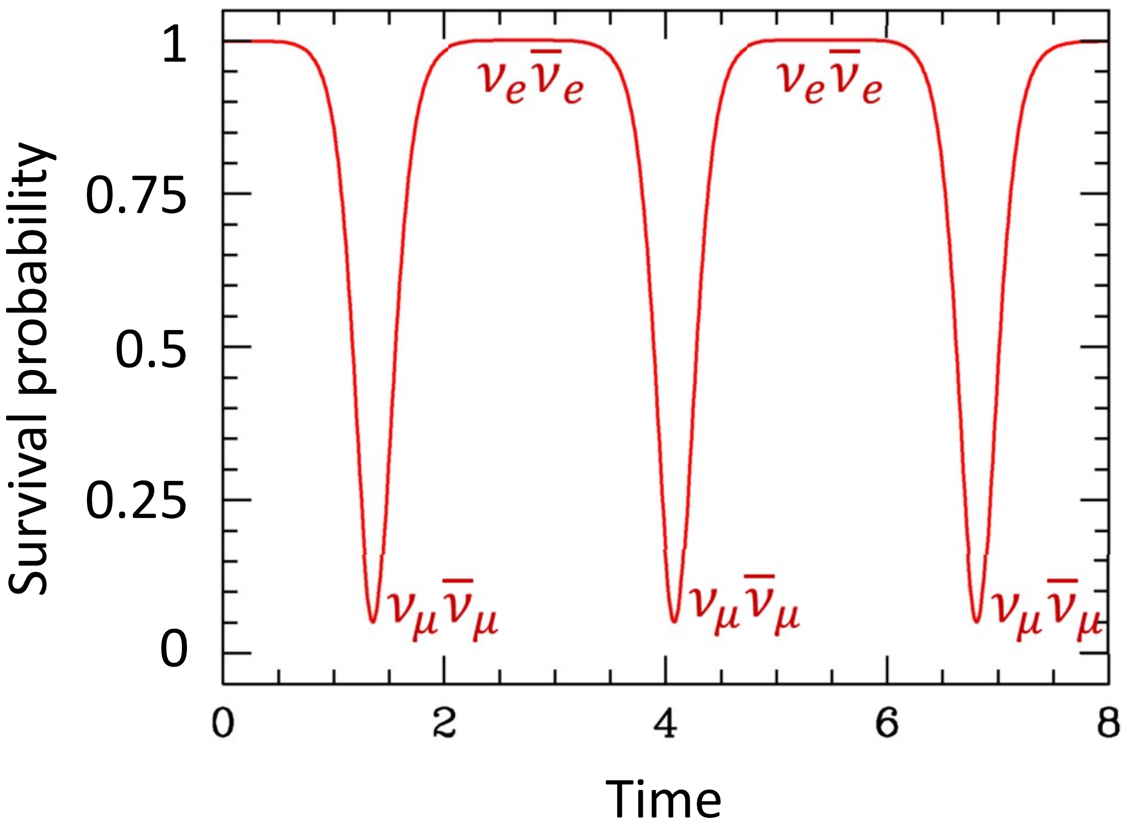
Collective Supernova Nu Oscillations since 2006

Two seminal papers in 2006 triggered a torrent of activities
Duan, Fuller, Qian, astro-ph/0511275, Duan et al. astro-ph/0606616

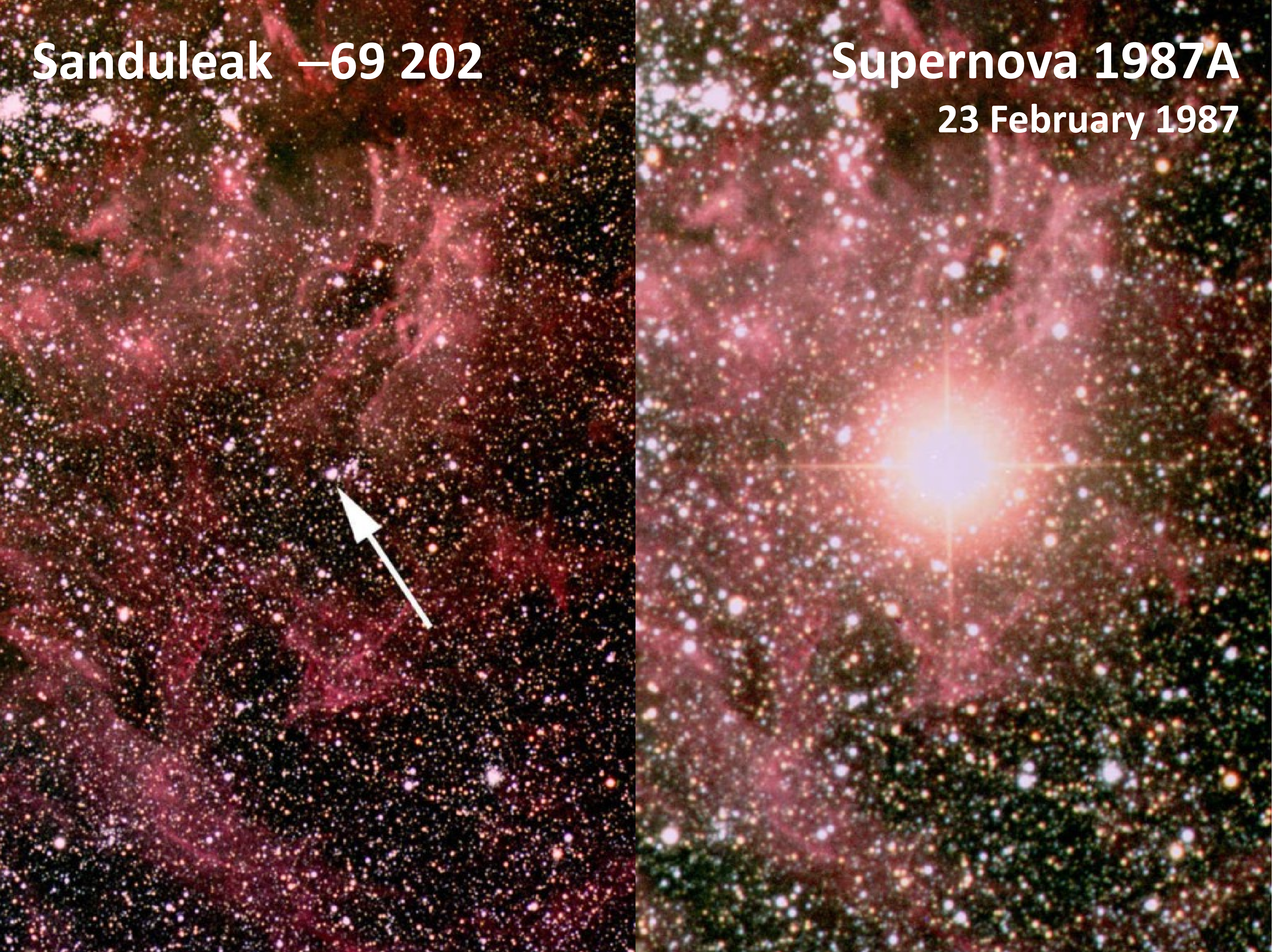
Balantekin, Gava & Volpe, arXiv:0710.3112. Balantekin & Pehlivan, astro-ph/0607527. Blennow, Mirizzi & Serpico, arXiv:0810.2297. Cherry, Fuller, Carlson, Duan & Qian, arXiv:1006.2175. Chakraborty, Choubey, Dasgupta & Kar, arXiv:0805.3131. Chakraborty, Fischer, Mirizzi, Saviano, Tomàs, arXiv:1104.4031, 1105.1130. Choubey, Dasgupta, Dighe & Mirizzi, arXiv:1008.0308. Dasgupta & Dighe, arXiv:0712.3798. Dasgupta, Dighe & Mirizzi, arXiv:0802.1481. Dasgupta, Dighe, Mirizzi & Raffelt, arXiv:0801.1660, 0805.3300. Dasgupta, Mirizzi, Tamborra & Tomàs, arXiv:1002.2943. Dasgupta, Dighe, Raffelt & Smirnov, 0904.3542. Dasgupta, Raffelt, Tamborra, arXiv:1001.5396. Duan, Fuller, Carlson & Qian, astro-ph/0608050, 0703776, arXiv:0707.0290, 0710.1271. Duan, Fuller & Qian, arXiv:0706.4293, 0801.1363, 0808.2046, 1001.2799. Duan, Fuller & Carlson, arXiv:0803.3650. Duan & Kneller, arXiv:0904.0974. Duan & Friedland, arXiv:1006.2359. Duan, Friedland, McLaughlin & Surman, arXiv:1012.0532. Esteban-Pretel, Pastor, Tomàs, Raffelt & Sigl, arXiv:0706.2498, 0712.1137. Esteban-Pretel, Mirizzi, Pastor, Tomàs, Raffelt, Serpico & Sigl, arXiv:0807.0659. Fogli, Lisi, Marrone & Mirizzi, arXiv:0707.1998. Fogli, Lisi, Marrone & Tamborra, arXiv:0812.3031. Friedland, arXiv:1001.0996. Gava & Jean-Louis, arXiv:0907.3947. Gava & Volpe, arXiv:0807.3418. Galais, Kneller & Volpe, arXiv:1102.1471. Galais & Volpe, arXiv:1103.5302. Gava, Kneller, Volpe & McLaughlin, arXiv:0902.0317. Hannestad, Raffelt, Sigl & Wong, astro-ph/0608695. Wei Liao, arXiv:0904.0075, 0904.2855. Lunardini, Müller & Janka, arXiv:0712.3000. Mirizzi, Pozzorini, Raffelt & Serpico, arXiv:0907.3674. Mirizzi & Tomàs, arXiv:1012.1339. Pehlivan, Balantekin, Kajino, Yoshida, arXiv:1105.1182. Raffelt, arXiv:0810.1407, 1103.2891. Raffelt & Tamborra, arXiv:1006.0002. Raffelt & Sigl, hep-ph/0701182. Raffelt & Smirnov, arXiv:0705.1830, 0709.4641. Sawyer, hep-ph/0408265, 0503013, arXiv:0803.4319, 1011.4585. Sarikas, Raffelt, Hüdepohl & Janka, arXiv:1109.3601. Wu & Qian, arXiv:1105.2068.

Collective Pair Conversion

Gas of equal abundances of ν_e and $\bar{\nu}_e$, inverted mass hierarchy
Small effective mixing angle (e.g. made small by ordinary matter)

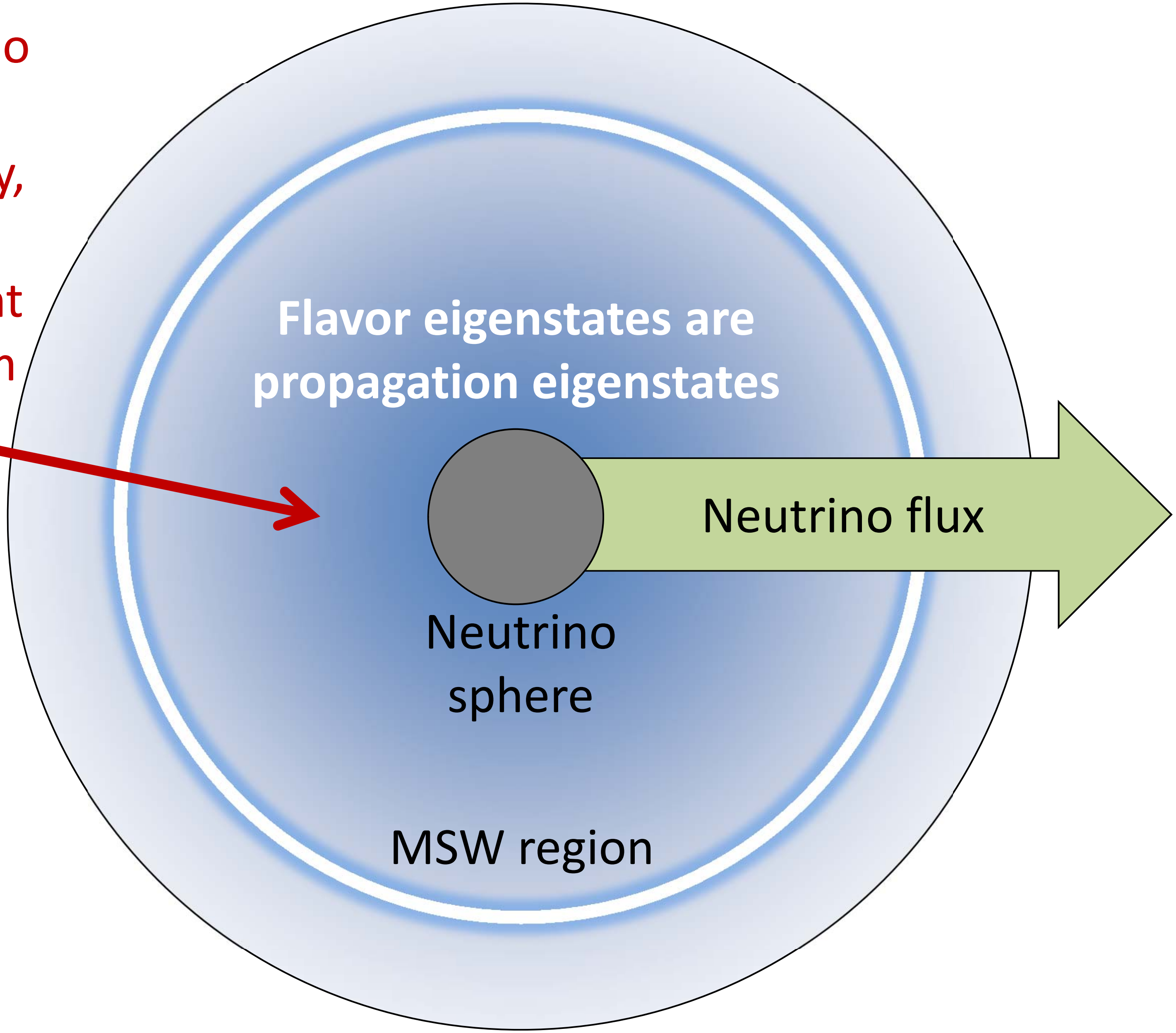


Dense neutrino gas unstable in flavor space: $\nu_e\bar{\nu}_e \leftrightarrow \nu_\mu\bar{\nu}_\mu$
Complete pair conversion even for a small mixing angle


