LII Krakow School of Theoretical Physics

Lorentz force corrections and dark energy in Strong (Nonlinear) Electromagnetic Fields

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work supported by US DoE Grant: DE-FG02-04ER41318

Outline

1. Strong Electromagnetic Fields in Astrophysics
2. Nonlinear Dynamics in Strong External Fields
    - Nonlinear Euler-Heisenberg Effective Potential
    - Euler-Heisenberg induced force
    - Radiation Reaction
3. Trace of the Energy-momentum tensor
4. Vacuum Structure and Dark Energy

Strong magnetic fields in the universe

Collapsing stellar cores
$R \sim 10^3\,\mathrm{km} \rightarrow 10\,\mathrm{km}$

- Flux conservation:
dipole $B \sim 10^3 - 10^6\,\mathrm{T}$

- Rotation pulls field lines into
azimuthal $B_\phi \sim 10^6 - 10^9\,\mathrm{T}$

- Magnetohydrodynamic instability
$|B| \sim 10^{11}\,\mathrm{T}$

[Balbus, Psaltis, Akiyama, Burrows]

Observations: Magnetars, soft gamma repeaters, anomalous x-ray pulsars...

NEWS 23 APRIL 2004 VOL 304 SCIENCE www.sciencemag.org

Crushed by Magnetism

The strongest—and strangest—magnetic fields in the galaxy stop some pulsars dead in their tracks and literally fracture their surfaces

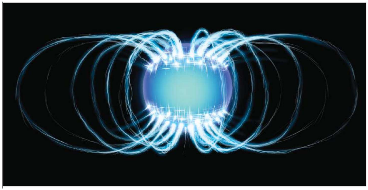
Mutations are the spice of life, but the most freakish mutants usually die at a tender age. This biological rule holds true in astrophysics: Some of the strangest mutations in space create superenergetic but short-lived cousins of pulsars, called magnetars.

Like a pulsar, a magnetar is a neutron star forged at the center of a supernova when a massive star explodes. But something odd happens during a magnetar's birth. An unknown process—perhaps ultrafast rotation within the dying star's collapsing core—endows each magnetar with a crushing magnetic field. This magnetism, up to 1000 times more intense than that of a typical pulsar, is the strongest known in space.

Flight Center (MSFC) in Huntsville, Alabama. Indeed, some proponents think the objects might not be mutants at all but common offspring of supernovas. "It's quite possible that a majority of neutron stars are magnetars rather than radio pulsars," says astrophysicist Robert Duncan of the University of Texas, Austin.

A hundred billion MRI scans

That's a grand claim, but Duncan and fellow theorist Christopher Thompson of the Canadian Institute for Theoretical Astrophysics in Toronto, Ontario, have swayed skeptics before. They first calculated that powerful magnetic fields could lace through newborn neutron stars in 1987,



Must know charged particle dynamics in strong external magnetic fields – can the Lorentz force be modified?

QED Strong Magnetic Fields

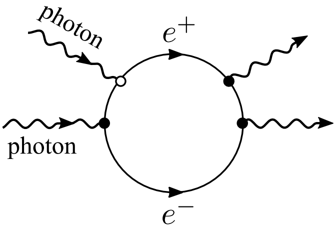
Study **Q**uantum **E**lectro-**D**ynamics in strong slowly-varying external fields:

$$\frac{|\vec{\nabla} B|}{B} \ll \frac{1}{\lambda\!\!\!^{-}_e} = m_e$$

scale set by electron mass

- Electron quantum fluctuations important in fields near scale set by

$$B_c = \frac{{m_e}^2 c^2}{e\hbar} = 4.41 \times 10^9\,\mathrm{T}$$



In vacuum, integrate fluctuations
$\Rightarrow$ **effective potential**
$\Rightarrow$ (effective) nonlinear EM

Gravity competes with quantum fluctuations

Weakness of gravity means fluctuations important even for $B < B_c$

The Vacuum as Polarizable Medium

$|0\rangle \equiv$ Vacuum, zero particle, zero charge state

Electromagnetic field polarizes electron-positron fluctuations: changed density of states

