Baryogenesis, Dark Matter and the Maximal Temperature of the Early Universe

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Baryogenesis & Dark Matter strongly depend on ‘reheating temperature’ $T_R$ in early universe; relevant temperature scales are determined by microscopic physics:

- **0.1 eV** [$10^{13}$ s]: decoupling of photons, CMB
- **0.1...10 MeV** [$10^2...10^{-2}$ s]: primordial nucleosynthesis (BBN)
- **$\sim 10$ GeV** [$10^{-8}$ s]: WIMP decoupling, standard SUSY dark matter
- **100 GeV** [$10^{-10}$ s]: electroweak transition, sphaleron processes
- **$10^6...10^{10}$ GeV** [$10^{-18}...10^{-26}$ s]: leptogenesis, gravitino DM
- **$\sim 10^{12}$ GeV** [$10^{-30}$ s]: ‘maximal’ temperature?
Evidence for $T_{\text{min}}$: CMB; evidence for $T_{\text{max}}$: gravitational waves?
OUTLINE

- Example I: Moduli Decay
- Example II: Electroweak Baryogenesis
- Example III: Majorana Neutrinos
- Example IV: Thermal Leptogenesis
- Supersymmetry & Spontaneous B-L Breaking
Example I: Moduli Decay (Kitano, Murayama, Ratz ’08)

Theoretical framework: supersymmetry, special initial state (condensate of ‘moduli field’, motivated by string theory); very low reheating temperature \( T_R \sim 100 \text{ MeV} \).

Consider ‘moduli’ superfield \( \Phi = (\phi, \tilde{\phi}, F_\phi) \), coupled to matter fields via nonrenormalizable interaction in superpotential (\( M_P \): Planck mass),

\[
W \supset \frac{1}{M_P} \Phi U D D ,
\]

with \( U = (\tilde{u}^c, u^c, F_u) \) and \( D = (\tilde{d}^c, d^c, F_d) \) denoting up- and down-type quark superfields; \( \phi \) charge density (difference between number densities \( n_\phi \) and \( n_{\phi^*} \))

\[
q_\phi = i \left( \dot{\phi}^* \phi - \phi^* \dot{\phi} \right) .
\]
Time evolution of moduli field and charge density ($H$: Hubble rate, $\Gamma_\phi$: decay rate),

\[
\ddot{\phi} + (3H + \Gamma_\phi) \dot{\phi} + \frac{\partial V}{\partial \phi^*} = 0 ,
\]

\[
\dot{q}_\phi + 3H q_\phi = -i \left( \phi \frac{\partial V}{\partial \phi} - \phi^* \frac{\partial V}{\partial \phi^*} \right) ,
\]

with nonrenormalizable potential induced by supersymmetry breaking

\[
V = m_\phi^2 |\phi|^2 + m_{3/2}^2 M_p^2 F(|\phi|^2/M_p^2)
+ \left( \kappa \frac{m_{3/2}^2}{M_p^4} \phi^6 + \text{h.c.} \right) + \ldots ;
\]

$\kappa = \mathcal{O}(1)$, $m_{3/2}$: gravitino mass (supersymmetry breaking scale),
$F(x)$: some polynomial.

Dynamical generation of charge density $q_\phi \neq 0$: $\phi_{\text{ini}} \sim M_P$ for $H \gg m_\phi$ (after inflation); integration of equation of motion:

$$q_\phi(t = m_\phi^{-1}) \sim |\kappa| \frac{m_\phi^{2/2} M_P^4}{2 m_\phi M_P^4} \phi_{\text{ini}}^6.$$ 

Number density of $\phi$ and $\phi^*$: $\rho_\phi/m_\phi \simeq \left( m_\phi^2 |\phi|^2 + |\dot{\phi}|^2 \right) / m_\phi \simeq n_\phi + n_{\phi^*}$, and corresponding $\phi$ asymmetry:

$$\varepsilon = \frac{q_\phi}{n_\phi + n_{\phi^*}} \sim |\kappa| \left( \frac{m_3/2}{m_\phi} \right)^2.$$ 

For $\phi < M_P$, approximate baryon number conservation and asymmetry:

$$\varepsilon = \frac{q_\phi}{n_\phi + n_{\phi^*}} = \frac{n_\phi - n_{\phi^*}}{n_\phi + n_{\phi^*}}.$$
Baryogenesis in $\phi$ decays, $\phi \rightarrow qq\tilde{q}$,

$$\Gamma_{\phi} = \xi \frac{m_{\phi}^3}{M^2}, \quad \xi = 10^{-3} \ldots 10^{-2};$$

‘decay temperature’ $T_d$ of the thermal bath after $\phi$ decay ($H = \Gamma_{\phi}$, $\sqrt{\pi^2 g_*/90} \simeq 1$),

$$T_d \simeq 120 \text{ MeV} \left( \frac{\xi}{10^{-2}} \right)^{1/2} \left( \frac{m_{\phi}}{1500 \text{ TeV}} \right)^{3/2}. $$