Sanduleak –69 202
Supernova 1987A
23 February 1987

Flavor Oscillations in Core-Collapse Supernovae

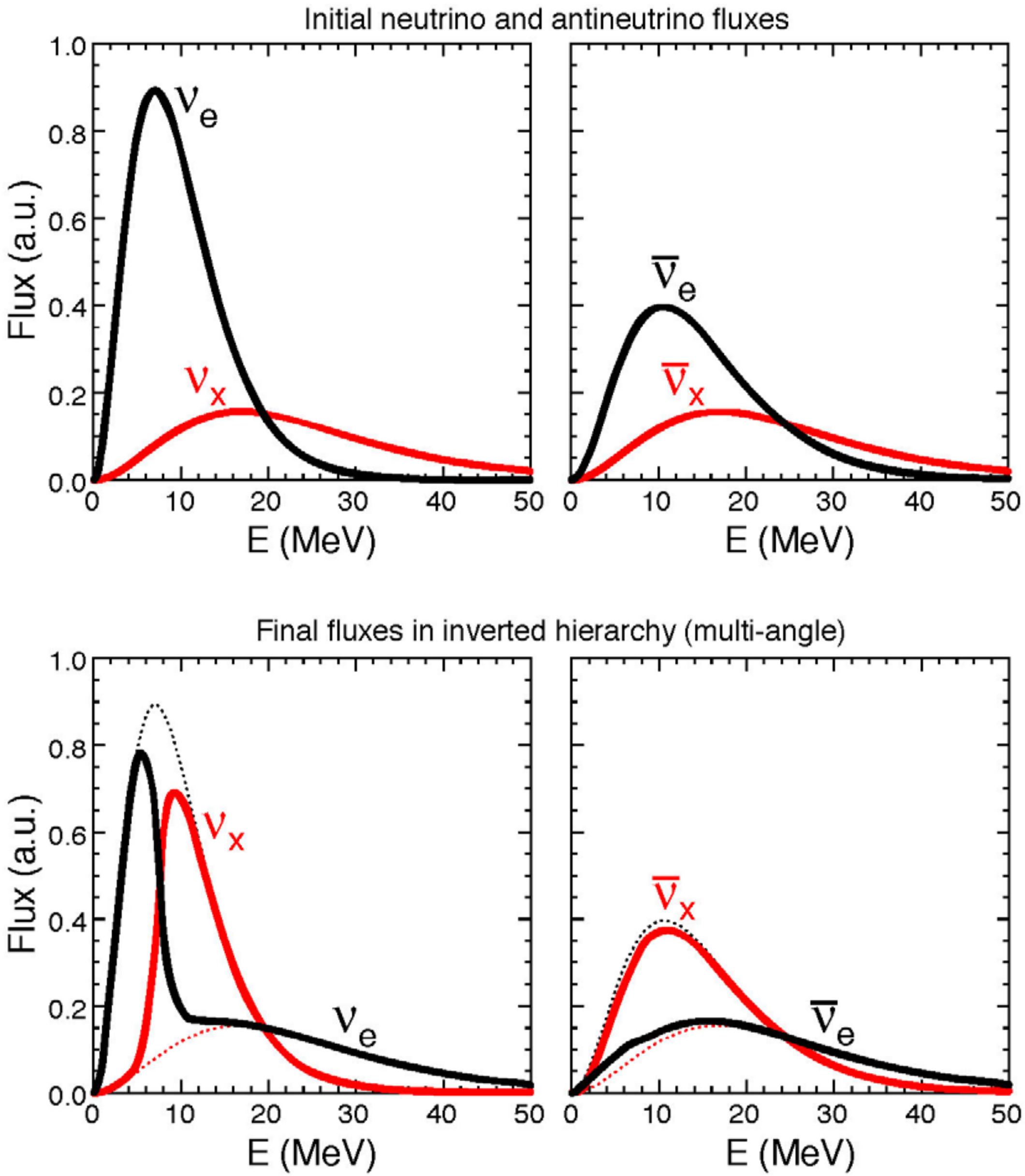
Neutrino-neutrino refraction causes a flavor instability, flavor exchanged between different parts of spectrum



Spectral Split

Initial fluxes at neutrino sphere

After collective trans-formation



Figures from

Fogli, Lisi, Marrone & Mirizzi, arXiv:0707.1998

Explanations in

Raffelt & Smirnov arXiv:0705.1830 and 0709.4641

Duan, Fuller, Carlson & Qian arXiv:0706.4293 and 0707.0290

Three Ways to Describe Flavor Oscillations

Schrödinger equation in terms of "flavor spinor"

$$\mathrm{i}\partial_t \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = H \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{\Delta m^2}{2E} \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Neutrino flavor density matrix

$$\rho = \begin{pmatrix} \langle \nu_e | \nu_e \rangle & \langle \nu_e | \nu_\mu \rangle \\ \langle \nu_\mu | \nu_e \rangle & \langle \nu_\mu | \nu_\mu \rangle \end{pmatrix}$$

Equivalent commutator form of Schrödinger equation

$$\mathrm{i}\partial_t \rho = [H, \rho]$$

Expand 2×2 Hermitean matrices in terms of Pauli matrices

$$\rho = \mathrm{Tr}(\rho) + \frac{1}{2}\mathbf{P}\cdot\boldsymbol{\sigma} \quad \text{and} \quad H = \frac{\Delta m^2}{2E}\mathbf{B}\cdot\boldsymbol{\sigma} \quad \text{with} \quad \mathbf{B} = (\sin 2\theta, 0, \cos 2\theta)$$

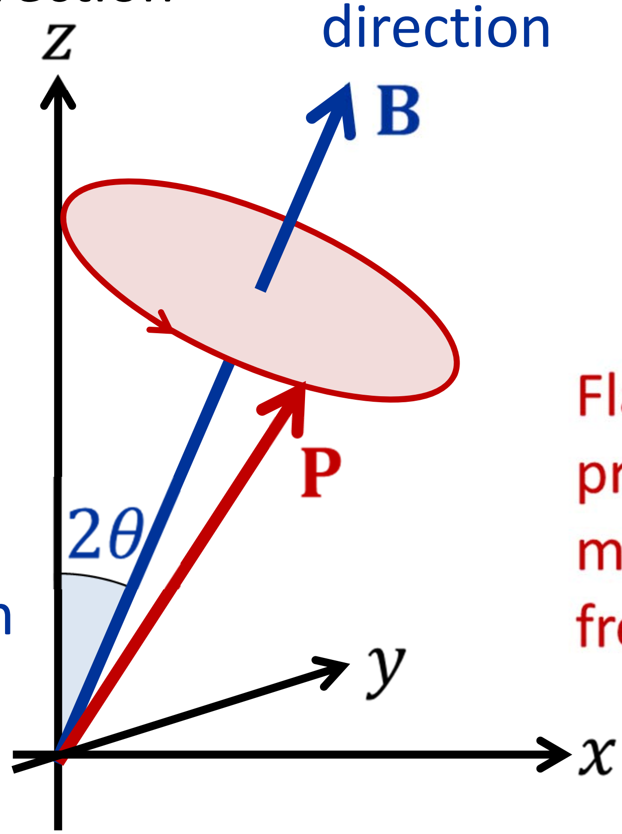
Equivalent spin-precession form of equation of motion

$$\dot{\mathbf{P}} = \omega \mathbf{B} \times \mathbf{P} \quad \text{with} \quad \omega = \frac{\Delta m^2}{2E}$$

$\mathbf{P}$ is "polarization vector" or "Bloch vector"

Flavor Oscillation as Spin Precession

$\uparrow$ Spin up ν_e
$\downarrow$ Spin down ν_μ



Twice the vacuum mixing angle

Flavor polarization vector precesses around the mass direction with frequency $\omega = \Delta m^2/2E$

Adding Matter

Schrödinger equation including matter

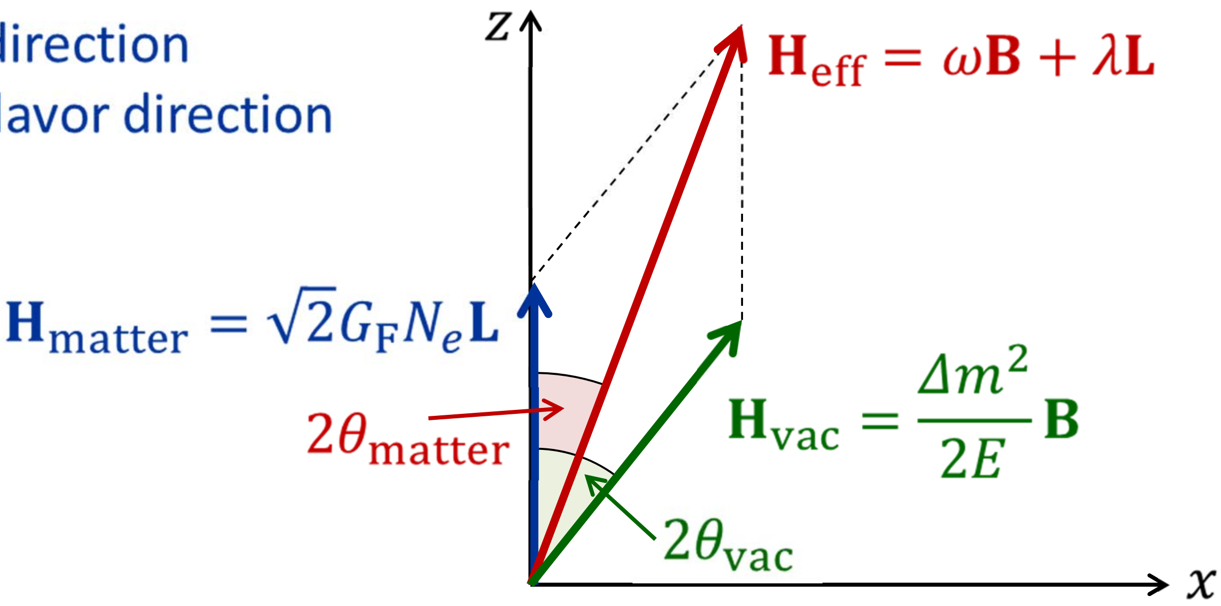
$$i\partial_t \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \left[\frac{\Delta m^2}{2E} \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} + \sqrt{2} G_F \begin{pmatrix} N_e - \frac{N_n}{2} & 0 \\ 0 & -\frac{N_n}{2} \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Corresponding spin-precession equation

$$\dot{\mathbf{P}} = \underbrace{(\omega \mathbf{B} + \lambda \mathbf{L})}_{\mathbf{H}_{\rm eff}} \times \mathbf{P} \quad \text{with} \quad \omega = \Delta m^2/2E \quad \text{and} \quad \lambda = \sqrt{2} G_{\rm F} N_e$$

$\mathbf{B}$ unit vector in mass direction

$\mathbf{L} = \mathbf{e}_z$ unit vector in flavor direction

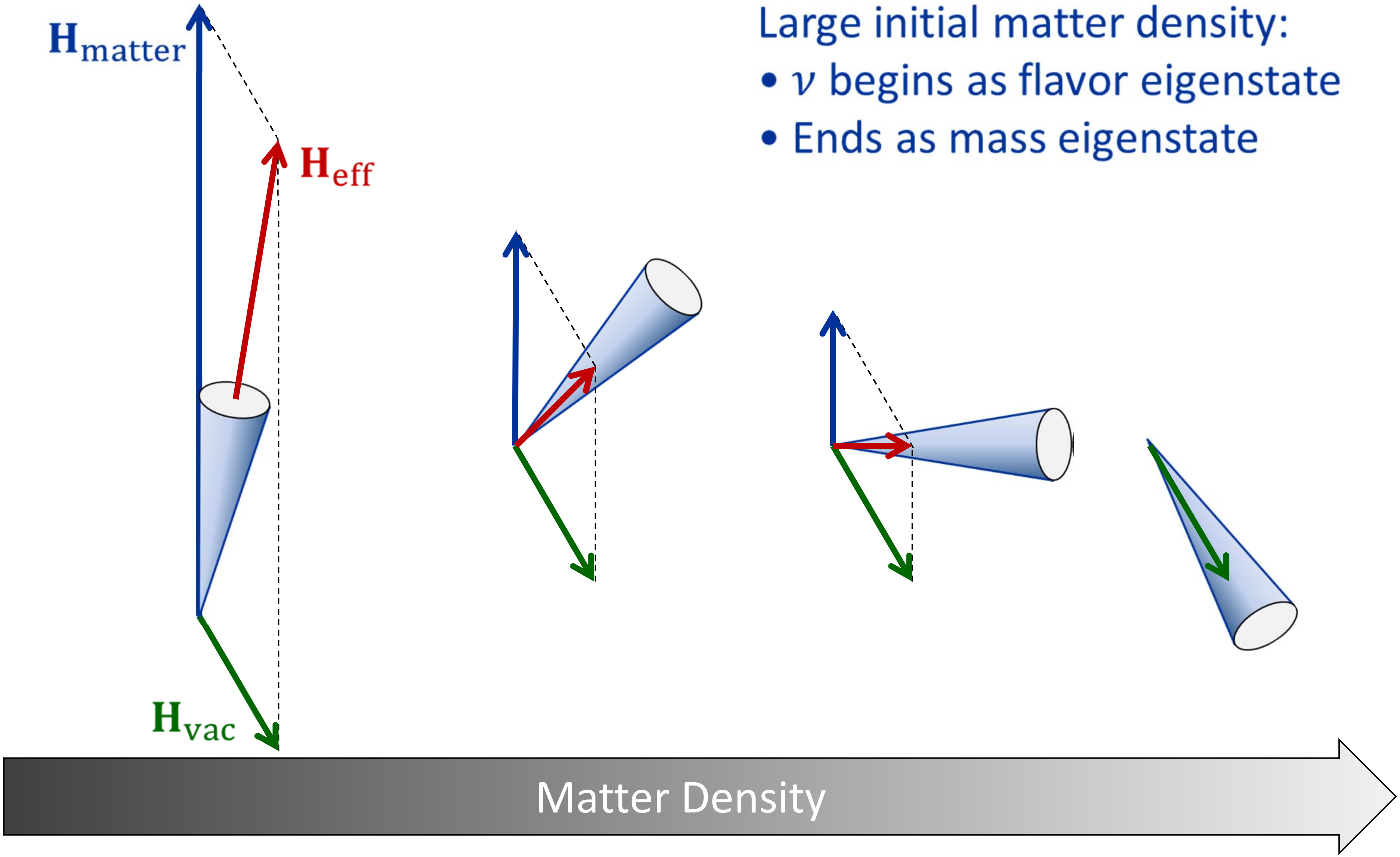


MSW Effect

Adiabatically decreasing density: Precession cone follows $\mathbf{H}_{\mathrm{eff}}$

Large initial matter density:

- ν begins as flavor eigenstate
- Ends as mass eigenstate



Adding Neutrino-Neutrino Interactions

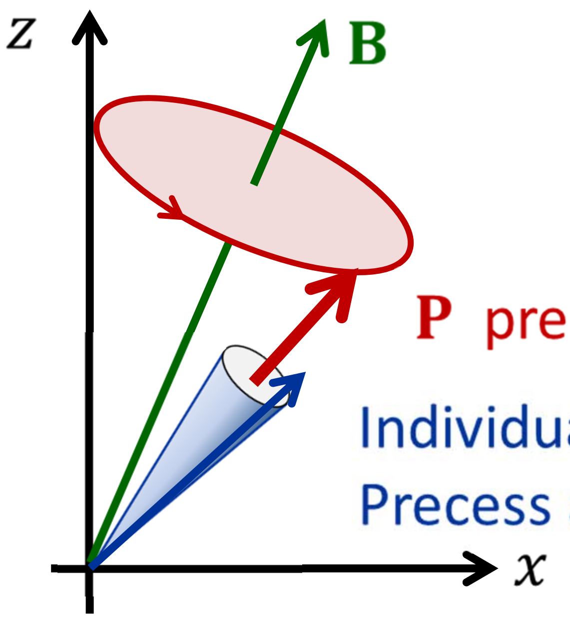
Precession equation for each ν mode with energy E, i.e. $\omega = \Delta m^2/2E$

$$\dot{\mathbf{P}}_\omega = \underbrace{(\omega\mathbf{B} + \lambda\mathbf{L} + \mu\mathbf{P})}_{\mathbf{H}_{\rm eff}} \times \mathbf{P}_\omega \quad \text{with} \quad \lambda = \sqrt{2}G_{\rm F}N_e \quad \text{and} \quad \mu = \sqrt{2}G_{\rm F}N_\nu$$

