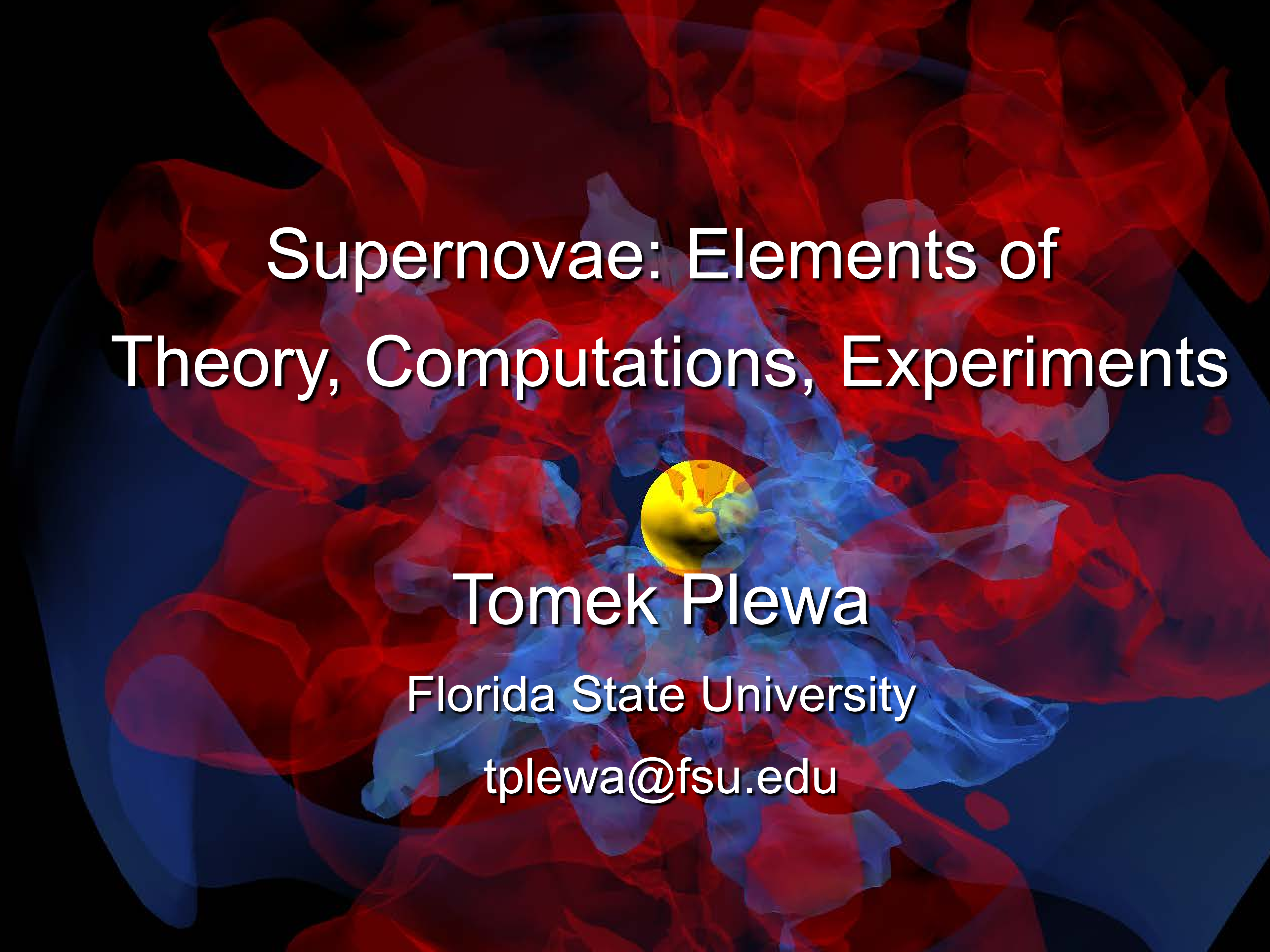

Supernovae: Elements of Theory, Computations, Experiments

Tomek Plewa

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Lecture 2

From SN 1987A to Experiments

Addendum I

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \mathbf{u} \right) = 0$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \left(\rho \mathbf{u}\mathbf{u} - \frac{\mathbf{B} \cdot \mathbf{B}}{4\pi} \right) + \nabla P = \rho \mathbf{g}$$

$$\frac{\partial \rho E}{\partial t} + \nabla \cdot \left[\left(\rho E + P \right) \mathbf{u} - \frac{\mathbf{B} \cdot \mathbf{B}}{4\pi} \cdot \mathbf{u} \right] = \rho \mathbf{u} \cdot \mathbf{g} + \nabla \cdot \left(\mathbf{q}_e + \mathbf{q}_i \right) + Q$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot \left(\mathbf{u}\mathbf{B} - \mathbf{B}\mathbf{u} \right) = \frac{c}{e} \left[\nabla \times \frac{\nabla P_e}{n_e} - \nabla \times \frac{\left(\nabla \times \mathbf{B} \right) \times \mathbf{B}}{4\pi n_e} - \nabla \times \frac{\mathbf{R}_T + \mathbf{R}_U}{n_e} \right]$$

Addendum II

- The main concern are applications of very large and complex computer codes by essentially untrained users
- There exist now a large number of such codes in public domain
- Those are frequently presented (for marketing purposes) as very stable, well-documented, proven and easy-to-use science application tools
- Large database of users is meant to offer evidence to the sponsor justifying continued funding
- It is crucial that those codes are not used blindly

Addendum III

- **Consistent discretization**

  For the consistent discretization, the truncation error goes to zero with increased mesh resolution for all sufficiently smooth solutions.

- **Stable discrete scheme**

  The scheme is stable in the norm and for a given refinement path if there exists a constant K such that for two solutions obtained from different initial data,

$$\left\| V^n - W^n \right\| \leq K \left\| V^0 - W^0 \right\|$$

  and K is independent of initial states and of refinement path stepping.

- **Convergent approximation**

  The discrete solution is said to provide a convergent approximation in a certain norm if in that norm the discrete solution approaches exact solution as the mesh resolution increases.

Lax Equivalence Theorem

If a linear PDE has been approximated by a consistent discretization method, then the discrete solution is convergent if and only if the method is stable.

Convergence = consistency + stability

- If the LET applies but no convergence is observed then either
  1. The algorithm is unstable, or
  2. It is inconsistent, or
  3. (for codes) implementation is wrong

The Method of Manufactured Exact Solutions

$$\partial_t T = \frac{1}{\rho C_p} \nabla \cdot \kappa \nabla T$$

$$D(u) = \rho C_p \partial_t u - \nabla \cdot \kappa \nabla u$$

Given

- a manufactured function u representing the exact solution, and
- a differential operator D

Find

the corresponding source term g such that

$$g = D(u)$$

That is, first we define the solution, then we calculate the corresponding source term and examine the result of the modified operator (residuals).


Southern Cross
Carina Nebula
Beta Centauri
Alpha Centauri
Coalsack Nebula
Sirius
Canopus
South Celestial Pole
Large Magellanic Cloud
Small Magellanic Cloud