Assume, $\phi$ dominates the energy density of universe before its decay,

$$m_{\phi} (n_{\phi} + n_{\phi^*}) \simeq \frac{\pi^2}{30} g_* T_d^4. $$
Charge density of scalar condensate yields baryon asymmetry

\[ \frac{n_b}{s} \sim \frac{3}{4} \varepsilon \frac{T_d}{m_\phi} \]

\[ \sim 10^{-10} \cdot |\kappa| \left( \frac{\xi}{10^{-2}} \right)^{1/2} \left( \frac{m_{3/2}}{50 \text{ TeV}} \right)^2 \left( \frac{m_\phi}{1500 \text{ TeV}} \right)^{-3/2}, \]

OK for heavy gravitino and very heavy modulus.

**Dark Matter:** about one LSP (higgsino or wino) per \( \phi \) decay; LSP density is reduced by pair annihilation; solution of Boltzmann equations yields DM density

\[ \Omega_\chi h^2 \sim 0.1 \left( \frac{3 \times 10^{-3}}{m_\chi^2 \langle \sigma v \rangle} \right) \left( \frac{10^{-2}}{\xi} \right)^{1/2} \left( \frac{m_\chi}{100 \text{ GeV}} \right)^3 \left( \frac{m_\phi}{1500 \text{ TeV}} \right)^{-3/2}. \]

Supersymmetry breaking via ‘anomaly mediation’: heavy gravitino with
\( m_\chi / m_{3/2} \sim g_2^2 / (16\pi^2) \), and therefore

\[
\frac{\Omega_\chi}{\Omega_b} \sim |\kappa|^{-1} \times 10^{-2} \times \frac{m_\chi}{m_{\text{nucleon}}} ;
\]

hence \( \Omega_{\text{CDM}}/\Omega_b \approx 5 \) ‘naturally’ realized. Predictions: WIMP DM inconsistent with standard freeze-out, relation between LSP mass and modulus mass, ...

Maximal temperature needed in moduli decay scenario:

\[ T_R = \mathcal{O}(100 \text{ MeV}) \]
Example II: Electroweak Baryogenesis (see Rubakov, Shaposhnikov '96, …)

Matter-antimatter asymmetry can be dynamically generated if particle interactions and cosmological evolution satisfy Sakharov’s conditions [1967],

- baryon number violation,
- $C$ and $CP$ violation,
- deviation from thermal equilibrium.

[Sakharov’s model for baryogenesis: $CP$ violating decays of ‘maximons’ with mass $\mathcal{O}(M_P)$ at temperature $T_i \sim M_P$, connection with $CP$ violation in $K^0$-meson system,…,proton lifetime $\tau_p > 10^{50}$ years predicted. In general, baryogenesis provides important relationship between the standard model of cosmology and the standard model of particle physics as well as its extensions.]
EWBG requires strong jump of Higgs expectation value \( v = \sqrt{\phi^+ \phi} \) at critical temperature (baryon asymmetry not erased in Higgs phase),

\[
\frac{\Delta v(T_c)}{T_c} > 1 .
\]

Lattice and perturbative calculations consistent for Higgs masses below 60 GeV (see Jansen ’96); transition too weak?! Now tested at LHC!
For large Higgs masses nonperturbative effects important; at critical Higgs mass $m^c_H = \mathcal{O}(m_W)$, first-order transition turns into crossover (WB, Philipsen '94; Kajantie et al '96; Csikor, Fodor '99); lattice simulations: $m^c_H = 72.1 \pm 1.4$ GeV; critical Higgs mass determined by magnetic mass $m_{SM} = Cg^2T$ ($C \simeq 0.35$),

$$m^c_H = \left(\frac{3}{4\pi C}\right)^{1/2} m_W \simeq 74 \text{ GeV}.$$
Baryon and lepton number violating sphaleron processes

\[^\text{'}t\text{ Hooft '76; Kuzmin, Rubakov, Shaposhnikov '85, ...}\]

\[ O_{B+L} = \prod_i (q_Li q_Li q_Li L_L), \]

\[ \Delta B = \Delta L = 3\Delta N_{CS}, \]

\[ B - L \text{ conserved} \]

Processes are in thermal equilibrium above electroweak phase transition, for temperatures

\[ T_{EW} \sim 100\text{GeV} < T < T_{SPH} \sim 10^{12}\text{GeV}. \]

Theory changed, result invariant; experimental evidence?
Sphaleron processes have a profound effect on the generation of cosmological baryon asymmetry. Analysis of chemical potentials of all particle species in the high-temperature phase yields relation between the baryon asymmetry ($B$) and $L$ and $B - L$ asymmetries,

$$\langle B \rangle_T = c_S \langle B - L \rangle_T = \frac{c_S}{c_S - 1} \langle L \rangle_T,$$

with $c_S$ number $\mathcal{O}(1)$; in standard model $c_s = 28/79$.

