The Early Universe III: From Quark-Gluon Plasma to Neutrino Decoupling – pion +lepton

Yield equilibration of mesons and leptons

Jan Rafelski, prepared with **Inga Kuznetsova**

Department of Physics, Univ. of Arizona

Work supported by a grant from: the U.S. Department of Energy DE-FG02-04ER41318

Outline.

We study kinetic equations for reactions:

$$1+2 \rightarrow 3 \rightarrow 1+2$$

We established* speed of chemical equilibration of resonances;

- We show how the medium can affect particle decay rate;
- We consider reactions $\pi^0 \leftrightarrow \gamma + \gamma$, $(\rho \leftrightarrow \pi + \pi,\ \varphi \leftrightarrow K+K,)$

$$\pi^\pm \leftrightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu) \quad \text{and} \quad \mu^\pm \leftrightarrow e^\pm + \nu_e(\bar{\nu}_e) + \bar{\nu}_\mu(\nu_\mu)$$

- Investigate freeze-out from chemical equilibrium of decaying particles with universe expansion.

* I. Kuznetsova and J. Rafelski, Phys. Rev. C 79, 014903 (2009) [arXiv:0811.1409 [nucl-th]]

I. Kuznetsova and J. Rafelski, Phys.Lett. B 668, 105 (2008)

Time evolution equation

$\text{boson}_3 \leftrightarrow \text{boson}_1 + \text{boson}_2$

$$\text{boson}_3 \leftrightarrow \text{fermion}_1 + \overline{\text{fermion}_2}$$

$$\text{fermion}_3 \leftrightarrow \text{fermion}_1 + \text{boson}_2$$

$$\frac{1}{V}\frac{dN_3}{dt} = \sum_i \frac{dW^i_{1+2\to 3}}{dtdV} - \sum_j \frac{dW^j_{3\to 1+2}}{dtdV}$$

Particle 3 yield change = Particle 3 formation –Particle 3 decay

$$m_3 > m_1 + m_2$$

Quantum reaction rates

Lorentz invariant rates are:

$$\frac{dW^i_{3\to 1+2}}{dt dV} = \frac{1}{8(2\pi)^5}\int\frac{d^3p_3}{E_3}\int\frac{d^3p_2}{E_2}\int\frac{d^3p_1}{E_1}\delta(p_1+p_2-p_3)\sum_{spin}\left|\left\langle p_3\right|M\left|p_1p_2\right\rangle\right|^2\times$$

$$\times(f_1\pm 1)(f_2\pm 1)f_3$$

$$\frac{dW^i_{1+2\to 3}}{dt dV} = \frac{1}{(2\pi)^5}\int\frac{d^3p_3}{E_3}\int\frac{d^3p_2}{E_2}\int\frac{d^3p_1}{E_1}f_1f_2\delta(p_1+p_2-p_3)\sum_{spin}\left|\left\langle p_1p_2\right|M\left|p_3\right\rangle\right|^2\times$$

$$\times(f_3\pm 1)f_2f_1$$

Fugacity definition

$$f_j = \frac{1}{\Upsilon_j^{-1}(t)\exp((u\cdot p_j)\beta(t))-1}, \text{ meson (bose)};$$

$$f_i = \frac{1}{\Upsilon_i^{-1}(t)\exp((u\cdot p_i)\beta(t))+1}, \text{baryons (fermi)};$$

$u\cdot p_i = E_i$ for $u = (1,\vec{0})$ in the rest frame of heat bath

We assume chemical potential μ=0, particle-antiparticle symmetry

Detailed balance condition

$$Y_1 Y_2 \frac{dW_{3\to 1+2}}{dtdV} = Y_3 \frac{dW_{1+2\to 3}}{dtdV} \equiv R$$

Equilibrium condition:

$$Y_3 = Y_1 Y_2 (=1)$$

Detailed balance conditions

$$\frac{1}{Y_3 Y_4}\frac{dW_{43\to 12}}{dtdV} = \frac{1}{Y_1 Y_2}\frac{dW_{12\to 34}}{dtdV} \equiv R$$

and

$$\frac{1}{Y_3}\frac{dW_{3\to 1+2}}{dtdV} = \frac{1}{Y_1 Y_2}\frac{dW_{1+2\to 3}}{dtdV} \equiv R$$

Means that in Equilibrium: $Y_1 Y_2 = Y_3 Y_4 (=1)$ and $Y_3 = Y_1 Y_2 (=1)$

$$\text{Reactions}: 3 \leftrightarrow 1 + 2$$

$$\text{boson}_3 \leftrightarrow \text{fermion}_1 + \overline{\text{fermion}_2}$$

$$\text{fermion}_3 \leftrightarrow \text{fermion}_1 + \text{boson}_2$$

$$\frac{1}{V}\frac{dN_3}{dt} = \sum_i \frac{dW^i_{1+2\to 3}}{dtdV} - \sum_j \frac{dW^j_{3\to 1+2}}{dtdV}$$

