# Two texture zeros in neutrino mass matrices

Patrick Otto Ludl

Faculty of Physics, University of Vienna

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# Lepton mixing

Lepton mass terms (we assume Majorana neutrinos):

$$\mathcal{L} = -\bar{\ell}'_R \mathcal{M}_\ell \ell'_L + \frac{1}{2} {\nu'_L}^T C^{-1} \mathcal{M}_\nu \nu'_L + \mathrm{H.c.} =$$
$$= -\bar{\ell}_R \hat{\mathcal{M}}_\ell \ell_L + \frac{1}{2} {\nu_L}^T C^{-1} \hat{\mathcal{M}}_\nu \nu_L + \mathrm{H.c.}$$

Diagonalization by a biunitary transformation:

$$\mathcal{M}_\ell = U_R^\ell \hat{\mathcal{M}}_\ell U_L^{\ell\dagger}, \quad \mathcal{M}_\nu = U_L^{\nu *} \hat{\mathcal{M}}_\nu U_L^{\nu\dagger}$$

$$\Rightarrow U_L^{\nu\dagger} \nu'_L = \nu_L, \quad U_L^{\ell\dagger} \ell'_L = \ell_L$$

Charged current interaction:

$$\mathcal{L}_{\mathrm{CC}} = -\frac{g}{\sqrt{2}} W_\mu^- \bar{\ell}'_L \gamma^\mu \nu'_L + \mathrm{H.c.} = -\frac{g}{\sqrt{2}} W_\mu^- \bar{\ell}_L \gamma^\mu \underbrace{U_L^{\ell\dagger} U_L^\nu}_{U_{\mathrm{PMNS}}} \nu_L + \mathrm{H.c.}$$

# Parameters of the lepton mixing matrix

$U_{\mathrm{PMNS}}$ can be parameterized by three mixing angles, the CP-violating phase $\delta$ and two physical Majorana phases.

| parameter | best fit | $2\sigma$ | HPS |
|---|---|---|---|
| $\sin^2\theta_{12}$ | $0.320^{+0.015}_{-0.017}$ | $0.29 - 0.35$ | $\frac{1}{3}$ |
| $\sin^2\theta_{23}$ | $0.49^{+0.08}_{-0.05}$ | $0.41 - 0.62$ | $\frac{1}{2}$ |
| | $0.53^{+0.05}_{-0.07}$ | $0.42 - 0.62$ | $\frac{1}{2}$ |
| $\sin^2\theta_{13}$ | $0.026^{+0.003}_{-0.004}$ | $0.019 - 0.033$ | 0 |
| | $0.027^{+0.003}_{-0.004}$ | $0.020 - 0.034$ | 0 |

M. Tórtola, J.W.F. Valle and D. Vanegas (2012)[1]

[1] arXiv: 1205.4018 (17 May 2012)

For a long time $U_{\mathrm{PMNS}}$ seemed to be compatible with the Harrison-Perkins-Scott (HPS) mixing matrix

$$U_{\mathrm{HPS}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

$\rightarrow$ Idea: **symmetries in the lepton sector**

Recent data from oscillation experiments (T2K, MINOS, Double Chooz, Daya Bay, RENO)

$$\Rightarrow \theta_{13} > 0 \quad \text{at} \quad > 6\sigma!$$

$$\text{Global fit}^2: \quad \sin^2\theta_{13} = 0.026^{+0.003}_{-0.004} \quad (0.027^{+0.003}_{-0.004}).$$

Although $\theta_{13} > 0$, the idea of symmetries in the lepton sector is still interesting.

[2] M. Tórtola, J.W.F. Valle and D. Vanegas (2012), arXiv: 1205.4018

# Texture zeros

A simple way to restrict the lepton mass matrices (and therefore also $U_{\mathrm{PMNS}}$) is to assume so-called texture zeros in the mass matrices:

$$\mathcal{M}_{ij} = 0 \quad \text{for some } i, j.$$

In the following we will assume:

- The charged lepton mass matrix $\mathcal{M}_\ell$ is diagonal.
- Neutrinos are Majorana particles ($\rightarrow \mathcal{M}_\nu$ symmetric).
- Texture zeros in the neutrino mass matrix $\mathcal{M}_\nu$.