Total flavor spin of entire ensemble

$$\mathbf{P} = \textstyle\sum_\omega \mathbf{P}_\omega \qquad \text{normalize} \qquad |\mathbf{P}_{\mathbf{t=0}}| = 1$$

Individual spins do not remain aligned – feel "internal" field $\mathbf{H}_{\nu\nu} = \mu\mathbf{P}$



Synchronized oscillations for large neutrino density $\mu \gg \delta\omega$

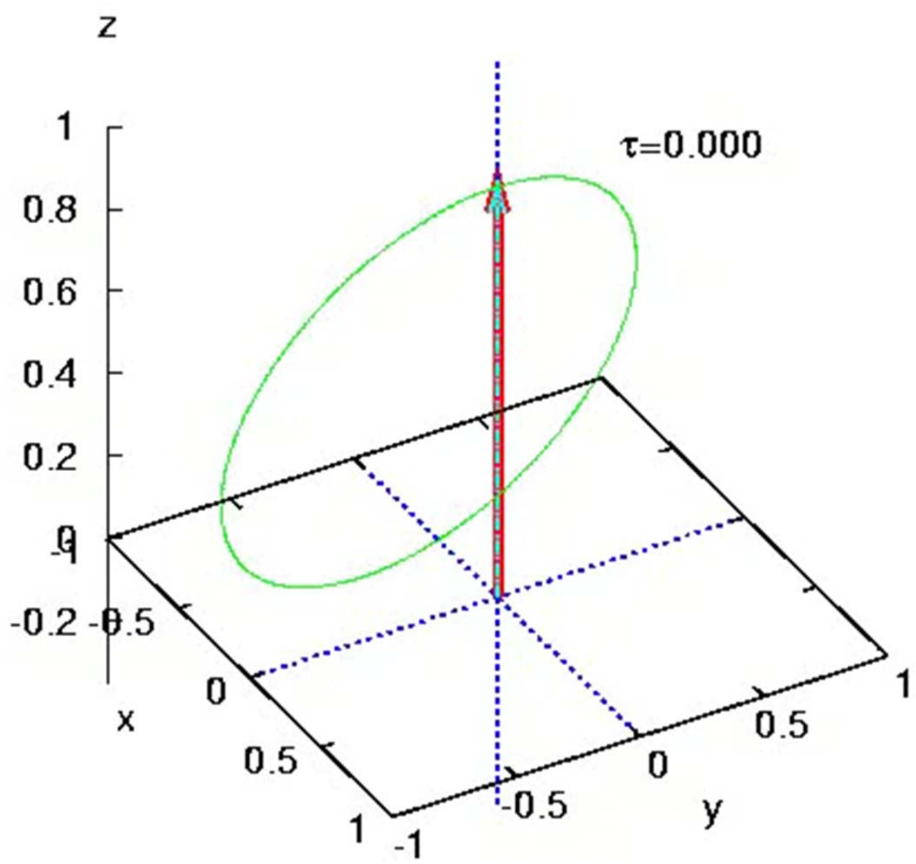
$\mathbf{P}$ precesses with $\omega_{\rm sync}$ for large ν density

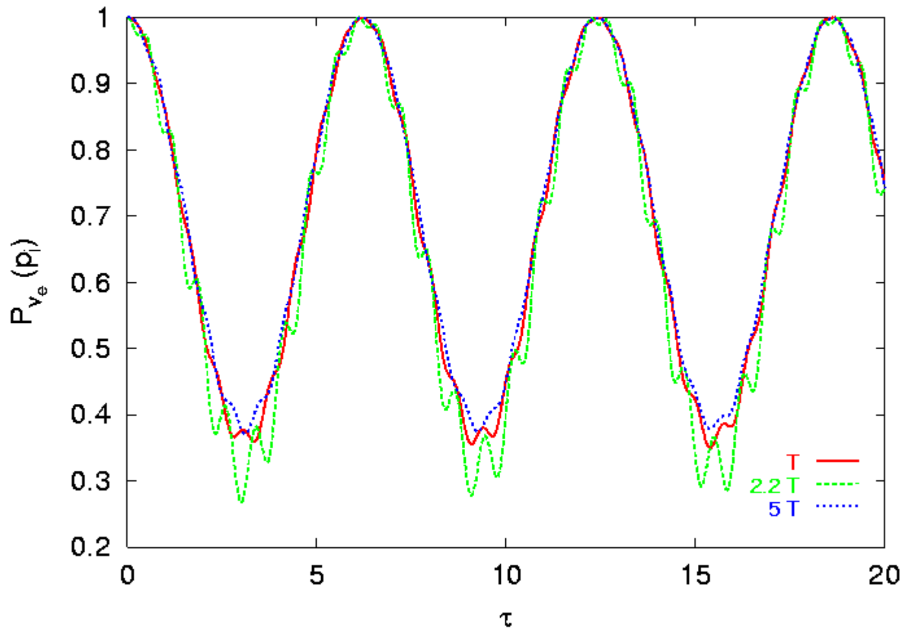
Individual $\mathbf{P}_\omega$ "trapped" on precession cones

Precess around $\mathbf{P}$ with frequency $\sim \mu$

Synchronized Oscillations by Nu-Nu Interactions

For large neutrino density, individual modes precess around large common dipole moment





Pastor, Raffelt & Semikoz, hep-ph/0109035

Two Spins Interacting with a Dipole Force

Simplest system showing ν-ν effects:
Isotropic neutrino gas with 2 energies E_1 and E_2, no ordinary matter

$$\dot{\mathbf{P}}_1 = (\omega_1\mathbf{B} + \mu\mathbf{P}) \times \mathbf{P}_1 \quad \text{with} \quad \mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2 \quad \text{and} \quad \omega_{1,2} = \Delta m^2/2E$$
$$\dot{\mathbf{P}}_2 = (\omega_2\mathbf{B} + \mu\mathbf{P}) \times \mathbf{P}_2$$

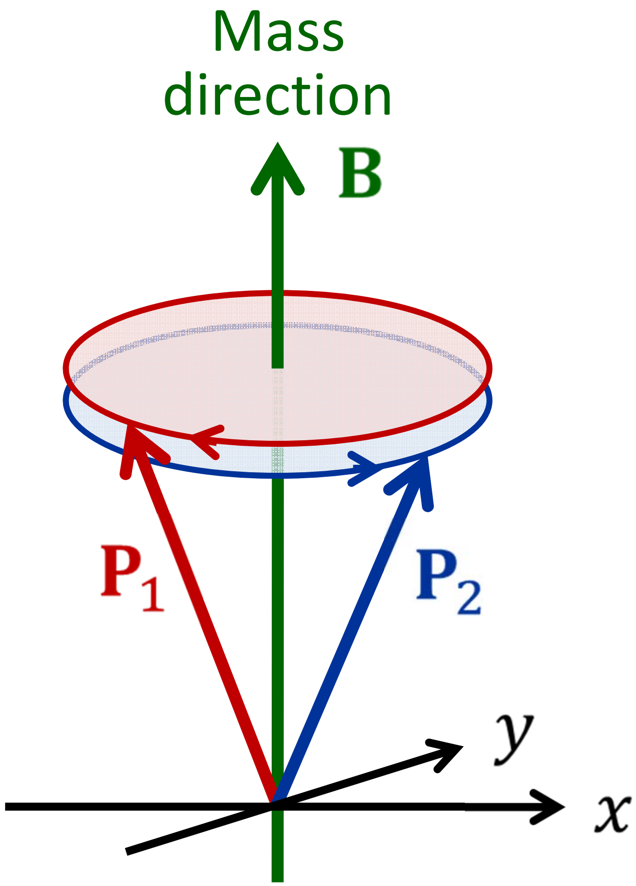
Go to "co-rotating frame" around $\mathbf{B}$ direction

$$\dot{\mathbf{P}}_1 = (\omega_c\mathbf{B} - \omega\mathbf{B} + \mu\mathbf{P}) \times \mathbf{P}_1$$
$$\dot{\mathbf{P}}_2 = (\omega_c\mathbf{B} + \omega\mathbf{B} + \mu\mathbf{P}) \times \mathbf{P}_2$$

with $\omega_c = \frac{1}{2}(\omega_2 + \omega_1)$ and $\omega = \frac{1}{2}(\omega_2 - \omega_1)$

No interaction ($\mu = 0$)
$P_{1,2}$ precess in opposite directions

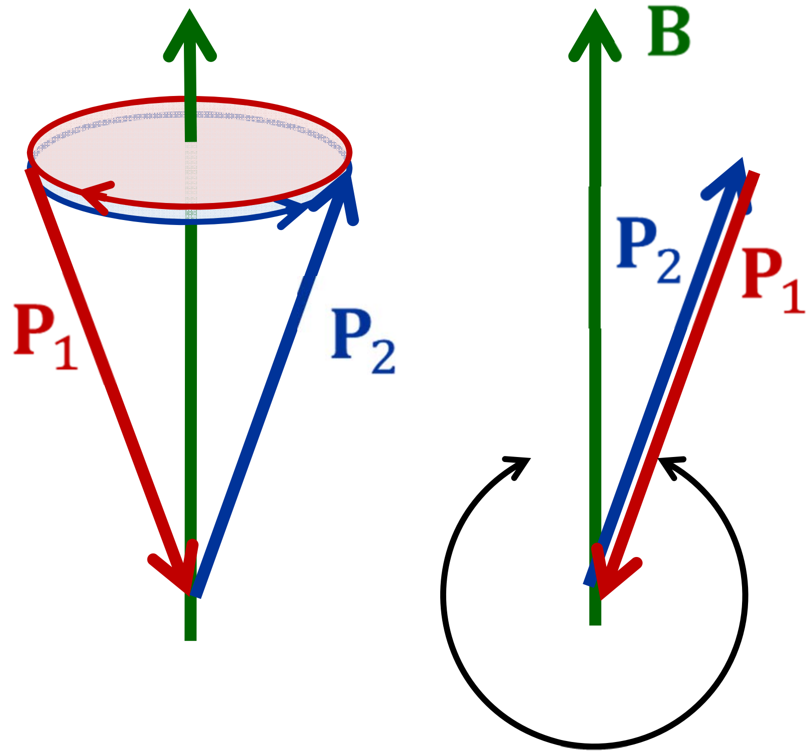
Strong interactions ($\mu \to \infty$)
$P_{1,2}$ stuck to each other
(no motion in co-rotating frame, perfectly synchronized in lab frame)



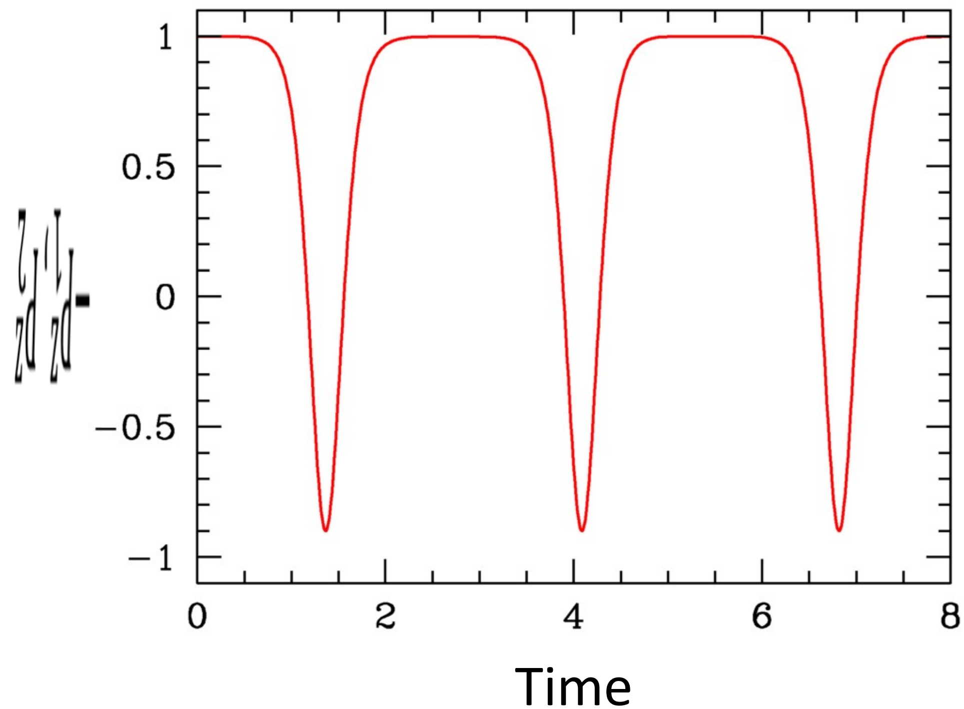
Two Spins with Opposite Initial Orientation

No interaction ($\mu = 0$)
Free precession in opposite directions

Even for very small mixing angle, large-amplitude flavor oscillations



Strong interaction
($\mu \to \infty$)
Pendular motion

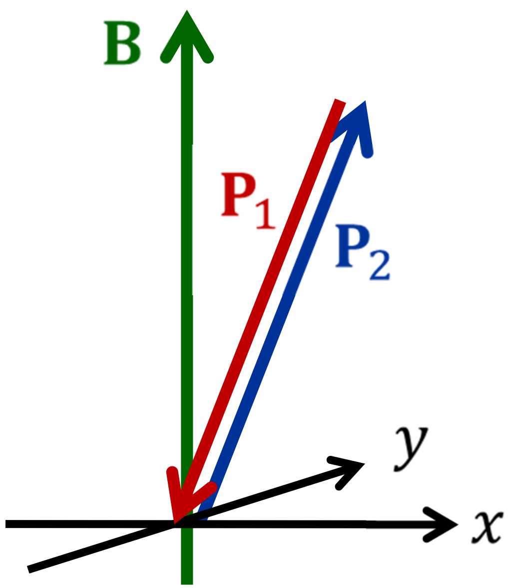


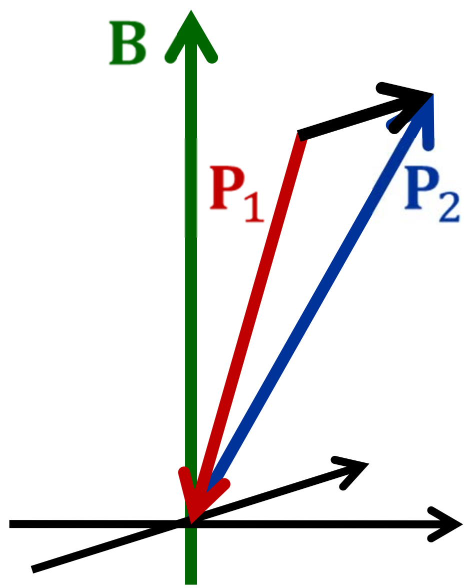
Instability in Flavor Space

Two-mode example in co-rotating frame, initially $\mathbf{P}_1 = \downarrow$, $\mathbf{P}_2 = \uparrow$ (flavor basis)

$$\dot{\mathbf{P}}_1 = [-\omega\mathbf{B} + \mu\,(\mathbf{P}_1 + \mathbf{P}_2)] \times \mathbf{P}_1$$

$$\dot{\mathbf{P}}_2 = [+\omega\mathbf{B} + \mu\,\underbrace{(\mathbf{P}_1 + \mathbf{P}_2)}_{0\text{ initially}}] \times \mathbf{P}_2$$





$\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2$

Matter effect transverse to mass and flavor directions

Both $\mathbf{P}_1$ and $\mathbf{P}_2$ tilt around $\mathbf{P}$ if μ is large

- Initially aligned in flavor direction and $\mathbf{P} = 0$
- Free precession $\pm\omega$