N ←
(The Bean Nebula)
N 11
N 76 >
< N 79
NGC 1712 >
NGC 1731
< NGC 1747
NGC 1727 >
< NGC 1711
NGC 1760-63 >
NGC 1737 >
< N 185
NGC 1783 >
NGC 1770 >
SL 186 >
< SL 111
N 186 >
< NGC 1791
< NGC 1806
< NGC 1818
< NGC 1820
< NGC 1809
< NGC 1831
< NGC 1829
< NGC 1837
NGC 1850 >
NGC 1856
< NGC 1848
NGC 1858 >
< NGC 1866
< NGC 1876
NGC 1869-71
< NGC 1892
NGC 1910 >
< NGC 1918
NGC 1914 >
N 44
< NGC 1939
N 43 >
NGC 1934-5 >
< NGC 1944
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N 200 >
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< NGC 1966
NGC 1972 >
N 206
< NGC 1983
< NGC 1994
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< NGC 1978
< NGC 2001
< NGC 2018
NGC 2002 >
< NGC 2011
< NGC 2015
< NGC 2014
< NGC 2031
N 55
< NGC 2020
N 57 >
NGC 2033
< SN1987a
< NGC 2032
(Dragon's Head)
NGC 2042 >
< NGC 2030
NGC 2080
The Tarantula Nebula >
(NGC 2070)
NGC 2027
< NGC 2083
NGC 2081
< NGC 2013
N 64 A & C
NGC 2085-6
NGC 2094 >
< NGC 2100
< NGC 2095
< LH120-No70
< NGC 2113
< NGC 2121
< NGC 2122
< NGC 2133
NGC 2134 >
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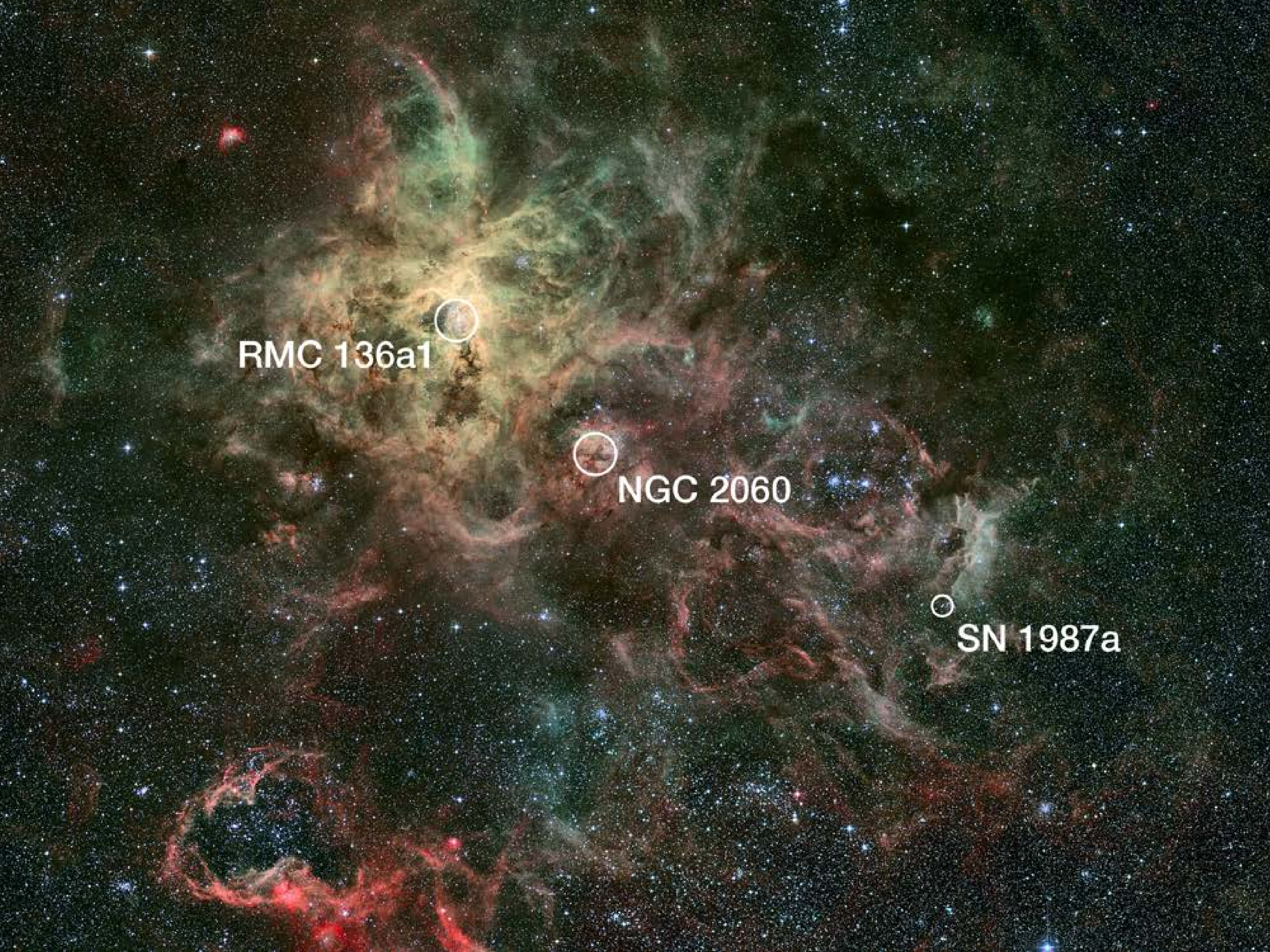
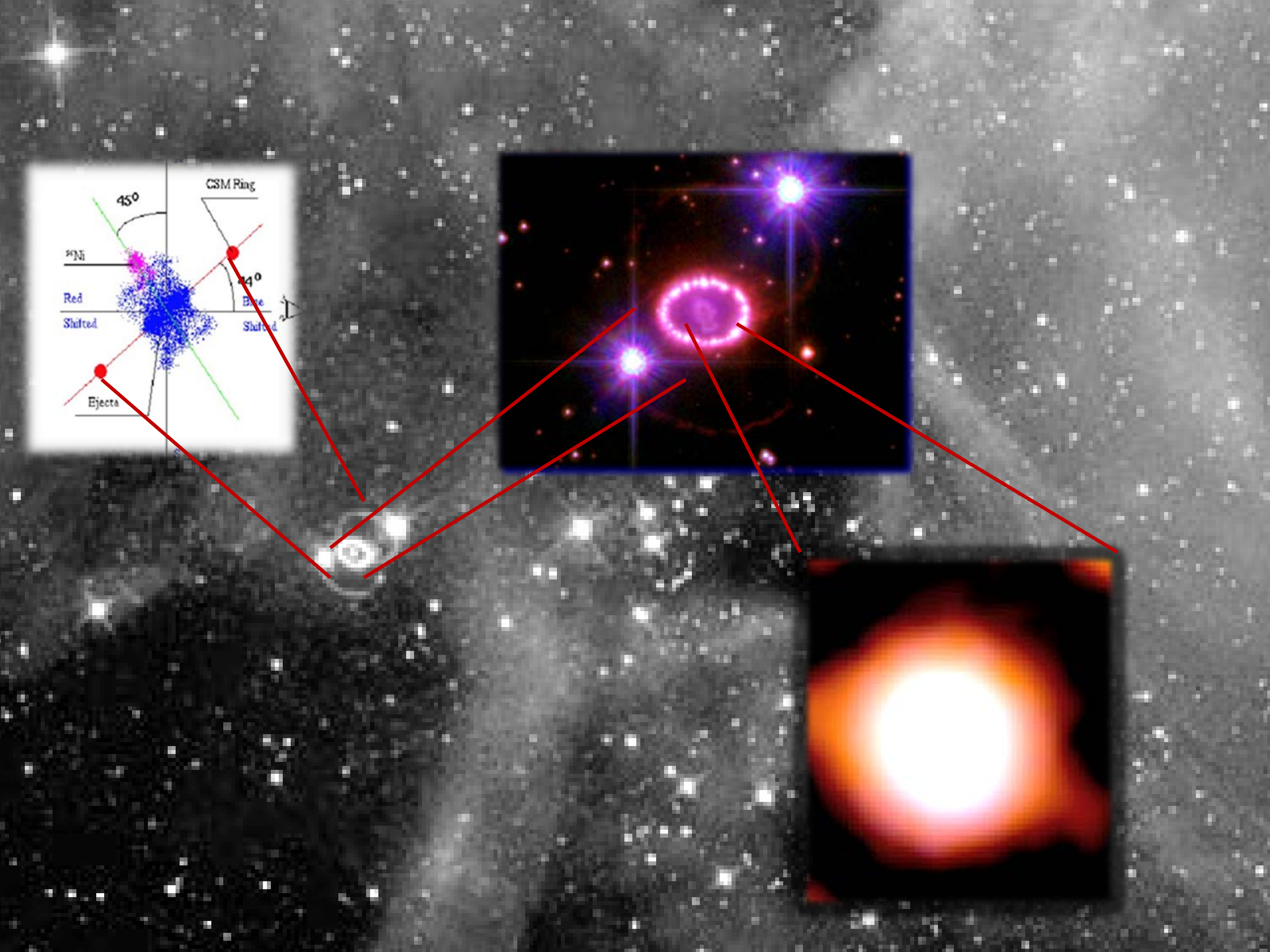
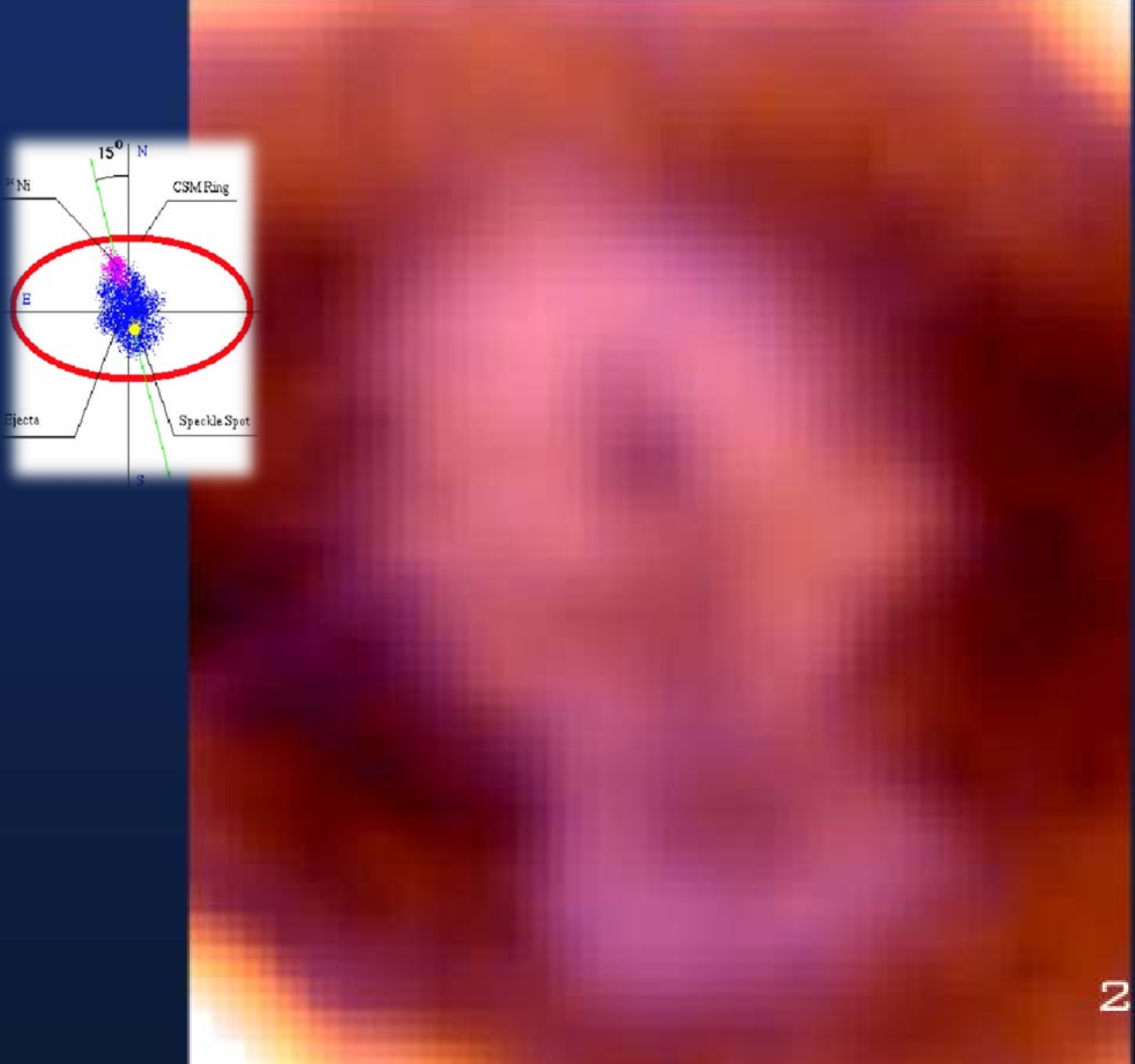
< NGC 2137
< NGC 2147
< NGC 2157
NGC 2156 >
< NGC 2159
NGC 2164


RMC 136a1
NGC 2060
SN 1987a

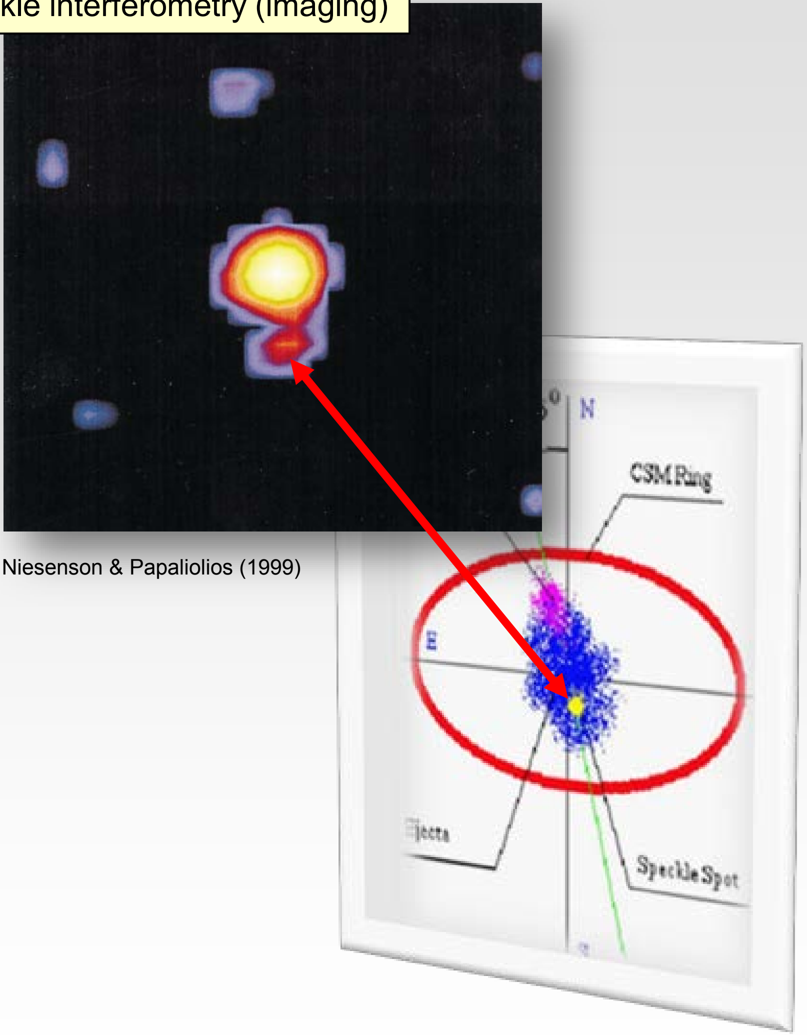

45°
CSM Ring
^{56}Ni
44°
Red Shifted
Blue Shifted
Ejecta



2006

Observational Evidence of Mixing

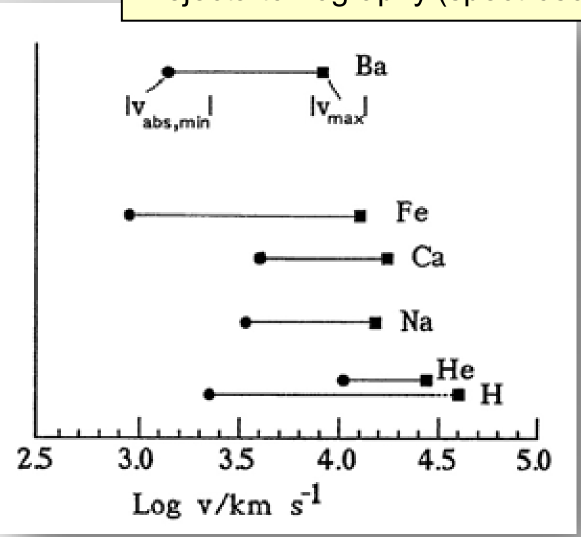
speckle interferometry (imaging)



