The Early Universe II: From Quark-Gluon Plasma to Neutrino Decoupling

Jan Rafelski
Department of Physics, The University of ARIZONA, Tucson

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[iii] **Particles in the UNIVERSE: time constant, constraints and chemical potentials, particle abundances, distillation**

[iv] **Validation of equilibrium: kinetic theory of strange particle production and equilibration**

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[v] **Validation of equilibrium: kinetic theory of pion, and lepton equilibration**

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Quark-Hadron-Lepton Universe

Work with Jean Letessier ('Hadrons from QGP' book) and Mike Fromerth, astro-ph/0211346.

Our objective is to study the conversion of Quark Universe into the hadronic phase, and to understand the dynamics of matter-antimatter annihilation and hadron disappearance in the range $300 < T < 3$ MeV and emergence of particle content as seen today.

There are a few tacit assumptions:

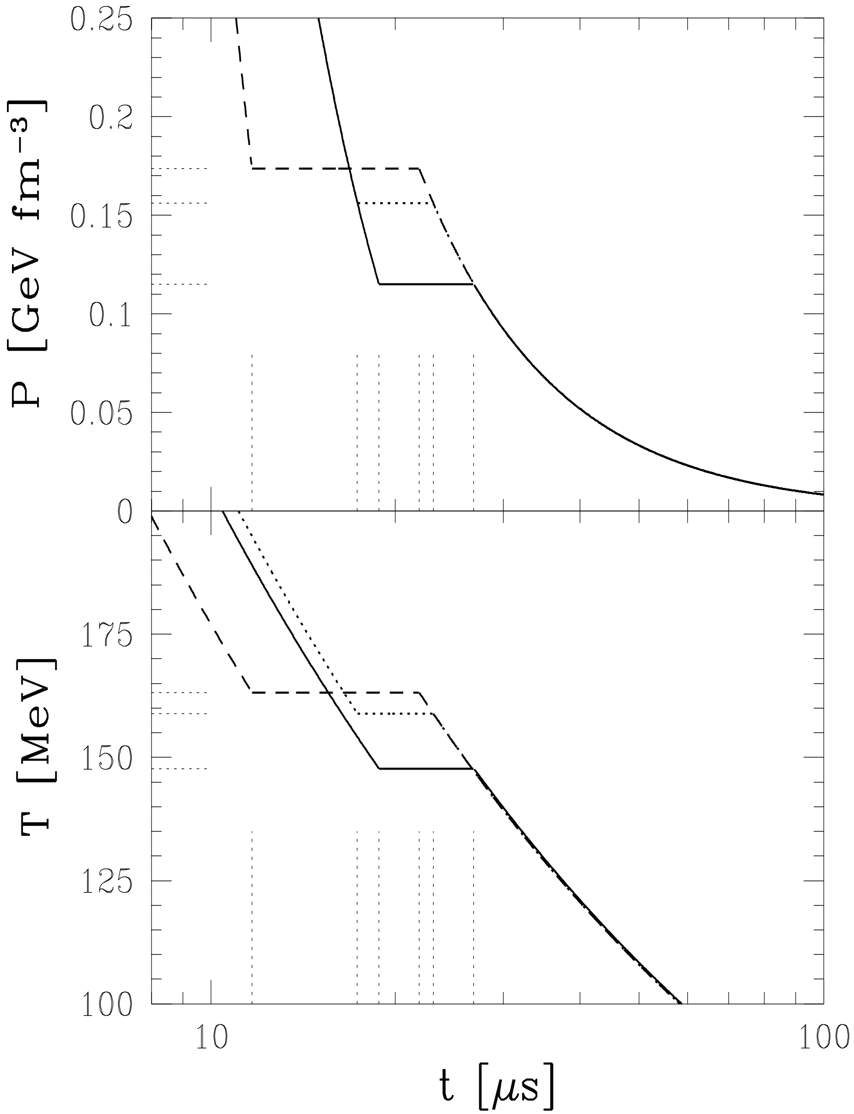
a) the dark energy does not matter in early Universe, e.g. it is Einstein's Λ

b) the dark matter does not populate visible matter

c) dark matter is either 'cold' or 'warm' and not 'luke warm'; mass outside range of our study

d) there are 3 neutrinos and antineutrinos which have each two components

Universe Hadronization: When and How?



Two dynamical equations:

Entropy conserving expansion:

$$dE + P\,dV = T\,dS = 0, \quad dE = d(\epsilon V),$$

$$\frac{dV}{V} = \frac{3\,dR}{R}, \quad \frac{3dR}{R} = -\frac{d\epsilon}{\epsilon + P}: \quad \dot\epsilon = -3\frac{\dot R}{R}(\epsilon + P)$$

Contraction of the Einstein equation

$$\mathcal{R}_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}\mathcal{R} + \Lambda_{\rm v} g_{\mu\nu} = 8\pi G T_{\mu\nu},$$

in Friedmann-Lemaître-Robertson-Walker coordinates,

$$\frac{\dot R^2}{R^2} = \frac{8\pi G}{3}\epsilon + \frac{\Lambda}{3} - \frac{k}{R^2}$$

$\Lambda = 0$ **and flat** $k = 0$ **universe, combine, use latent heat, only massless particles count:**

$$\epsilon_p = \epsilon - \mathcal{B} = 3P_p = 3P + 3\mathcal{B}, \quad \dot\epsilon^2 = \frac{128\pi G}{3}\,\epsilon\,(\epsilon - \mathcal{B})^2,$$

GENERAL ANALYTIC Solution:

in QGP book, original, not seen published:

$$\epsilon_{\rm QGP} = \mathcal{B}\coth^2(t/\tau_{\rm U}),$$

With time constant:

$$\tau_{\rm U} = \sqrt{\frac{3c^2}{32\pi G\mathcal{B}}} = 25\sqrt{\frac{\mathcal{B}_0}{\mathcal{B}}}\,\mu\text{s}, \quad \mathcal{B}_0 = 0.4\,\frac{\textbf{GeV}}{\textbf{fm}^3}$$

Pressure (upper) and temperature (lower part) in the Universe, as function of time, in the vicinity of the phase transition from the deconfined phase to the confined phase. Solid lines, $\mathcal{B}^{1/4} = 195$ **MeV; dotted lines,** $\mathcal{B}^{1/4} = 170$ **MeV (lower part) and** $\mathcal{B}^{1/4} = 220$ **MeV (upper part) all for** $\alpha_s = 0.6$**.**

PARTICLES IN THE EARLY UNIVERSE

The understanding of equations of state of QGP and Hadron Gas phase allow precise exploration of the conditions in which matter (protons, neutrons) formed. We need to obtain all chemical potentials μ Neutrino oscillations close system of equations!

Objectives

1) Describe in quantitative terms the chemical composition of the Universe before hadronization and at hadronization:

$$T \simeq 160\,\mathbf{MeV} \qquad t \simeq 40\mu s,$$

2) Understand the quark-hadron phase transformation dynamics, baryon number distillation;

3) Describe the composition of the Universe during evolution towards the condition of neutrino decoupling

$$T \simeq 1-3\,\mathbf{MeV} \qquad t \simeq 10\,s$$

Chemical potentials

- **Photons in chemical equilibrium, assume the Planck distribution, implying a zero photon chemical potential; i.e., $\mu_\gamma = 0$.**
- **Because reactions such as $f + \bar{f} \rightleftharpoons 2\gamma$ are allowed, where f and $\bar{f}$ are a fermion – antifermion pair, we immediately see that $\mu_f = -\mu_{\bar{f}}$ whenever chemical and thermal equilibrium have been attained.**
- **More generally for any reaction $\nu_i A_i = 0$, where ν_i are the reaction equation coefficients of the chemical species A_i, chemical equilibrium occurs when $\nu_i \mu_i = 0$, which follows from a minimization of the Gibbs free energy.**
- **Weak interaction reactions assure:**

$$\mu_e - \mu_{\nu_e} = \mu_\mu - \mu_{\nu_\mu} = \mu_\tau - \mu_{\nu_\tau} \equiv \Delta\mu_l, \qquad \mu_u = \mu_d - \Delta\mu_l, \qquad \mu_s = \mu_d \ ,$$

- **The experimentally-favored "large mixing angle" solution is correct, the neutrino oscillations $\nu_e \rightleftharpoons \nu_\mu \rightleftharpoons \nu_\tau$ imply that:**

$$\mu_{\nu_e} = \mu_{\nu_\mu} = \mu_{\nu_\tau} \equiv \mu_\nu,$$

and the mixing is occurring fast in 'dense' matter.