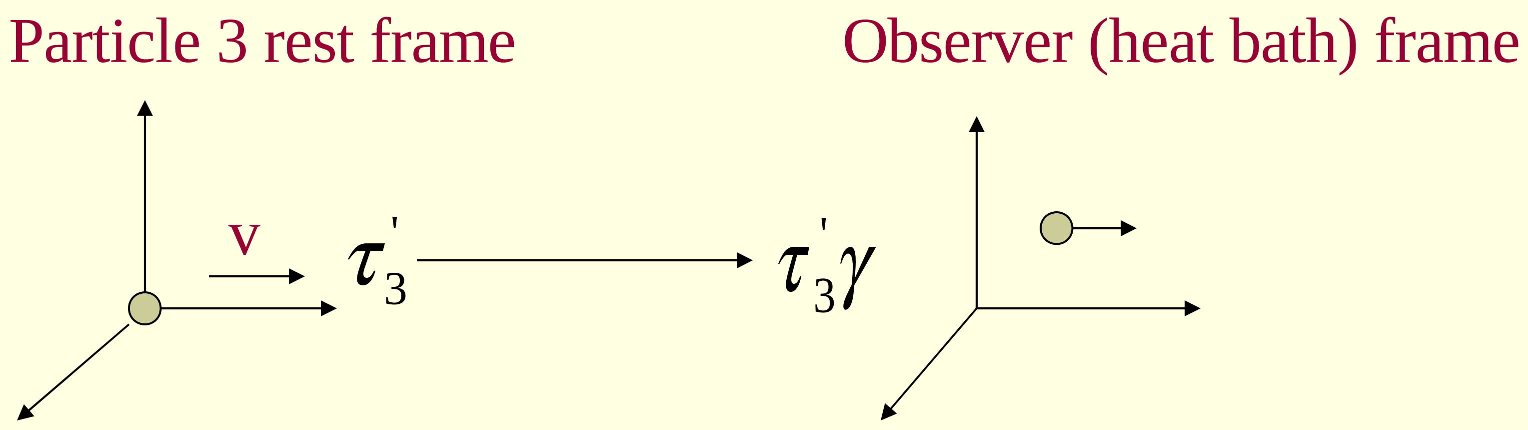
Particle 3 yield change = Particle 3 formation –Particle 3 decay

$$m_3 > m_1 + m_2$$

Time evolution equation

$\text{boson}_3 \leftrightarrow \text{boson}_1 + \text{boson}_2$

Calculation of particle 3 decay / production rate



$$\frac{dW_{3\to 1+2}}{dt\,dV} = \frac{g_3}{(2\pi)^3}\int d^3p_3\, f_{b,f}(\Upsilon_3, p_3)\frac{m_3}{E_3}\frac{1}{\tau_3'} = n_3\left\langle \frac{1}{\gamma\tau_3'}\right\rangle$$

τ_3' is particle 3 lifespan in its rest frame in medium of particles 1 and 2

$$\Upsilon_1\Upsilon_2\frac{dW_{3\to 1+2}}{dt\,dV} = \Upsilon_3\frac{dW_{1+2\to 3}}{dt\,dV} \equiv R$$

Calculation of particle 3 decay / production rate

Particle 3 decay / production rate in a medium can be calculated, using particle 3 decay time in the this particle rest frame.

$$\frac{dW_{3\leftrightarrow 1+2}}{dtdV} = \frac{g_3}{(2\pi)^3}\int d^3p_3 f_{b,f}(\Upsilon_3, p_3)\frac{m_3}{E_3}\frac{1}{\tau_3'} \qquad \frac{1}{\tau_3'} = \frac{1}{\tau_3^0}\frac{\exp(E_3/T)}{2}\Phi_3(p_3)$$

τ_3' is particle 3 decay time in its rest frame in medium of particles 1 and 2

$$\Phi_3(p_3) = \int_{-1}^{1} d\zeta \frac{\Upsilon_1^{-1}}{\Upsilon_1^{-1}\exp(a_1 - b\zeta) \pm 1}\frac{\Upsilon_2^{-1}}{\Upsilon_2^{-1}\exp(a_2 + b\zeta) \pm 1} = \frac{1}{e^{a_1+a_2} \pm \Upsilon_1\Upsilon_2}\ln\frac{(e^{-a_2} \pm \Upsilon_2 e^{b})(e^{a_1} \pm \Upsilon_1 e^{-b})}{(e^{-a_2} \pm \Upsilon_2 e^{-b})(e^{a_1} \pm \Upsilon_1 e^{b})}$$

τ_3^0 is particle 3 decay time in this particle rest frame in vacuum

$$a_1 = \frac{E_1E_3}{m_3T}, \quad a_2 = \frac{E_2E_3}{m_3T}, \quad b = \frac{pp_3}{m_3T}, \quad \zeta = \cos(\vec{p} \wedge \vec{p}_3)$$

$$E_{1,2} = \frac{m_3^2 \pm (m_1^2 - m_2^2)}{2m_3}, \quad \mathrm{p} = \mathrm{E}_{1,2}^2 - m_{1,2}^2$$ are in particle 3 rest frame