Within this framework: More than two texture zeros in $\mathcal{M}_\nu \Rightarrow$ incompatible with experimental data.

$\Rightarrow$ (At most) two texture zeros.

# Two texture zeros

P.H. Frampton, S.L. Glashow, D. Marfatia (2002)[3]:

In the basis where $\mathcal{M}_\ell$ is diagonal, there are seven cases of two texture zeros that are compatible with the experimental data.

$$A_1 = \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}, \quad B_1 = \begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix},$$

$$B_2 = \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}, \quad B_3 = \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}, \quad B_4 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix},$$

$$C = \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}.$$

[3] Phys.Lett. B536 (2002) 79-82 [hep-ph/0201008]

# Possible predictions of two texture zeros in $\mathcal{M}_\nu$

C.I. Low (2005)[4]:

**The only extremal mixing angle which can be enforced by texture zeros is $\theta_{13} = 0$.**

$\Rightarrow$ Problems:

- $\theta_{13} = 0$ is ruled out by experiment.
- $\theta_{23} = 45^\circ$ cannot be enforced by texture zeros.

However, even if texture zeros do not restrict the mixing angles to certain values:

Texture zeros imply relations among the observables ($m_i$,$\Delta m^2_{ij}$, $\sin^2\theta_{ij}$, $\delta, \sigma_i$).

[4] Phys. Rev. D71 (2005) 073007 [hep-ph/0501251].

# Near maximal atmospheric mixing from texture zeros

Texture zeros cannot enforce $\theta_{23} = 45^\circ$

$\rightarrow$ Can $\theta_{23} \approx 45^\circ$ be achieved with an additional assumption?

W. Grimus, POL (2011)[5]:

1. Texture zeros of type $B_3$ or $B_4$
2. Quasi-degenerate neutrino mass spectrum: $m_i \gg \sqrt{|\Delta m^2_{ij}|}$

$$\Rightarrow \theta_{23} \approx 45^\circ.$$

Note: If one assumes a quasi-degenerate neutrino mass spectrum one has to take into account the cosmological bounds: PDG (2010):

$$\sum_i m_i < 1\,\mathrm{eV} \Rightarrow \text{quasi-degenerate spectrum not excluded yet.}$$

[5] Phys. Lett. B700 (2011) 356-361 [arXiv:1104.4340]

# Near maximal atmospheric mixing from texture zeros

$$B_3:\ \mathcal{M}_\nu \sim \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix},\quad B_4:\ \mathcal{M}_\nu \sim \begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix}$$

$\mathcal{M}_\ell$ diagonal $\Rightarrow U_{\mathrm{PMNS}} = U_L^\nu =: U$.

$$\mathcal{M}_\nu = U^* \operatorname{diag}(m_1,\, m_2,\, m_3)\, U^\dagger$$

Two texture zeros in $\mathcal{M}_\nu \Rightarrow$ two equations of the form

$$(\mathcal{M}_\nu)_{ij} = \sum_k m_k U_{ik}^* U_{jk}^* = 0.$$

Input: $m_1,\, m_2,\, m_3,\, \theta_{12},\, \theta_{13} \Rightarrow$ predictions: $\theta_{23},\, \delta,\, \sigma_{1,2}$

$\rightarrow$ One obtains a cubic equation for $\lambda = \tan^2\theta_{23}$.

# The limit of a quasi-degenerate spectrum

In the limit of a quasi-degenerate neutrino mass spectrum the cubic equation becomes

$$\lambda^3 + \lambda^2 - \lambda - 1 = 0$$

$$\Rightarrow \lambda = \tan^2\theta_{23} = 1 \Rightarrow \sin^2\theta_{23} = \frac{1}{2} \Rightarrow \theta_{23} = 45^\circ.$$