- **There are three chemical potentials which are 'free' and we choose to follow the following: μ_d, μ_e, and μ_ν.**

- **Quark chemical potentials can be used also in the hadron phase, e.g. $\Sigma^0\,(uds)$ has chemical potential $\mu_{\Sigma^0} = \mu_u + \mu_d + \mu_s$**

- **The baryochemical potential is:**

$$\mu_b = \frac{1}{2}(\mu_p + \mu_n) = \frac{3}{2}(\mu_d + \mu_u) = 3\mu_d - \frac{3}{2}\Delta\mu_l = 3\mu_d - \frac{3}{2}(\mu_e - \mu_\nu).$$

Chemical Conditions

Three chemical potentials obtained solving the three available constraints:

i. *Charge neutrality* **($Q = 0$) is required to eliminate Coulomb energy. This implies that:**

$$n_Q \equiv \sum_i Q_i \, n_i(\mu_i, T) = 0,$$

where Q_i and n_i are the charge and number density of species i.

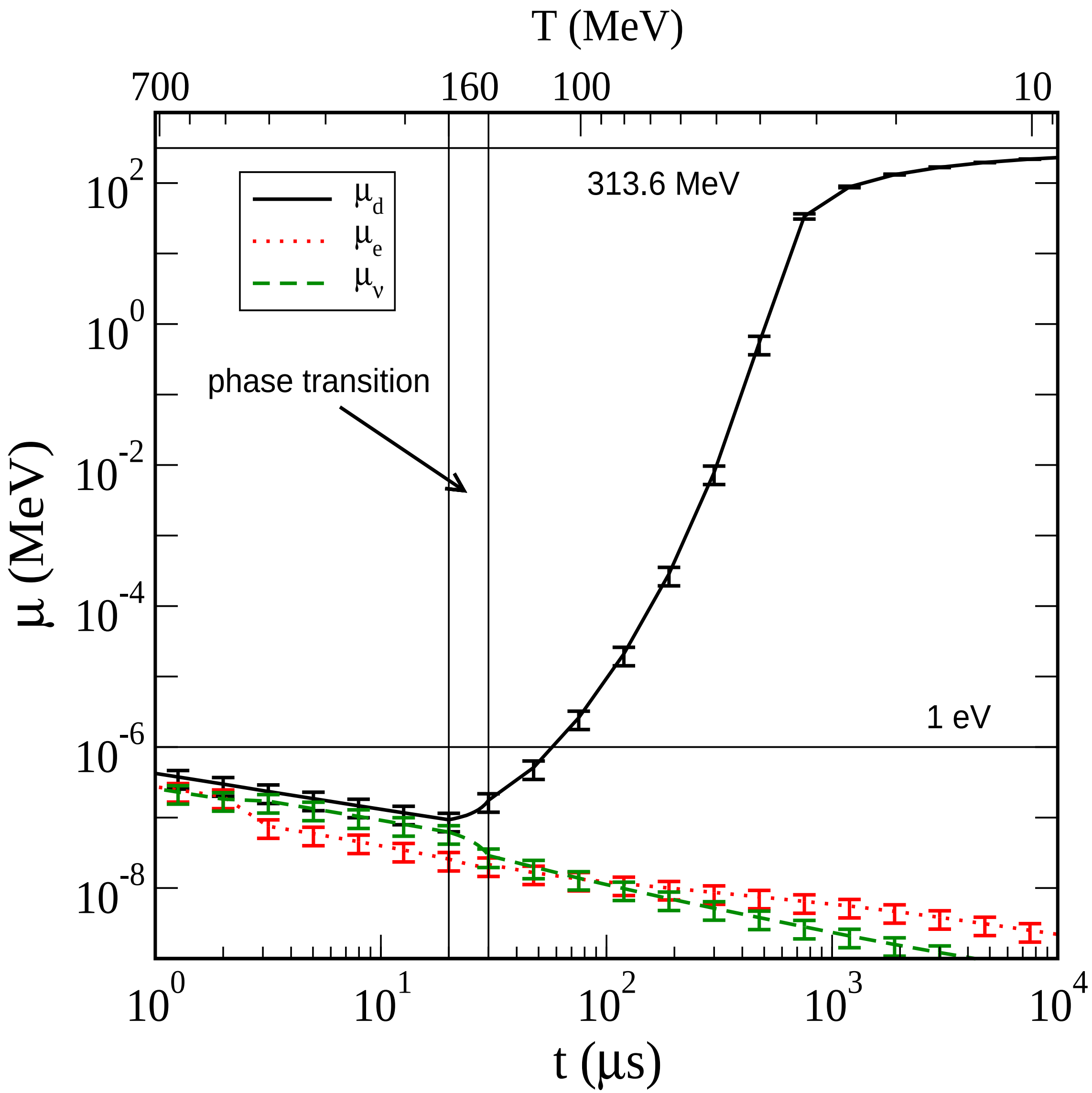
ii. *Net lepton number equals net baryon number* **($L = B$): often used condition in baryogenesis:**

$$n_L - n_B \equiv \sum_i (L_i - B_i) \, n_i(\mu_i, T) = 0,$$

iii. *Constant in time entropy-per-baryon* **(S/B) i.e. the Universe evolves adiabatically,**

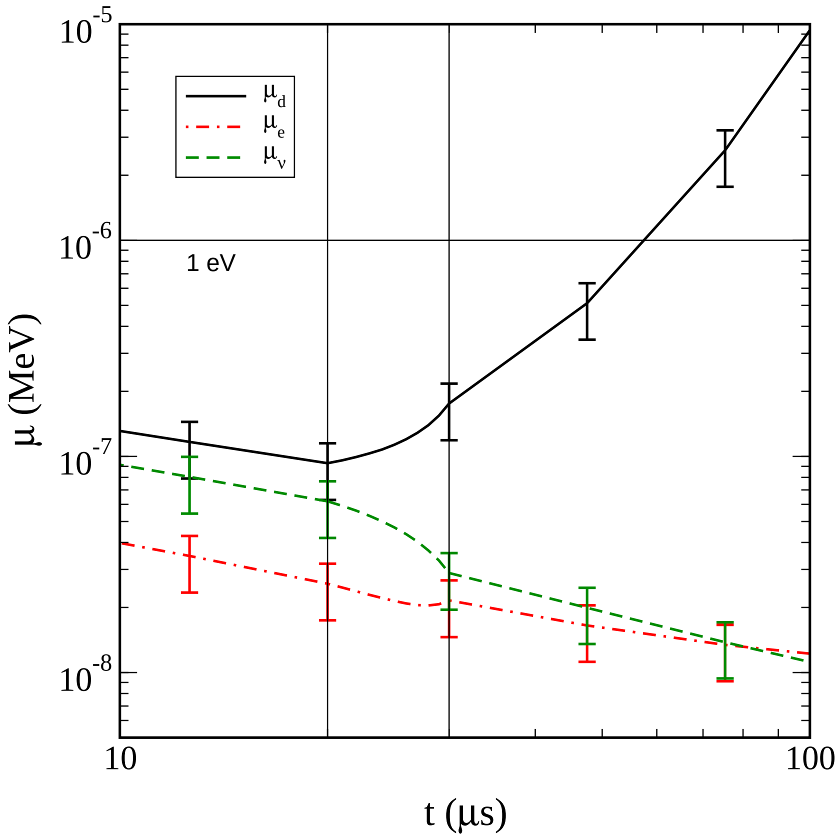
$$\frac{\sigma}{n_B} \equiv \frac{\sum_i \sigma_i(\mu_i, T)}{\sum_i B_i \, n_i(\mu_i, T)} = 4.5^{+1.4}_{-1.1} \times 10^{10}$$