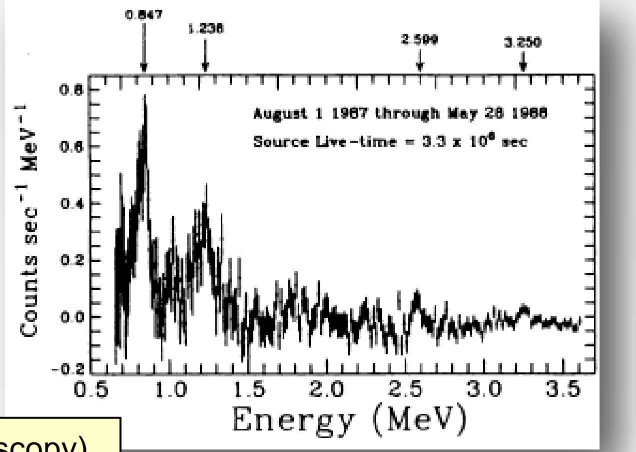
Niesenson & Papaliolios (1999)

Wang et al. (2002)

ejecta tomography (spectroscopy)



Hanuschik et al. (1991)



gamma rays (spectroscopy)

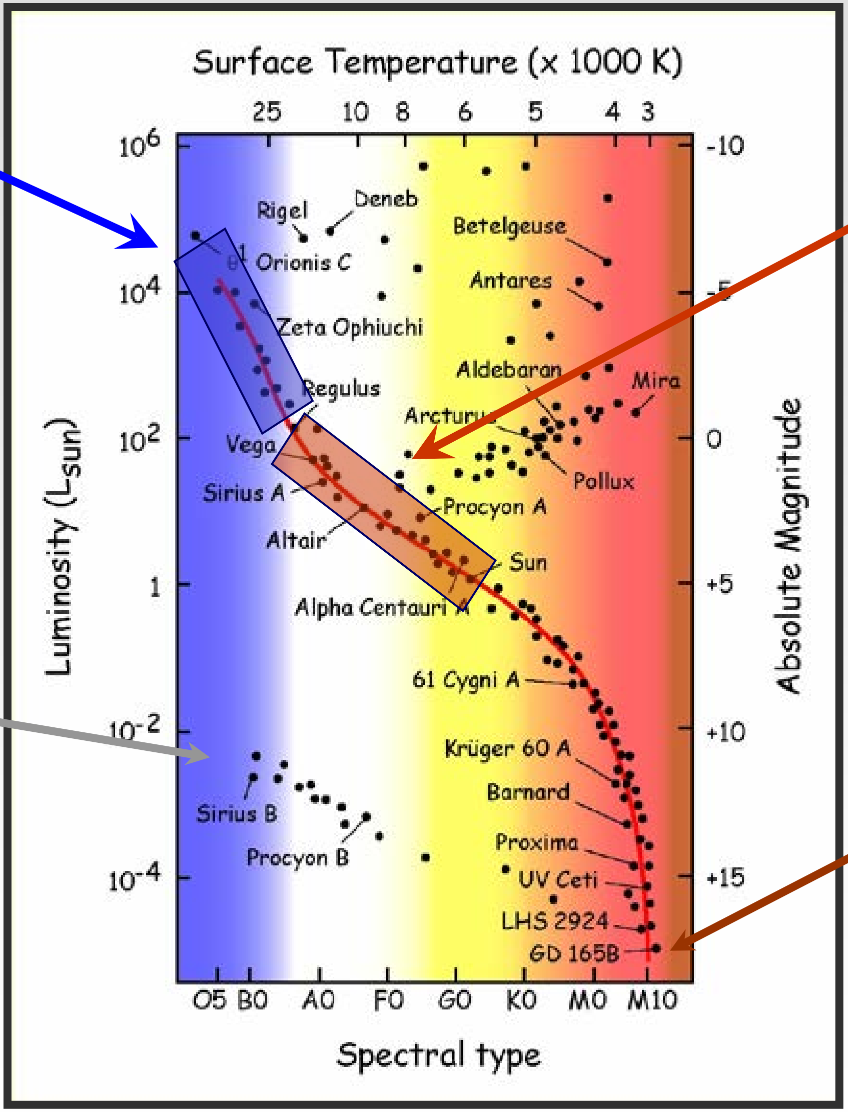
Leising & Share (1990)

Our Stellar Neighbours

Type II

Massive
Single
H-rich

White Dwarfs
m < 1.4 Msun

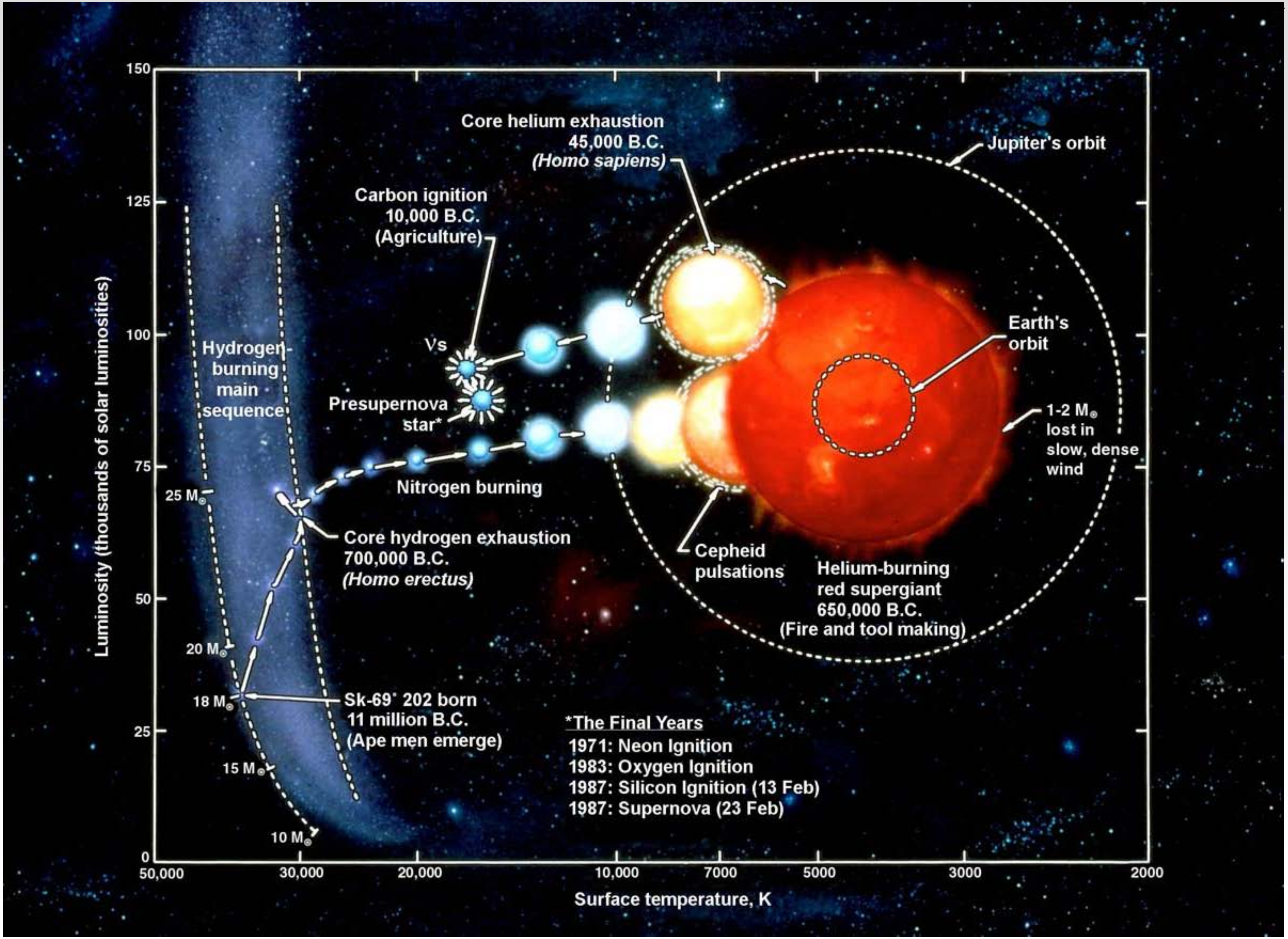


Type Ia

Medium
mass
Binary
H/He-free

Brown Dwarfs
m > 0.075 Msun

Supernovae From Single Stars



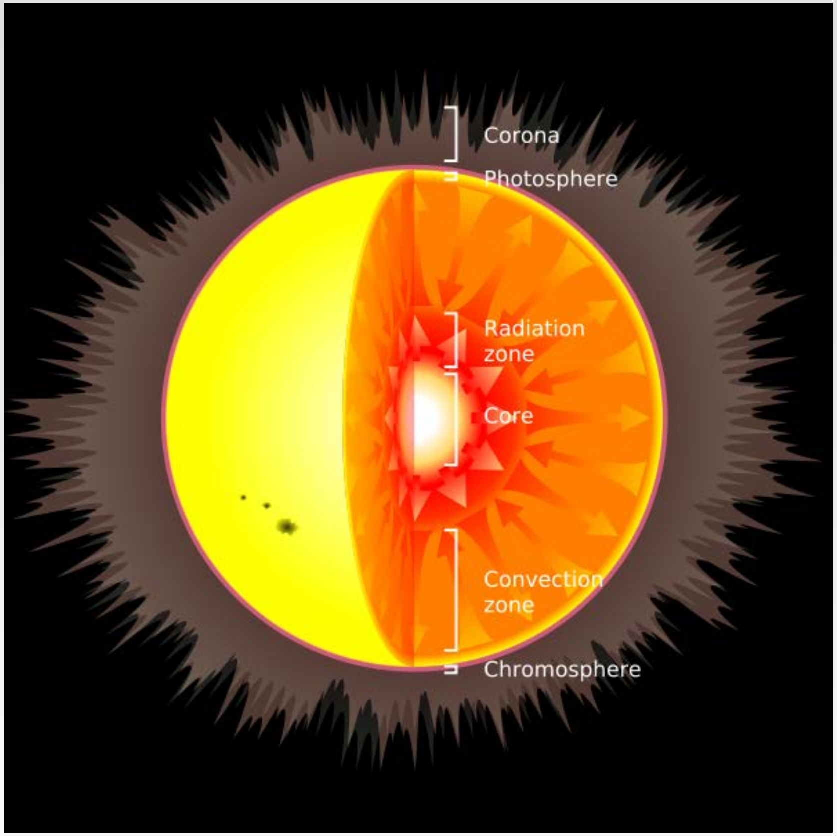
Computations in Stellar Evolution

$$\partial_t \mathbf{U} + \nabla \mathbf{F}(\mathbf{U}) = \mathbf{S}(\mathbf{U})$$

$$\nabla^2 \Phi = 4\pi G\rho$$

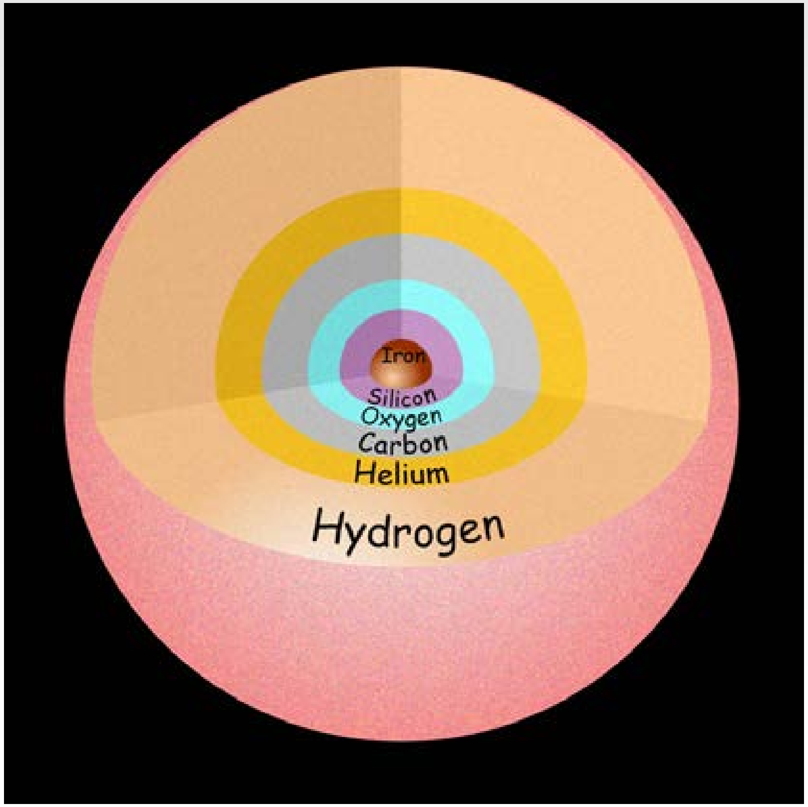
- **PDEs** of every possible type
- **ODEs** frequently stiff
- complex equation of state (first closure relation)
- **strongly coupled**
- **multidimensional** (4D...7D, more closure relations)
- various discretization methods (finite volume solvers, multigrid, particles, subgrid, front tracking)
- adaptive in space and time
- prone to produce demonstration runs
- unlimited computing resources (“**tree barking**”)