In nonequilibrium process, sphaleron transitions can generate baryon asymmetry (EWBG, depends on rate $\Gamma_{\text{sph}}$). In thermal equilibrium, sphaleron transitions can convert lepton asymmetry to baryon asymmetry (leptogenesis). Then lepton number violation is needed to explain the cosmological baryon asymmetry. However, it can only be weak, since otherwise any baryon asymmetry would be washed out. The interplay of these conflicting conditions leads to important contraints on neutrino properties.
Composite Higgs Model  (Espinoza, Gripaios, Konstandin, Riva '11; ...)

Consider Higgs(es) as pseudo-Goldstone bosons in strongly interacting theory, related to coset space \( SO(6)/SO(5) \), yields additional singlet; finite-temperature effective potential

\[
V(h, s, T) = \frac{\lambda_h}{4} \left[ h^2 - v_c^2 + \frac{v_c^2}{w_c^2} s^2 \right]^2 + \frac{\kappa}{4} s^2 h^2 \\
+ \frac{1}{2} (T^2 - T_c^2) (c_h h^2 + c_s s^2) ;
\]

strong first-order transition at tree level for realistic Higgs masses.

<table>
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<th>( m_h )</th>
<th>( m_s )</th>
<th>( v_c )</th>
<th>( f/b )</th>
<th>( L_w v_c )</th>
<th>( v_c/T_c )</th>
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<td>81 GeV</td>
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<td>1.88 TeV</td>
<td>7.1</td>
<td>2.0</td>
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<tr>
<td>S2</td>
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<td>139.2 GeV</td>
<td>177.8 GeV</td>
<td>1.185 TeV</td>
<td>3.5</td>
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Baryogenesis via CP violating couplings of top quarks,

\[ \frac{s}{f} H \bar{Q}_3(a + ib\gamma_5) t + h.c. , \]

bubbles produced by nucleation in first-order transition (see Bernreuther '02)
CP violating top quark couplings lead to electric dipole moments of electron and neutron:

\[ \frac{d_e}{e} < 1.05 \times 10^{-27} \text{cm} , \]
\[ \frac{d_n}{e} < 2.9 \times 10^{-26} \text{cm} ; \]

also constraint from EWPO; consistent for Higgs masses between 100 GeV and 150 GeV

**Dark Matter:** scalar WIMP, additional singlet, ‘inert’ doublet .... Maximal temperature needed in composite Higgs models: \( T_R = \mathcal{O}(100 \text{ GeV}). \)

**EWBG in MSSM:** very difficult to reconcile with LHC results; excluded? \( (m_{\tilde{t}_R} < 110 \text{ GeV}, m_{\tilde{t}_L} > 30 \text{ TeV}, ...) \)
Example III: Majorana Neutrinos (Canetti, Drewes, Shaposhnikov '12)

Neutrino oscillations, baryogenesis and dark matter can be explained by just the Standard Model with 3 right-handed (sterile) neutrinos ($\nu_{\text{MSM}}$) (Asaka, Blanchet, Shaposhnikov '05; ...),

$$L_{\nu_{\text{MSM}}} = L_{\text{SM}} + i \bar{\nu}_R \partial \nu_R - \bar{L}_L F \nu_R \tilde{\Phi} - \bar{\nu}_R F^\dagger L_L \tilde{\Phi}^\dagger - \frac{1}{2} (\bar{\nu}_R^c M_M \nu_R + \bar{\nu}_R M_M^\dagger \nu_R^c).$$

Mass eigenstates (seesaw): $\nu_i$ with masses $m_i$, $N_I$ with masses $M_I$; active-sterile mixings: $\theta = m_D M_M^{-1} (U^2 = \text{tr}(\theta^\dagger \theta))$.

Dark Matter: $N_1$; $1 \text{ keV} < M_1 \lesssim 50 \text{ keV}$, $10^{-13} \lesssim \sin^2(2\theta_{\alpha_1}) \lesssim 10^{-7}$ (constraints from X-ray observations)

