Screening in two-dimensional lattice gauge theories

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- Start with abelian $U(1)$ gauge group: QED_2:
  - Free electrodynamics in 1+1 dimensions – no true dynamics but confining linear potential for probe charges
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  - Schwinger, 1962: QED$_2$ with single massless flavour: solvable using bosonisation trick, exhibits charge screening $\forall Q_{ext}$.
  - Coleman, Jackiw, Susskind, 1975: perturbative addition of small mass. Only integer charges Q_{ext} are screened for non-zero mass (string breaking).
  - Plethora of numerical studies (though only for integer charges) both using bosonisation & standard lattice methods (and also DLCQ). Still field of active research (e.g. Dürr, 2012).

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- Adjoint fermions analysed theoretically (e.g. Kutasov, 1994) and by DLCQ (e.g. Bhanot, Demeterfi, Klebanov, 1993) but hardly any lattice calculation in 1+1 dimensions.

Lattice discretization of QED_2

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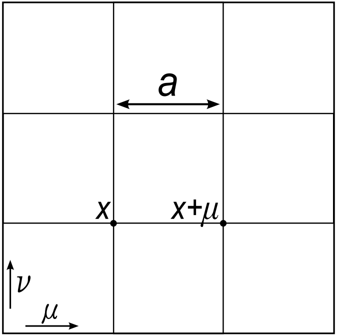

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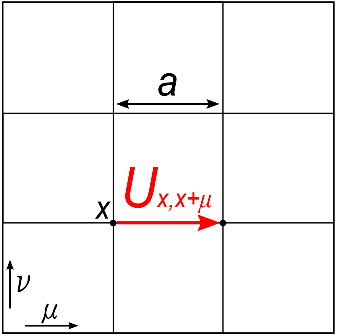

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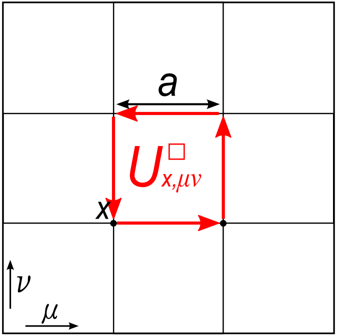

$$\mathcal{Z} = \int (\prod_{x,\mu} dU_{x,x+\mu}) e^{-S_{\text{gauge}}[U]}$$

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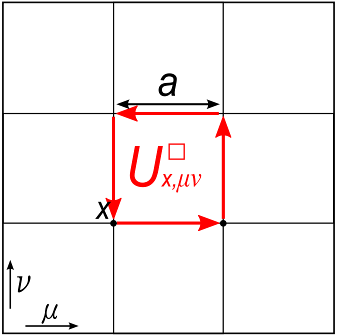

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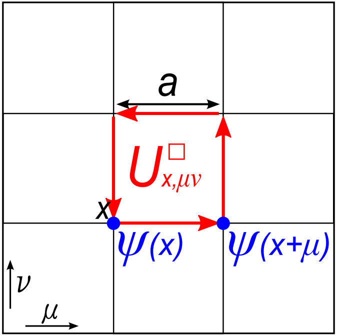

$$\mathcal{Z} = \int \left(\prod_{x,\mu} dU_{x,x+\mu}\right) e^{-\#g^{-2}\sum_{\square}(1-\mathrm{Re}U^{\square}_{x,\mu\nu}) - \sum_{\psi}\bar{\psi}(D^{\mathrm{lat}}+am)\psi}$$

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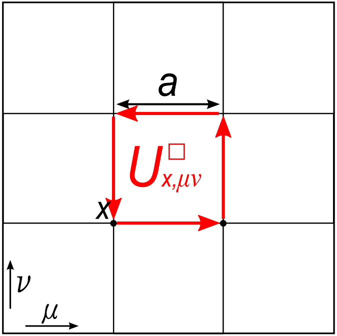

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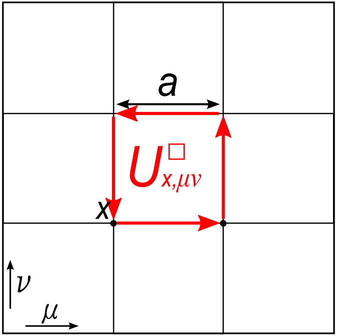

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- We choose Wilson fermions for $D^{\rm lat}$ and use standard Hybrid Monte Carlo numerical setup.

Confinement vs. charge screening

- For closed contour Γ: $W_\Gamma = \exp\{ig \oint_\Gamma dl_\mu A_\mu(l)\}$ – Wilson loop. On the lattice: $W_\Gamma = \prod_{U_i \in \Gamma} U_i$
- Wilson loop for square contour of sizes $R \times T$ – interpreted as pair of opposite static charges, $W(R,T) \cong \sum_i C_i e^{-E_i(R)T}$

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- String breaking – hard to observe on the lattice. Conjecture: small overlap of Wilson loops onto broken-string ground state. Including other observables necessary (Bali et al., 2005).