Internal Structure of Stars



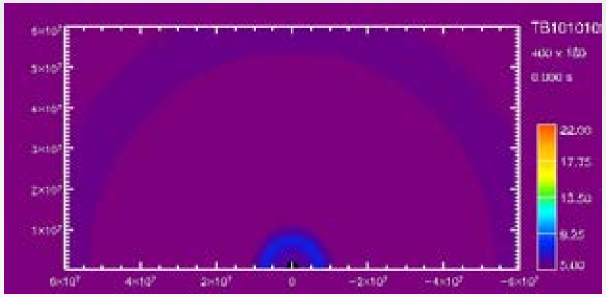
solar-type star

evolved massive star

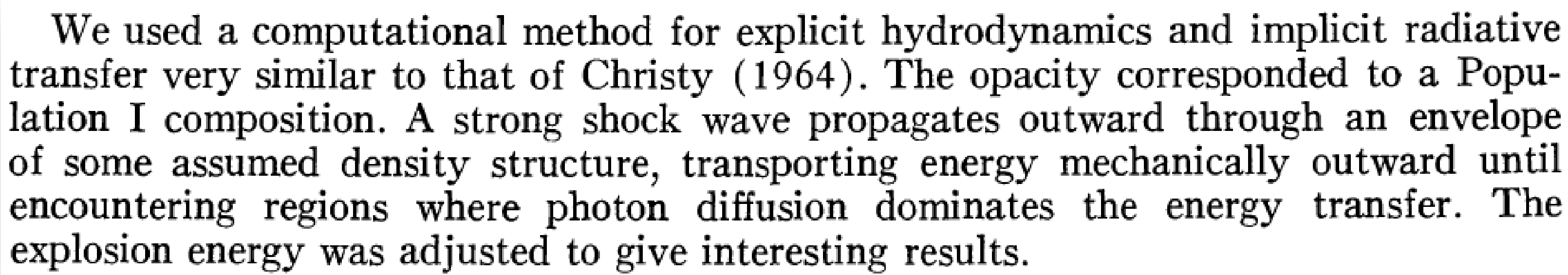


Core-Collapse SN Explosion Theory

- Massive stars
- Gravity bombs
- Energy extracted by neutrinos
- Accretion shock originally too weak
- Revived by neutrino heating of the post-shock matter



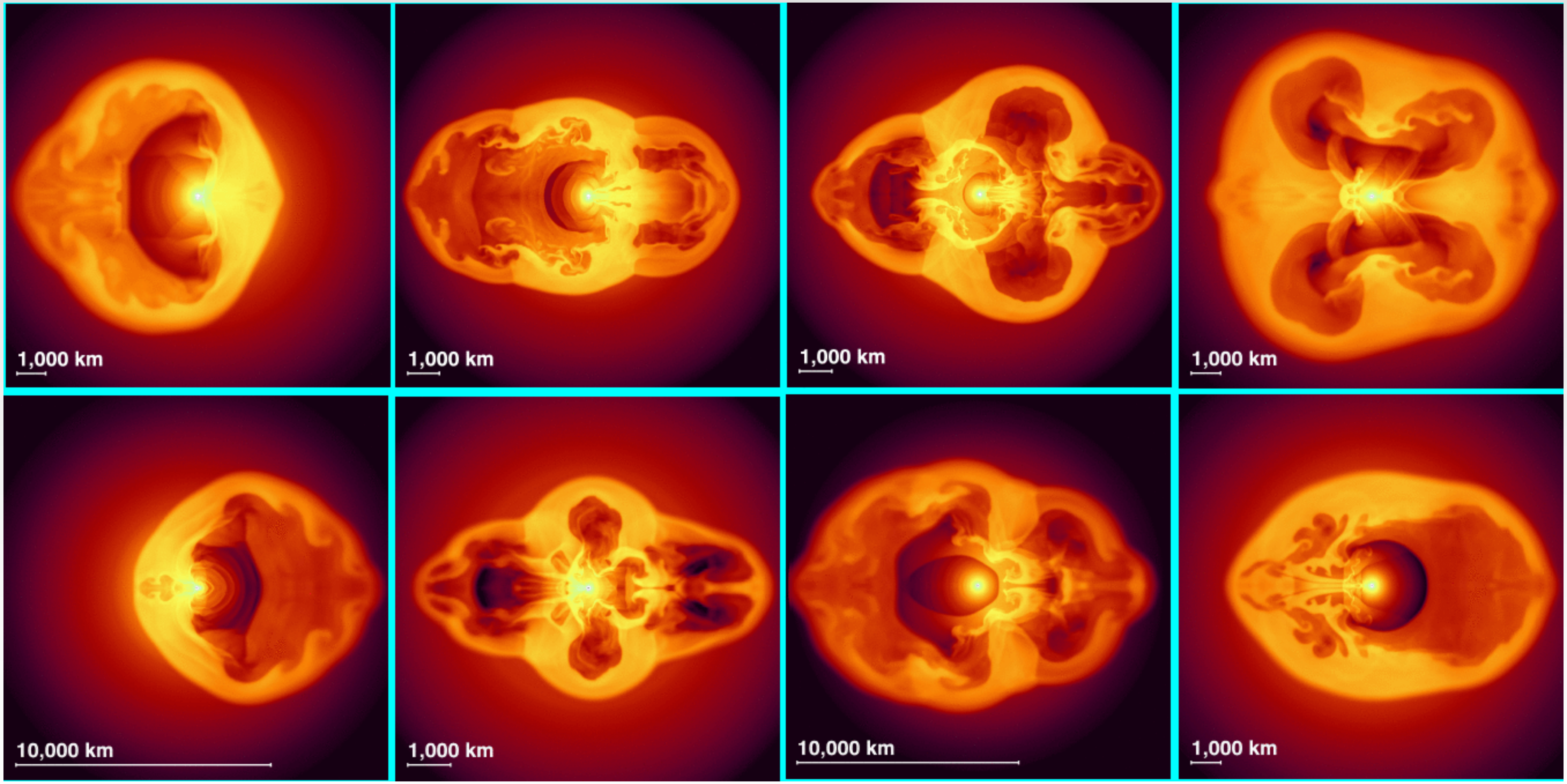
- Once the shock is launched…



We used a computational method for explicit hydrodynamics and implicit radiative transfer very similar to that of Christy (1964). The opacity corresponded to a Population I composition. A strong shock wave propagates outward through an envelope of some assumed density structure, transporting energy mechanically outward until encountering regions where photon diffusion dominates the energy transfer. The explosion energy was adjusted to give interesting results.

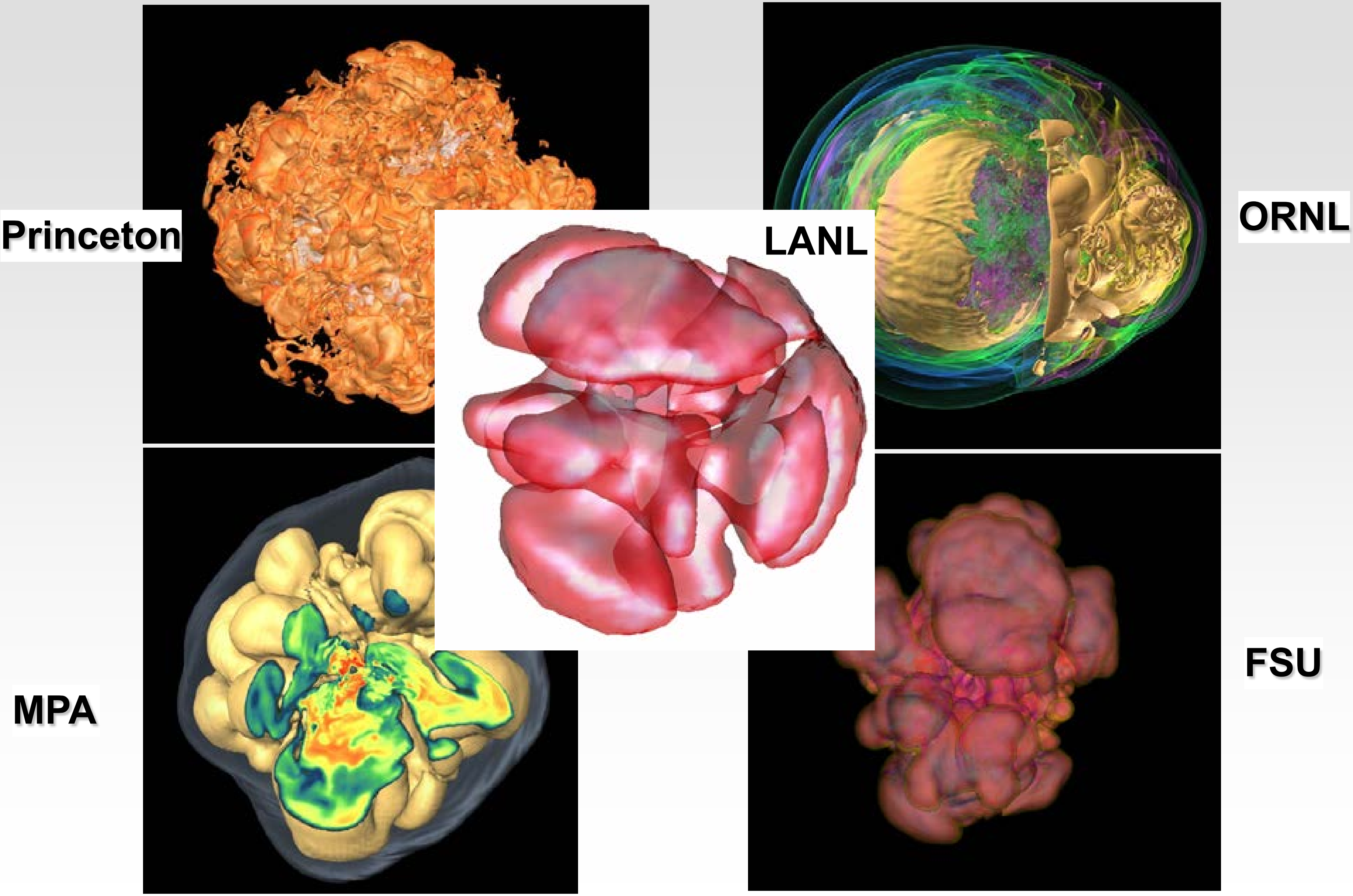
Falk & Arnett (1973)