**Limit independent of the values of $\theta_{12}$ and $\theta_{13}$!**

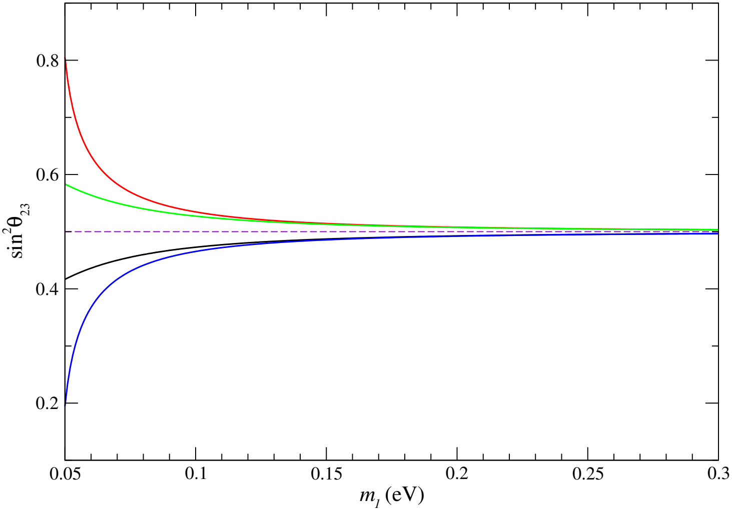
Approximate solution of the exact equation:

$$\sin^2\theta_{23} \simeq \frac{1}{2} \mp \frac{1}{8}\frac{\Delta m^2_{31}}{m_1^2}(1 + \sin^2\theta_{13})$$

# The limit of a quasi-degenerate spectrum

($\Delta m^2_{21}$, $\Delta m^2_{31}$, $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$ fixed to best fit values.)

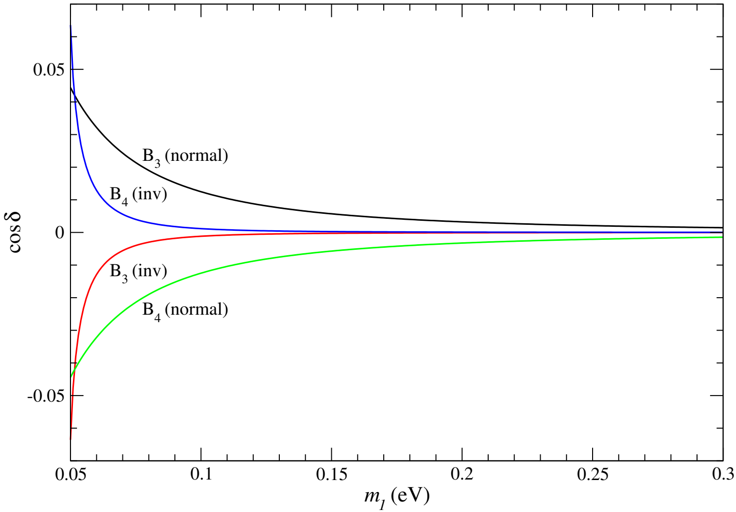
$B_3$ (inverted) $B_4$ (normal) $B_3$ (normal) $B_4$ (inverted)



# The limit of a quasi-degenerate spectrum

In the limit of a quasi-degenerate mass spectrum:

$$\tan\theta_{12} \sin\theta_{13} \cos\delta \to 0.$$



# Two texture zeros and the relation between $\theta_{13}$ and $\delta$

- $\sin^2\theta_{13}$ well known by now.
- CP-phase $\delta$ almost not restricted at a significant level.

$\Rightarrow$ Interesting question: Correlation between $\theta_{13}$ and $\delta$ in models with two texture zeros?

POL, S. Morisi, E. Peinado (2011)[6]:

Two texture zeros $A_1, \ldots, C$ ($\mathcal{M}_\ell$ diagonal)

$\Rightarrow$ two equations of the form

$$(\mathcal{M}_\nu)_{ij} = \sum_k m_k U^*_{ik} U^*_{jk} = 0.$$

Leads (in the seven viable cases) to an at most cubic equation for $\cos\delta$ as a function of the ratio $\Delta m^2_{21}/\Delta m^2_{31}$ and the three mixing angles.

[6] Nucl. Phys. B857 (2012) 411-423 [arXiv:1109.3393]

$\Rightarrow$ Vary $\Delta m^2_{21}/\Delta m^2_{31}$, $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$ in their $n\sigma$-ranges.