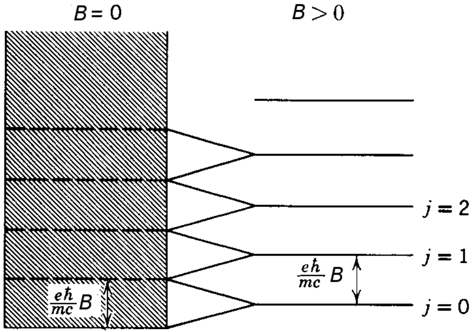
Constant field solvable *exactly*

- Magnetic Landau levels:

$\epsilon^2_{\sigma,j} = m^2 + p_z^2 + e|\vec{B}|(j + 1/2 \pm \sigma)$

for $j = 0, 1, 2...$ and $\sigma = \pm 1/2$

$$\int \frac{d^2 p_\perp}{(2\pi)^2} \rightarrow \frac{e|\vec{B}|}{2\pi}$$



[adapted from Huang (2007)]

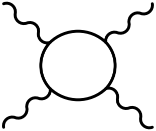
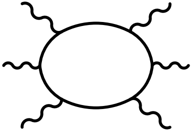
Comparing energy with field to without field → **effective potential**

Heisenberg/Euler, Schwinger

Effective potential

Integrate fluctuations $\to$ Effective theory of EM $\mathcal{L}_{\rm eff} = \frac{1}{2}(E^2 - B^2) + V_{\rm eff}$

$$V_{\rm eff} \simeq \frac{\alpha}{90\pi}\frac{e^2}{m_e^4}\left((\vec{E}^2 - \vec{B}^2)^2 + 7(\vec{E}\cdot\vec{B})^2\right)$$
$$+ \frac{2\alpha}{315\pi}\frac{e^4}{m_e^8}(\vec{E}^2 - \vec{B}^2)\left(2(\vec{E}^2 - \vec{B}^2)^2 + 13(\vec{E}\cdot\vec{B})^2\right) + \ldots$$

=  +  +...

- Applicable for slowly-varying fields, expansion in $\frac{|\vec{\nabla} B|}{B} \ll m_e$
- higher terms suppressed by field scale

$$|\vec{B}_c| = \frac{m_e^2}{e} = 4.41\,10^9\ \mathrm{T} \qquad |\vec{E}_c| = 1.32\,10^{18}\ \mathrm{V/m}$$

- shows energy cost of increasing field strength,

implies energy cost for new sources

A Lorentz Invariant (Polarized) Medium

Constant field preserves Lorentz symmetry of vacuum state

$\Rightarrow \mathcal{L}_{\rm eff} = -\mathcal{S} + V_{\rm eff}(\mathcal{S},\mathcal{P})$ function only of Lorentz invariants

$$V_{\rm eff} \simeq \frac{\alpha}{90\pi}\frac{e^2}{m_e^4}\left(4\mathcal{S}^2 + 7\mathcal{P}^2\right) + \frac{2\alpha}{315\pi}\frac{e^4}{m_e^8}\mathcal{S}\left(8\mathcal{S}^2 + 13\mathcal{P}^2\right)$$

$$\mathcal{S} = \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} = \text{scalar}, \quad \mathcal{S} = F^{\mu\nu}\tfrac{1}{2}\varepsilon_{\mu\nu\kappa\lambda}F^{\kappa\lambda} = \text{pseudoscalar}$$

- First order in α (1 loop calculation)
- ALL orders in external field (semi-convergent series in $\frac{eF^{\mu\nu}}{m_e^2}$)
- principle of superposition no longer applies to $F^{\mu\nu}$
  $\Rightarrow$ energy cost of superposing EM fields e.g. from new sources

Light-light scattering and new force

Effective Lagrangian EM+**current** $\mathcal{L}_{\rm eff} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + V_{\rm eff} + qeu^\nu A_\nu$

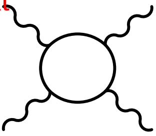
Currents determine displacement fields

$$qeu^\nu \equiv j^\nu = \partial_\mu K^{\mu\nu} \quad K^{\mu\nu} = -\frac{\partial\mathcal{L}_{\rm eff}}{\partial F^{\mu\nu}}$$

- Linear in currents and displacement fields

$$j_{\rm tot} = j_1 + j_2 \qquad K^{\mu\nu}_{\rm tot} = K^{\mu\nu}_1 + K^{\mu\nu}_2$$