Standing Accretion Shock Instability



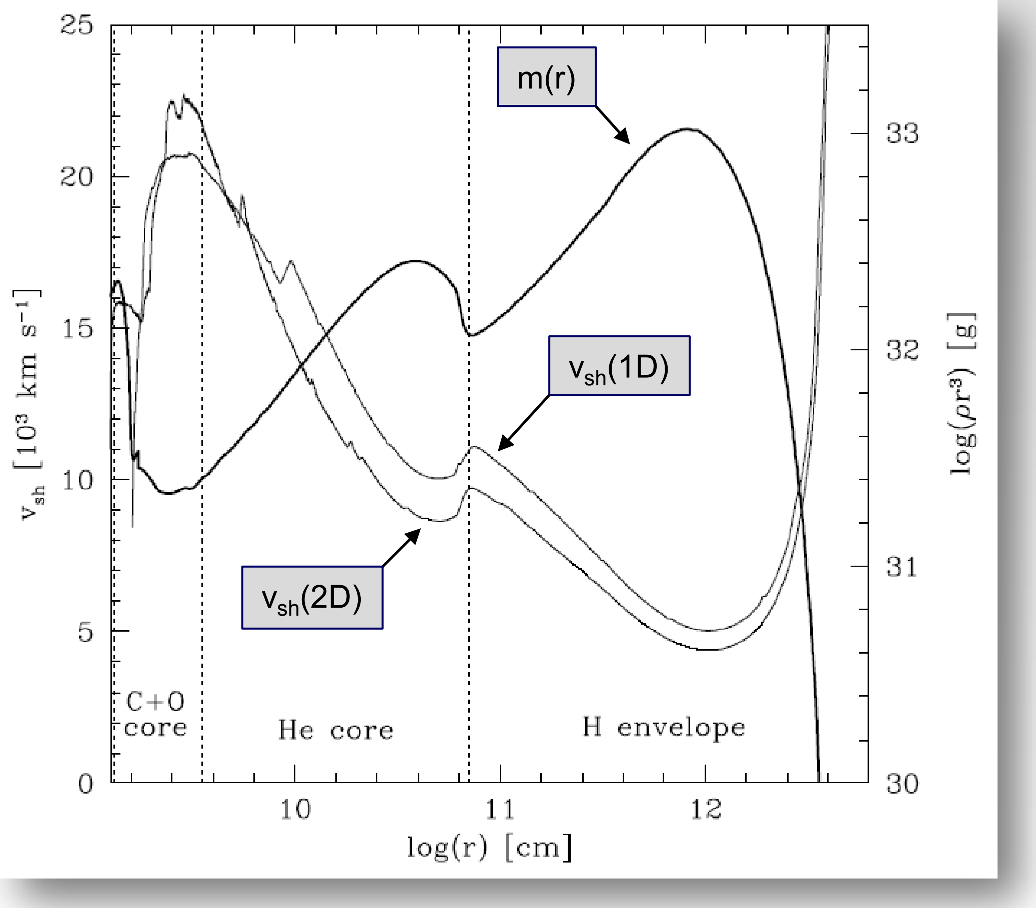
Janka et al. (2006)

ccSN Shock Revival in 3D



Origins of the ccSN RT Mixing

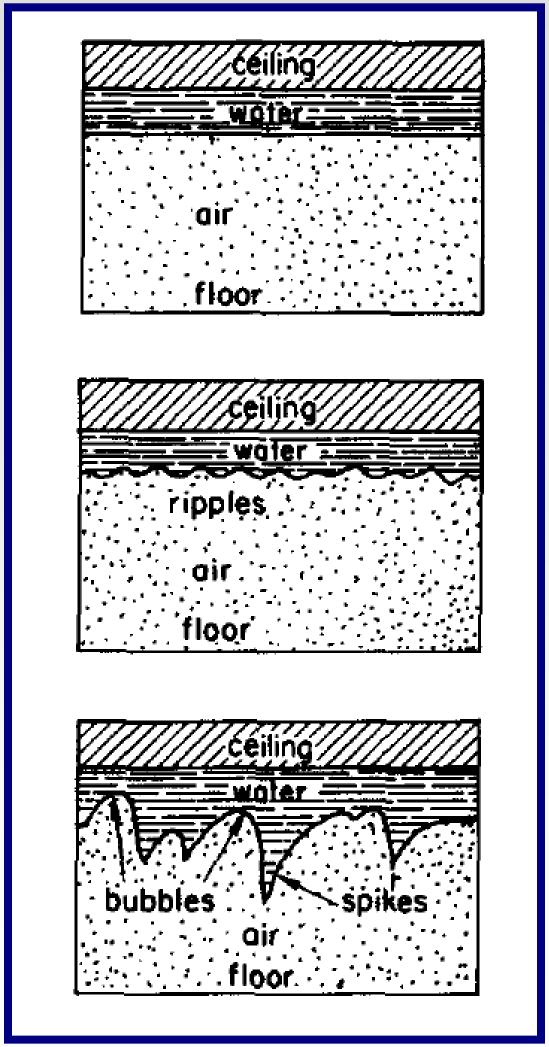
- Time-dependent deceleration of dense layers due to unsteady supernova shock motion though the progenitor envelope



Gawryszczak et al. (2010)

Accelerating material interface

- Column of air can easily support a thin flat layer of water (actually layers up to 10 meters thick!)
- However, in reality the interface will not be perfectly flat but slightly perturbed
- Water will "leak" down in form of spikes and be replaced with air bubbles



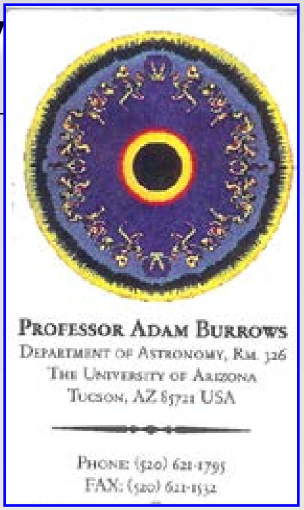
Sharp (1984)

Local interface dynamics

- Presence of small ripples translates into variable pressure support along the interface
- Parcels of fluid lying higher than average will experience higher pressure than needed to support them
- They will start rising pushing aside neighboring fluid elements
- The opposite applies to elements lying lower than average height: these will not have the sufficient support and will fall down

Rayleigh-Taylor instability

- Heavy fluid pushing against lighter fluid (bubble of rising ashes)
- Growth in time is exponential.
- There exists the most unstable mode.



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Department of Astronomy, Rm. 326
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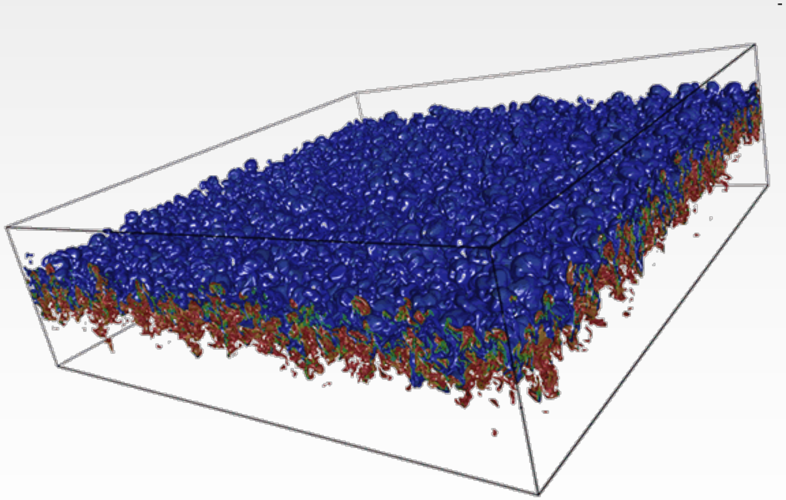
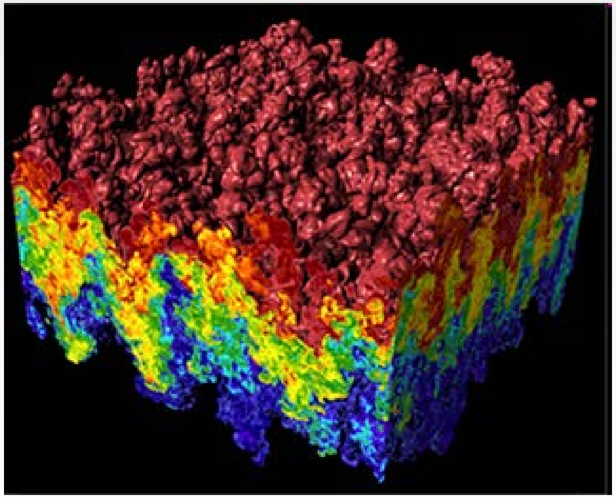
Tucson, AZ 85721 USA

Phone: (520) 621-1795
FAX: (520) 621-1532

A. Burrows



Livescu et al.

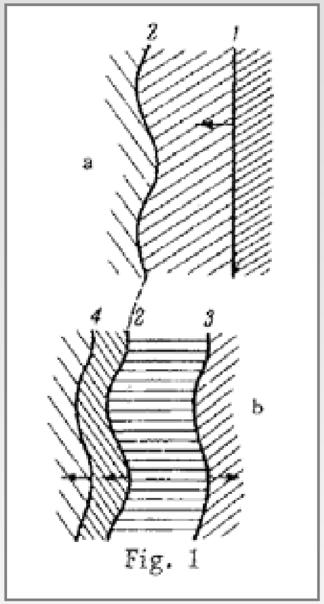


Cabot & Cook (2006)

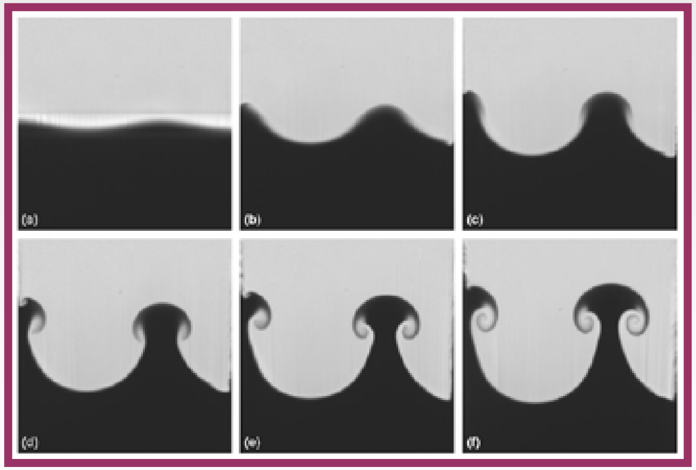
US DOE

Richtmyer-Meshkov instability

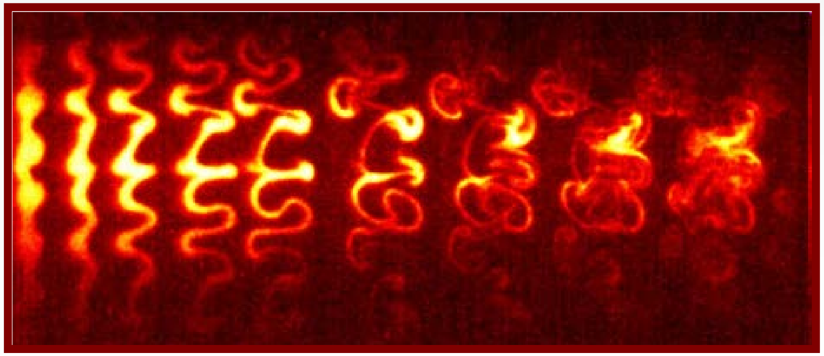
- RMI is similar to RTI but involves impulsive acceleration. Growth in time is linear. There exists the most unstable mode.



Meshkov (1969)



Jacobs & Krivets (2005)



Rightley et al. (1996)

Kelvin-Helmholtz instability

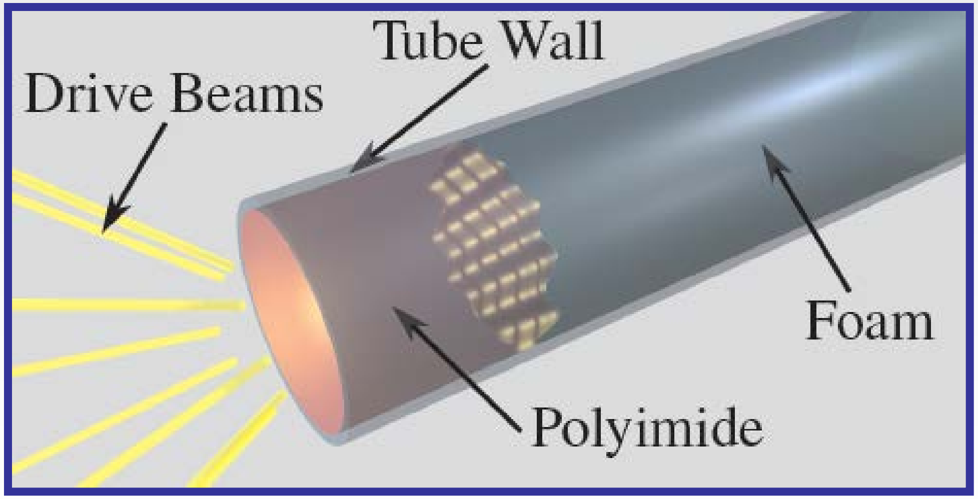
- The interface between two fluids is unstable if there is a jump in the tangential component of the velocity across the interface. All wavelengths are unstable.