Fugacity Y(t) computation

Relaxation time: $$\tau_3^i = \frac{1}{V}\frac{dN_3}{d\Upsilon_3}\Upsilon_3 \Bigg/ \frac{dW^i_{3\to 1+2}}{dtdV}$$

$$\frac{d\Upsilon_3}{d\tau} = \sum_i \Upsilon_1^i \Upsilon_2^i \frac{1}{\tau_3^i} + \Upsilon_3\left(\frac{1}{\tau_T} + \frac{1}{\tau_S} - \sum_j \frac{1}{\tau_3^j}\right)$$

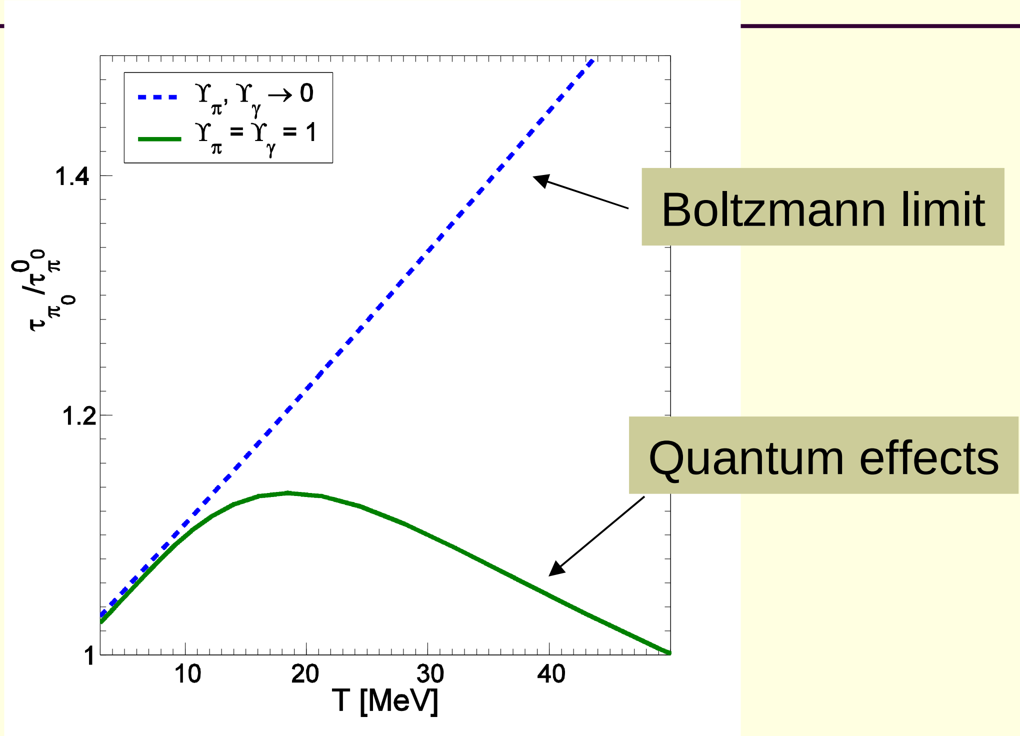
τ is time in fluid element co-moving frame.

$$\frac{1}{\tau_T} = \frac{d\ln(x_\Delta^2 K_2(x_\Delta))}{dT}\frac{dT}{d\tau}, \qquad \frac{1}{\tau_S} = \frac{d\ln(VT^3)}{d\tau} \approx 0$$

the entropy is conserved

The relaxation times for the decay and backward reactions are the same.