After a short time, transverse $\mathbf{P}$ develops by free precession

Flavor Pendulum

Classical Hamiltonian for two spins interacting with a dipole force μ

$$H = \omega \mathbf{B} \cdot (\mathbf{P}_2 - \mathbf{P}_1) + \frac{\mu}{2} \mathbf{P}^2$$

Angular-momentum Poisson brackets

$$\{P_i, P_j\} = \epsilon_{ijk} P_k$$

Total angular momentum

$$\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2$$

Precession equations of motion

$$\dot{\mathbf{P}}_{1,2} = (\mp \omega \mathbf{B} + \mu \mathbf{P}) \times \mathbf{P}_{1,2}$$

Lagrangian top (spherical pendulum with spin), moment of inertia I

$$H = \omega \mathbf{B} \cdot \mathbf{Q} + \frac{\mathbf{P}^2}{2I}$$

Total angular momentum $\mathbf{P}$, radius vector $\mathbf{Q}$, fulfilling

$$\{P_i, P_j\} = \epsilon_{ijk} P_k, \quad \{Q_i, Q_j\} = 0$$

$$\{P_i, Q_j\} = \epsilon_{ijk} Q_k$$

Pendulum EoMs

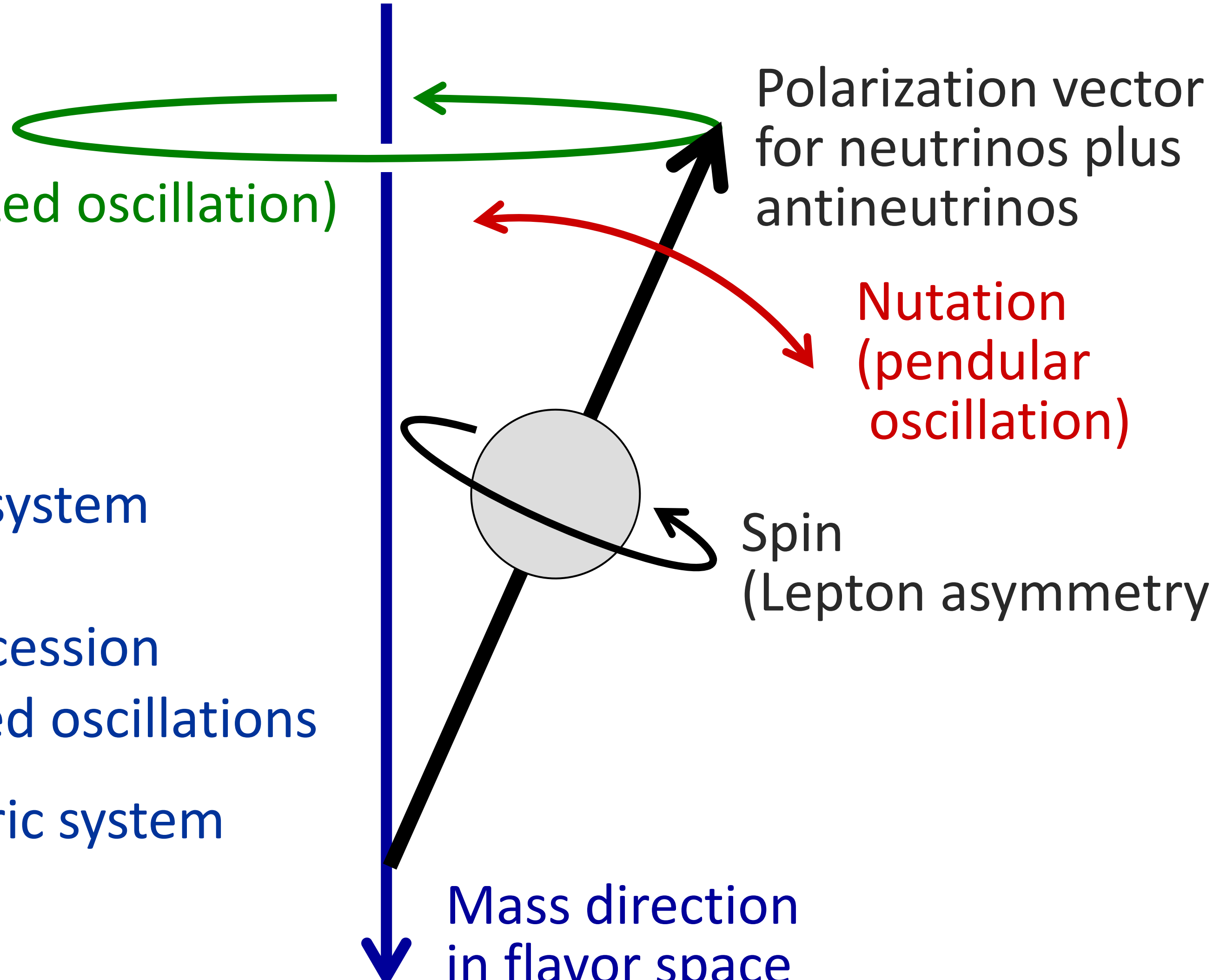
$$\dot{\mathbf{Q}} = I^{-1} \mathbf{P} \times \mathbf{Q} \quad \text{and} \quad \dot{\mathbf{P}} = \omega \mathbf{B} \times \mathbf{Q}$$

EoMs and Hamiltonians identical (up to a constant) with the identification

$$\mathbf{Q} = \mathbf{P}_2 - \mathbf{P}_1 - \frac{\omega}{\mu} \mathbf{B} \quad \text{and} \quad \mu = I^{-1}$$

Constants of motion: $\mathbf{P}_1^2$, $\mathbf{P}_2^2$, $\mathbf{B} \cdot \mathbf{P}$, $\mathbf{P} \cdot \mathbf{Q}$, $\mathbf{Q}^2$ and H

Pendulum in Flavor Space

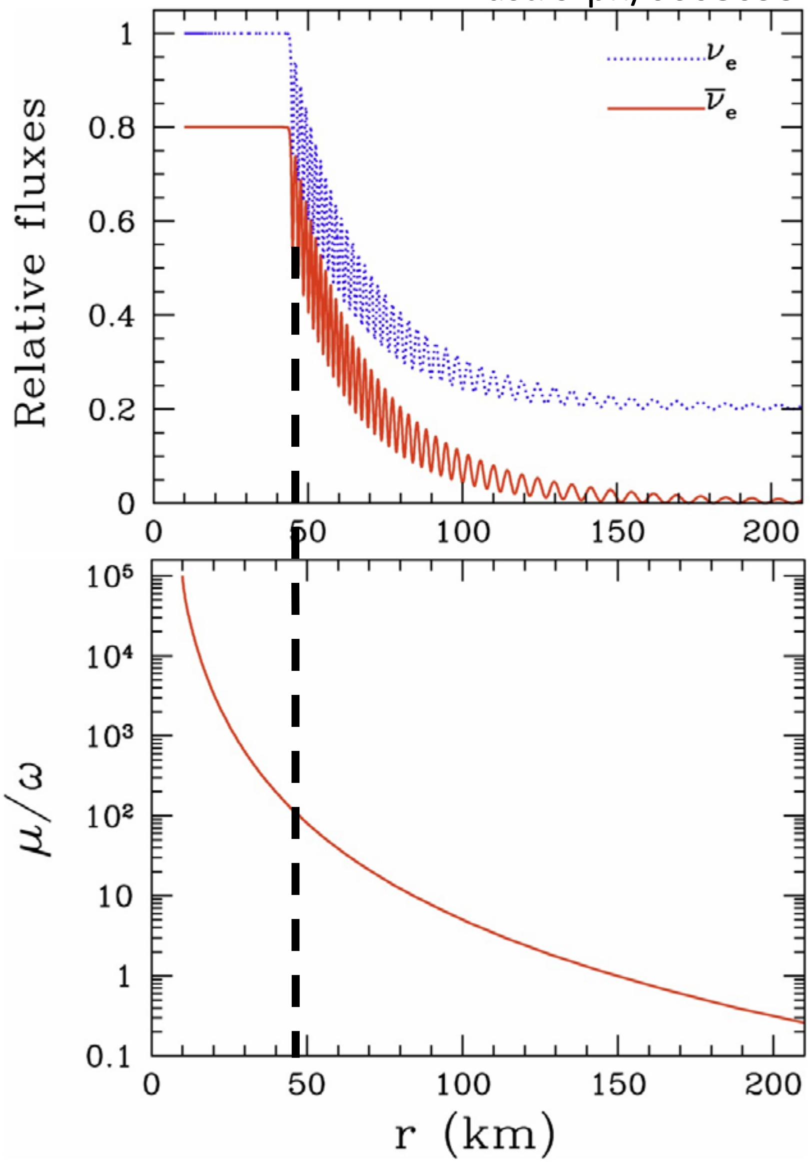


- Very asymmetric system
  - Large spin
  - Almost pure precession
  - Fully synchronized oscillations
- Perfectly symmetric system
  - No spin
  - Plane pendulum

[Hannestad, Raffelt, Sigl, Wong: astro-ph/0608695]

Flavor Conversion in a Toy Supernova

astro-ph/0608695



- Two modes with $\omega = \pm 0.3\ \mathrm{km}^{-1}$
- Assume 80% anti-neutrinos

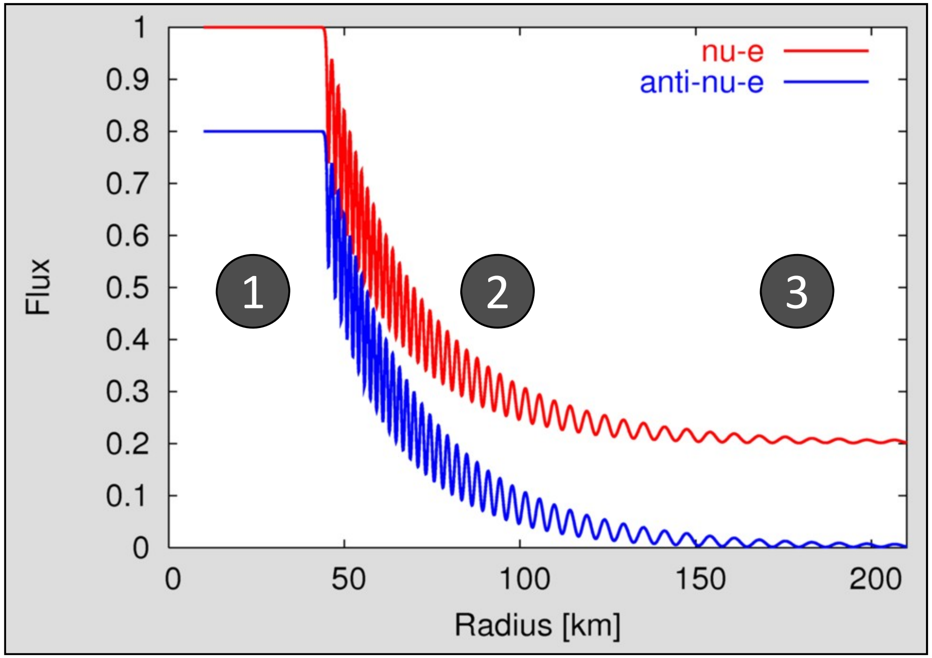
- Sharp onset radius
- Oscillation amplitude declining

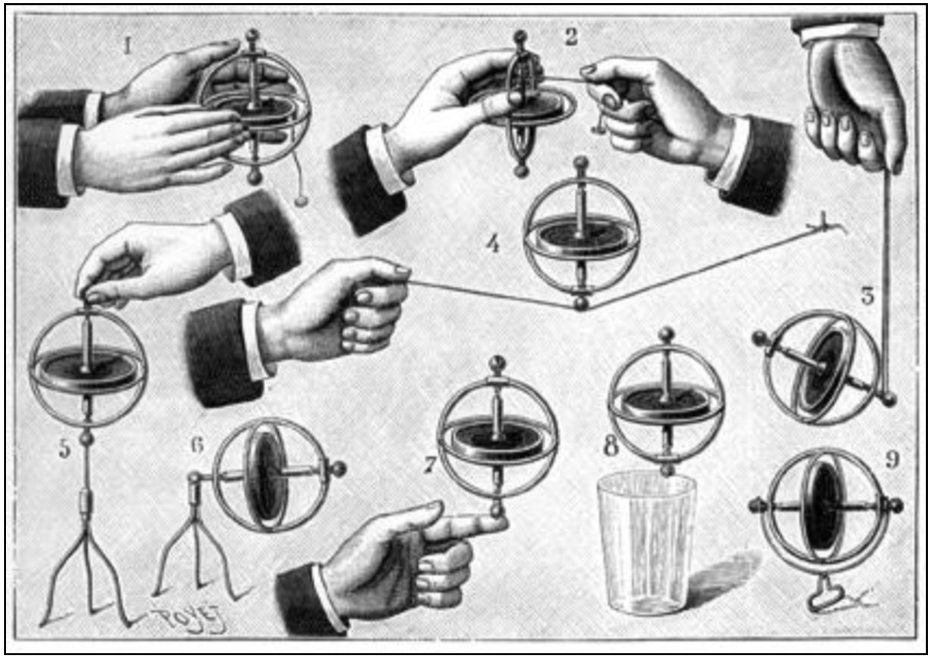
- Neutrino-neutrino interaction energy at nu sphere $(r = 10\ \mathrm{km})$

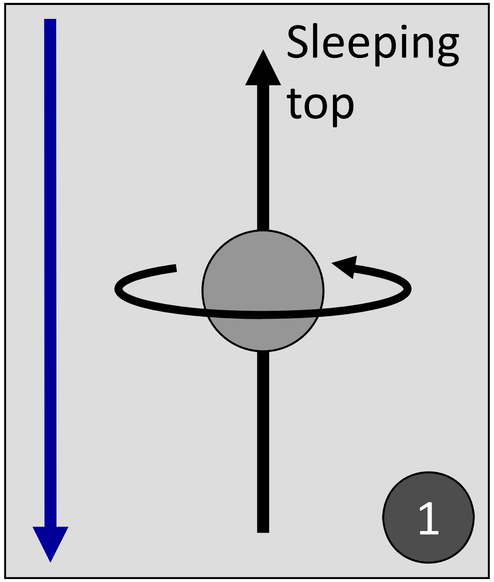
$$\mu = 0.3 \times 10^5\ \mathrm{km}^{-1}$$

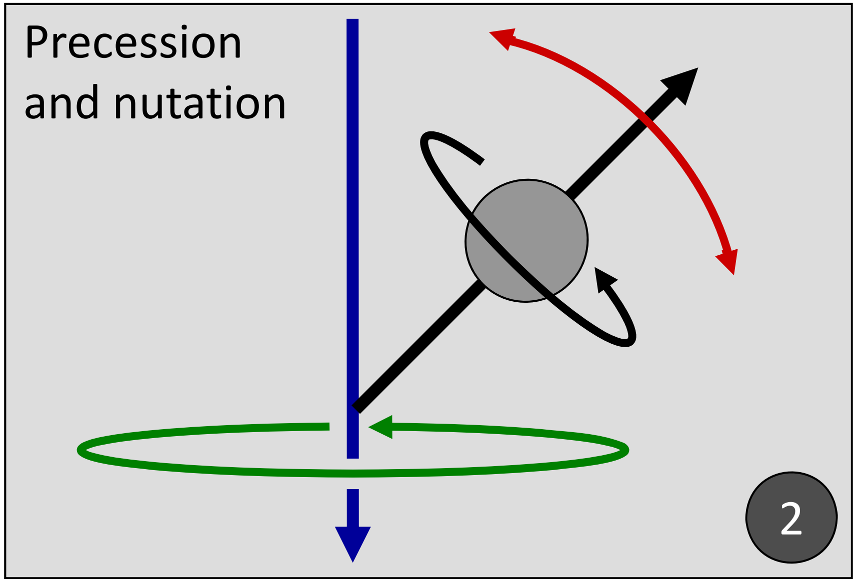
- Falls off approximately as r^{-4} (geometric flux dilution and nus become more co-linear)