- $F^{\mu\nu}$ do not sum $\Leftrightarrow$ field nonlinearity



Charged **p**article has Coulomb field $j^\nu_{\rm p} = \partial_\mu K^{\mu\nu}_{\rm p}$

Nonlinear potential for charged **p**article in **ext**ernal field
Schematically: 3 attachments to external field $\rightarrow$ 3 powers of $K^{\mu\nu}_{\rm ext}$

Other Nonlinear Electrodynamics

1. Regulating divergence of electron self-energy:

Born, M. and Infeld, L. (1934).Proc.Roy.Soc.Lond., **A144** 425–451.

$$\mathcal{L}_{\rm eff}^{\rm (BI)} = M_{\rm BI}^4 \left(1 - \sqrt{1 + 2\mathcal{S}/M_{\rm BI}^4 - \mathcal{P}^2/M_{\rm BI}^8}\right) \simeq -\mathcal{S} + \frac{1}{2M_{\rm BI}^4}\mathcal{S}^2 + \frac{1}{2M_{\rm BI}^4}\mathcal{P}^2$$

- Arises as Effective theory in some string theories, inflation models,
- Studies of nonlinear field-field interaction, photon propagation effects

fewer of particle dynamics with nonlinear $\mathcal{L}_{\rm eff}$

2. Particle dynamics nonlinear in field: Landau-Lifshitz equation

$$M\frac{du^\mu}{d\tau} = -eF^{\mu\nu}u_\nu - \frac{2e^3}{3M}\left(\partial_\eta F^{\mu\nu}u_\nu u^\eta - \frac{e}{M}F^{\mu\nu}F^\eta_\nu u_\eta\right) + \frac{2e^4}{3M^2}F^{\eta\nu}F_{\eta\delta}u_\nu u^\delta u^\mu$$

- No action principle!
- Studied for electron-laser collisions

[DiPiazza arXiv:0801.1751; Hadad et al, arXiv:1005.3980]

(More) Complete Story of Force in Nonlinear EM

Effective Lagrangian: Nonlinear EM +current+**inertia**

$$S_{\rm eff} = \int d^4x \, -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \int d^4x V_{\rm eff} + \int_{\rm path} d\tau \; qeA_\nu u^\nu + \frac{M}{2}\int_{\rm path} d\tau \,(u^2 - 1)$$

Standard procedure: Particle and field dynamics derived from *separate* variations

$\delta x(\tau) \rightarrow$ Lorentz force	$\delta A^\mu(x) \rightarrow$ Maxwell equations
$-qeF^{\mu\nu}u_\nu - M\dfrac{du^\mu}{d\tau} = 0$	$qeu_\nu - \partial_\mu K^{\mu\nu} = 0$
	$K^{\mu\nu} = F^{\mu\nu} - \dfrac{\partial V_{\rm eff}}{\partial F_{\mu\nu}}$

(Current conservation $\Rightarrow$ also have homogeneous Maxwell: $\partial_\mu \widetilde{F}^{\mu\nu} = 0$)

Problem: current generates displacement $K^{\mu\nu}$ but feels force from $F^{\mu\nu}$

Competing force: Radiation Reaction

In fact, there is a problem even with $V_{\rm eff} \rightarrow 0$:

Here, Lorentz force $\dfrac{dp^\mu}{d\tau} = -qeF^{\mu\nu}u_\nu$, and Maxwell $j^\nu = \partial_\mu F^{\mu\nu}$

1. Fields accelerate charges
2. Accelerated charges radiate $\left.\dfrac{dE}{dt}\right|_{\rm lab} = -\dfrac{2}{3}(qe)^2\dot{u}^\nu\dot{u}_\nu$

But Lorentz force is conservative–it must not know about energy lost to radiation....How can we have a consistent solution?