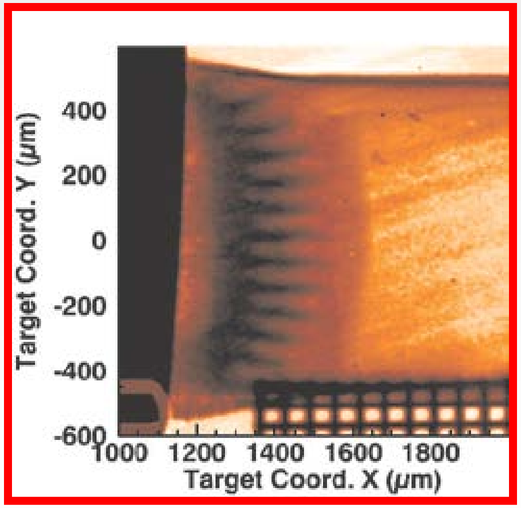
Denver, CO

Mt.Duval, Australia

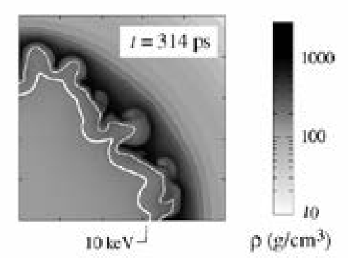
Combined RTI & RMI

- Passage of a laser-driven shock initially leads to RMI
- Rarefaction follows leading to a long-term deceleration of the interface and RTI growth



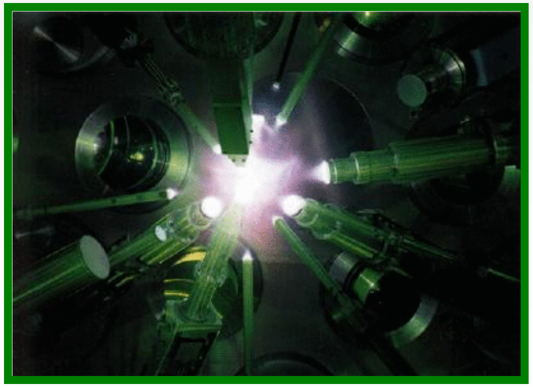


Importance/Examples

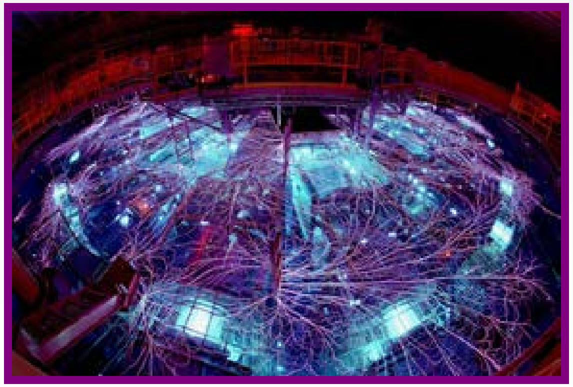


Atzeni et al. (2005)

- Inertial Confinement Fusion (RMI+RTI)
- Electromagnetic implosions (RTI)
- Core-collapse supernovae (RMI+RTI)
- Thermonuclear supernovae (RTI)
- Exhibits transition to turbulence



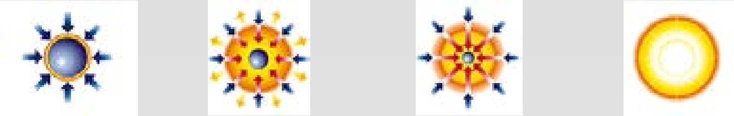
Omega/LLE Rochester



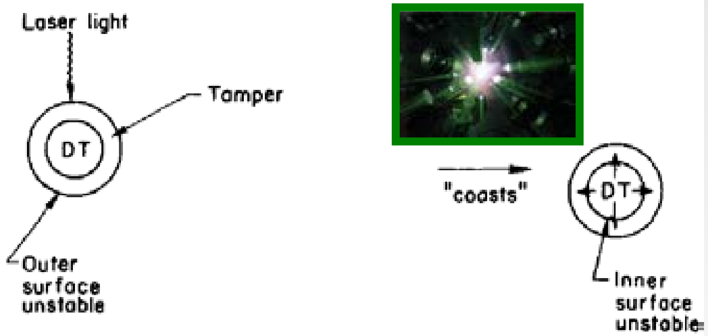
Z Machine/SNL

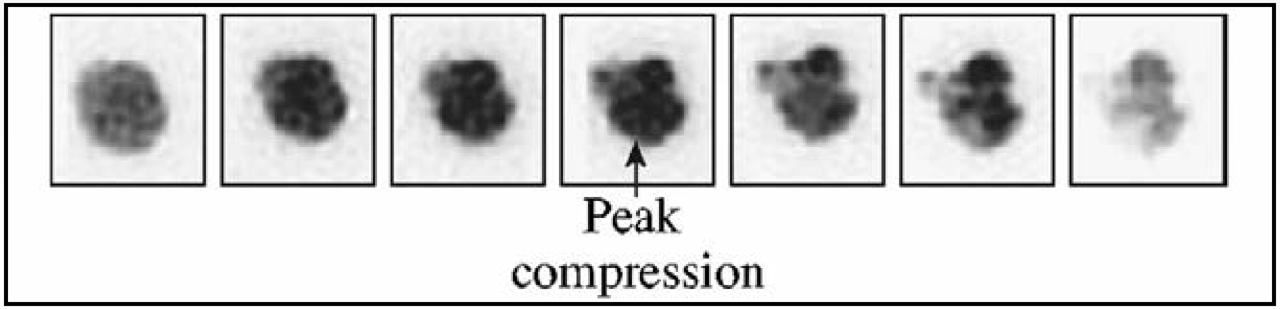
NIF/LLNL

Inertial Confinement Fusion



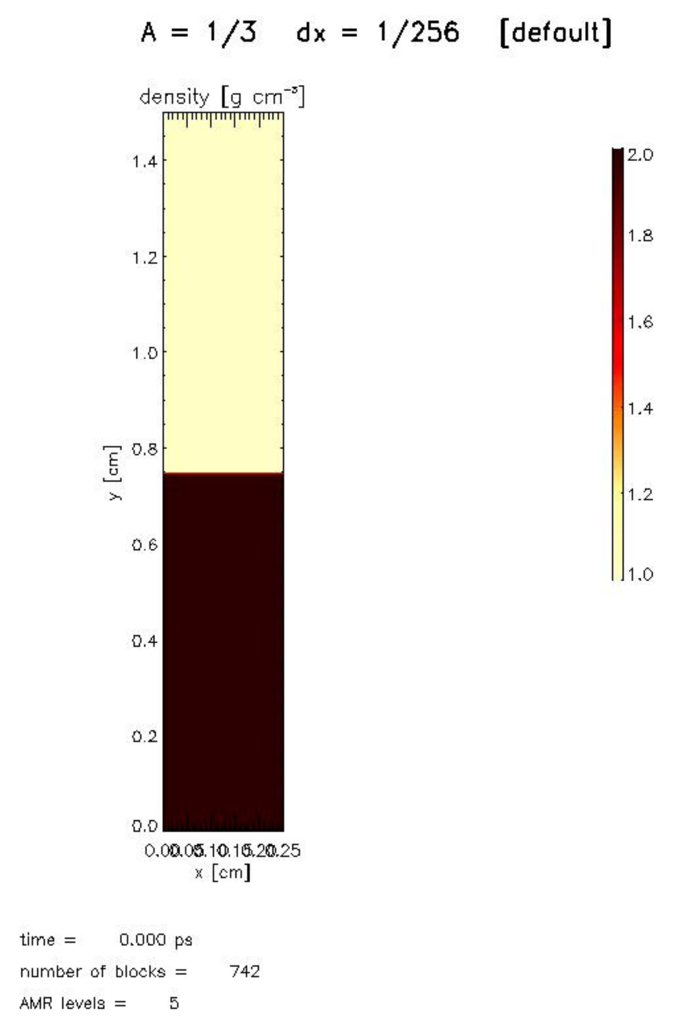
- Severely limits efficiency of the burn by decreasing density and temperature





Smalyuk et al. (2007)