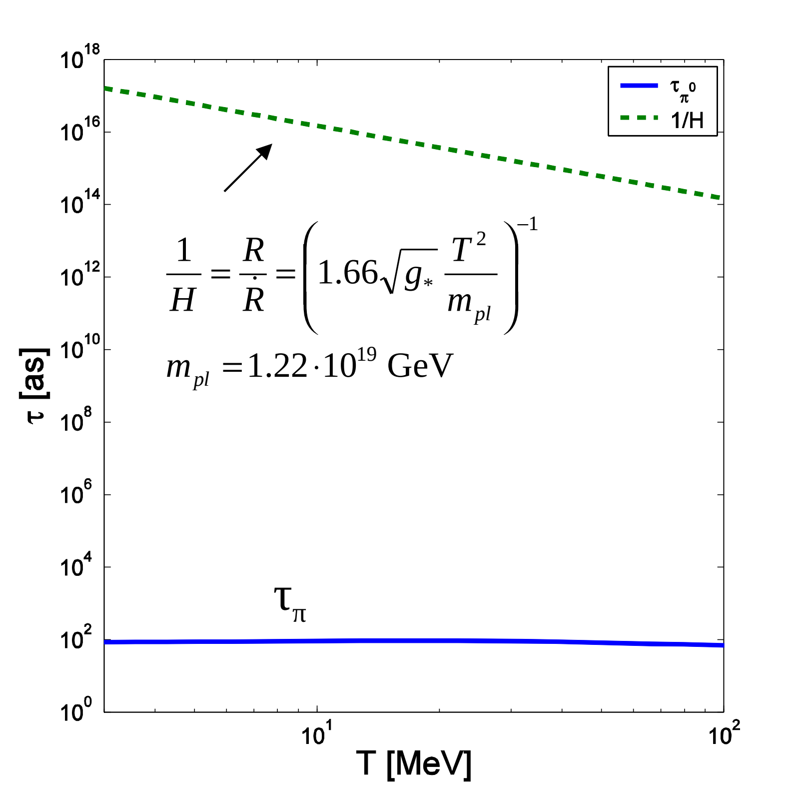
Example: $\gamma+\gamma \to \pi_0$

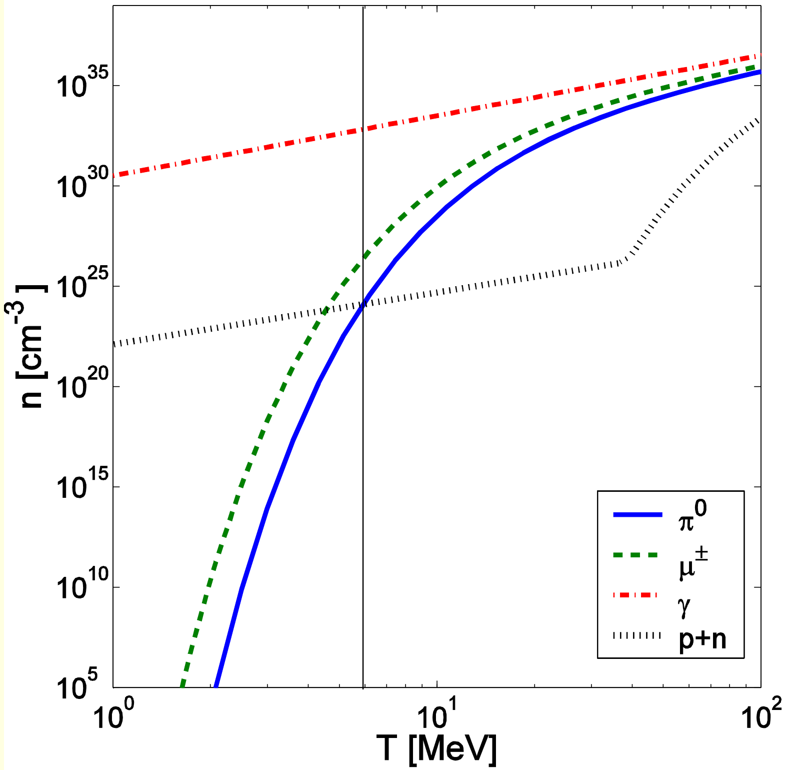


In relativistic Boltzmann limit (no medium effects) in rest of heat bath frame:

$$\tau_{\pi^0} \approx \frac{\tau^0_{\pi^0}}{\langle 1/\gamma \rangle} = \tau^0_{\pi^0} \frac{K_2(m_{\pi^0}/T)}{K_1(m_{\pi^0}/T)} \qquad \tau^0_{\pi^0} \approx 8.4 \cdot 10^{-17}\ \mathrm{s}$$