Baryogenesis: CP-violating oscillations of $N_{2,3}$ during thermal production at $T \gtrsim T_{EW} \sim 140 \text{ GeV}$ (for Higgs mass $m_H = 126 \text{ GeV}$)
Time evolution described by kinetic equations for density matrices ($\mu_\alpha$: lepton chemical potentials),

\[
\begin{align*}
    i\frac{d\rho_N}{dT} &= [H, \rho_N] - \frac{i}{2}\{\Gamma_N, \rho_N - \rho_{eq}\} + \frac{i}{2}\mu_\alpha\tilde{\Gamma}_N^\alpha, \\
    i\frac{d\rho_{\bar{N}}}{dT} &= [H^*, \rho_{\bar{N}}] - \frac{i}{2}\{\Gamma_{\bar{N}}^*, \rho_{\bar{N}} - \rho_{eq}\} - \frac{i}{2}\mu_\alpha\tilde{\Gamma}_{\bar{N}}^{\alpha*}, \\
    i\frac{d\mu_\alpha}{dT} &= -i\Gamma_L^\alpha\mu_\alpha + i\text{tr}\left[\tilde{\Gamma}_L^\alpha(\rho_N - \rho_{eq})\right] \\
    &\quad - i\text{tr}\left[\tilde{\Gamma}_L^{\alpha*}(\rho_{\bar{N}} - \rho_{eq})\right].
\end{align*}
\]

Resonant enhancement of CP violation required, together with wanted DM abundance: \[|M_2 - M_3|/|M_2 + M_3| \sim 10^{-11}\]! Needed chemical potentials:

i) $\mu_\alpha \sim 10^{-10}$ at $T \sim T_{EW}$ (for BAU)

ii) $|\mu_\alpha| \gtrsim 8 \cdot 10^{-6}$ at $T \sim 100$ MeV (for DM)
Lower bound on heavy neutrino mass from Dark Matter abundance: $M_{2,3} > 2 \text{ GeV}$; experimental discovery conceivable up to 10 GeV (missing energy in meson decays, beam-dump experiments, ...)

Astrophysical tests: search for X-ray lines!

Maximal temperature needed in $\nu$MSM: $T_R = \mathcal{O}(100 \text{ GeV})$
Example IV. Thermal Leptogenesis \,(Fukugita, Yanagida '86)

The \textit{seesaw mechanism} explains smallness of the light neutrino masses by largeness of the heavy Majorana masses; predicts six mass eigenstates,

\[ N \simeq \nu_R + \nu_R^c : \quad m_N \simeq M ; \]
\[ \nu \simeq \nu_L + \nu_L^c : \quad m_\nu = -m_D \frac{1}{M} m_D^T . \]

For Yukawa couplings of third generation \( O(1) \), as in some SO(10) GUT models, one obtains heavy and light neutrino masses,

\[ M_3 \sim \Lambda_{GUT} \sim 10^{15} \text{ GeV} , \quad m_3 \sim \frac{v^2}{M_3} \sim 0.01 \text{ eV} ; \]

light neutrino mass \( m_3 \) compatible with \( (\Delta m_{atm}^2)^{1/2} \sim 0.05 \text{ eV} \) from \( \nu \)-oscillations \( \rightarrow \) GUT scale physics ?! (also other interpretations).
Interactions of $N \equiv N_1$, the lightest heavy Majorana neutrino, with Higgs doublet $\phi$ and lepton doublets $l_{Li}$, described by effective Lagrangian

$$\mathcal{L} = \bar{l}_{Li}\tilde{\phi}\lambda^*_i N + N^T\lambda_i C l_{Li}\phi - \frac{1}{2}M N^T C N$$

$$+ \frac{1}{2} \eta_{ij} l^T_{Li} \phi C l_{Lj} \phi + \frac{1}{2} \eta^{*}_{ij} \bar{l}_{Li} \tilde{\phi} C ar{l}^T_{Lj} \tilde{\phi} ;$$

$C$: charge conjugation matrix, $\tilde{\phi} = i\sigma_2 \phi^*$; effective coupling

$$\eta_{ij} = \sum_{k>1} \lambda_{ik} \frac{1}{M_k} \lambda^T_{kj} ,$$

after integrating out heavy Majorana neutrinos $N_{k>1}$ with $M_{k>1} \gg M_1 \equiv M$ (takes care of self-energy and vertex corrections); small Yukawa couplings: $\lambda_{i1} \ll 1$, i.e. decay width $\Gamma$ of $N$ much smaller than mass $M$. 

22
Ideal candidate for baryogenesis: lightest (heavy) Majorana neutrino $N_1$, no SM gauge interactions (out-of-equilibrium condition !) $N_1$ decays to lepton-Higgs pair $\rightarrow$ lepton asymmetry $\langle L \rangle_T \neq 0$, partially converted to baryon asymmetry $\langle B \rangle_T \neq 0$ (work by several groups since about 1996; expectation $m_i < 1$ eV before experimental results on $\Delta m_{\text{atm}}^2$; afterwards leptogenesis boom).