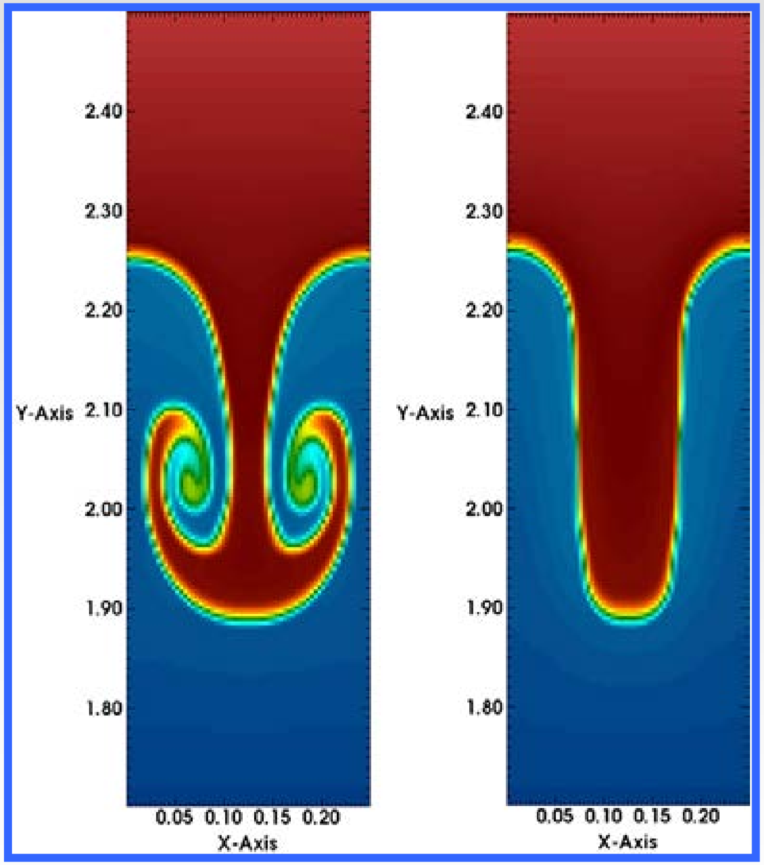
Single mode RTI in 2D



Magnetized RTI Model

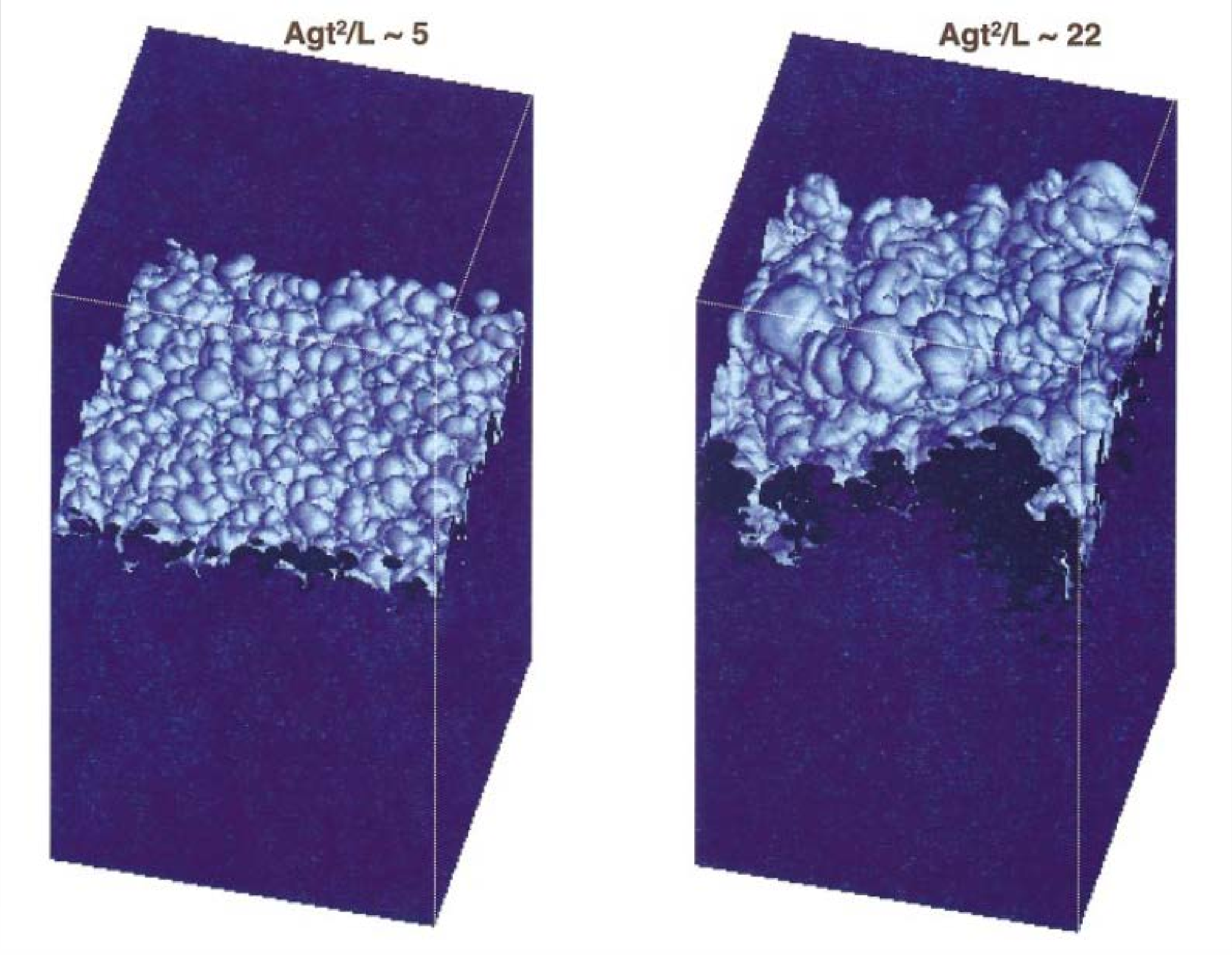
negligible field:
beta=250,000

modest field:
beta=2,500



See also early work by Jun, Norman, & Stone (1995).

Multimode RTI in 3D



RTI mixing layer growth

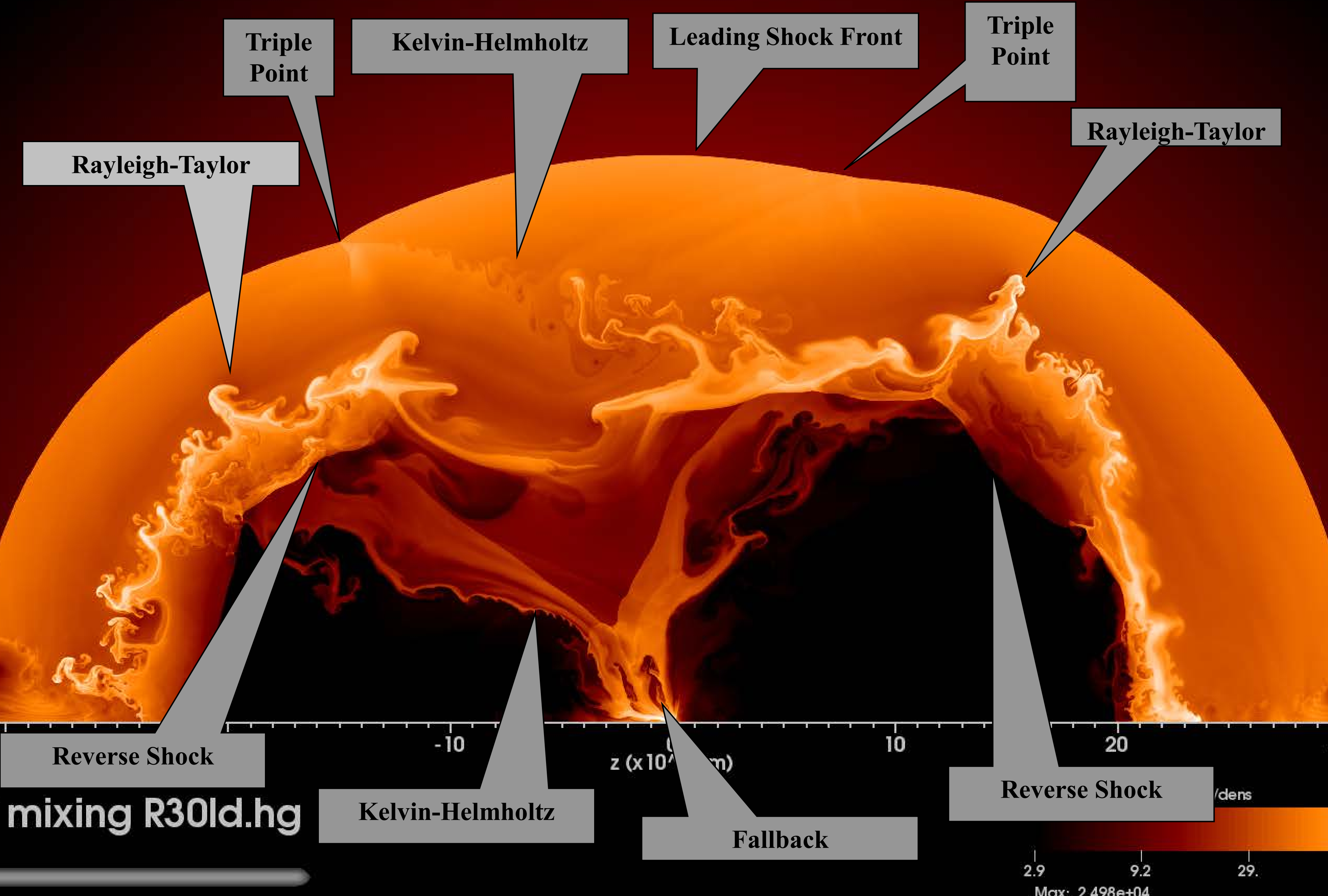
- This yields the famous “alpha”:

$$h(t) = \alpha Agt^2 + 2(\alpha Agh_0) + h_0$$

- Usually only the leading term is retained

$$\boxed{\alpha = \frac{h}{Agt^2}} - \left(\frac{ah_0}{Ag}\right)^{1/2} \frac{2}{t} - \frac{h_0}{Agt^2}$$

Complex Post-Explosion ccSN Dynamics



Triple Point

Kelvin-Helmholtz

Leading Shock Front

Triple Point

Rayleigh-Taylor

Rayleigh-Taylor

Reverse Shock

Kelvin-Helmholtz

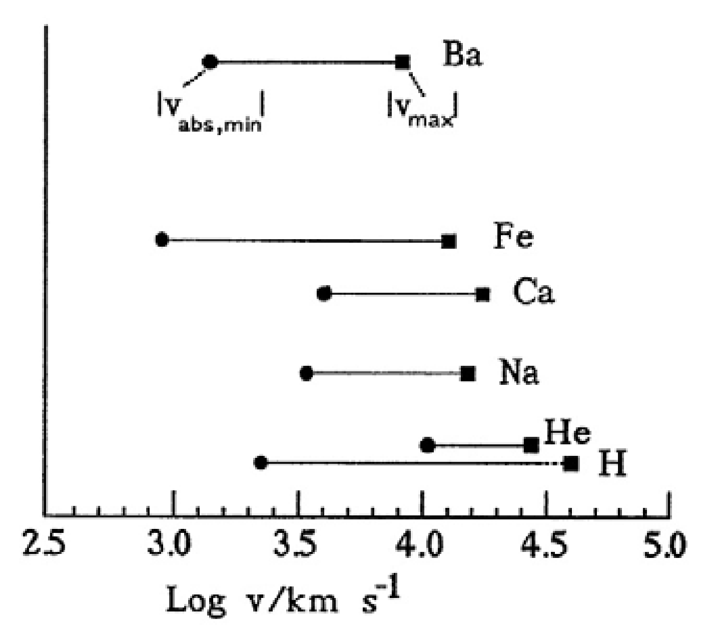
Fallback

Reverse Shock

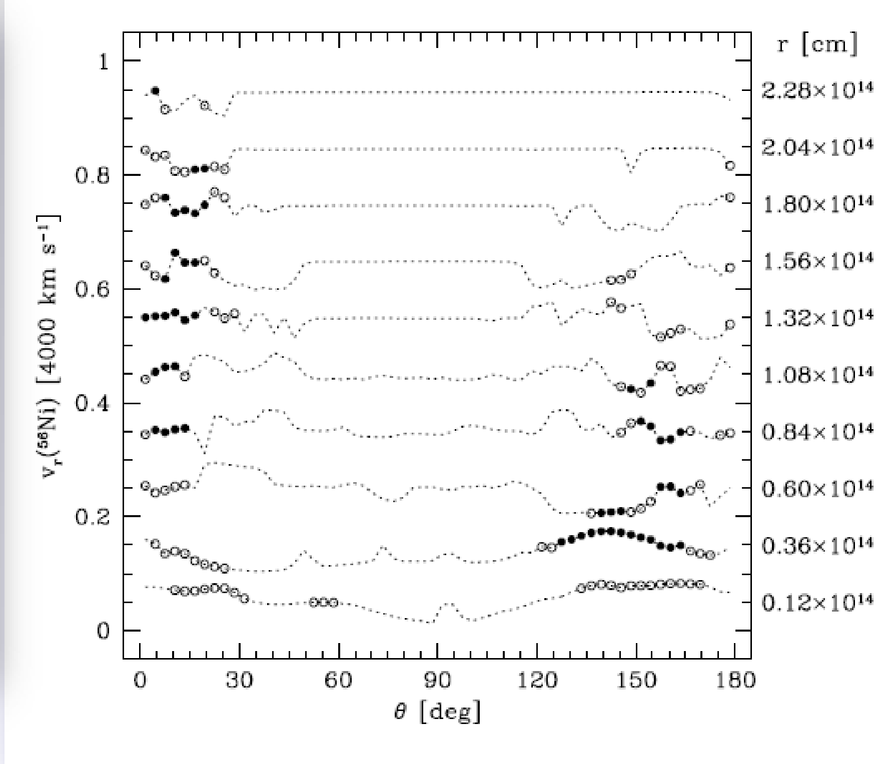
Model Validation: How Much ^{56}Ni? How Fast?

ejecta tomography (spectroscopy)

- Following SN 1987A observations, ^{56}Ni distribution evolution is one of the primary model evaluation criteria



Hanuschik et al. (1991)

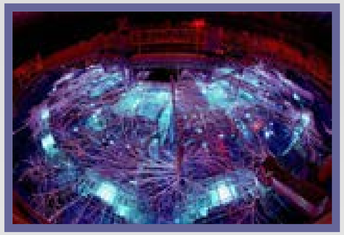


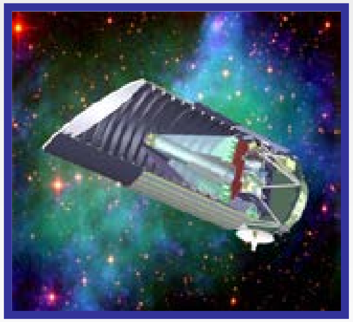
Gawryszczak et al. (2010)

Supernovae Do Not Love Us Back!