Early Universe

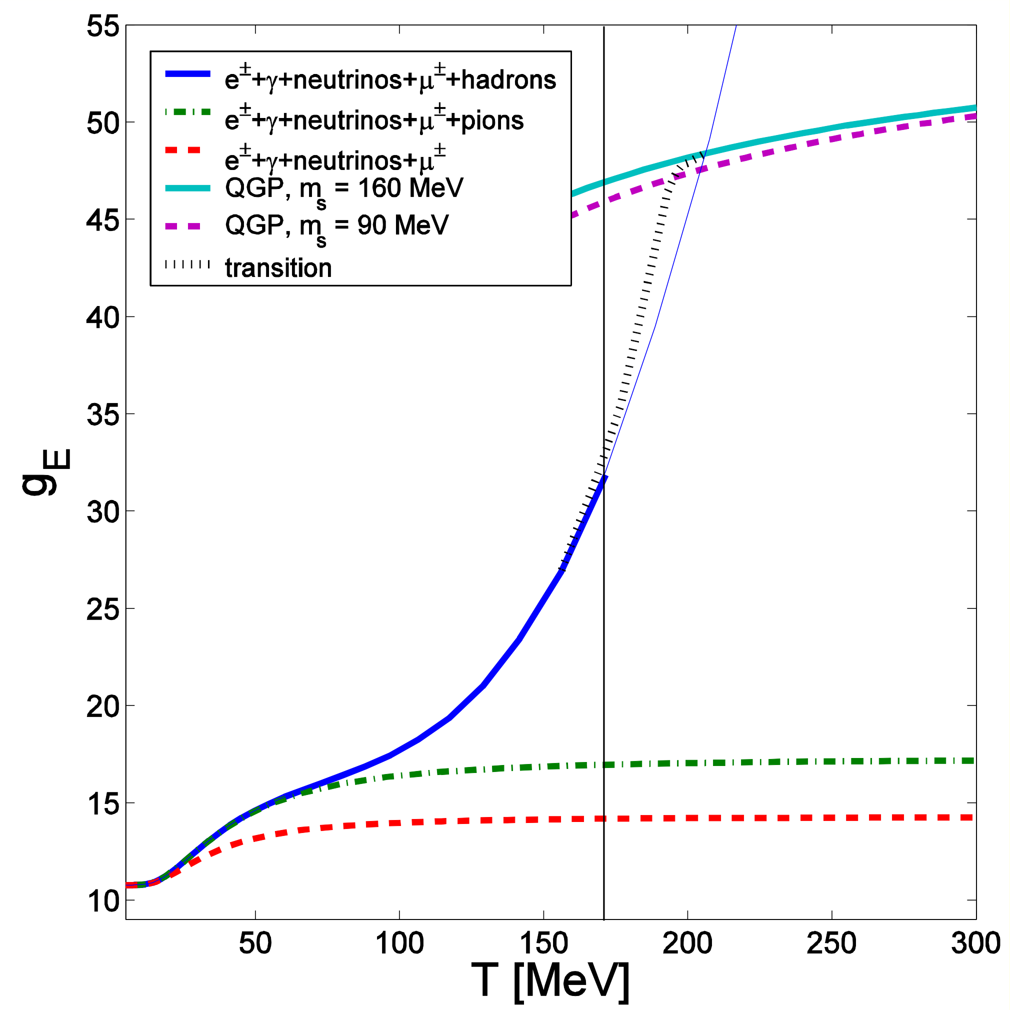




π^0 stay in chemical equilibrium; $n_\pi > n_N$ until T on the order few MeV

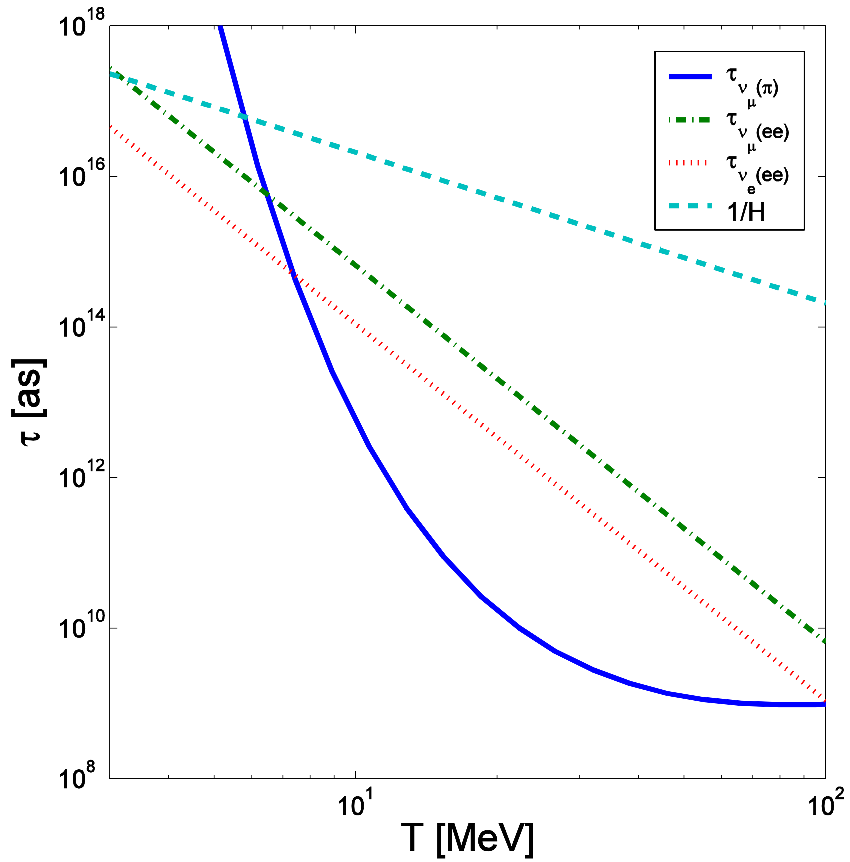
π^0 can intensively participate in reaction with nucleons

Effective degeneracy in the Universe



$$g_E = \frac{\varepsilon}{\sigma T^4}$$

Lepton equilibration



$$\pi^{\pm} \leftrightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$$

$\tau_0 = 2.6\ 10^{-8}$ s

$\tau_\pi \sim \tau_0\, n_\nu / n_\pi$

$$e^{+} + e^{-} = \nu_{e,\mu} + \bar{\nu}_{e,\mu}$$

$$\tau_{\nu_e} = (0.1 G_F^2 T^5)^{-1};\quad \tau_{\nu_\mu} = (0.6 G_F^2 T^5)^{-1};$$

$$G_F = 1.166 \cdot 10^{-5}\ \mathrm{GeV}^{-2}$$

$\pi^{\pm}$ has dominant influence on neutrino equilibration between 7-100 MeV

This can be important if $\mu_\nu > 0$, neutrinos freeze out earlier.