TRACING μ_d IN THE UNIVERSE



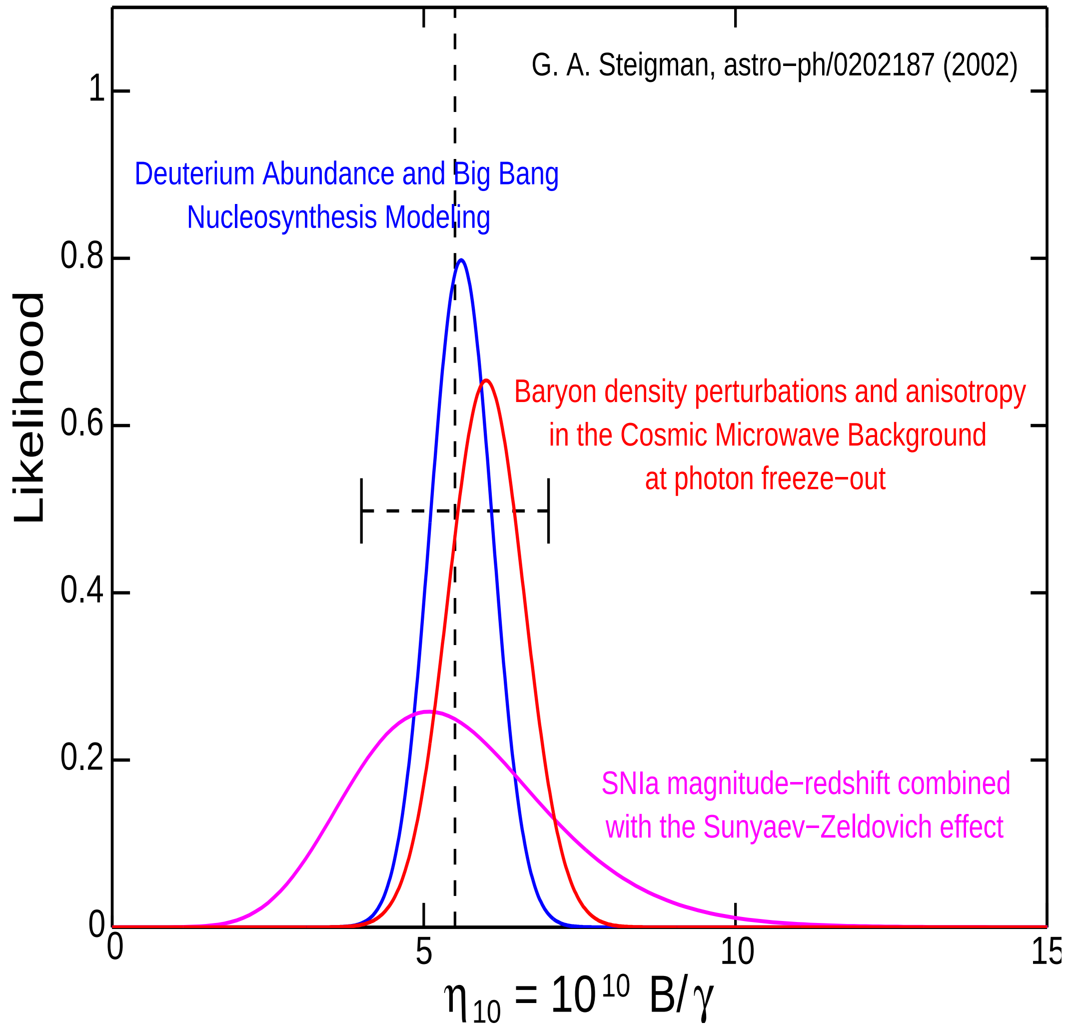
Minimum $\mu_b = 0.33^{+0.11}_{-0.08}$ eV.

μ_b relevant at final hadron $(\pi, \overline{N})$ freeze-out.



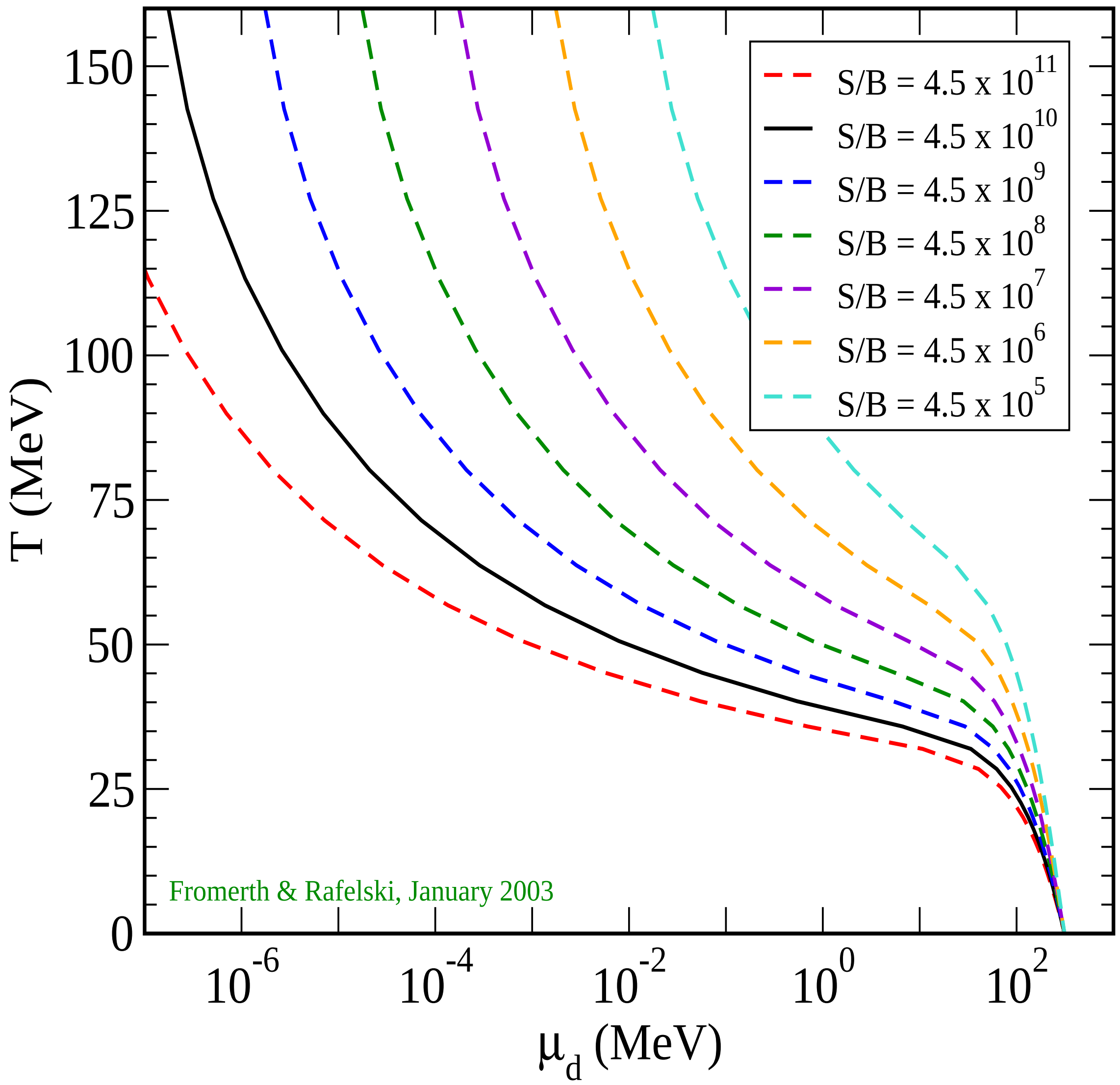
Entropy per Baryon in the Universe

$$\eta \equiv n_{\rm B}/n_\gamma = 5.5 \pm 1.5 \times 10^{-10}$$



This yields $S/b \simeq 4.5\,10^{10}$ – note that the information is for 'small' T. So if 'free' baryon number was smaller/larger in the QGP Universe, S/b was larger/smaller.

TRACING μ_d IN A UNIVERSE



Mixed Phase

Many properties of the Universe jump as one compares QGP with Hadron Phase. Thus we introduce the mixed hadron-quark phase and parametrize the partition function during the phase transformation as

$$\ln Z_{\rm tot} = f_{\rm HG} \ln Z_{\rm HG} + (1 - f_{\rm HG}) \ln Z_{\rm QGP}$$

$f_{\rm HG}$ represents the fraction of total phase space occupied by the HG phase.

The three constraints are accordingly modified, e.g.:

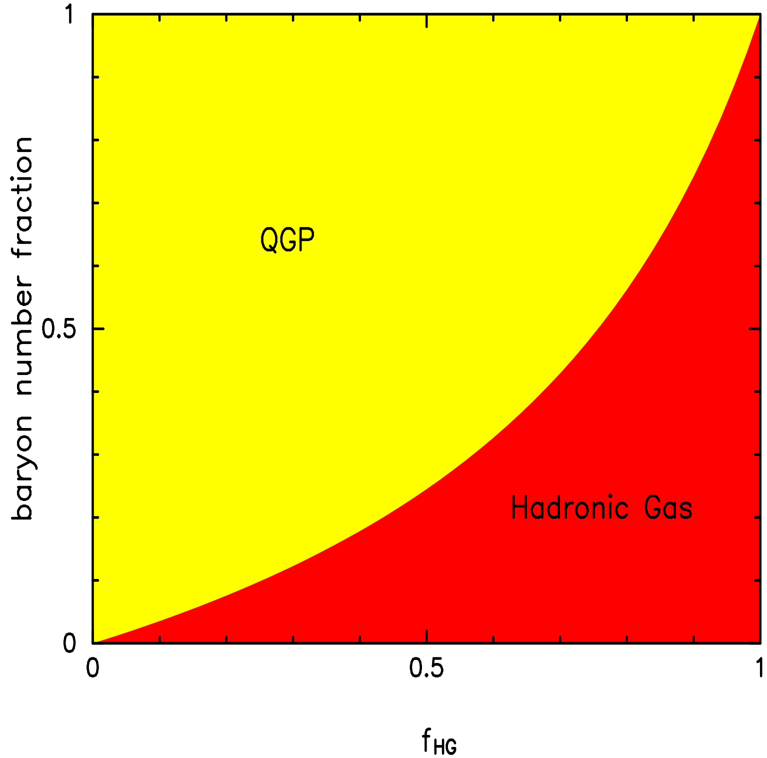
$$Q = 0 = n_Q^{\rm QGP} V_{\rm QGP} + n_Q^{\rm HG} V_{\rm HG} = V_{\rm tot} \left[(1 - f_{\rm HG})\, n_Q^{\rm QGP} + f_{\rm HG}\, n_Q^{\rm HG} \right[$$

where the total volume $V_{\rm tot}$ is irrelevant to the solution. Analogous expressions can be derived for $L - B$ and S/B constraints.