Generated baryon asymmetry is proportional to the $CP$ asymmetry in $N_1$-decays (simplest case, rough estimate) (Covi, Roulet, Vissani '96; Davidson, Ibarra '02; ...)

$$\epsilon_1 = \frac{\Gamma(N_1 \rightarrow l\phi) - \Gamma(N_1 \rightarrow \bar{l}\bar{\phi})}{\Gamma(N_1 \rightarrow l\phi) + \Gamma(N_1 \rightarrow \bar{l}\bar{\phi})}$$

$$= -\frac{3}{16\pi} \frac{\text{Im}(m_D^\dagger m_\nu m_D)_{11}}{(m_D^\dagger m_D)_{11}} M_1 v^2$$

$$\sim \frac{3}{16\pi} \frac{m_3 M_1}{v^2} \sim 0.1 \frac{M_1}{M_3}.$$
Order of magnitude of $CP$ asymmetry is given by the mass hierarchy of the heavy Majorana neutrinos, e.g., $\epsilon_1 \sim 10^{-5} \ldots 10^{-6}$ for $M_1/M_3 \sim 10^{-4} \ldots 10^{-5}$.

**Baryon asymmetry** for given $CP$ asymmetry $\epsilon_1$,

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = -d \epsilon_1 \kappa_f \sim 10^{-9} \ldots 10^{-10},$$

with dilution factor $d \sim 0.01$ (increase of photon number density), efficiency factor $\kappa_f \sim 10^{-2}$ (Boltzmann equations, competition between production and washout processes); baryogenesis temperature $T_B \sim M_1 \sim 10^{10}$ GeV.

The observed value of the baryon asymmetry, $\eta_B \sim 10^{-10}$, is obtained as consequence of the hierarchy of the heavy neutrino masses, leading to a small $CP$ asymmetry, and the kinematical factors $d$ and $\kappa_f$ (WB, Plüümacher '96).

In recent years many detailed studies, including GUT models, flavour dependence, connection to dark matter, ... (see Davidson, Nardi, Nir, 2008)
THERMAL PROCESSES ...

\[ \Delta L = 2 \text{ processes (}\mathcal{N}_i\text{ virtual)} \]

\[
\begin{align*}
    l & \leftrightarrow N_i l & l & \leftrightarrow l \phi & \phi \\
    \phi & \leftrightarrow \phi l & & & \\
    l \phi & \leftrightarrow \phi \phi (N) \\
    \phi & \leftrightarrow \phi \phi (N, t)
\end{align*}
\]

\[ \Delta L = 1 \text{ processes (}\mathcal{N}_i\text{ real)} \]

\[
\begin{align*}
    l & \leftrightarrow l \phi & l & \phi \\
    l & \leftrightarrow q & N_i & \phi \\
    l & \leftrightarrow t & t & q
\end{align*}
\]

decays (D), inverse decays (ID) quantum interference !!
Boltzmann Equations

Decays and inverse decays of heavy Majorana neutrinos sufficient for relevant range of neutrino masses; simplest case: hierarchical heavy neutrinos \((N_1 \equiv N)\), "one-flavour" approximation; dynamics described by set of Boltzmann equations:

\[
\frac{dn_N}{dt} + 3Hn_N = - (n_N - n_N^{eq}) \Gamma_N ,
\]

\[
\frac{dn_L}{dt} + 3Hn_L = -\epsilon_1 (n_N - n_N^{eq}) \Gamma_N + \text{washout};
\]

number densities and distribution functions:

\[
n_N(t) = \int \frac{d^3q}{(2\pi)^3} f_N(t, \omega) , \quad n_L(t) = \int \frac{d^3k}{(2\pi)^3} f_L(t, k) ;
\]

CP asymmetry \(\epsilon_1\): quantum interference; washout terms: tree level
Leptogenesis in Expanding Universe

For zero initial $N$-abundance: initial asymmetry has comparable magnitude as final asymmetry (for simplificity: initial $|N_{B-L}|$ at fixed temperature)

Maximal temperature needed for leptogenesis: $T_R = \mathcal{O}(10^{10} \text{ GeV})$
V. Supersymmetry & Spontaneous B-L breaking

(WB, Domcke, Schmitz, Vertongen '10 - '12)

- Light neutrino masses can be explained by mixing with Majorana neutrinos with GUT scale masses from $B - L$ breaking (seesaw mechanism)

- Decays of heavy Majorana neutrinos natural source of baryon asymmetry (leptogenesis)

- Heavy neutrinos can be produced thermally (Fukugita, Yanagida '86) or nonthermally (Lazarides, Shafi '91)

- What are the implications for dark matter?

- In supersymmetric models natural connection with hybrid inflation (Copeland et al '94; Dvali, Shafi, Schaefer '94)

- Consistent picture of inflation, baryogenesis and dark matter?
Leptogenesis and gravitinos: thermal leptogenesis ($T_L$) and thermal gravitino production ($T_R$) can yield observed amount of DM (see lecture of Laura Covi),

$$\Omega_{\tilde{G}}h^2 = C \left( \frac{T_R}{10^{10} \text{ GeV}} \right) \left( \frac{100 \text{ GeV}}{m_{\tilde{G}}} \right) \left( \frac{m_{\tilde{g}}}{1 \text{ TeV}} \right)^2 , \quad C \sim 0.5 ;$$

$\Omega_{\text{DM}}h^2 \sim 0.1$ is natural value; but why $T_R \sim T_L$?