- Theory ultimately insufficient…☹
- Computations not terribly successful… ☹☹☹
- Experiments…? :-\

Experiments





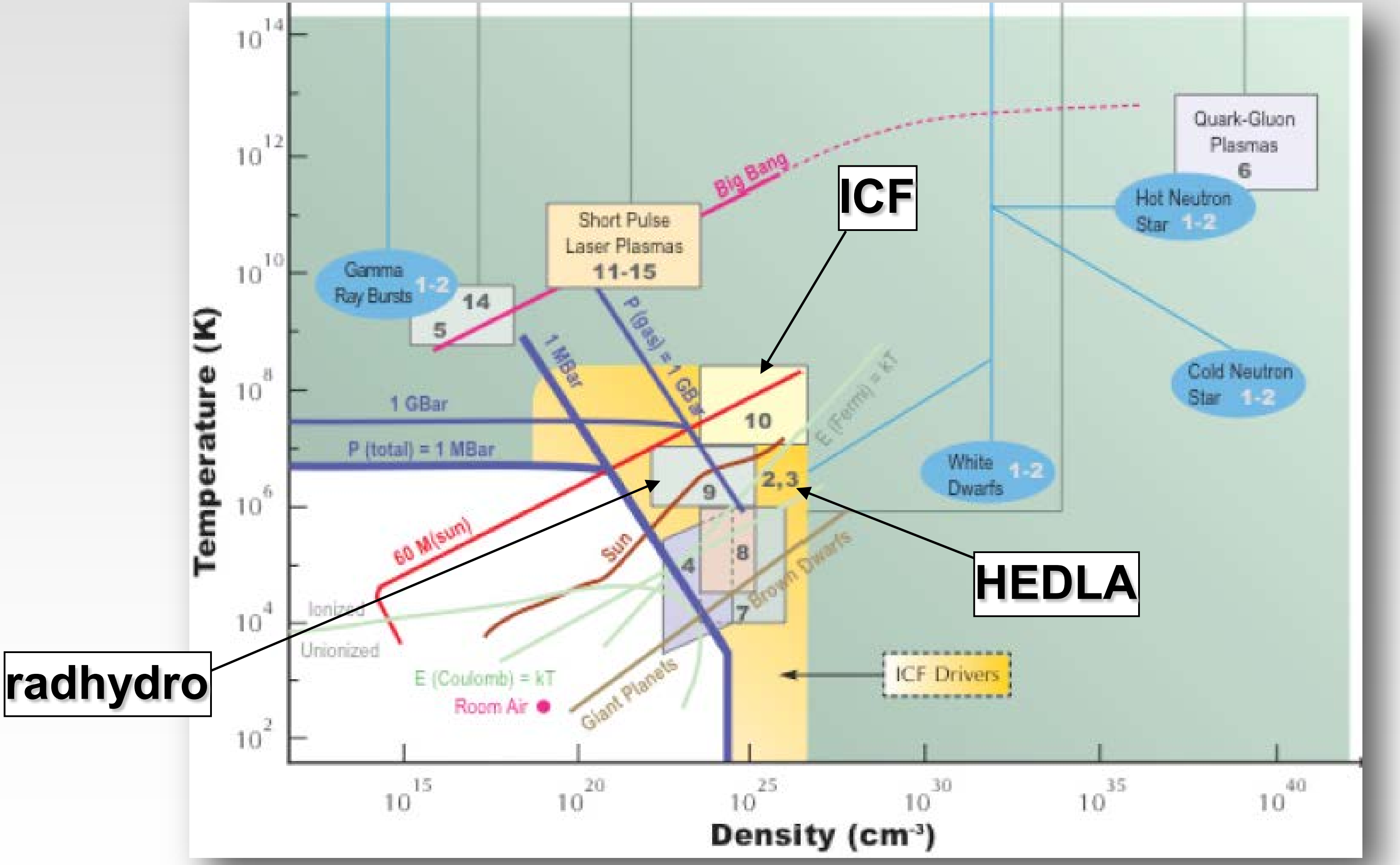
- Heavy ion and particle accelerators (RHIC, RIA…)
- Fluid dynamics and plasma experiments (stellar formation and collapse, jets)
- High-Energy Density Physics experiments (U.S., France, UK, Japan, China, EU)
  - Nuclear plasma physics

Dark-energy related astronomical missions:

- SDSS, SNLS, SN Factory, ESSENCE, HST…
- LSST, Pan-STARRS, South Pole Telescope, GSMT…
- JDEM (2014?), Constellation-X
- JWST (201x)

HED Laboratory Astrophysics

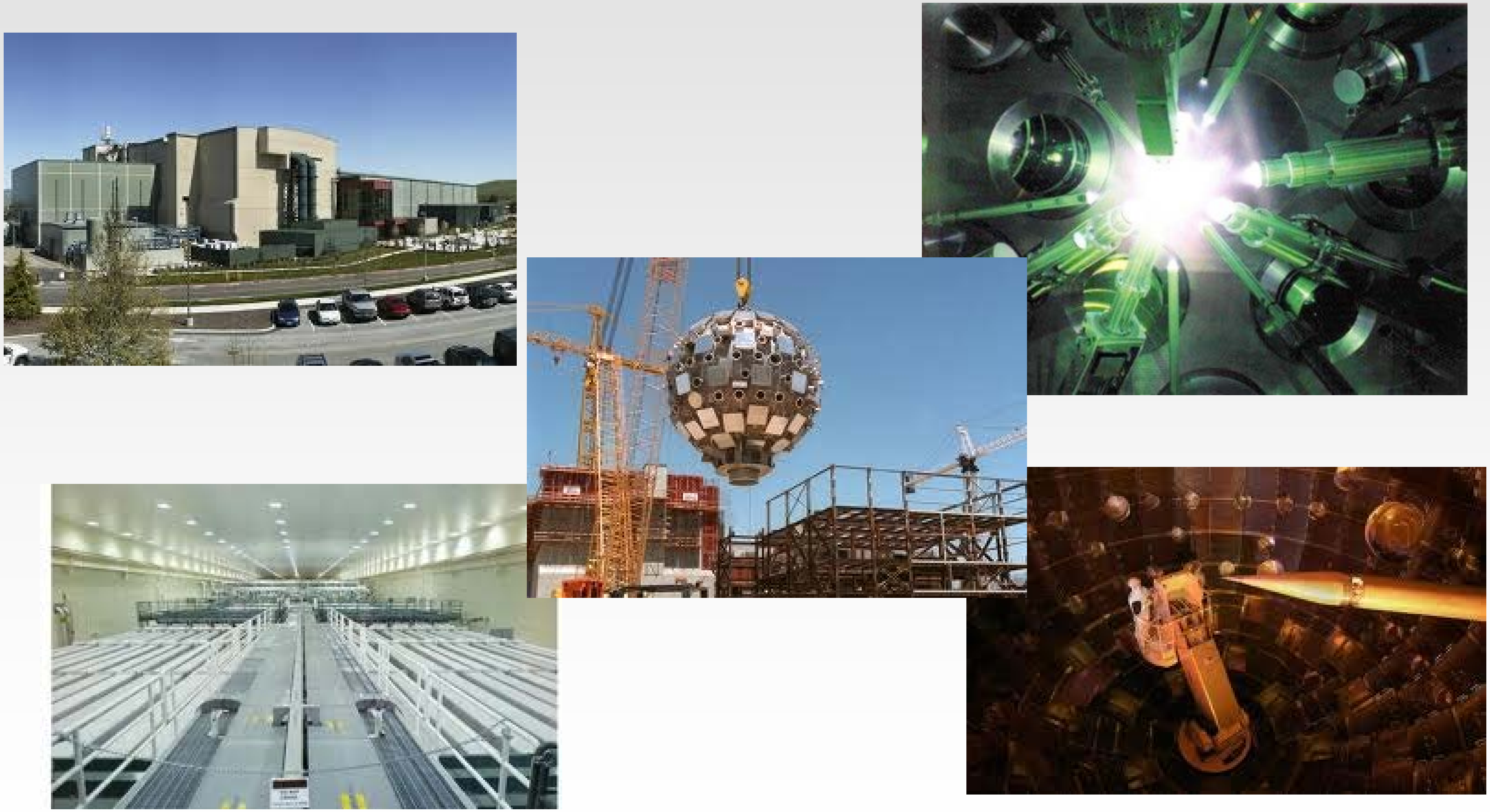
- **High Energy Density**: pressure > 1 Mbar (100 Mbar in IFC), energies > 10^{11} J/m^3 or > 10^{12} erg/cm^3, temperatures > 5×10^6 K or > 400 eV



Davidson et al. (2004)

National Ignition Facility (NIF)

- 2 MJ = 400 Twatts = 500 W per 24 hr = food for 10,000 men for 1 year = energy of a 1 tonne car at 100 mph



High-Energy Density Physics References

Connecting Quarks with the Cosmos (NRC 2002)

High Energy Density Physics: The X-Games of Contemporary Science (NRC 2003)

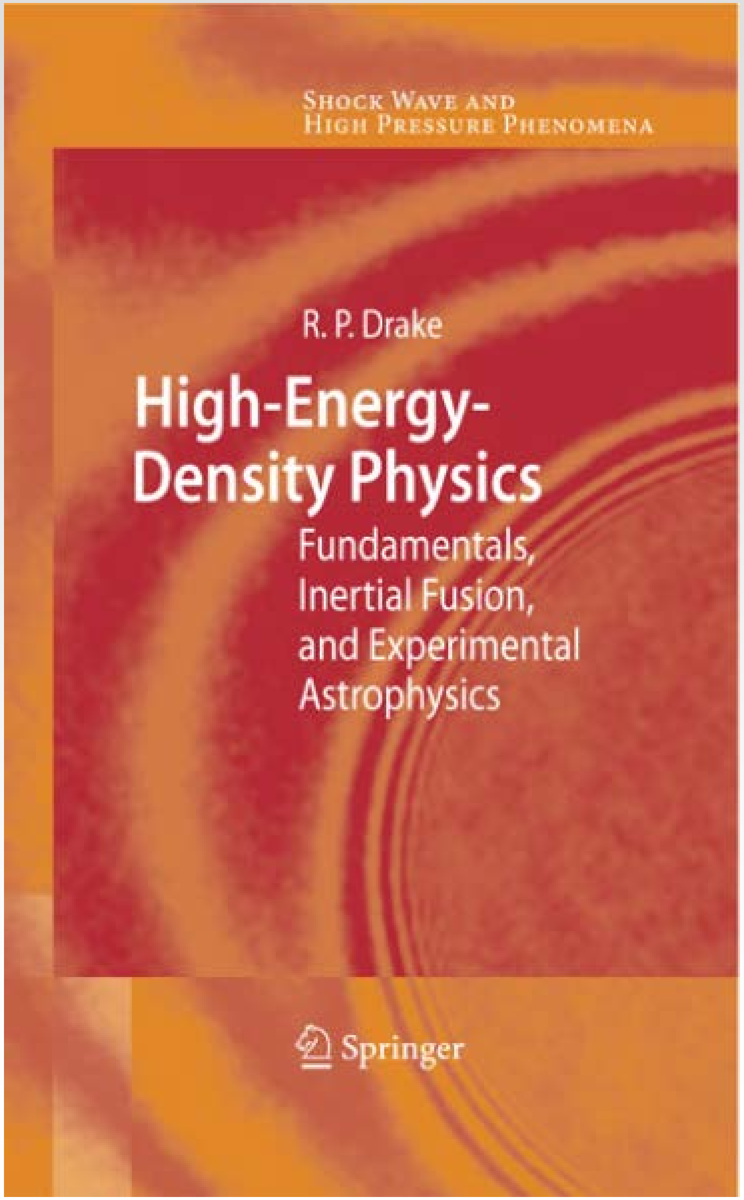
The Physics of the Universe (NSTC 2004)

Frontiers for Discovery in High Energy Density Physics (OSTP 2004)

Report of the Dark Energy Task Force (DOE/NASA/NSF 2006)

High Energy Density Physics textbook

- Drake, R. P. *High-Energy Density Physics – Fundamentals, Inertial Fusion, and Experimental Astrophysics* (Springer)



SHOCK WAVE AND
HIGH PRESSURE PHENOMENA

R. P. Drake

High-Energy-
Density Physics

Fundamentals,
Inertial Fusion,
and Experimental
Astrophysics

Springer