JR & LL, arXiv:1010.1970, arXiv:1204.4923 [hep-ph])

- Lorentz, Abraham, Dirac add effective *radiation reaction* force
- Landau, Lifshitz: Perturbative in acceleration $\dot{u}$ iteration

In terms of Energy-momentum tensor and Thomson cross-section

$$\frac{dp^\mu}{d\tau} = -qeF^{\mu\nu}u_\nu + g^\mu$$

$$g^\mu = \sigma_T\left(T^{\mu\nu}_{\rm ext}u_\nu - u^\mu(u_\kappa T^{\kappa\nu}_{\rm ext}u_\nu)\right) \qquad \sigma_T = \frac{8\pi}{3}\left(\frac{(qe)^2}{M}\right)^2$$

(Padmanabhan 1997)

Compare nonlinear corrections to Lorentz

Radiation reaction force

$$|g^\mu| \sim \frac{(qe)^2}{M^2}(qeF_{\text{ext}})^2$$

Vacuum flucutation correction

$$|\delta f^\mu| \sim e^2 \left(\frac{eF_{\text{ext}}}{m_e^2}\right)^2 qeF_{\text{ext}}$$

$$\frac{|g^\mu|}{|\delta f^\mu|} \sim \frac{m_e^2}{M^2}\frac{q^3}{eF_{\text{ext}}/m_e^2}$$

As long as $\frac{m_e^2}{M^2} \ll \frac{eF_{\text{ext}}}{m_e^2}$

vacuum fluctuation-induced force dominates

Vacuum fluctuation Induced Force

In addition to Lorentz force field-field interaction induces nonlinear correction

$$\frac{dp^\mu}{d\tau} = f^\mu = -\partial_\nu T^{\mu\nu}_{\rm int} \; = \; j_\nu F^{\mu\nu}_{\rm ext} + \delta f^\mu$$

In rest frame of point charge,

$$\frac{1}{Ze}\delta\vec{f} \approxeq \left(\left(\frac{\partial V_{\rm eff}}{\partial \mathcal{S}}\right)^2 + \frac{\varepsilon - 2}{2}\frac{\partial T^\mu_\mu}{\partial \mathcal{S}} \right)\Bigg|_{\rm ext} \vec{E}_{\rm ext} - \mathbf{C}_{\rm e}\, \Phi_{\rm ext} \vec{\nabla}\mathcal{S}_{\rm ext},$$

NEW: gradient of scalar invariant $\vec{\nabla}\mathcal{S} = \vec{\nabla}(\frac{1}{4}F^{\mu\nu}F_{\mu\nu})$

$$\mathbf{C}_{\rm e} \approxeq \frac{8\alpha}{45\pi}\frac{1}{B_c^2}\left(1 - \left(\frac{B_{\rm ext}}{B_c}\right)^2\left(\frac{6}{7} + \frac{2\alpha}{45\pi}\right) + \dots\right) \qquad B_c = \frac{m_e^2}{e}$$

- Works to eject particle from strong field region

Comparison to gravity $\quad B_c = 4.4 \times 10^9\ \mathrm{T}$

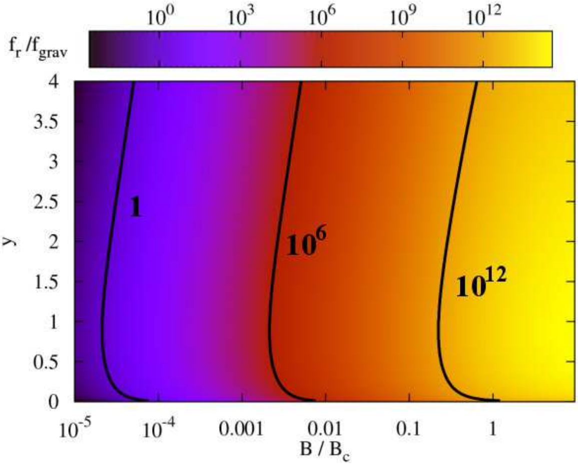
LL & J. Rafelski, arXiv:0810.1323

Example

Flux conservation
$\Rightarrow B \gtrsim 10^{-6} B_c$

Radial gradient in B^2
$\rightarrow$ radial force

Surface gravity
12 km, $1.5 M_\odot$ star



Radially moving charge with rapidity $\cosh y = \gamma \qquad V_{\rm grav} \rightarrow \gamma^2 V_{\rm grav}$