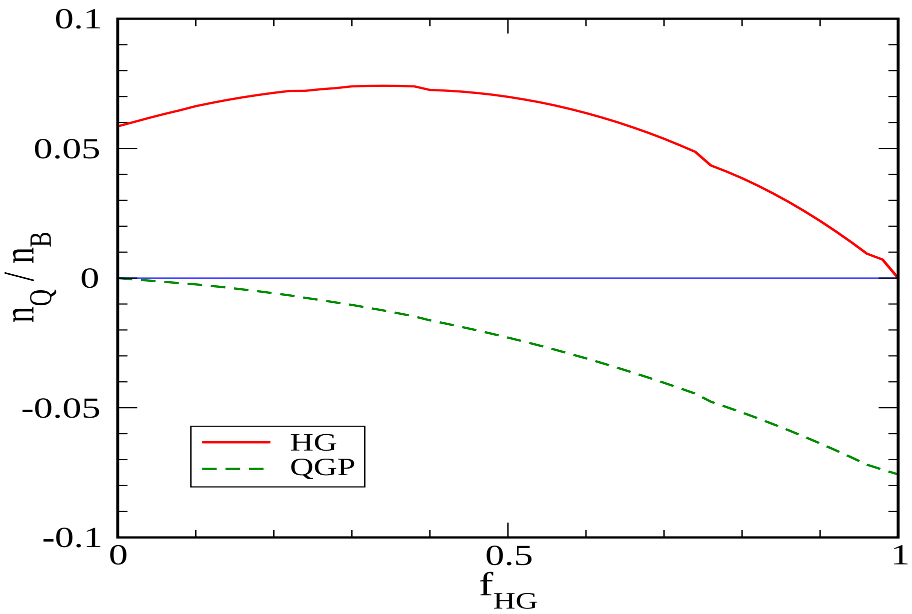
We assume that mixed phase exists $10\,\mu s$ and that $f_{\rm HG}$ changes linearly in time. Actual values will require dynamic nucleation transport theory description.

Charge and baryon number distillation

Initially at $f_{\mathrm{HG}} = 0$ all matter in QGP phase, as hadronization progresses with $f_{\mathrm{HG}} \to 1$ the baryon component in hadronic gas reaches 100%.

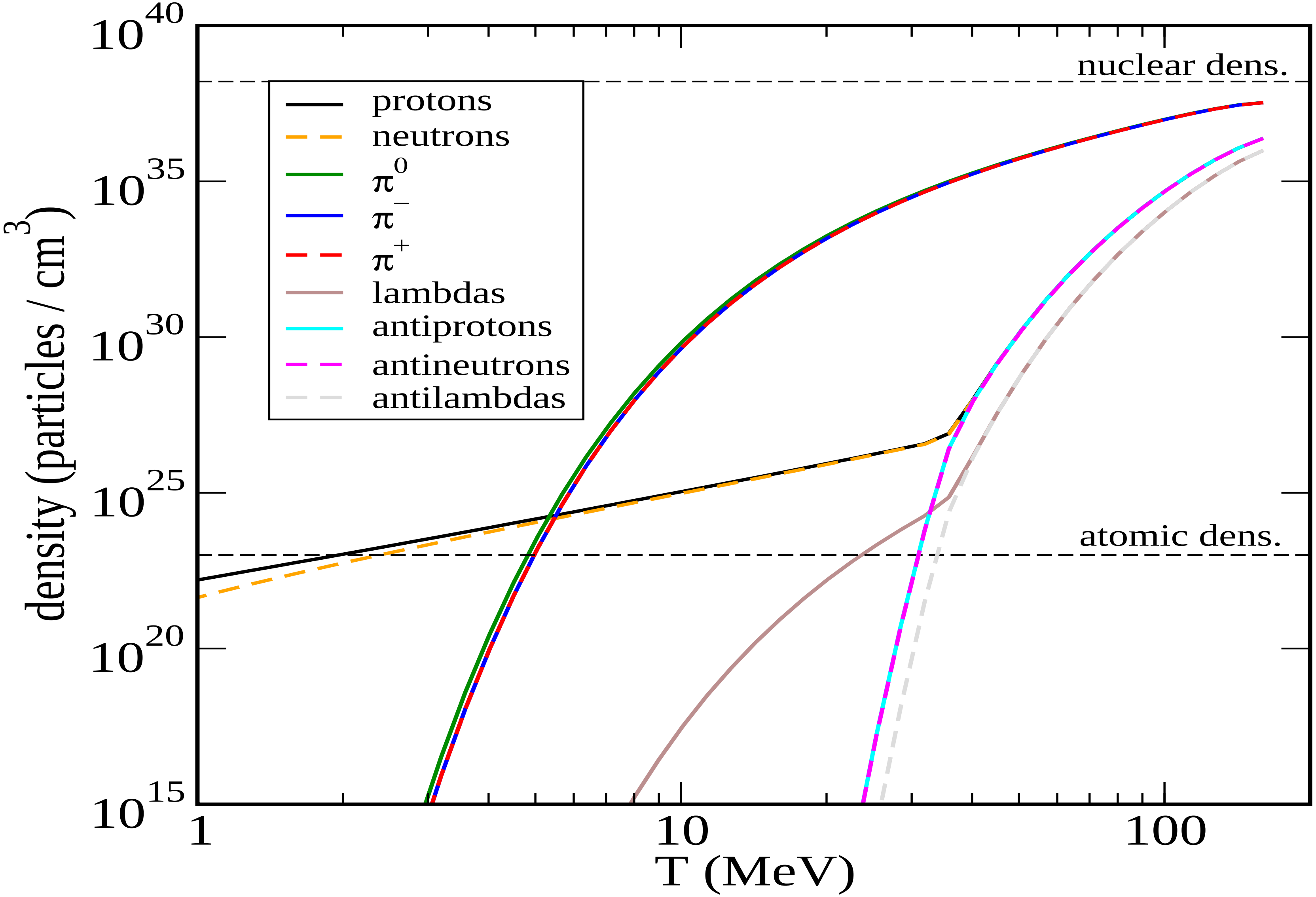


The constraint to a charge neutral universe conserves charge in both fractions. Charge in each fraction can be finite.