Starting point simple observation: heavy neutrino decay width

$$\Gamma_{N_1}^0 = \frac{\tilde{m}_1}{8\pi} \left( \frac{M_1}{v_{\text{EW}}} \right)^2 \sim 10^3 \text{ GeV} , \quad \tilde{m}_1 \sim 0.01 \text{ eV} , \quad M_1 \sim 10^{10} \text{ GeV} .$$

yields reheating temperature (for decaying gas of heavy neutrinos)

$$T_R \sim 0.2 \cdot \sqrt{\Gamma_{N_1}^0 M_P} \sim 10^{10} \text{ GeV} ,$$

wanted for gravitino DM. Intriguing hint or misleading coincidence?
V.1 B-L breaking and false vacuum decay

Supersymmetric SM with right-handed neutrinos ($SU(5)$ notation),

$$W_M = h_{ij}^u 10_i 10_j H_u + h_{ij}^d 5_i^* 10_j H_d + h_{ij}^{\nu} 5_i^* n_j^c H_u + h_i^n n_i n_i^c S_1,$$

with electroweak breaking, $\langle H_{u,d} \rangle = v_{u,d}$, and $B-L$ breaking,

$$W_{B-L} = \frac{\sqrt{\lambda}}{2} \Phi (v_{B-L}^2 - 2 S_1 S_2),$$

$$\langle S_{1,2} \rangle = v_{B-L} / \sqrt{2};$$ Yukawa’s from FN flavour symmetry (e.g. WB, Yanagida '98),

$$h_{ij} \propto \eta^{Q_i + Q_j}, \quad \eta \simeq 1 / \sqrt{300}.$$
Lagrangian is determined by low energy physics: quark, lepton, neutrino masses etc, but it contains all ingredients wanted in cosmology: inflation, leptogenesis, dark matter,..., all related!

Parameters of $B-L$ breaking sector ($\overline{m}_\nu = \sqrt{m_2m_3} = 3 \times 10^{-2}$ eV):

\[
m_\nu = -m_D \frac{1}{M} m_T^D , \quad m_D = h^\nu v_{EW} , \]
\[
M_1 \simeq \eta^{2d} v_{B-L} , \quad M_1 \ll M_{2,3} \simeq m_S ,
\]
\[
v_{B-L} \simeq \eta^{2a} \frac{v_{EW}^2}{m_\nu} , \quad \tilde{m}_1 = \frac{(m_D^\dagger m_D)_{11}}{M_1} \sim \eta^{2a} \frac{v_{EW}^2}{v_{B-L}}.
\]

Spontaneous symmetry breaking: consider Abelian Higgs model in unitary gauge ($\rightarrow$ massive vector multiplet, no Wess-Zumino gauge!),

\[
S_{1,2} = \frac{1}{\sqrt{2}} S' \exp(\pm iT) , \quad V = Z + \frac{i}{2g} (T - T^*) .
\]
Shift around time-dependent background, $s' = \frac{1}{\sqrt{2}}(\sigma' + i\tau)$, $\sigma' \rightarrow \sqrt{2v(t)} + \sigma$ with $v(t) = \frac{1}{\sqrt{2}} \langle \sigma'^2(t, \vec{x}) \rangle^{1/2}$; masses of fluctuations:

\begin{align*}
    m_{\sigma}^2 &= \frac{1}{2} \lambda (3v^2(t) - v_{B-L}^2) , \\
    m_{\tau}^2 &= \frac{1}{2} \lambda (v_{B-L}^2 + v^2(t)) , \\
    m_{\phi}^2 &= \lambda v^2(t) , \\
    m_{\psi}^2 &= \lambda v^2(t) , \\
    m_Z^2 &= 8g^2v^2(t) , \\
    M_i^2 &= (h_i^n)^2 v^2(t) .
\end{align*}
Constraints from cosmic strings and inflation: upper bound on string tension (Battye et al '10, Dvorkin et al '11)

\[ G\mu \lesssim 5 \times 10^{-7}, \quad \mu = 2\pi B(\beta)v_{B-L}^2, \]

with \( \beta = \lambda/(8g^2) \) and \( B(\beta) = 2.4 [\ln(2/\beta)]^{-1} \) for \( \beta < 10^{-2} \); further constraint from CMB (cf. Nakayama et al '10), yields

\[ 3 \times 10^{15} \text{ GeV} \lesssim v_{B-L} \lesssim 7 \times 10^{15} \text{ GeV}, \]

\[ 10^{-4} \lesssim \sqrt{\lambda} \lesssim 10^{-1}; \]

fixes FN charges \( a = 0, d \sim 2 \); final range of parameters:

\[ v_{B-L} = 5 \times 10^{15} \text{ GeV}, \quad 10^{-5} \text{ eV} \leq \tilde{m}_1 \leq 1 \text{ eV}, \]

\[ 10^{9} \text{ GeV} \leq M_1 \leq 3 \times 10^{12} \text{ GeV}. \]