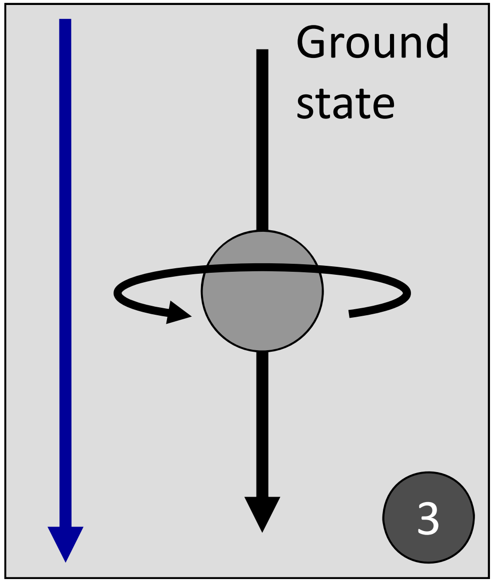
Neutrino Conversion and Flavor Pendulum











Fermi-Dirac Spectrum

Fermi-Dirac energy spectrum

$$\frac{dN}{dE} \propto \frac{E^2}{e^{E/T-\eta}+1}$$

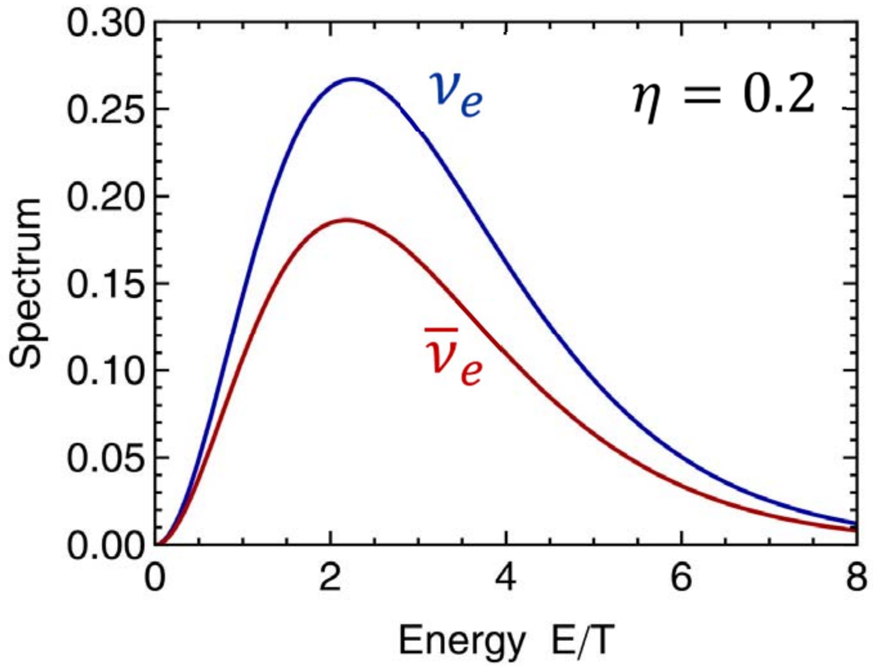
η degeneracy parameter, $-\eta$ for $\overline{\nu}$

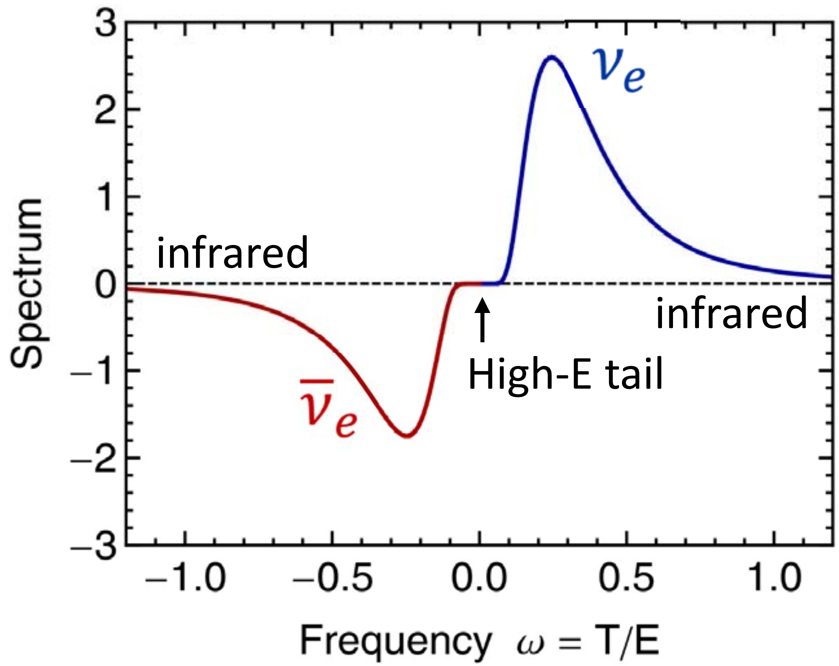
Same spectrum in terms of $\omega = T/E$

- Antineutrinos $E \to -E$
- and dN/dE negative (flavor isospin convention)

$\omega > 0$: $\nu_e = \uparrow$ and $\nu_\mu = \downarrow$

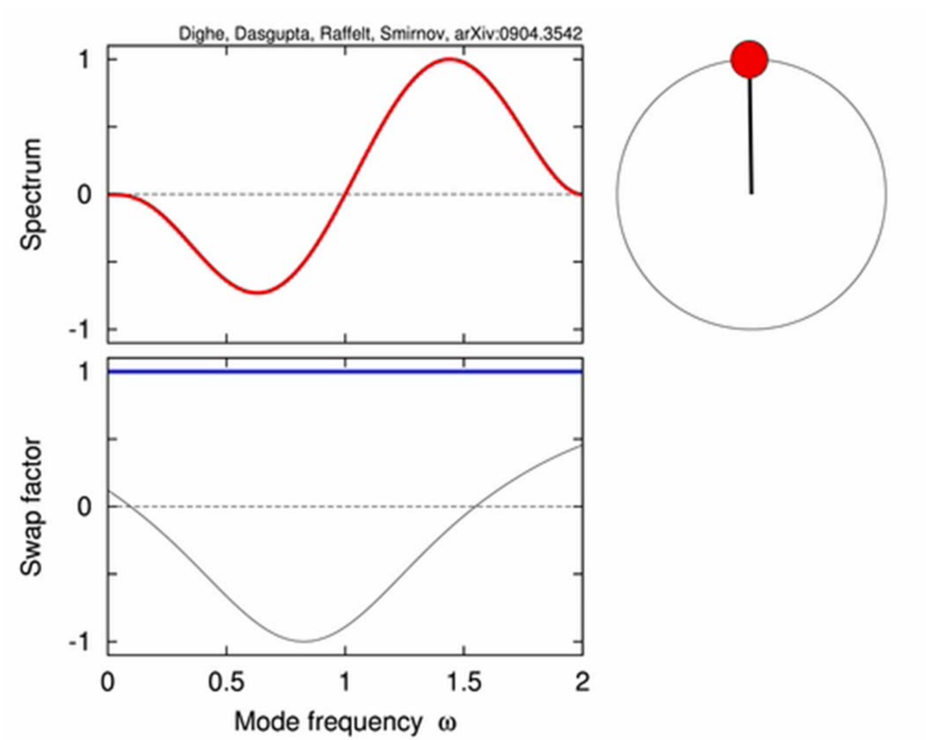
$\omega < 0$: $\overline{\nu}_e = \downarrow$ and $\overline{\nu}_\mu = \uparrow$



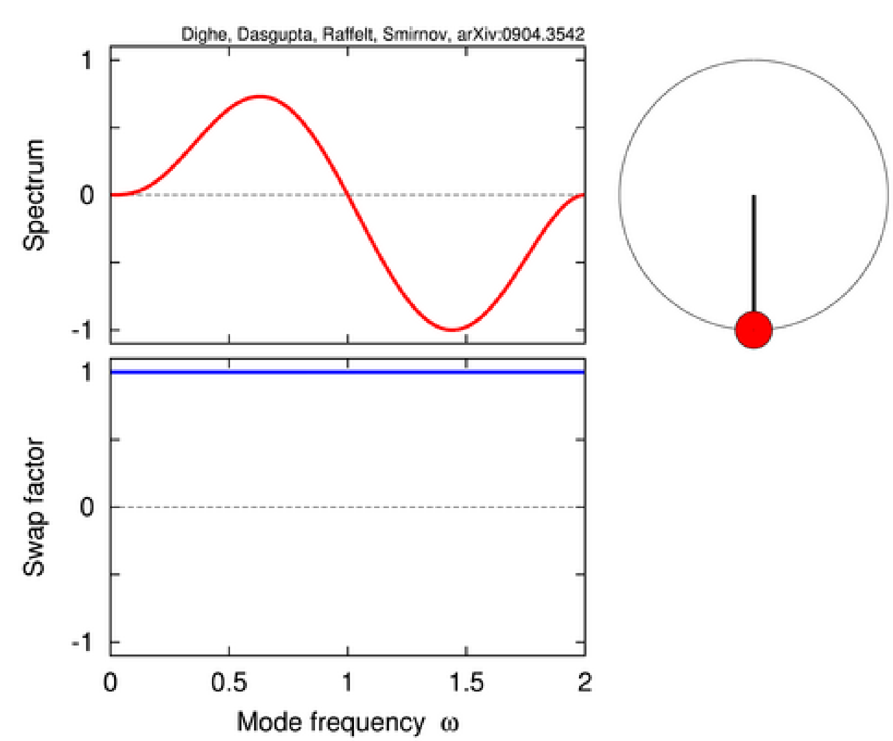


Flavor Pendulum

Single "positive" crossing
(potential energy at a maximum)



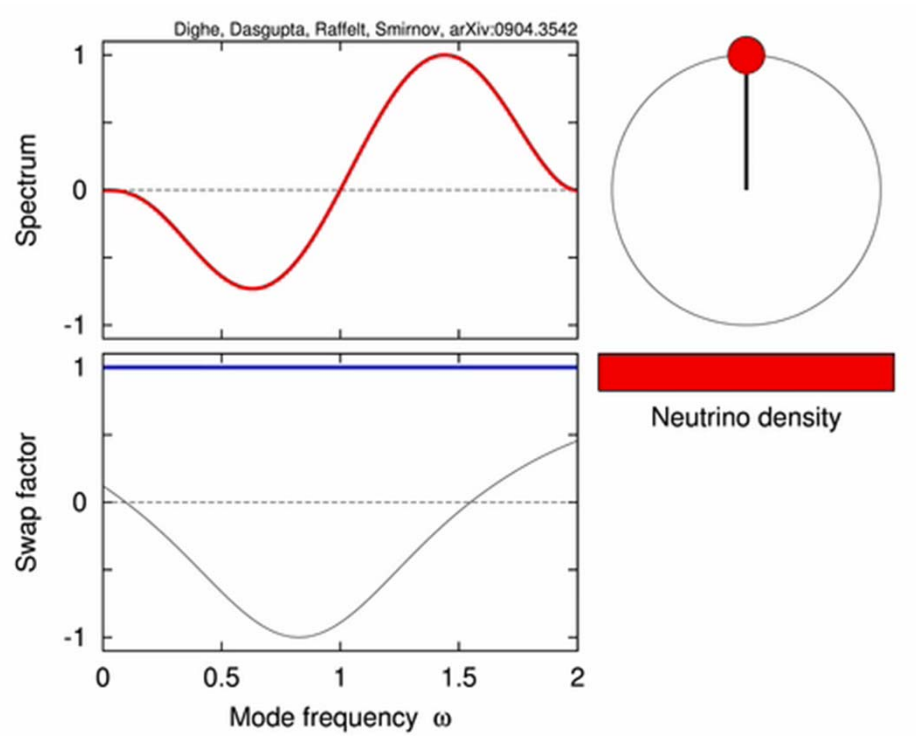
Single "negative" crossing
(potential energy at a minimum)



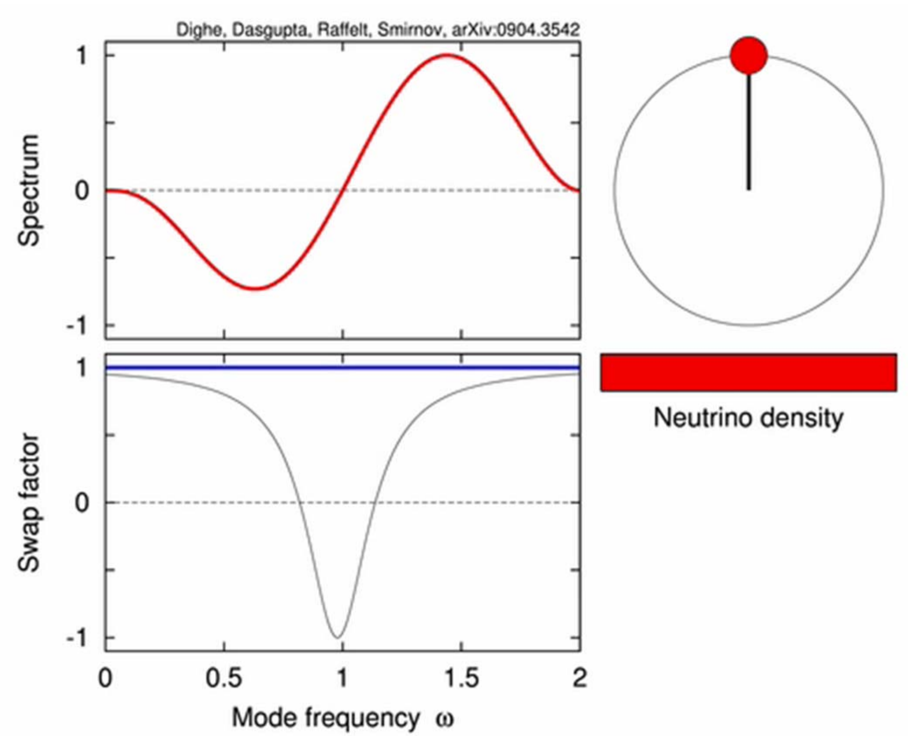
Dasgupta, Dighe, Raffelt & Smirnov, arXiv:0904.3542
For movies see http://www.mppmu.mpg.de/supernova/multisplits

Decreasing Neutrino Density

Certain initial neutrino density



Four times smaller initial neutrino density



Dasgupta, Dighe, Raffelt & Smirnov, arXiv:0904.3542

For movies see http://www.mppmu.mpg.de/supernova/multisplits

Multi-Angle Matter Effect

Precession equation in a homogeneous ensemble

$$\partial_t \mathbf{P}_{\omega,\mathbf{v}} = (\omega\mathbf{B} + \lambda\mathbf{L} + \mu\mathbf{P}) \times \mathbf{P}_{\omega,\mathbf{v}},$$ where $\lambda = \sqrt{2}G_{\rm F}N_e$ and $\mu = \sqrt{2}G_{\rm F}N_\nu$