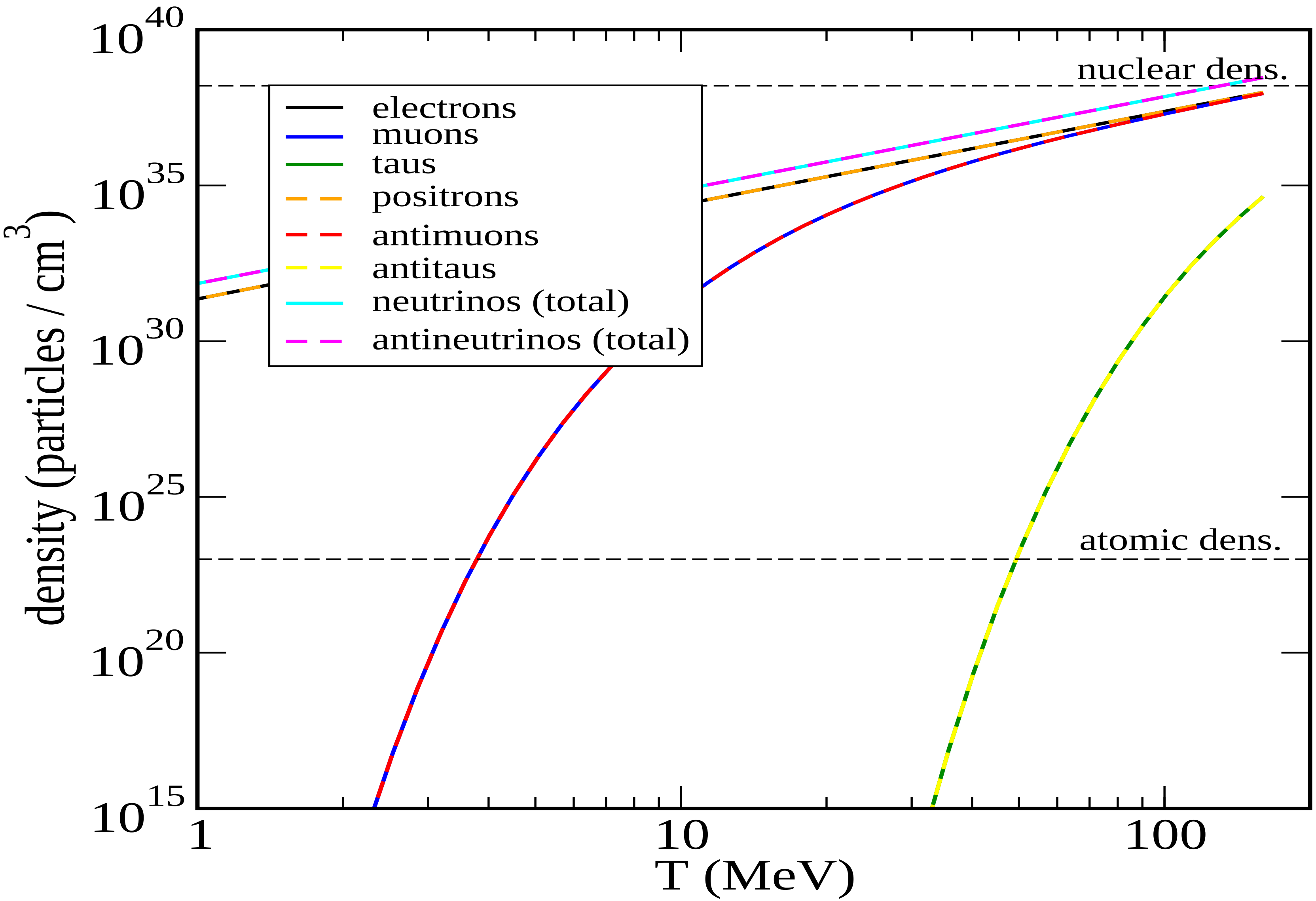


Even a small charge separation introduces a finite non-zero Coulomb potential and this amplifies the existent baryon asymmetry. This mechanism noticed by Witten in his 1984 paper, and exploited by A. Olinto for generation of magnetic fields.

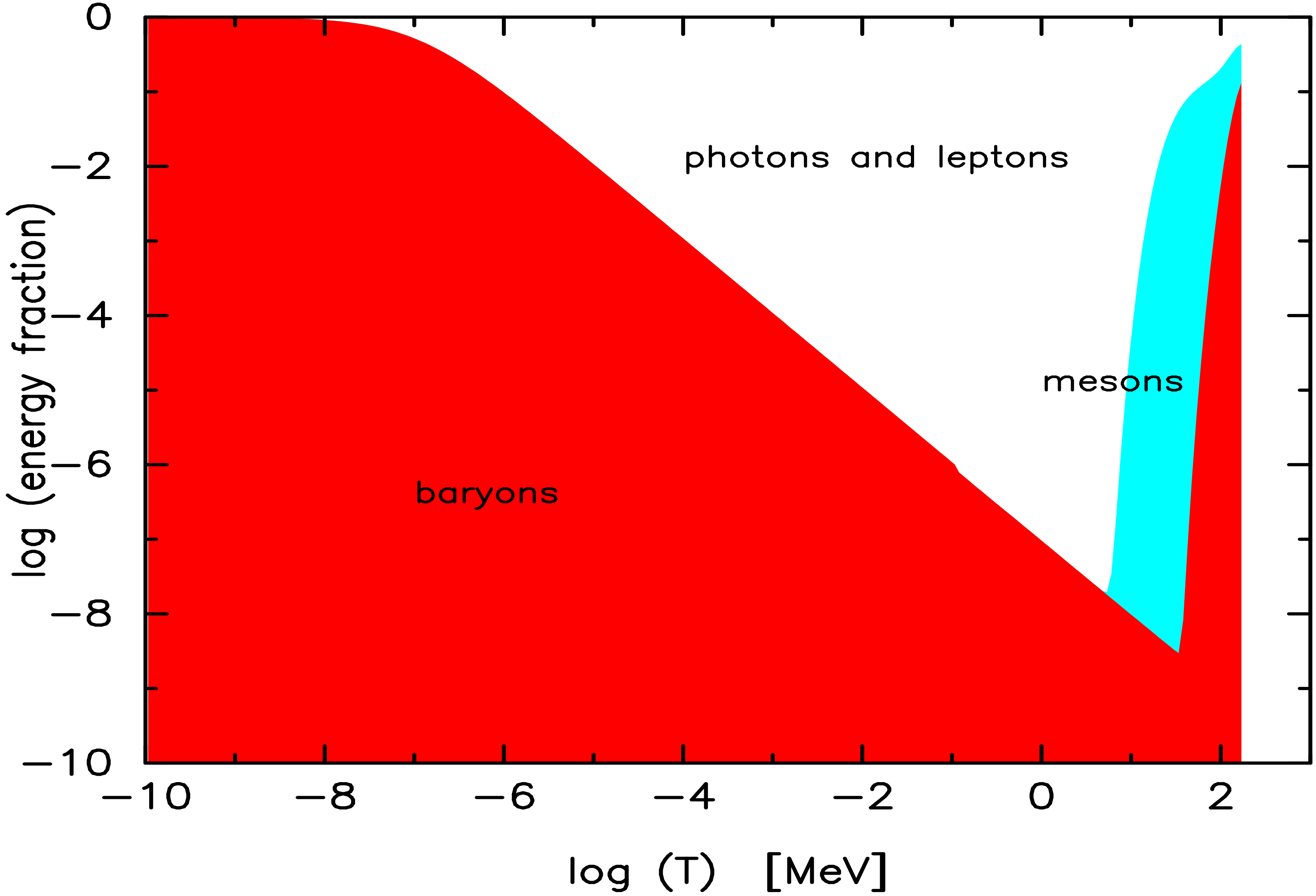
Hadronic Universe Hadron Densities



Universe Lepton Densities



Energy in luminous hadronic Universe

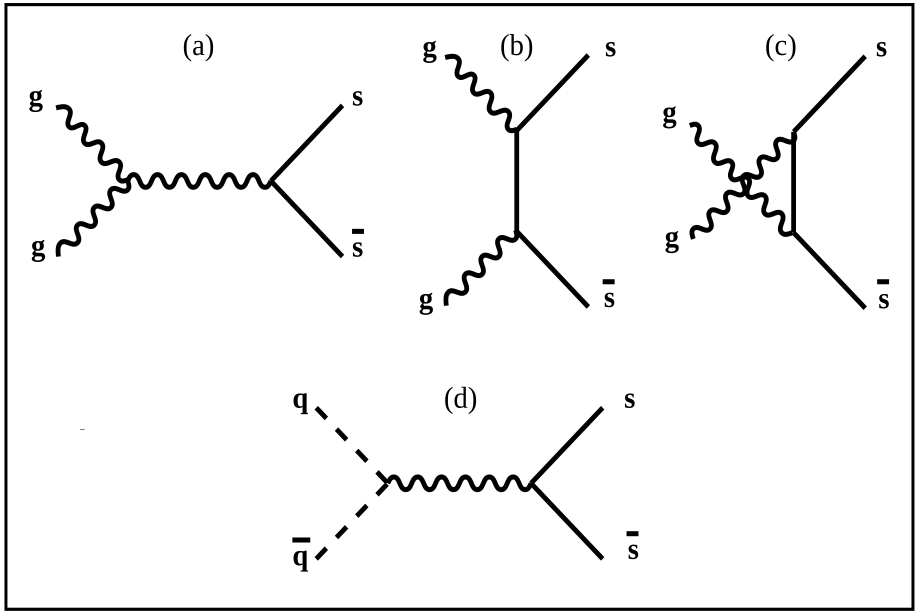


NEXT: Kinetic theory to establish time constants for particle yield equilibration.