Muon equilibration

- $\gamma + \gamma \leftrightarrow \mu^+ + \mu^-$
- $e^+ + e^- \leftrightarrow \mu^+ + \mu^-$

Muon lifespan is $2.197 \cdot 10^{-6}$ s $\quad \mu^{\pm} \leftrightarrow e^{\pm} + \nu_e(\bar{\nu}_e) + \bar{\nu}_\mu(\nu_\mu)$

$$\Upsilon_\mu^{-2} \frac{dW_{\mu^+\mu^- \to \gamma\gamma(e^+e^-)}}{dtdV} = \Upsilon_{\gamma(e)}^{-2} \frac{dW_{\gamma\gamma(e^+e^-) \to \mu^+\mu^-}}{dtdV} \equiv R_{\gamma\gamma(e^+e^-) \leftrightarrow \mu^+\mu^-}$$

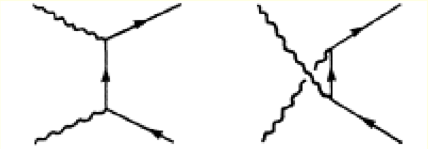
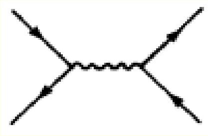
Rate for muon production

- Equation for $R_{\gamma\gamma(e^+e^-)\leftrightarrow\mu^+\mu^-}$ is similar to that for strangeness production in QGP, see review *Koch et al, Physics Reports, 142, 4 (1986)* and *I. Kuznetsova and J. Rafelski, Phys.Lett. B 668, 105 (2008)*

$$R_{\gamma\gamma(e^+e^-)\leftrightarrow\mu^+\mu^-} = \frac{\Upsilon_{\gamma(e)}^{-2}\Upsilon_{\mu}^{-2}}{8(2\pi)^6(1+I)}\int\frac{d^3p_{\mu^+}}{2E_{\mu^+}}f_{\mu^+}\int\frac{d^3p_{\mu^-}}{2E_{\mu^-}}f_{\mu^-}\int\frac{d^3p_1}{2E_1(2\pi)^3}f_1\int\frac{d^3p_2}{2E_2(2\pi)^3}f_2\times$$

$$\times(2\pi)^4\delta(p_1+p_2-p_{\mu^+}-p_{\mu^-})\sum_{spin}\left|\left\langle p_1p_2\middle|M\middle|p_{\mu^+}p_{\mu^-}\right\rangle\right|^2 e^{\frac{u\cdot(p_1+p_2)}{T}}$$

Matrix elements for muons production near threshold

$\gamma+\gamma \to \mu^+ + \mu^-$:  $e^+ + e^- \to \mu^+ + \mu^-$ 

- Near threshold we find:

$$\left|\left\langle p_{1\gamma} p_{2\gamma} \middle| M \middle| p_{\mu^+} p_{\mu^-} \right\rangle\right|^2 = 64\pi^2\alpha^2; \quad \left|\left\langle p_{e^+} p_{e^-} \middle| M \middle| p_{\mu^+} p_{\mu^-} \right\rangle\right|^2 = 32\pi^2\alpha^2$$

The reaction $e^+ + e^- \to \mu^+ + \mu^-$ involves a single photon, and thus it is more constrained (by factor 2) compared to the photon fusion, which is governed by two Compton type Feynman diagrams. Indistinguishability of the two photons introduces an additional factor 1/2, so that both reactions differ only by the difference in the quantum Bose and Fermi distributions:

$$R_{\gamma\gamma\to\mu^+\mu^-} \approx R_{e^+e^-\to\mu^+\mu^-}, \text{ for } T << m_\mu$$

Time evolution of μ$^{\pm}$

$$\frac{1}{V}\frac{dN_{\mu}}{dt} = \left(\Upsilon_{\gamma}^{2} - \Upsilon_{\mu}^{2}\right) R_{\gamma\gamma\to\mu^{+}\mu^{-}} + \left(\Upsilon_{e}^{2} - \Upsilon_{\mu}^{2}\right) R_{e^{+}e^{-}\to\mu^{+}\mu^{-}}$$

$\Upsilon_{\mu} = \Upsilon_{e} = \Upsilon_{\gamma}$ (=1) is chemical equilibrium

Muon production relaxation time is: $\tau_{\mu} = \frac{1}{2}\frac{dn_{\mu}/d\Upsilon_{\mu}}{(R_{\gamma\gamma\leftrightarrow\mu^{+}\mu^{-}} + R_{e^{+}e^{-}\leftrightarrow\mu^{+}\mu^{-}})}$

$\pi^{\pm}$ equilibration

- $\pi^0 + \pi^0 \leftrightarrow \pi^+ + \pi^-$
- $\gamma + \gamma \leftrightarrow \pi^+ + \pi^-$
- $e^+ + e^- \leftrightarrow \pi^+ + \pi^-$