(range of \( \tilde{m}_1 \): uncertainty of \( \mathcal{O}(1) \) parameters)
Tachyonic Preheating

Hybrid inflation ends at critical value $\Phi_c$ of inflaton field $\Phi$ by rapid growth of fluctuations of $B-L$ Higgs field $S'$ (‘spinodal decomposition’):

particles with couplings to $S'$ are produced by rapid increase of ‘waterfall field’ (Garcia-Bellido, Morales ’02); nonperturbative effects (Berges, Gelfand, Pruschke ’10)
Decay of false vacuum produces long wave-length $\sigma$-modes, true vacuum reached at time $t_{PH}$ (even faster decay with inflaton dynamics),

$$\langle \sigma'{}^2 \rangle \bigg|_{t=t_{PH}} = 2v_{B-L}^2, \quad t_{PH} \simeq \frac{1}{2m_\sigma} \ln \left( \frac{32\pi^2}{\lambda} \right).$$

Initial state: nonrelativistic gas of $\sigma$-bosons, $N_{2,3}$, $\tilde{N}_{2,3}$, $A$, $\tilde{A}$, $C$ (contained in superfield $Z$), ... ; energy fractions ($\alpha = m_X/m_S$, $\rho_0 = \lambda v_{B-L}^4/4$):

$$\rho_B/\rho_0 \simeq 2 \times 10^{-3} \ g_s \ \lambda f(\alpha, 1.3), \quad \rho_F/\rho_0 \simeq 1.5 \times 10^{-3} \ g_s \ \lambda f(\alpha, 0.8).$$

Time evolution: rapid $N_{2,3}$, $\tilde{N}_{2,3}$, $A$, $\tilde{A}$, $C$ decays, yields initial radiation, thermal $N_1$'s and gravitinos; $\sigma$ decays produce nonthermal $N_1$'s; $N_1$ decays produce most of radiation and baryon asymmetry; details of evolution described by Boltzmann equations.
Reheating Process

**Major work:** solve network of Boltzmann equations for all (super)particles; treat nonthermal and thermal contributions differently, varying equation of state; result: detailed time resolved description of reheating process, prediction of baryon asymmetry and gravitino density (possibly dark matter).

**Illustrative example** for parameter choice

\[
M_1 = 5.4 \times 10^{10} \text{ GeV} , \quad \tilde{m}_1 = 4.0 \times 10^{-2} \text{ eV} , \\
m_{\tilde{G}} = 100 \text{ GeV} , \quad m_{\tilde{g}} = 1 \text{ TeV} ;
\]

fixes within FN flavour model all other masses, CP asymmetries etc. **Note:** emergence of temperature plateau at intermediate times; final result:

\[
\eta_B \approx 3.7 \times 10^{-9} \approx \eta_B^{nt} , \quad \Omega_{\tilde{G}} h^2 \approx 0.11 ,
\]

i.e., dynamical realization of original conjecture.
Thermal and nonthermal energy densities

Comoving energy densities of thermal and nonthermal $N_1$s,..., gravitinos and radiation as functions of scale factor $a$. 

Inverse temperature $M_1 / T$
Thermal and nonthermal number densities

Comoving number densities of thermal and nonthermal $N_1, ..., B - L$, gravitinos and radiation as functions of scale factor $a$. 
Time evolution of temperature: intermediate plateau

Gravitino abundance can be understood from ‘standard formula’ and effective ‘reheating temperature’ (determined by neutrino masses).
V.2 Leptogenesis & Dark Matter

• Baryogenesis is mixture of nonthermal and thermal leptogenesis; effective reheating temperature determined by neutrino masses

• Allowed range in $M_1 - \tilde{m}_1$ plane is significantly extended compared to thermal leptogenesis

• Inflation and cosmic strings constrain heavy neutrino mass

• Gravitino production is dominated by thermal processes (contribution from inflaton decay negligible (cf. Nakayama et al '10))

• Gravitino dark matter possible in mass range $10 \text{ GeV} \lesssim m_{\tilde{G}} \lesssim 700 \text{ GeV}$ (assuming $m_{\tilde{G}} = 1 \text{ TeV}$, otherwise simple rescaling)

• Gravitino dark matter further constrains heavy neutrino mass
(Non)thermal leptogenesis in $M_1 - \tilde{m}_1$ plane

Upper bound on $M_1$ from inflation; lower bound from baryogenesis
Gravitino Dark Matter vs leptogenesis