Towards particle yield equilibration

Tutorial – Strangeness production (with Berndt Muller, Jean Letessier and others)

- production of strangeness in gluon fusion $GG \to s\bar{s}$
  strangeness linked to gluons from QGP;



- coincidence of scales:

$$m_s \simeq T_c \to \tau_s \simeq \tau_{\rm QGP} \to$$

  strangeness a clock for reaction

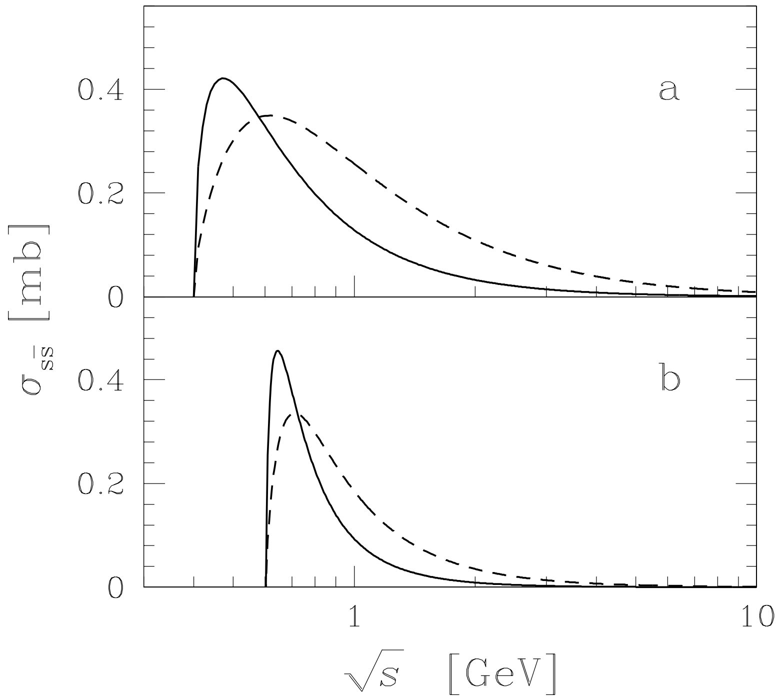
- Often $\bar{s} > \bar{q} \to$
  strange antibaryon enhancement and
  at LHC also (anti)hyperon dominance of (anti)baryons.

Kinetic description of strangeness production

The generic angle averaged cross sections for (heavy) flavor s, $\bar{s}$ production processes $g+g\rightarrow s+\bar{s}$ and $q+\bar{q}\rightarrow s+\bar{s}$, are:

$$\bar{\sigma}_{gg\rightarrow s\bar{s}}(s)=\frac{2\pi\alpha_s^2}{3s}\left[\left(1+\frac{4m_s^2}{s}+\frac{m_s^4}{s^2}\right)\tanh^{-1}W(s)-\left(\frac{7}{8}+\frac{31m_s^2}{8s}\right)W(s)\right],$$

$$\bar{\sigma}_{q\bar{q}\rightarrow s\bar{s}}(s)=\frac{8\pi\alpha_s^2}{27s}\left(1+\frac{2m_s^2}{s}\right)W(s)\,.\qquad W(s)=\sqrt{1-4m_s^2/s}$$

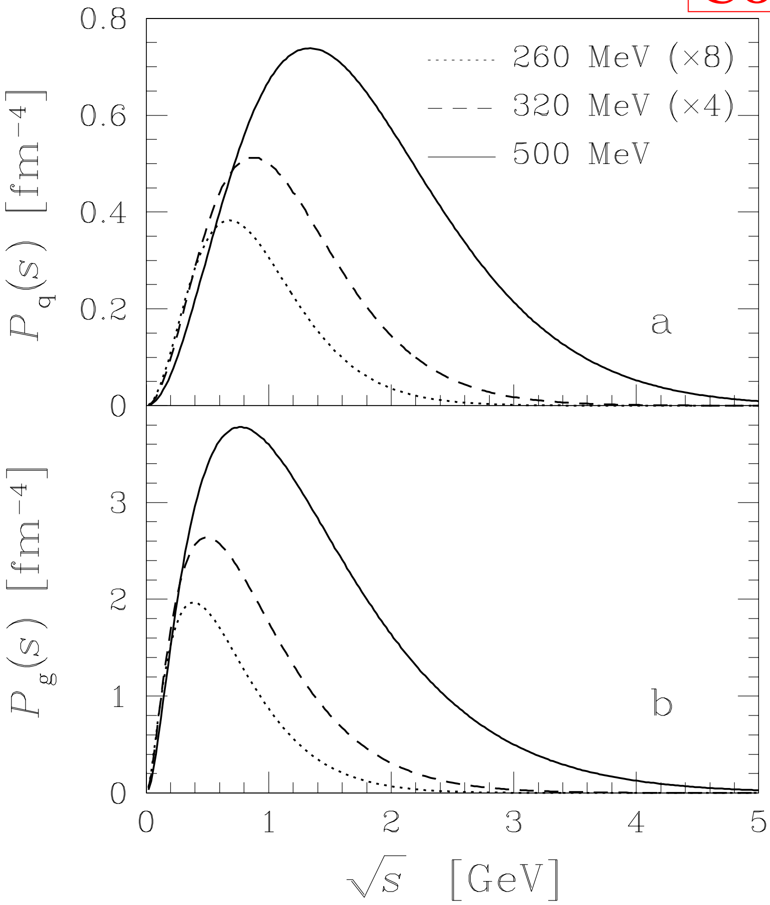


QGP Strangeness production cross sections: Solid lines $q\bar{q}\rightarrow s\bar{s}$; dashed lines $gg\rightarrow s\bar{s}$.

a) TOP for fixed $\alpha_s=0.6$, $m_s=200\,$MeV;
b) BOTTOM: for running $\alpha_s(\sqrt{s})$ and $m_s(\sqrt{s})$,

with $\alpha_{s(M_Z)}=0.118$. $m_s(M_Z)=90\pm20\%$ MeV, $m_s(1\mathbf{GeV})\simeq2.1m_s(M_Z)\simeq200$MeV.

Collision rate



The collision distribution functions as function $\sqrt{s}$:

a) for quarks,
b) for gluons.

Computed for temperature $T = 260\,$MeV, $\lambda_q = 1.5$ (dotted lines, amplified by factor 8); $T = 320\,$MeV, $\lambda_q = 1.6$ (dashed lines, amplified by factor 4); and $T = 500\,$MeV, $\lambda_q = 1.05$ (solid lines). In all cases $\gamma_q, \gamma_g = 1$.

Gluon collisions dominate!

$$P_g = \frac{4Ts^{3/2}}{\pi^4} \sum_{l,n=1}^{\infty} \frac{1}{\sqrt{nl}} K_1\left(\frac{\sqrt{nl\,s}}{T}\right) .$$

$$P_q|_{\mu_q=0} = \frac{9Ts^{3/2}}{4\pi^4} \sum_{l=1}^{\infty} (-)^{l+1} \frac{\gamma_q^l}{l\lambda_q^l} \int_0^{\infty} \frac{dp_1}{\sqrt{s}} \frac{e^{-l\frac{s}{4Tp_1}}}{\gamma_q^{-1}\lambda_q^{-1}e^{p_1/T}+1} .$$

Thermal average of reactions

Kinetic (momentum) equilibration is faster than chemical, use thermal particle distributions $f(\vec{p}_1, T)$ to obtain average rate:

$$\langle \sigma v_{\mathrm{rel}} \rangle_T \equiv \frac{\int d^3p_1 \int d^3p_2 \sigma_{12} v_{12} f(\vec{p}_1, T) f(\vec{p}_2, T)}{\int d^3p_1 \int d^3p_2 f(\vec{p}_1, T) f(\vec{p}_2, T)} \, .$$

Invariant reaction rate in medium:

$$A^{gg\to s\bar{s}} = \frac{1}{2}\rho_g^2(t)\,\langle\sigma v\rangle_T^{gg\to s\bar{s}}\,, \quad A^{q\bar{q}\to s\bar{s}} = \rho_q(t)\rho_{\bar{q}}(t)\langle\sigma v\rangle_T^{q\bar{q}\to s\bar{s}}\,, \quad A^{s\bar{s}\to gg,q\bar{q}} = \rho_s(t)\,\rho_{\bar{s}}(t)\,\langle\sigma v\rangle_T^{s\bar{s}\to gg,q\bar{q}}.$$

$1/(1+\delta_{1,2})$ **introduced for two gluon processescompensates the double-counting of identical particle pairs, arising since we are summing independently both reacting particles.**

This rate enters the momentum-integrated Boltzmann equation which can be written in form of current conservation with a source term

$$\partial_\mu j_s^\mu \equiv \frac{\partial \rho_s}{\partial t} + \frac{\partial \vec{v} \rho_s}{\partial \vec{x}} = A^{gg\to s\bar{s}} + A^{q\bar{q}\to s\bar{s}} - A^{s\bar{s}\to gg,q\bar{q}}$$

Strangeness density time evolution

in local rest frame ($\vec{v}$) we have :

$$\frac{d\rho_s}{dt} = \frac{d\rho_{\bar{s}}}{dt} = \frac{1}{2}\rho_g^2(t)\,\langle\sigma v\rangle_T^{gg\to s\bar{s}} + \rho_q(t)\rho_{\bar{q}}(t)\langle\sigma v\rangle_T^{q\bar{q}\to s\bar{s}} - \rho_s(t)\,\rho_{\bar{s}}(t)\,\langle\sigma v\rangle_T^{s\bar{s}\to gg,q\bar{q}}$$