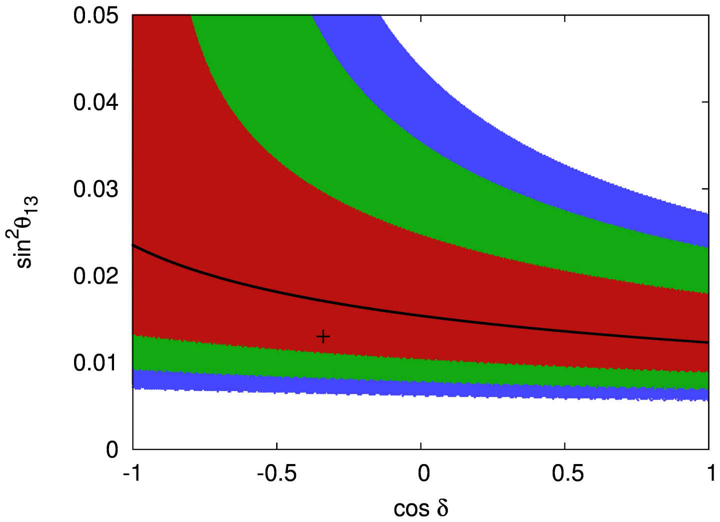
$\Rightarrow$ One obtains plots of the allowed regions for $\sin^2\theta_{13}$ and $\cos\delta$.

Colors: black: best fit, red: $1\sigma$, green: $2\sigma$, blue: $3\sigma$.

The best fit points[7] for $\sin^2\theta_{13}$ and $\cos\delta$ are indicated by black crosses.

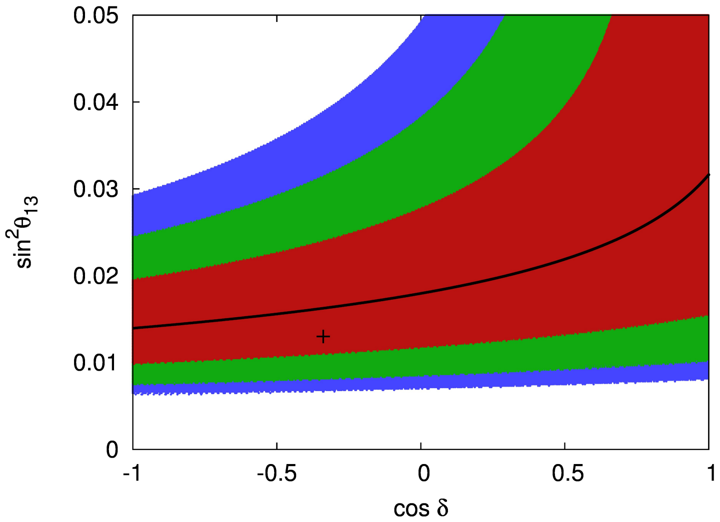
[7] T. Schwetz, M. Tórtola and J.W.F. Valle, New J. Phys. 13 (2011) 109401 [arXiv:1108.1376].

$A_1$: only normal spectrum allowed:



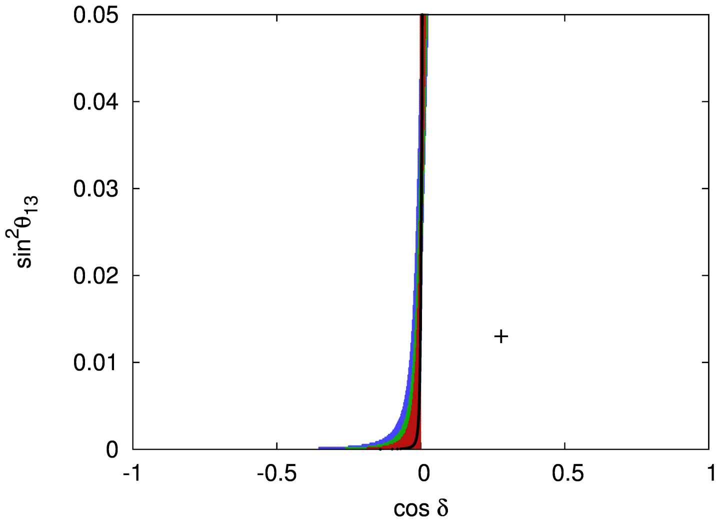

"Predicts" $\sin^2\theta_{13} > 0$.