Repulsive Force of T_μ^μ

Lowest order $(\alpha \frac{e^2}{m_e^4})$ correction due to electron fluctuations:

$$\delta f^\mu = \left(\frac{\partial V_{\rm eff}}{\partial \mathcal{S}}\right)^2 j_\nu^{\rm part} F_{\rm ext}^{\mu\nu} - T_{\rm ep}^{\mu\nu} \partial_\nu \left(\frac{\partial V_{\rm eff}}{\partial \mathcal{S}}\right)^2 + \partial^\mu \frac{\varepsilon - 2}{4} \frac{\partial T_\nu^\nu}{\partial \mathcal{S}}\bigg|_{\rm ext} F_{\alpha\beta}^{\rm ext} K_{\rm part}^{\alpha\beta}$$

$$\text{Scalar invariant } \mathcal{S} = \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \qquad \varepsilon - 1 \equiv -\frac{\partial V_{\rm eff}}{\partial \mathcal{S}}$$

- Leading contribution due to energy-momentum trace $T_\mu^\mu = g_{\mu\nu} T^{\mu\nu}$
- Works to eject particle from strong field region
- *Repulsive* effect is a signature of T_μ^μ

Energy-momentum trace (anomaly)

Energy momentum tensor of general electromagnetic theory

$$T^{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}} \sqrt{-g}(-\mathcal{S} + V_{\rm eff})$$
$$= \varepsilon(g^{\mu\nu}\mathcal{S} - F^{\mu\lambda}F^{\nu}{}_{\lambda}) - g^{\mu\nu}\left(V_{\rm eff} - \mathcal{S}\frac{\partial V_{\rm eff}}{\partial \mathcal{S}} - \mathcal{P}\frac{\partial V_{\rm eff}}{\partial \mathcal{P}}\right)$$

- 1st term is traceless Maxwell energy-momentum tensor, modified by $\varepsilon \equiv 1 - \dfrac{\partial V_{\rm eff}}{\partial \mathcal{S}}$
- 2nd term $4\left(V_{\rm eff} - \mathcal{S}\frac{\partial V_{\rm eff}}{\partial \mathcal{S}} - \mathcal{P}\frac{\partial V_{\rm eff}}{\partial \mathcal{P}}\right) \equiv T_\kappa^\kappa$ quantum anomaly – not present in Maxwell EM

T_κ^κ arises from nonlinearity of $V_{\rm eff}$ [Adler, Collins, Duncan, PRD **15** 1977]

$$\boxed{T^{\mu\nu} = T^{\mu\nu}_{\rm (Max)}\varepsilon + g^{\mu\nu}\frac{1}{4}T_\kappa^\kappa}$$

Vacuum Energy and T_μ^μ

Constant, homogeneous field:
- modifies vacuum fluctuations,
- finite shift in energy $\to V_{\text{eff}}$
- retains Lorentz symmetry

Observable is energy-momentum tensor $T^{\mu\nu}$

$\lambda g^{\mu\nu}$ is only Lorentz invariant constant

▶ Expected excited vacuum state has $\lambda g^{\mu\nu}$ component in $T^{\mu\nu}$

For effect of T_μ^μ, compare:

Perfect fluid $T_\nu^\mu = \text{diag}(\rho, -p, -p, -p) \to T_\mu^\mu = \rho - 3p \geq 0$

- Classical Electromagnetism: $p = \rho/3 \Rightarrow T_\mu^\mu \equiv 0$
- Dust matter: $\rho > 0, p \equiv 0 \Rightarrow T_\mu^\mu = \rho$
- $\lambda g^{\mu\nu} \leftrightarrow \rho = -p$, i.e. energy with a negative pressure

'External' true vacuum presses inward to *decrease* volume of excited state: pressure < 0

$$\rho - 3p \to -2\lambda$$

T^{μ}_{μ} in local domains

For local domain, consider effect in Oppenheimer-Volkov:

$$\frac{dp}{dr} = -\frac{G}{c^2}(\rho + p)\frac{m + 4\pi r^3 p}{r(r - 2Gm)}$$

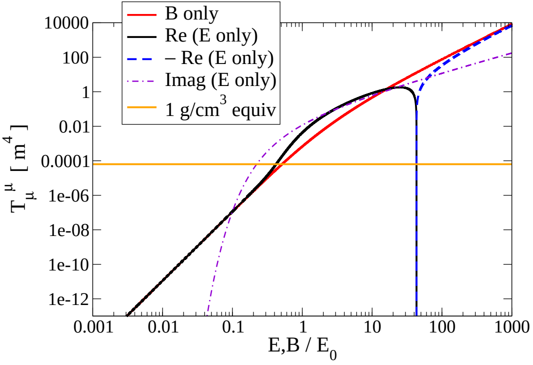
$\rho + \Lambda, p - \Lambda$ softens pressure gradient $\Rightarrow$ supports heavier stars