Evolution for s and $\bar{s}$ identical, which allows to set $\rho_s(t) = \rho_{\bar{s}}(t)$.
Use detailed balance to simplify

$$\frac{d\rho_s}{dt} = A\left(1 - \frac{\rho_s^2(t)}{\rho_s^2(\infty)}\right), \qquad A = A^{gg\to s\bar{s}} + A^{q\bar{q}\to s\bar{s}}$$

The generic solution at fixed T ($\rho \propto \tanh$) implies that in all general cases there is an exponential approach to chemical equilibrium

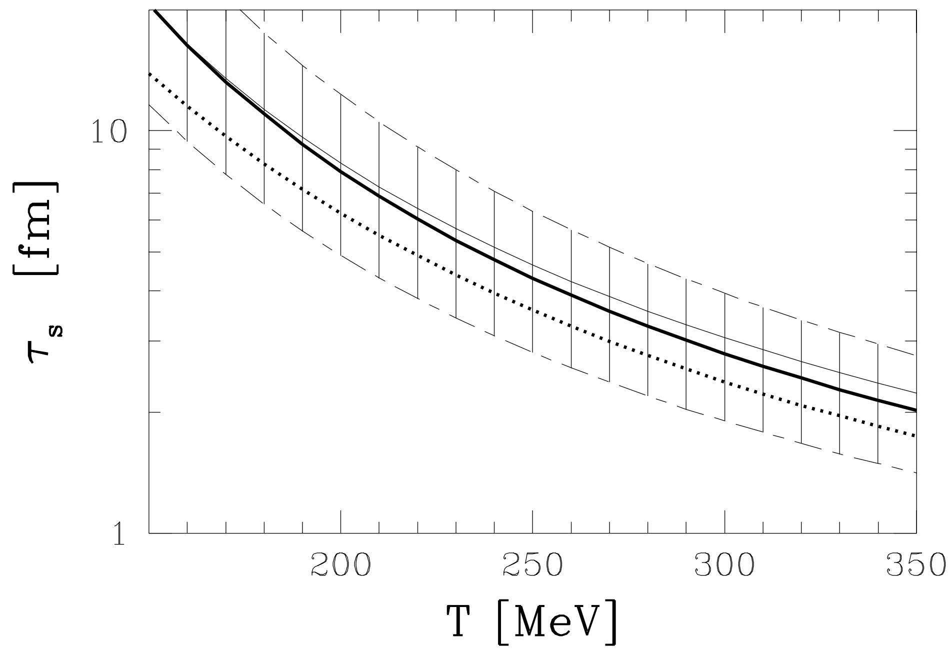
$$\frac{\rho_s(t)}{\rho_s^\infty} \longrightarrow 1 - e^{-t/\tau_s}$$

with the characteristic time constant τ_s:

$$\tau_s \equiv \frac{1}{2}\frac{\rho_s(\infty)}{(A^{gg\to s\bar{s}} + A^{q\bar{q}\to s\bar{s}} + \ldots)}$$

$$A^{12\to 34} \equiv \frac{1}{1+\delta_{1,2}}\rho_1^\infty\rho_2^\infty\langle\sigma_s v_{12}\rangle_T^{12\to 34}.$$

Characteristic time constant and γ_s-evolution



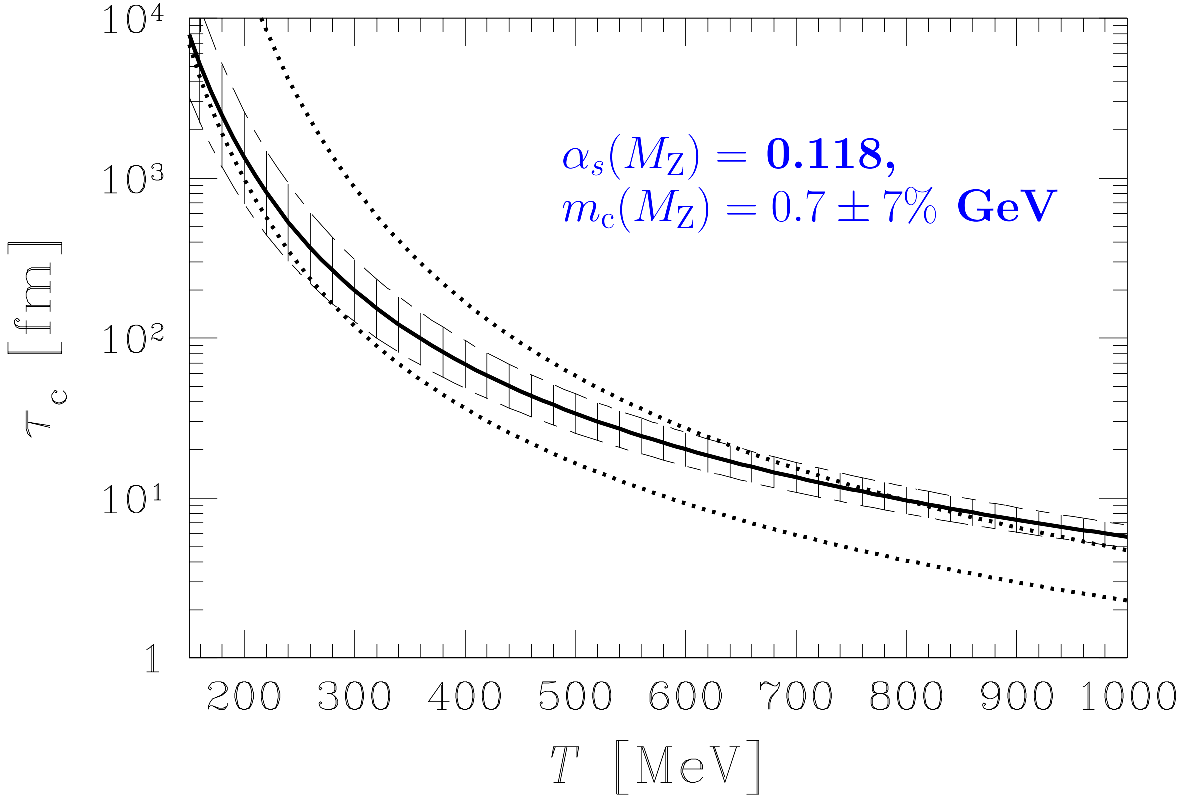
$\sigma_{\rm QCD}^{\to s\bar{s}}$ gives τ_s similar to lifespan of the plasma phase!
Strange quark pair production dominated by gluon fusion: $G+G\to s\bar{s}$, also some (10%) $q\bar{q}\to s\bar{s}$, present; this is due to gluon collision rate.
ENTROPY CONSERVING expansion i.e. at SPS T^3V =Const. (not yet long. scaling):

$$2\tau_s\frac{dT}{dt}\left(\frac{d\gamma_s}{dT}+\frac{\gamma_s}{T}z\frac{K_1(z)}{K_2(z)}\right)=1-{\gamma_s}^2\,,\quad \gamma_s(t)\equiv n_s(t)/n_s^\infty\,,\quad z=\frac{m_s}{T}\,,\quad K_i:\mathrm{Besself.}$$

Once γ_s known, $\langle\rho_s(t)\rangle=\langle\bar{\rho}_s(t)\rangle=\int dx^3\rho_s^\infty(T(t,x))\gamma_s(T(t,x),\dot{T}(t,x))$;
evolution till $t\to t_f$,but effectively production stops for $T<180$ MeV.