Wilson loops and string breaking in QED_2

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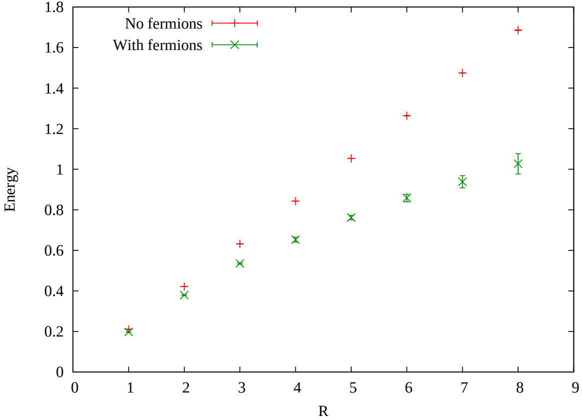

Wilson loop energies, $g^{-2} = 1.5, \kappa = 0.245, V = 24 \times 24$.

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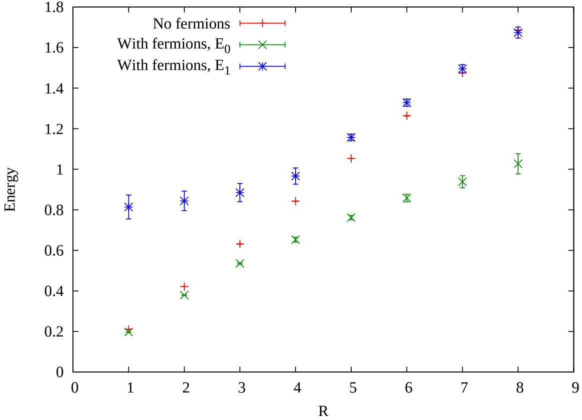

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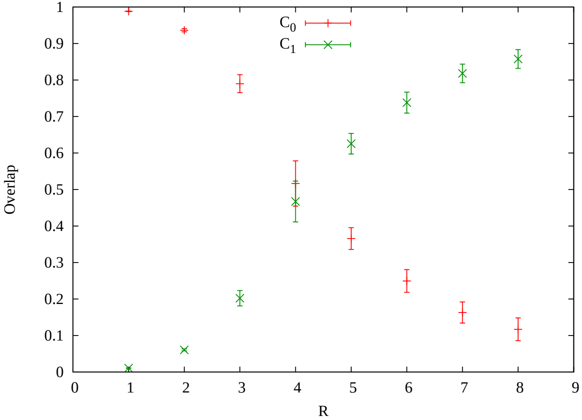

Overlap coefficients, $g^{-2} = 1.5$, $\kappa = 0.245$, $V = 24 \times 24$.

Fractional charges

- Despite numerous analyses for $Q_{ext} \in \mathbb{Z}$, equation (CJS, 1975)

$$\sigma = \# mg \left(1 - \cos(2\pi Q_{ext})\right)$$

  has never been verified on the lattice when Q_{ext} is non-integer.
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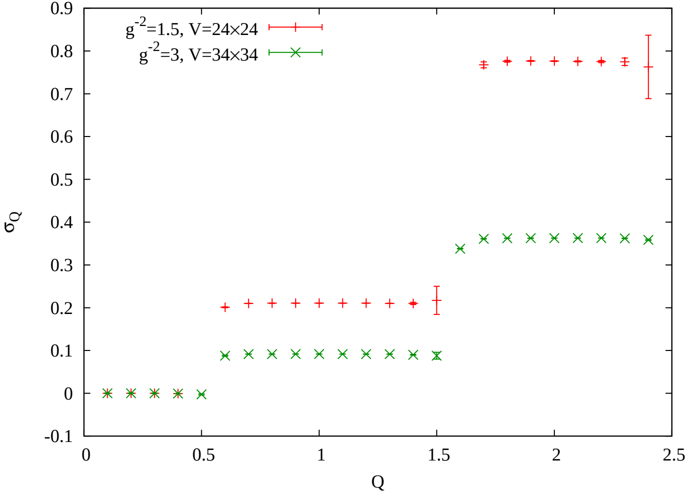

String tension vs. external charge in pure gauge QED_2

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- σ_Q with $Q \notin \mathbb{Z}$ is "cast" to the closest integer value.
- Manton, 1984: QED_2 on a (spatial) circle – extra topologically non-trivial gauge transformations – fulfilled only for integer charges.

Conclusions & outlook

- Two-dimensional theories share many intrinsic features with those in 4 dimensions and can be used as a test bed for concepts relating to QCD_4.
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Plans for future:

- Move to non-abelian theories, in particular with adjoint matter, which are of great interest recently (technicolor, large-N equivalences).