$T^{\kappa}_{\kappa} \propto m^4$
too small to affect internal structure of compact stars

$$\rho_{\rm nuc} \sim 0.1\frac{\rm GeV}{\rm fm^3} = 10^{10} m^4$$

$$p_{\rm nuc} \sim 1\frac{\rm MeV}{\rm fm^3} = 10^{8} m^4$$

(Lattimer & Prakash 2006)



Vacuum Energy, T^μ_μ, and Cosmology

T^κ_κ has same effect as *cosmological constant*, Λ

$$\frac{1}{8\pi G}\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right)=-\widehat{T_{\mu\nu}}-\left(\frac{1}{4}T^\mu_\mu+\frac{\Lambda_U}{8\pi G}\right)g_{\mu\nu},$$

(separating T^μ_μ, denoting $\widehat{T_{\mu\nu}}$ the traceless remainder)

Verify effect in Friedman cosmological metric, scale factor $a(t)$

$$3\frac{\ddot{a}}{a}=-4\pi G(\rho-3p)+2\pi G T^\kappa_\kappa+\Lambda$$

Dark energy has characteristics of Λ (Serra, et al. 2009)

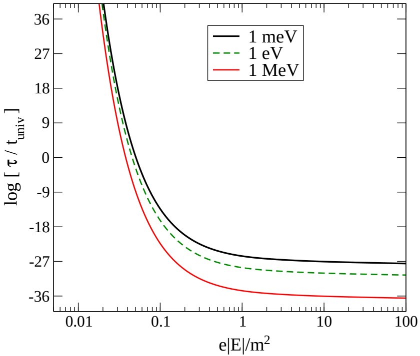
$$\frac{\Lambda_U}{4\pi G}\to\frac{\Lambda_d}{4\pi G}=(2.325\ \mathrm{meV})^4\to\frac{T^\kappa_\kappa}{2}$$

from Euler-Heisenberg: $|\vec{E}|_d=32.4\,10^9\ \mathrm{V/m},\ \ |\vec{B}|_d=108\ \mathrm{T}$

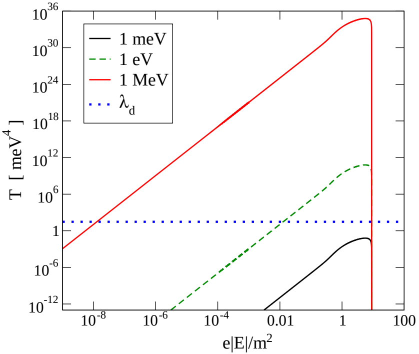
Vacuum Structure and Dark Energy

Hypothesis: dark energy due to excited vacuum state

LL & JR arXiv:1011.3497, also JR, LL, Y. Hadad, P. Chen, C09-06-08.3 [arXiv:0909.2989]



Decay time



$T^{\kappa}_{\kappa} \rightarrow$ dark energy

Fixed coupling $\alpha_{\rm QED} = 1/137$,
suggests $m \rightarrow 1 - 10^3\ \rm meV$ as interesting energy scale,
near to Standard Model: meV neutrino and axion masses

Conclusions

Nonlinear field-field interaction mediated by electron fluctuations:

1. Additional to Lorentz force, repulsive component from T_{μ}^{μ}

2. For strong fields $B > 10^{-}6B_c$, fluctuation-induces force dominates gravity and radiation reaction forces

3. Present for magnetic-only fields and more general configurations

Applications in accretion and cataclysmic stellar evolution

Nonlinearity implies energy-momentum trace T_{μ}^{μ}:

1. Has character of excitation energy of QED vacuum in strong external fields

2. QED-like theory can generate cosmological dark energy if mass scale is $m \sim 1\mathrm{meV} - 1\mathrm{eV}$

Thank you!