Matter term is "achromatic", disappears in a rotating frame

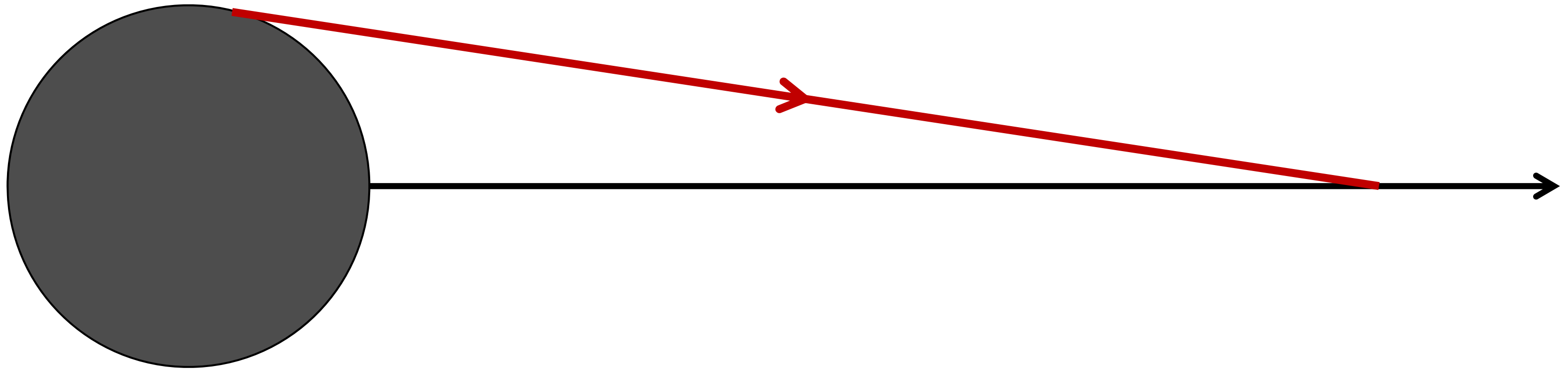
Neutrinos streaming from a SN core, evolution along radial direction

$$(\mathbf{v}\cdot\boldsymbol{\nabla}_r)\,\mathbf{P}_{\omega,\mathbf{v}} = (\omega\mathbf{B} + \lambda\mathbf{L} + \mu\mathbf{P}) \times \mathbf{P}_{\omega,\mathbf{v}}$$

Projected on the radial direction, oscillation pattern compressed:

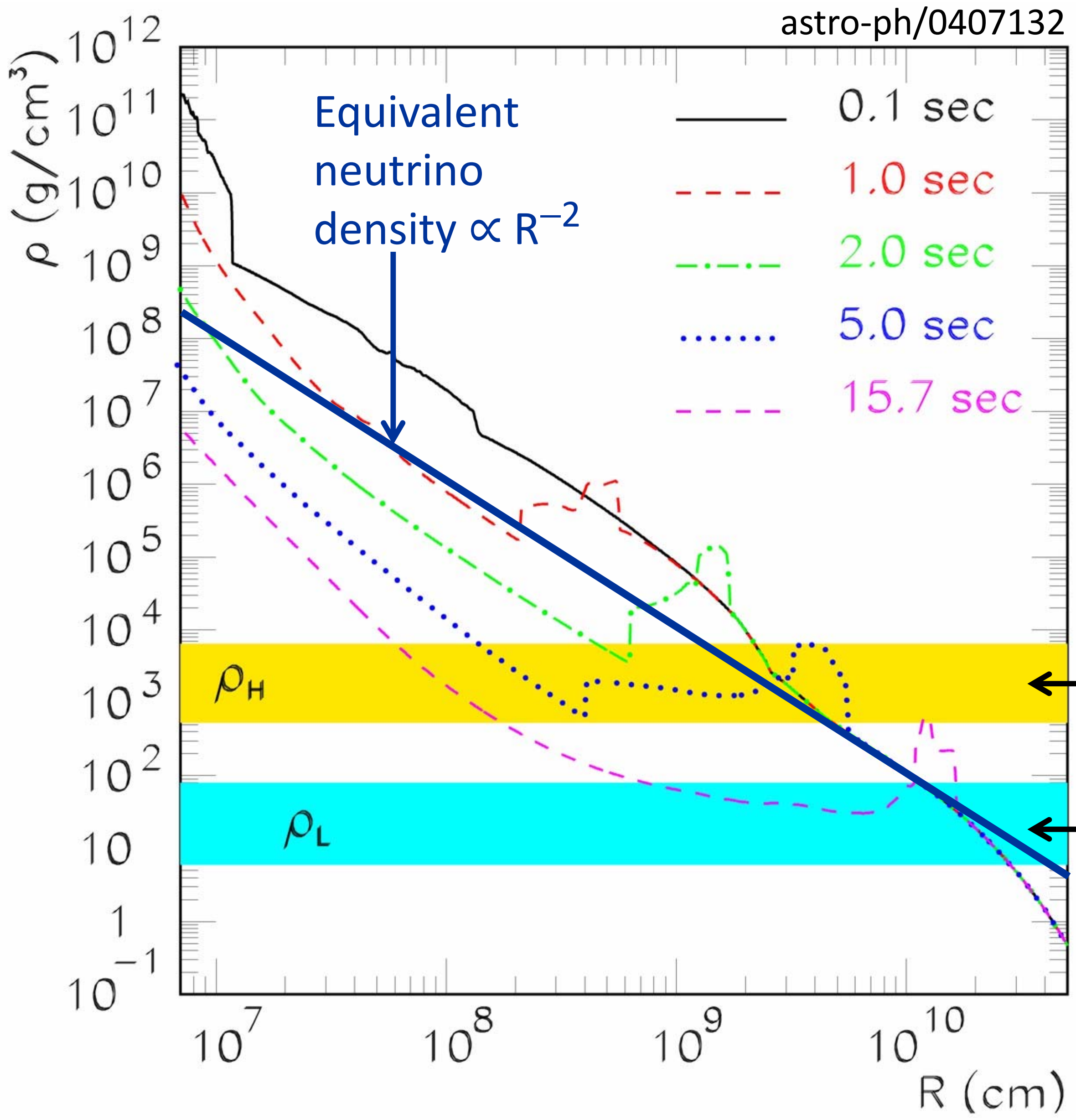
Accrues vacuum and matter phase faster than on radial trajectory

Matter effect can suppress collective conversion unless $N_\nu \gtrsim N_e$



Esteban-Pretel, Mirizzi, Pastor, Tomàs, Raffelt, Serpico & Sigl, arXiv:0807.0659

Snap Shots of Supernova Density Profiles



H-resonance (13 splitting)

L-resonance (12 splitting)

Accretion-phase luminosity

$$L_{\bar{\nu}_e} \sim 3 \times 10^{52}\,\mathrm{erg/s}$$

Corresponds to a neutrino number density of

$$3 \times 10^{33}\,\mathrm{cm}^{-3} \left(\frac{10\ \mathrm{km}}{R}\right)^2$$

General Stability Condition

Spin-precession equations of motion for modes with $\omega = \Delta m^2/2E$

$$\dot{\mathbf{P}}_\omega = \omega \mathbf{B} \times \mathbf{P}_\omega + \mu \mathbf{P} \times \mathbf{P}_\omega$$

Small-amplitude expansion: x-y-component described as complex number S (off-diagnonal ρ element), linearized EoMs

$$-\mathrm{i}\dot{S}_\omega = \omega S_\omega - \mu \int d\omega'\, g_{\omega'} S_{\omega'}$$

Fourier transform $S_\omega = Q_\omega e^{i\Omega t}$, with $\Omega = \gamma + i\kappa$ a complex frequency

$$(\omega - \Omega) Q_\omega = \mu \int d\omega'\, g_{\omega'} S_{\omega'}$$

Eigenfunction is $Q_\omega \propto (\omega - \Omega)^{-1}$ and eigenvalue $\Omega = \gamma + i\kappa$ is solution of

$$\mu^{-1} = \int d\omega \frac{g_\omega}{\omega - \Omega}$$

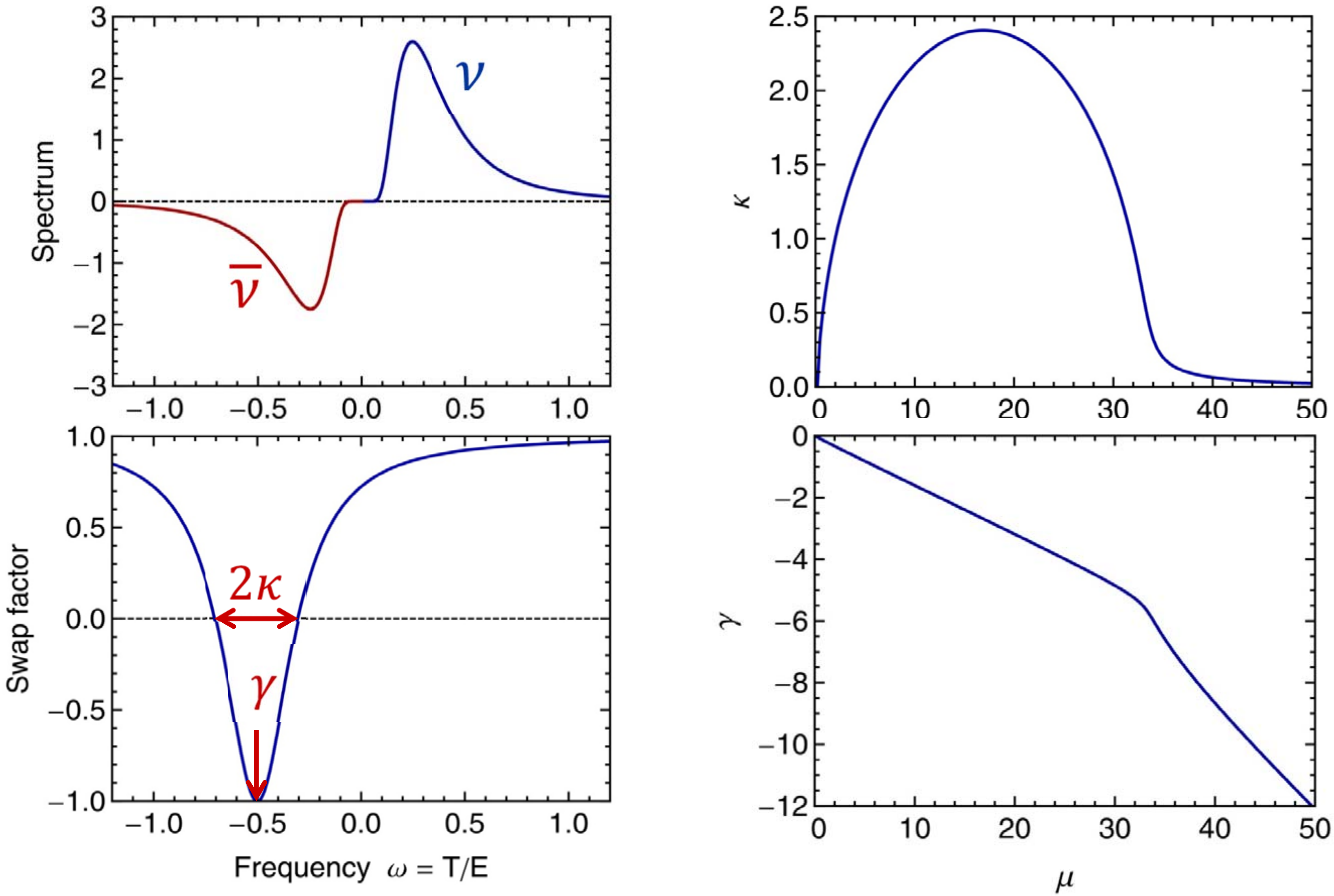
Instability occurs for

$$\kappa = \mathrm{Im}\, \Omega \neq 0$$

Exponential run-away solutions become pendulum for large amplitude.

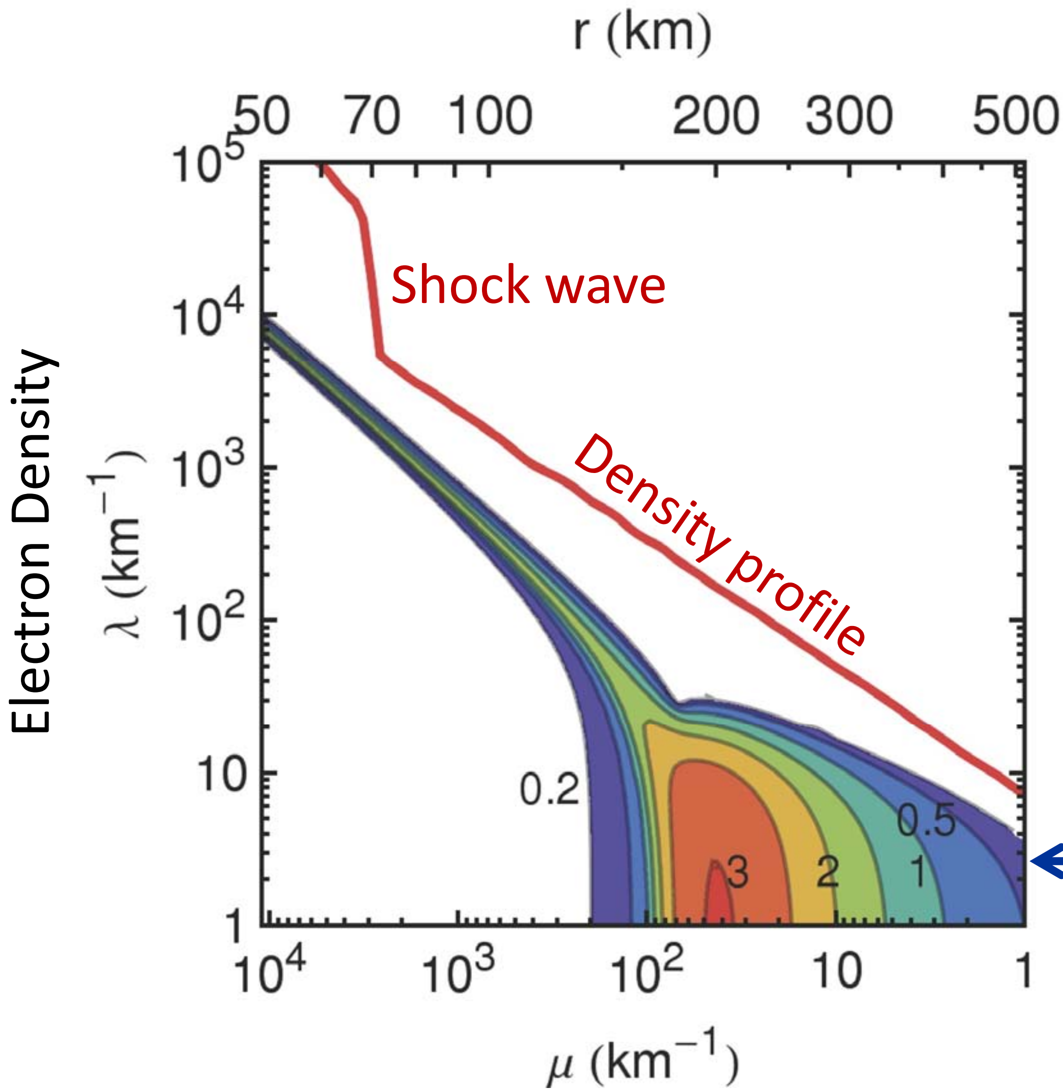
Banerjee, Dighe & Raffelt, arXiv:1107.2308