Gravitino mass range: $10 \text{ GeV} \lesssim m_{\tilde{G}} \lesssim 700 \text{ GeV}$; heavy neutrino mass range: $2 \times 10^{10} \text{ GeV} \lesssim M_1 \lesssim 2 \times 10^{11} \text{ GeV}$ (more stringent than inflation)
Higgsino/wino Dark Matter

Mass spectrum of superparticles motivated by anomaly mediation and the present hints for Higgs boson mass from LHC,

$$m_{\text{LSP}} \ll m_{\text{squark, slepton}} \ll m_{\tilde{G}}.$$  

LSP is typically ‘pure’ wino or higgsino (bino disfavoured, overproduction in thermal freeze-out), almost mass degenerate with chargino. Thermal abundance of wino ($\tilde{w}$) or higgsino ($\tilde{h}$) LSP significant for masses above 1 TeV, well approximated by (Arkani-Hamed et al '06; Hisano et al '07, Cirelli et al '07)

$$\Omega_{\tilde{w},\tilde{h}}^\text{th} h^2 = c_{\tilde{w},\tilde{h}} \left( \frac{m_{\tilde{w},\tilde{h}}}{1 \text{ TeV}} \right)^2 , \quad c_{\tilde{w}} = 0.014 , \quad c_{\tilde{h}} = 0.10 ,$$

Heavy gravitinos (10 TeV ... $10^3$ TeV) consistent with BBN,

$$\tau_{\tilde{G}} = \Gamma_{\tilde{G}}^{-1} = \left( \frac{1}{32\pi} \left( n_v + \frac{n_m}{12} \right) \frac{m_{\tilde{G}}^3}{M_P^2} \right)^{-1} = 24 \left( \frac{10 \text{ TeV}}{m_{\tilde{G}}} \right)^3 \text{ sec} .$$
Total higgsino/wino abundance

\[ \Omega_{\tilde{w}, \tilde{h}} h^2 = \Omega_{\tilde{w}, \tilde{h}}^{\tilde{G}} h^2 + \Omega_{\tilde{w}, \tilde{h}}^{\text{th}} h^2, \]

\[ \Omega_{\tilde{G}}^{\tilde{G}} h^2 = \frac{m_{\text{LSP}}}{m_{\tilde{G}}} \Omega_{\tilde{G}} h^2 \approx 2.7 \times 10^{-2} \left( \frac{m_{\text{LSP}}}{100 \text{ GeV}} \right) \left( \frac{T_{\text{RH}}(M_1, \tilde{m}_1)}{10^{10} \text{ GeV}} \right), \]

with ‘reheating temperature’ determined by neutrino masses (takes reheating process into account),

\[ T_{\text{RH}} \approx 1.3 \times 10^{10} \text{ GeV} \left( \frac{\tilde{m}_1}{0.04 \text{ eV}} \right)^{1/4} \left( \frac{M_1}{10^{11} \text{ GeV}} \right)^{5/4}. \]

Requirement of LSP dark matter, i.e. \( \Omega_{\text{LSP}} h^2 = \Omega_{\text{DM}} h^2 \approx 0.11 \), yields upper bound on the reheating temperature, \( T_{\text{RH}} < 4.2 \times 10^{10} \text{ GeV} \); lower bound on \( T_{\text{RH}} \) from successful leptogenesis (depends on \( \tilde{m}_1 \)).
For each ‘reheating temperature’, i.e. pair \((M_1, \tilde{m}_1)\), lower bound on gravitino mass (taken from Kawasaki et al ’08) (left panel). Requirement of higgsino/wino dark matter puts upper bound on LSP mass, dependent on \(\tilde{m}_1\), ‘reheating temperature’ (right panel); more stringent for higgsino mass, since freeze-out contribution larger. E.g., \(m_1 = 0.05\) eV implies \(m_{\tilde{h}} \lesssim 900\) GeV, \(m_{\tilde{G}} \gtrsim 10\) TeV.

Maximal temperature for ‘false vacuum decay scenario’: \(T_R = \mathcal{O}(10^9\) GeV\))
Summary and Outlook

Comparison of models for baryogenesis and dark matter:

• Moduli Decay: $T_R = \mathcal{O}(100 \, \text{MeV})$ *

• Electroweak Baryogenesis: $T_R = \mathcal{O}(100 \, \text{GeV})$ ***

• Majorana Neutrinos: $T_R = \mathcal{O}(100 \, \text{GeV})$ **

• Thermal Leptogenesis: $T_R = \mathcal{O}(10^{10} \, \text{GeV})$ ***

• ‘False vacuum decay scenario’: $T_R = \mathcal{O}(10^9 \, \text{GeV})$ ***

interesting: * ; interesting & falsifiable: ** ; theoretically motivated & falsifiable: ***