Similar for $A_2$:



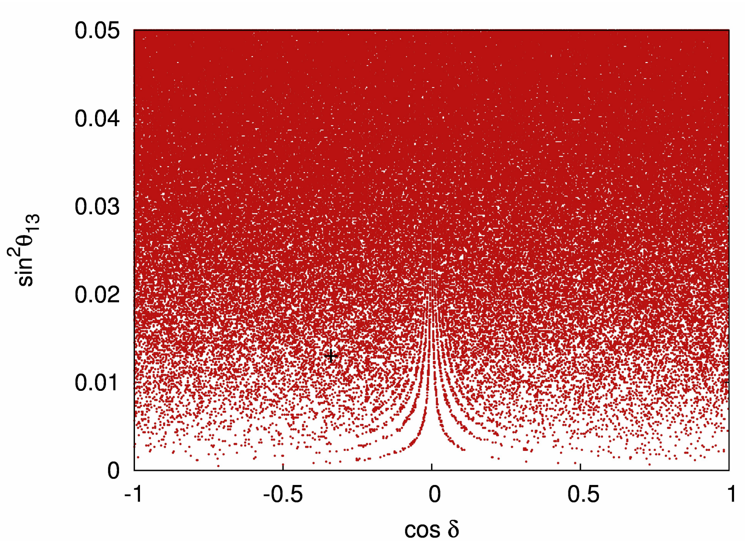

"Predicts" $\sin^2\theta_{13} > 0$.

Cases $B_1$ to $B_4$ all predict $\cos\delta \approx 0$.

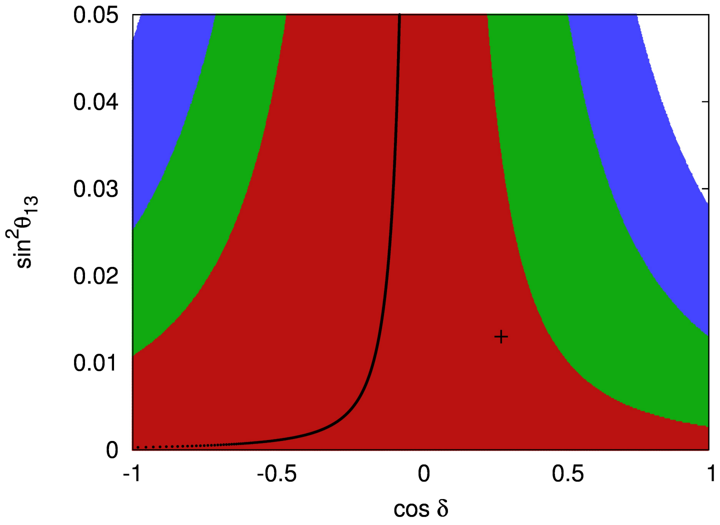



As an example: $B_1$ (inverted spectrum)

Case $C$ (normal): No correlation between $\sin^2\theta_{13}$ and $\cos\delta$.



Case $C$ (inverted):



# Conclusions

- We have found two instances in the framework of two texture zeros in $\mathcal{M}_\nu$ ($\mathcal{M}_\ell$ diagonal) which lead to maximal atmospheric neutrino mixing in the limit of a quasi-degenerate neutrino mass spectrum.
- In this scenario the limit

$$\sin^2\theta_{23} \to \frac{1}{2}$$

is independent of the values of $\theta_{13}$ and $\theta_{12}$!
- $\theta_{23} \approx 45^\circ$ and $\cos\delta \approx 0$ achievable by the use of an Abelian symmetry (texture zeros). $\to$ Alternative road to explain near maximal atmospheric neutrino mixing.
- Seven cases of two texture zeros in $\mathcal{M}_\nu$ are compatible with the experimental data.
- Two of these cases are compatible with the experimental data on $\sin^2\theta_{12}$, $\sin^2\theta_{23}$ and $\Delta m^2_{ij}$ only if $\sin^2\theta_{13} > 0$.