Stability of Fermi-Dirac Spectrum



Banerjee, Dighe & Raffelt, arXiv:1107.2308

Multi-Angle Multi-Energy Stability Analysis



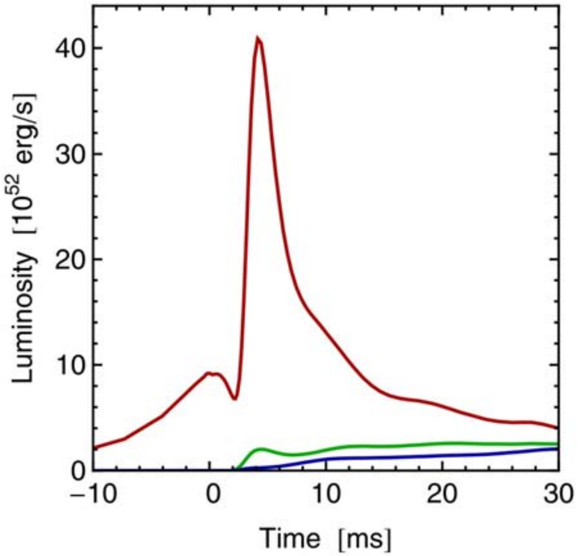
Tested 15 $M_\odot$ accretion-phase models (Garching) are stable against collective flavor conversion

Contours of growth rate $\kappa\ [\mathrm{km}^{-1}]$

Sarikas, Raffelt, Hüdepohl & Janka, arXiv:1109.3601

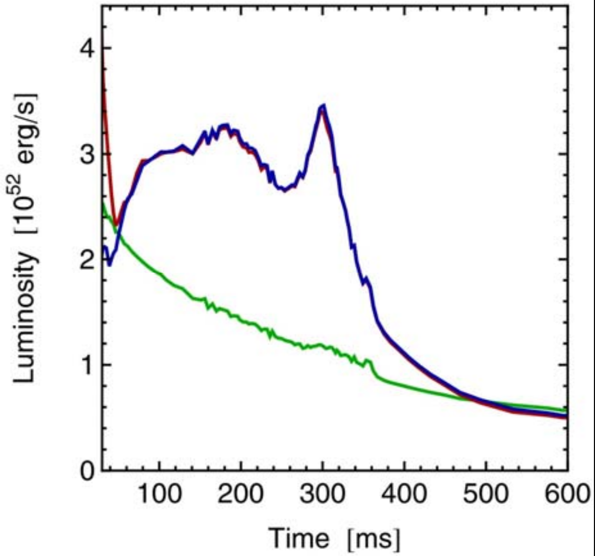
Three Phases of Neutrino Emission

Prompt ν_e burst



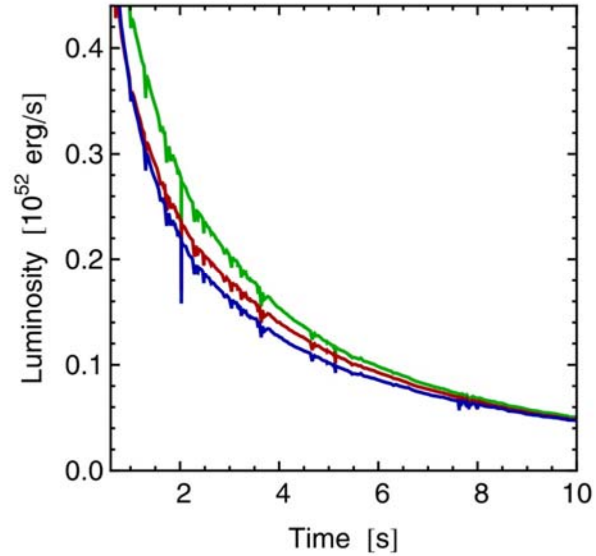
- Shock breakout
- De-leptonization of outer core layers

Accretion



- Shock stalls $\sim$ 150 km
- Neutrinos powered by infalling matter

Cooling



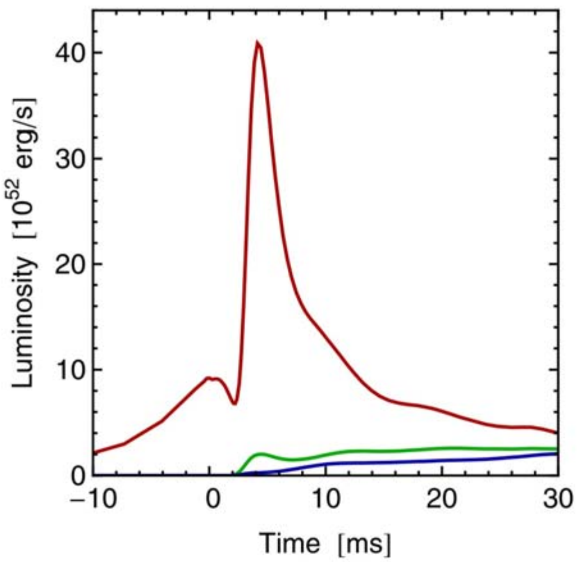
Cooling on neutrino diffusion time scale

- Spherically symmetric model (10.8 $M_\odot$) with Boltzmann neutrino transport
- Explosion manually triggered by enhanced CC interaction rate

Fischer et al. (Basel group), A&A 517:A80, 2010 [arxiv:0908.1871]

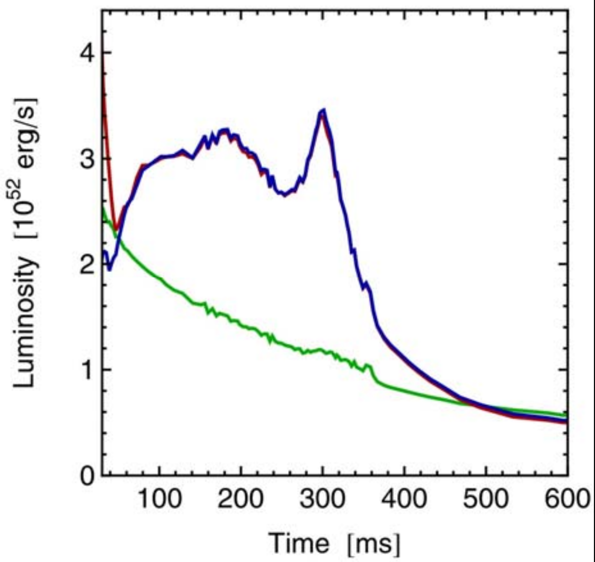
Three Phases of Neutrino Emission

Prompt ν_e burst



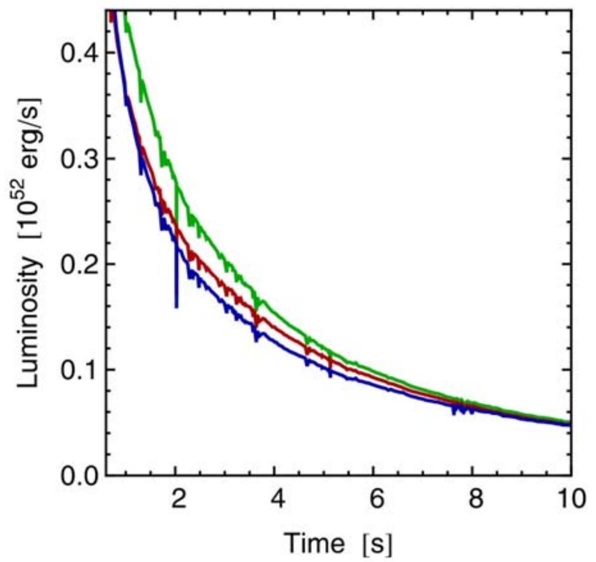
No large ν_e detector available

Accretion



Large fluxes and large $\bar{\nu}_e$-$\bar{\nu}_x$ flux differences

Cooling



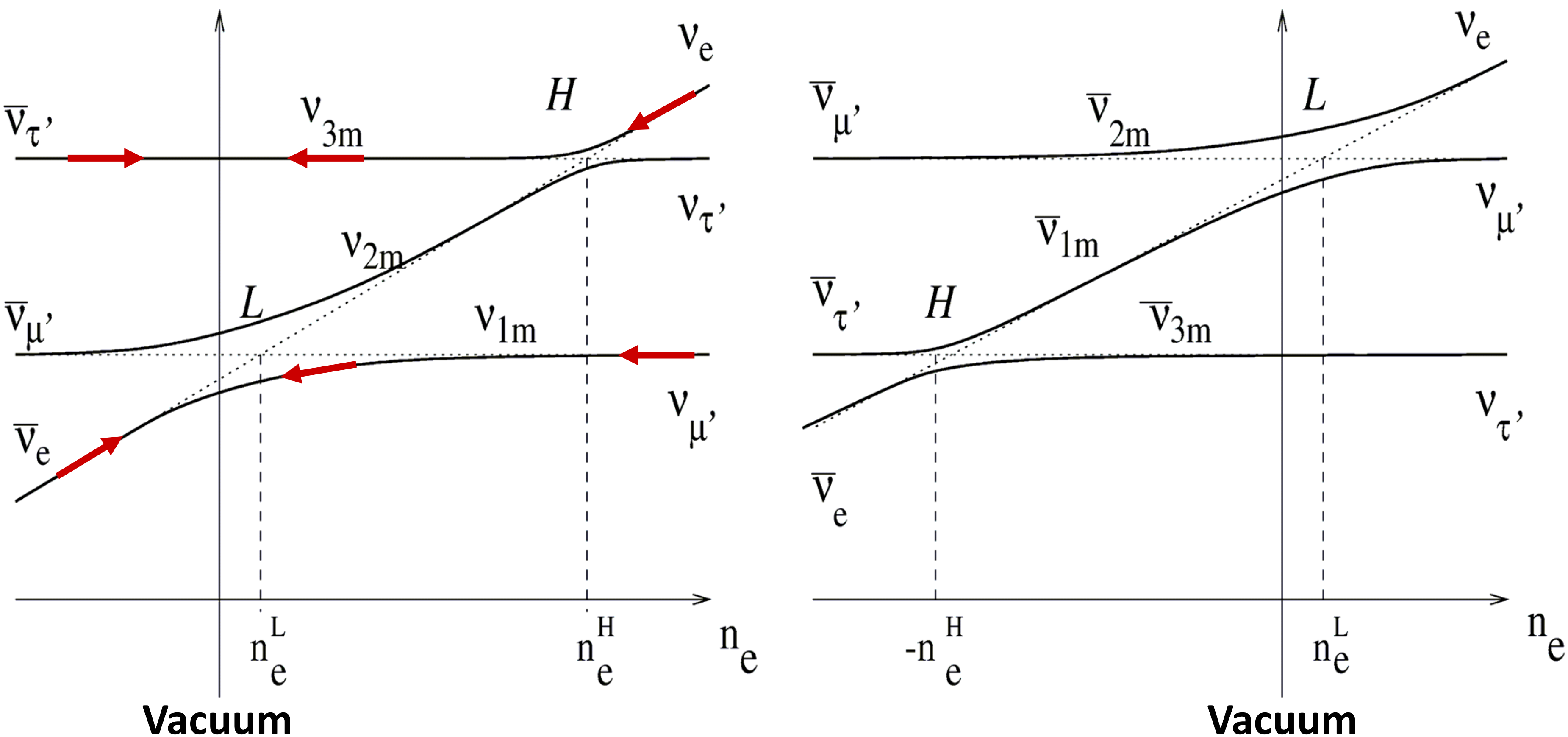
Smaller fluxes and small $\bar{\nu}_e$-$\bar{\nu}_x$ flux differences

- Spherically symmetric model (10.8 $M_\odot$) with Boltzmann neutrino transport
- Explosion manually triggered by enhanced CC interaction rate

Fischer et al. (Basel group), A&A 517:A80, 2010 [arxiv:0908.1871]

Level-Crossing Diagram in a Supernova Envelope

Normal mass hierarchy | **Inverted mass hierarchy**



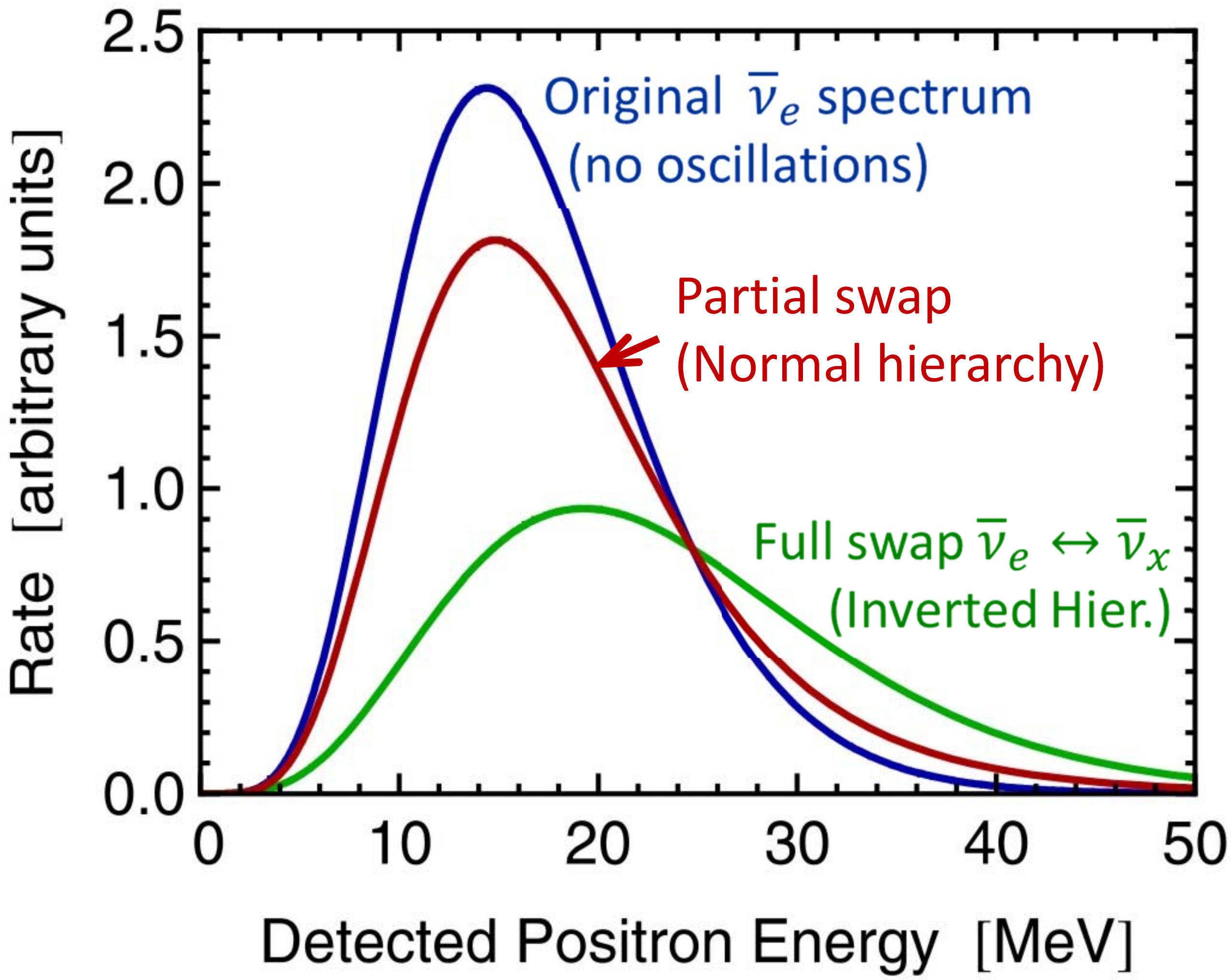
Dighe & Smirnov, Identifying the neutrino mass spectrum from a supernova neutrino burst, astro-ph/9907423