What about charm? $m_s \to m_c$

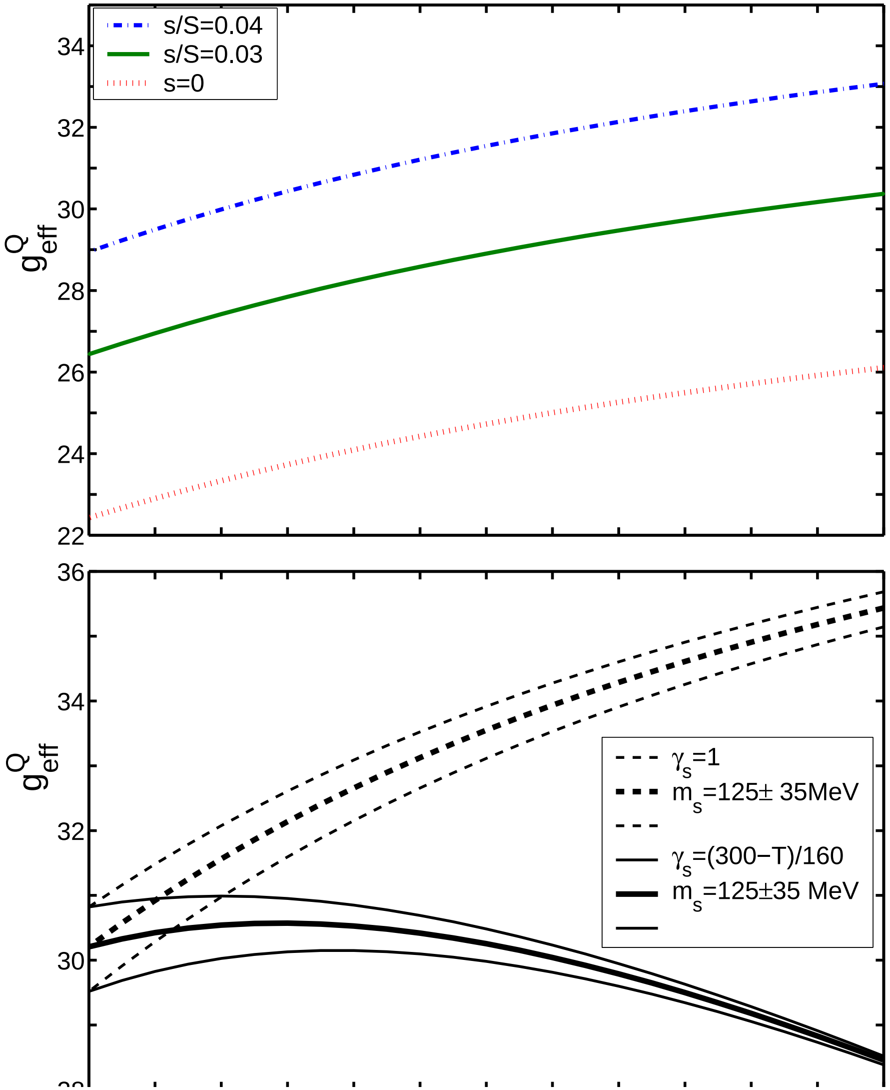
We expect that thermal charm production is of relevance only for $T \to m_c(1\,\mathrm{GeV}) \simeq 1.5\,\mathrm{GeV}$, probably not accessible.



Lower dotted line: for fixed $m_c = 0.9\,\mathrm{GeV}$, $\alpha_s = 0.35$;
upper doted line: for fixed $m_c = 1.5\,\mathrm{GeV}$, $\alpha_s = 0.4$.
Equilibrium density for $\rho_c^\infty(m_c \simeq 1.5\,\mathrm{GeV})$.

Charm is produced relatively abundantly in first parton collisions. Benchmark: 10 $c\bar{c}$ pairs in central Au–Au at RHIC-200. This yield is greater than the expected equilibrium yield at hadronization of QGP.

Another key property: entropy in QGP – degrees of freedom $g_{\rm eff}^Q$?



$g_{\rm eff}^Q$ in QGP

$$\sigma = \frac{4\pi^2}{90} g_{\rm eff}^Q T^3,$$

$$g_{\rm eff}^Q(T) = g_g(T) + \frac{7}{4} g_q(T) + 2g_s \frac{90}{\pi^4} + \frac{\mathcal{A}^{\rm pert}}{T^4}\frac{90}{4\pi^2}.$$

Upper frame: fixed s/S
green solid line $s/S = 0.03$
blue dot-dashed $s/S = 0.04$.
red dotted 2-flavor QCD $-u, d, G$;

Bottom:
2+1-flavor QCD with $m_s = 125 \pm 35$ MeV
dashed: equilibrated u, d, s, G system
solid lines: strangeness contents increasing with decreasing temperature
$\gamma_s = (300 - T)/160$

Measure of s-enhancement at RHIC and LHC: Strangeness / Entropy

s/S: ratio of the number of active degrees of freedom in QG plasma,

For chemical equilibrium IN PLASMA:

$$\frac{s^Q}{S^Q} \simeq \frac{1}{4}\frac{n_s}{n_s + n_{\bar{s}} + n_q + n_{\bar{q}} + n_G} = \frac{\frac{g_s}{2\pi^2}T^3(m_s/T)^2K_2(m_s/T)}{(g2\pi^2/45)T^3 + (g_s n_{\rm f}/6)\mu_q^2 T} \simeq \frac{1}{35} = 0.0286$$

with $\mathcal{O}(\alpha_s)$ interaction $s/S \to 1/31 = 0.0323$

CENTRALITY A, and ENERGY DEPENDENCE: $\gamma_s^{\rm Q} \to 1$

Chemical non-equilibrium occupancy of strangeness $\gamma_s^{\rm Q}$

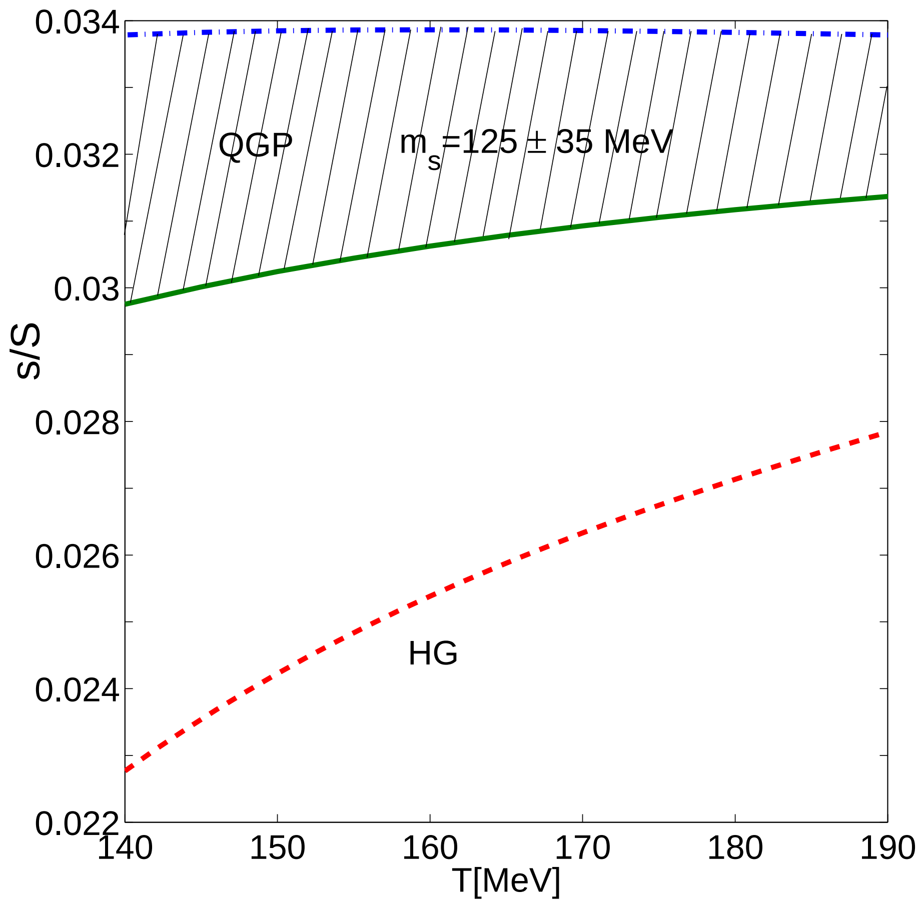
$$\frac{s^Q}{S^Q} = \frac{0.03\gamma_s^{\rm Q}}{0.4\gamma_{\rm G} + 0.1\gamma_s^{\rm Q} + 0.5\gamma_q^{\rm Q} + 0.05\gamma_q^{\rm Q}(\ln\lambda_q)^2} \to 0.03\gamma_s^{\rm Q}.$$

Analysis of experiment: we count all strange/nonstrange hadrons in final state, we use Fermi model (statistical hadronization) to extrapolate to unmeasured particle yields and/or kinematic domains, and evaluate resonance cascading:

$$\frac{s^Q}{S^Q} \simeq \frac{\text{count of primary strange hadrons}}{(\text{nonstrange} + \text{strange})\ \text{entropy} = 4\,\text{number of primary mesons} + \ldots}$$

s/S QGP and HG comparison in chem. equilibrium

We compare deconfined quark-gluon plasma with hadron gas at a common measured T. This is a phase enhancement of strangeness needed to understand hadronization, not an experimental enhancement.



Strangeness to entropy ratio $s/S(T;\mu_B = 0, \mu_S = 0)$ for the chemically equilibrated QGP (green, solid line for $m_s = 160$ MeV, blue dash-dot line for $m_s = 90$ MeV); and for chemically equilibrated HG (red, dashed). The excess of SPECIFIC strangeness not assured if QGP not chemically equilibrated. However, since QGP is a high entropy and strangeness density phase, in absolute terms, there is both entropy and strangeness excess ALWAYS when QGP is formed.

Note that much (30% at LHC!) of HG phase strangeness invisible, in hidden strangeness states η, η', ϕ

NEXT: SHOW MECHANISM OF CHEMICAL EQUILIBRATION FO

Work with Inga Kuznetsova