$\pi^{\pm}$ lifespan is $2.6 \cdot 10^{-8}$ s

$$\pi^{\pm} \leftrightarrow \mu^{\pm} + \nu_{\mu} (\bar{\nu}_{\mu})$$

Production rate, using cross section for two to two body processes:

$$R_{12 \rightarrow \pi^+\pi^-} = \frac{g_1 g_2}{32\pi^4} \frac{T}{1+I} \int_{s_{th}}^{\infty} ds\, \sigma(s) \frac{\lambda_2(s)}{\sqrt{s}} K_1(\sqrt{s}/T),$$

where $s=(p_1 + p_2)^2$, $\lambda_2(s) = (s-(m_1+m_2)^2)(s-(m_1-m_2)^2)$, m_1 and m_2, g_1 and g_2, Y_1 and Y_2 are masses, degeneracy and fugacities of initial interacting particles.

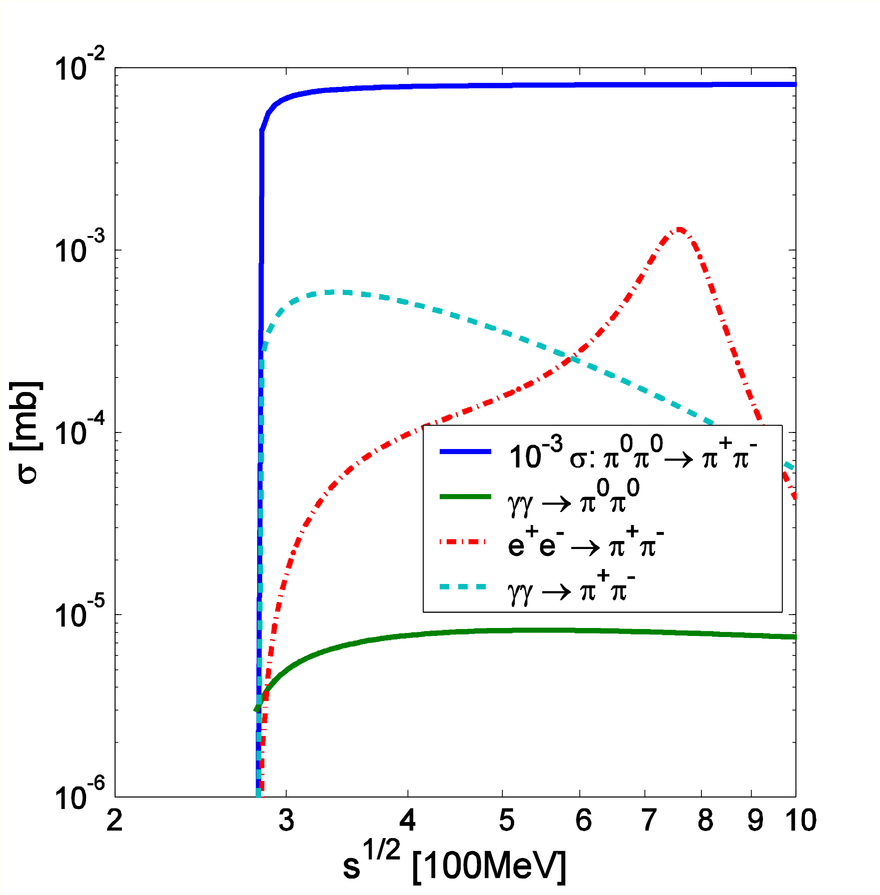
$\pi^{\pm}$ evolution equation

$$\frac{1}{V}\frac{dN_{\pi^{\pm}}}{dt} = \left(\Upsilon_{\pi^0}^2 - \Upsilon_{\pi^{\pm}}^2\right) R_{\pi^0\pi^0 \leftrightarrow \pi^+\pi^-} + \left(\Upsilon_{\gamma}^2 - \Upsilon_{\pi^{\pm}}^2\right) R_{\gamma\gamma \leftrightarrow \pi^+\pi^-} + \left(\Upsilon_{e}^2 - \Upsilon_{\pi^{\pm}}^2\right) R_{e^+e^- \leftrightarrow \pi^+\pi^-} + \left(\Upsilon_{\mu^{\pm}}\Upsilon_{\nu} - \Upsilon_{\pi^{\pm}}\right) R_{\pi^{\pm} \leftrightarrow \mu^{\pm} + \nu(\bar{\nu})}$$

Relaxation times are: $\tau_{12 \to \pi^+\pi^-} = \frac{1}{2}\frac{dn_{\pi^{\pm}} / d\Upsilon_{\pi^{\pm}}}{R_{12 \leftrightarrow \pi^+\pi^-}}$

$$\tau_{\pi^{\pm} \leftrightarrow \mu^{\pm} + \nu(\bar{\nu})} = \frac{dn_{\pi^{\pm}} / d\Upsilon_{\pi^{\pm}}}{R_{\pi^{\pm} \leftrightarrow \mu^{\pm} + \nu(\bar{\nu})}}$$