Signature of Flavor Oscillations (Accretion Phase)

	1-3-mixing scenarios		
	A	B	C
Mass ordering	Normal (NH)	Inverted (IH)	Any (NH/IH)
$\sin^2\theta_{13}$	$\gtrsim 10^{-3}$		$\lesssim 10^{-5}$
MSW conversion	adiabatic		non-adiabatic
ν_e survival prob.	0	$\sin^2\theta_{12} \approx 0.3$	$\sin^2\theta_{12} \approx 0.3$
$\bar{\nu}_e$ survival prob.	$\cos^2\theta_{12} \approx 0.7$	0	$\cos^2\theta_{12} \approx 0.7$
$\bar{\nu}_e$ Earth effects	Yes	No	Yes
	May distinguish mass ordering		

Assuming collective effects are not important during accretion phase (Chakraborty et al., arXiv:1105.1130, Sarikas et al. arXiv:1109.3601)

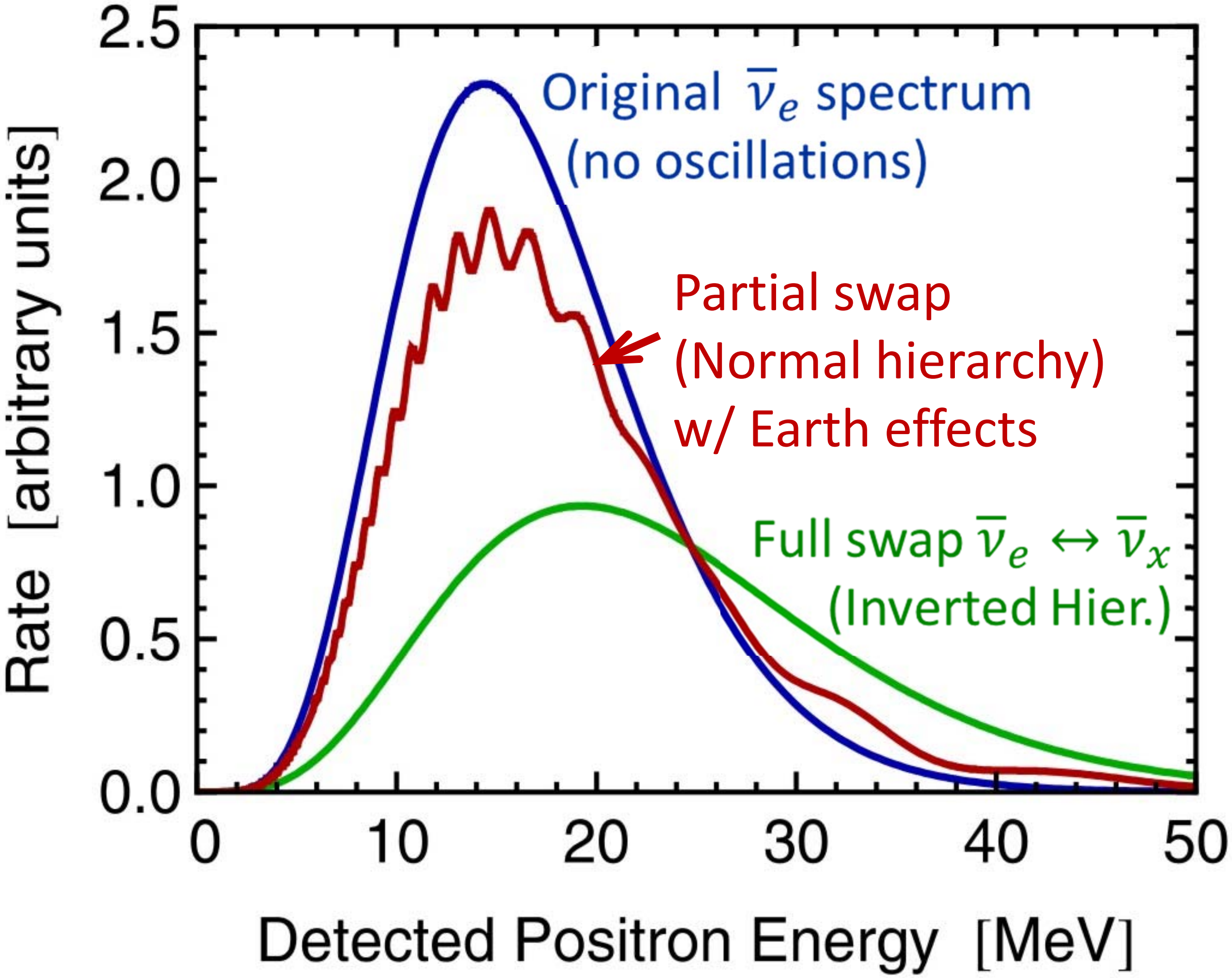
Oscillation of Supernova Anti-Neutrinos



Basel accretion phase model (10.8 $M_\odot$)

Detection spectrum by $\bar{\nu}_e + p \rightarrow n + e^+$ (water Cherenkov or scintillator detectors)

Oscillation of Supernova Anti-Neutrinos



Basel accretion phase model (10.8 $M_\odot$)

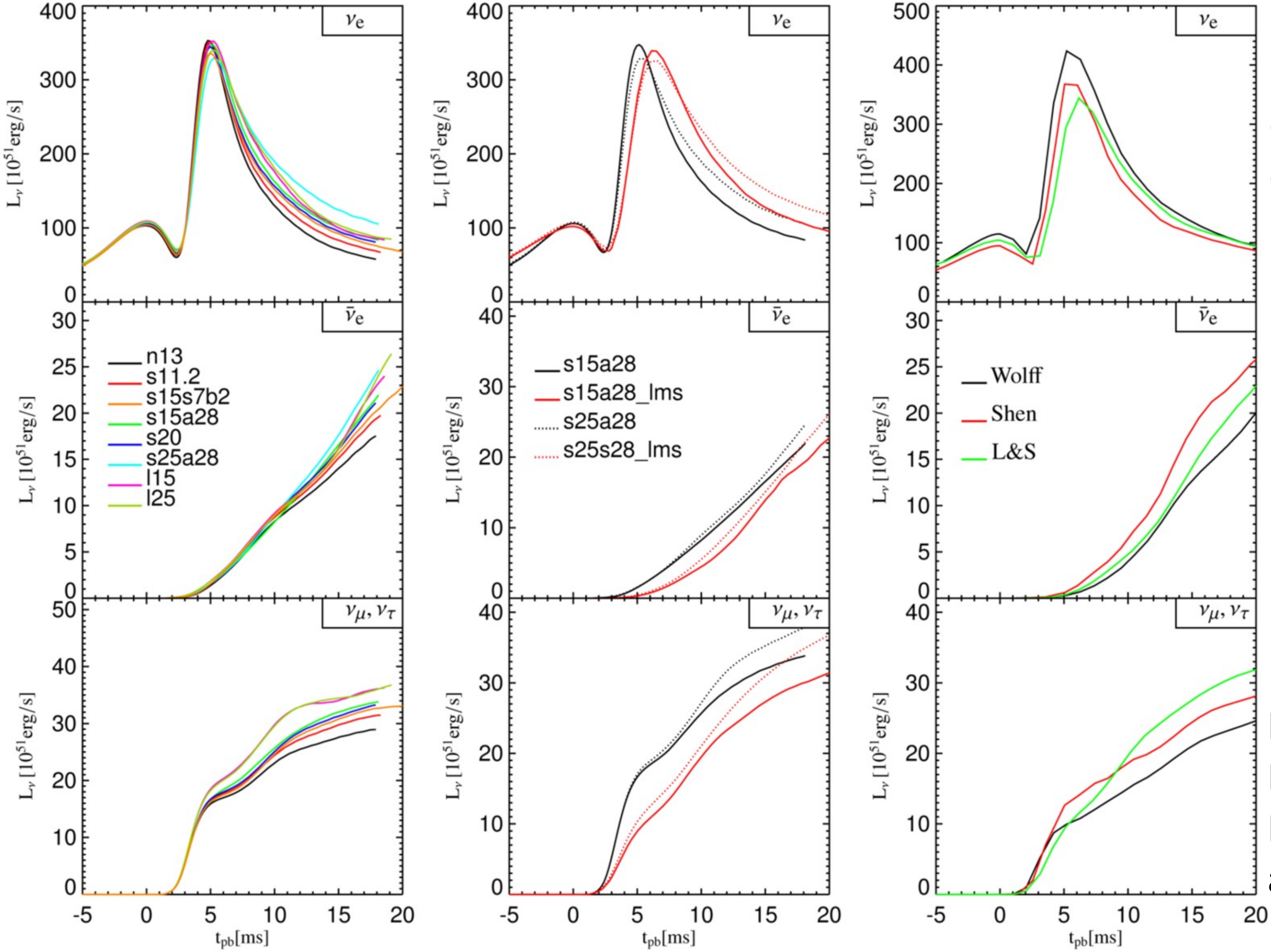
Detection spectrum by $\overline{\nu}_e + p \rightarrow n + e^+$ (water Cherenkov or scintillator detectors)

8000 km path length in Earth assumed

Detecting Earth effects requires good energy resolution
(Large scintillator detector, e.g. LENA, or megaton water Cherenkov)

Neutronization Burst as a Standard Candle

Different Mass | Neutrino Transport | Nuclear EoS

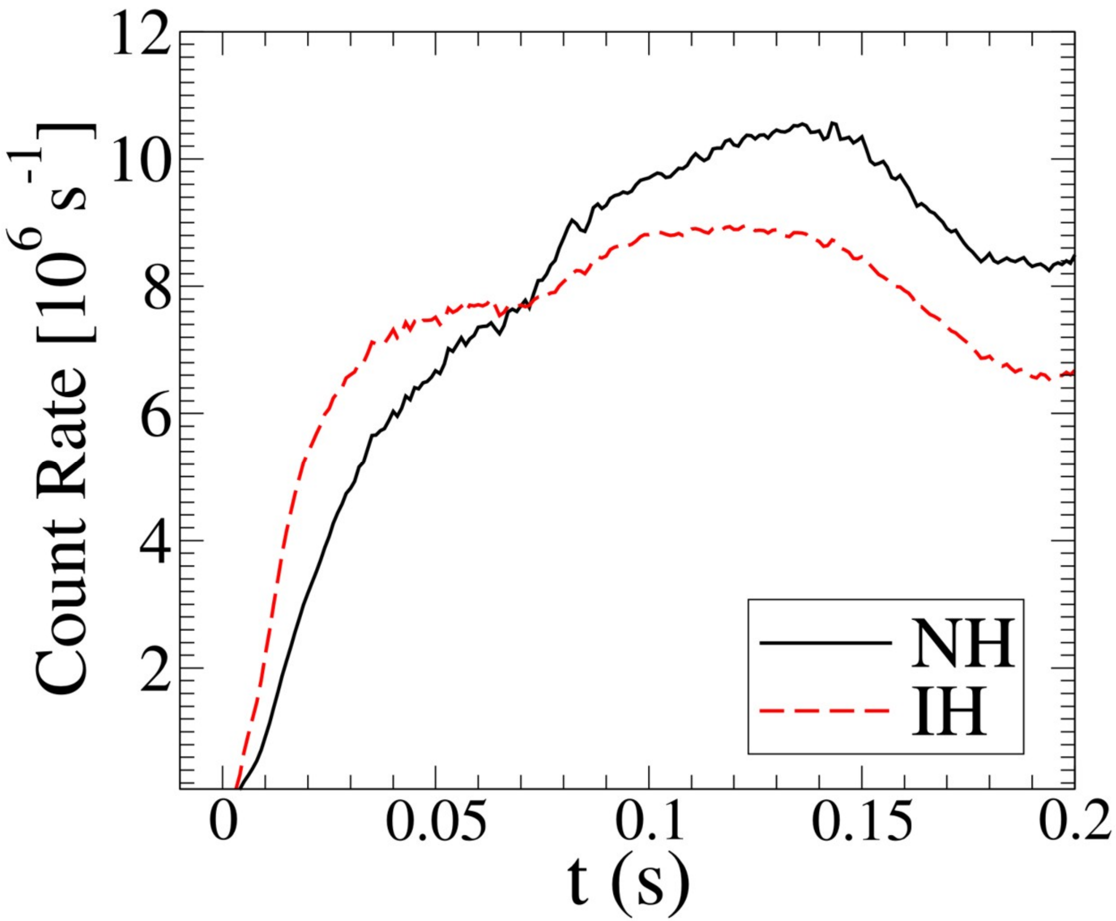


If mixing scenario is known, can determine SN distance (better than 5-10%)

Kachelriess, Tomàs, Buras, Janka, Marek & Rampp, astro-ph/0412082

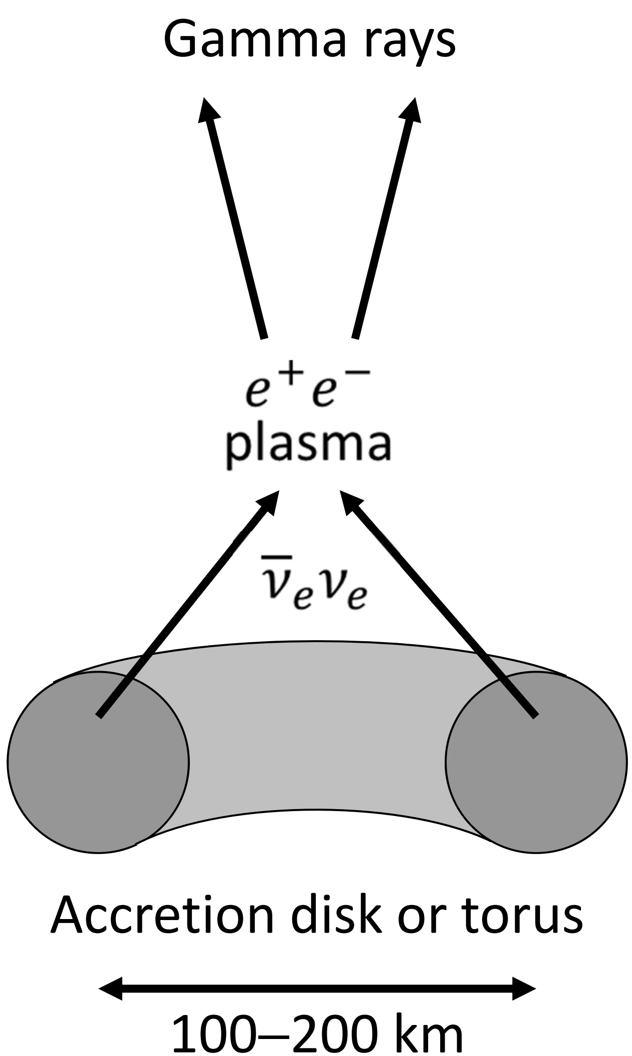
Rise Time as Hierarchy Discriminator

Rise time of counting rate in IceCube can distinguish hierarchy (for "large" θ_{13}), but depends on numerical model calibration



Chakraborty, Fischer, Hüdepohl, Janka, Mirizzi, Serpico, arXiv:1111.4483

Coalescing Neutron Stars and Short Gamma-Ray Bursts



- Annihilation rate strongly suppressed if $\nu_e\bar{\nu}_e$ pairs transform to $\nu_x\bar{\nu}_x$ pairs
- Collective effects important?

Density of torus relatively small:

- ν_μ and ν_τ not efficiently produced
- Large $\nu_e\bar{\nu}_e$ pair abundance