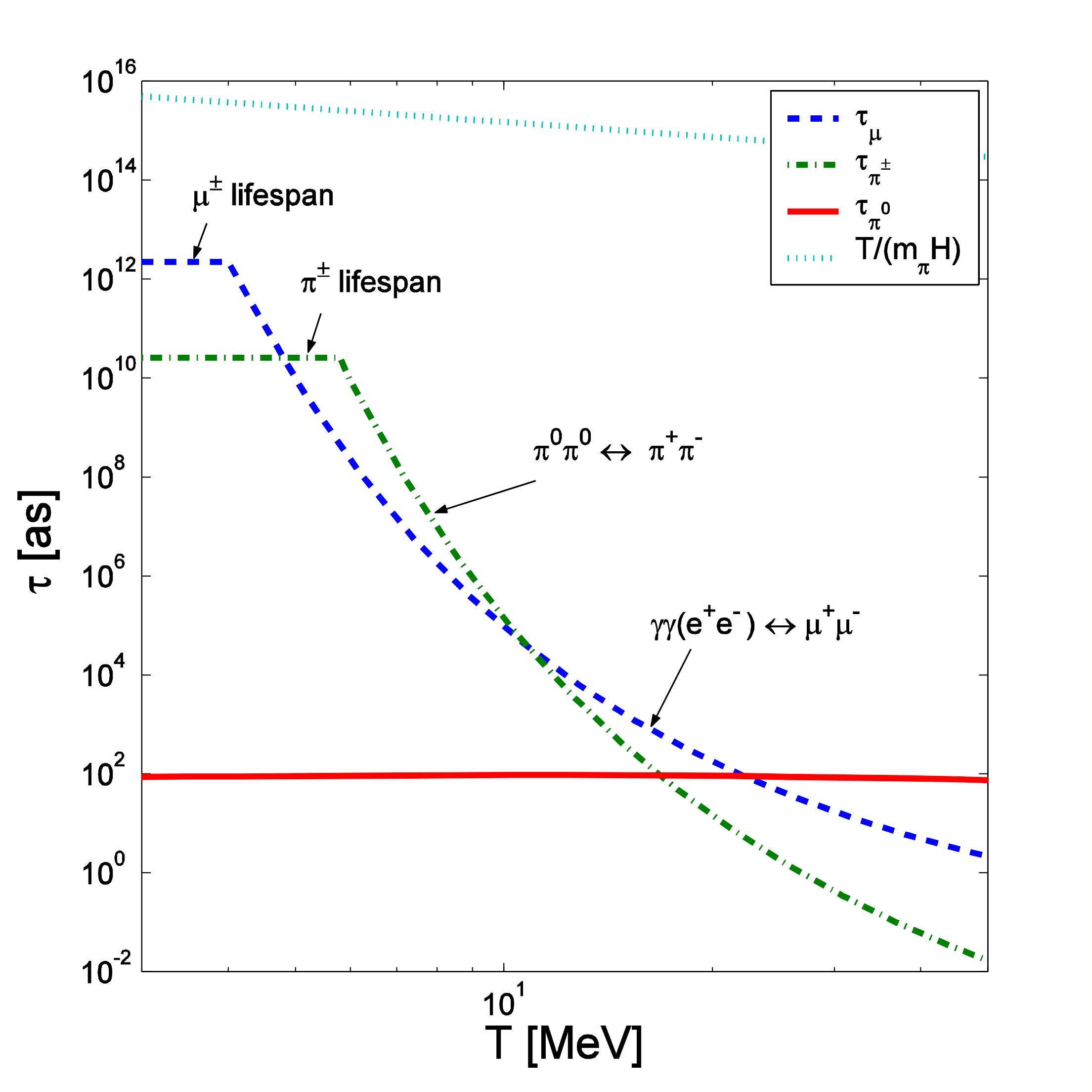
Relaxation times and cross sections for charged pions production



Cross sections for pions production are from:

R.Kaminski, et al, arXiv:0710.1150 [hep-ph];
H.Terazawa, Phys. Rev. D, 954 (1995);
G.J.Gounaris and J.J.Sakurai, Phys. Rev. Lett. 21, 244 (1968); G.~Mennessier et al, arXiv:0707.4511 [hep-ph]

Equilibration times for pions and muons in different reactions



Conclusions

- We studied here kinetic equations for 1 to 2 particle and backward reactions and derived how reaction rate in medium and in frame of radiation is connected to the reaction rate in the frame of decaying particle in vacuum.
- We showed how thermal medium influences the particle decay time and the backward reaction relaxation time.
- We showed that in expanded universe the pions, the muons and other hadrons (except n and p) stay in chemical equilibrium with radiation up to the few MeV. Until these temperatures their densities are larger than nucleon density and they can actively participate in the reactions with nucleons.
- We showed that reaction $\pi^{\pm} \leftrightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$ can be important for the neutrinos equilibration at T > 7 MeV.