Model of a Strong First Order Electroweak Phase Transition at Low Temperature

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Electroweak Phase Transition

- In the early universe at temperature on the order of 100 GeV the electroweak (EW) symmetry is restored.
- For standard model (SM) parameters, quantum corrections wash out the weak first order phase transition, as shown through non-perturbative methods and lattice simulations.[1,2]

[1] W. Buchmuller, O. Philipsen Nucl. Phys. B **443** (1995) 47.

[2] M. E. Shaposhnikov, Phys. Rec. Lett. **77** (1996) 2887.

Electroweak Phase Transition and Baryogenesis

- Sakharov Conditions for Baryogenesis
  - B-number violating process.
  - C, CP violation.
  - Out of equilibrium interactions.
- A strong first order phase transition is necessary in order to have the system out of equilibrium.
- A phase transition at lower temperature prevents system from returning to equilibrium during the transition.
- A low temperature phase transition would potentially allow for exploration of the electroweak phase transition in the laboratory.

Constraints on Higgs Potential

- Vacuum Expectation value $v_0 = 246/\sqrt{2}$ GeV.
- Mass constrained near $m_h \simeq 125$ GeV.
- Renormalizability.
- SM Potential in unitary gauge

$$V(h) = \frac{\lambda_h}{4}(h^2 - v_0^2)^2, \quad \lambda_h = \frac{m_h^2}{2v_0^2}.$$

Modifying the Higgs Potential

- Viewing the SM Higgs sector as an effective theory opens the path to modifying the potential.
- We view a modification as the result of integrating out other degrees of freedom. We currently have no model for this. The following is purely phenomenological.
- Renormalizability only constrains the potential in the neighborhood of different vacuum states where perturbation theory applies.
- Removes the condition that the potential contain only quadratic and quartic terms in h.

Modifying the Higgs Potential

- We consider the family of potentials

$$W(h) = f(V(h)), \qquad f(x) = \frac{x}{(1 + Bx/V(0))^k}.$$

- This preserves the known vacuum expectation value and mass of the Higgs, but modifies the potential near $h = 0$ and for large values of h.
- We could take f to be any smooth function with $f(0) = 0$ and $f'(0) = 1$ and still maintain the desired properties of the Higgs.

Modifying the Higgs Potential

- In contrast to[3], the expansion near the vacuum state contains only even powers in $h^2 - v_0^2$.

$$W(h) = V(h) - \frac{\lambda Bk}{4v_0^4}(h^2 - v_0^2)^4 + \mathcal{O}\left((h^2 - v_0^2)^6\right).$$

- The new physics is primarily due to modification near $h = 0$.

[3] C. Grojean, G. Servant and J. D. Wells, Phys. Rev. D **71** (2005).

Properties of Modified Potential

- The parameters B and k control the height of the potential at $h = 0$

  $$B = (V(0)/W(0))^{1/k} - 1.$$

- $W(0) < V(0)$, allowing for effects at lower energy scales than the standard potential.

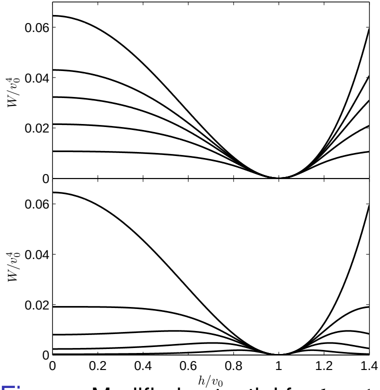


Figure: Modified potential for $k = 1$ (top) and $k = 3$ (bottom) and for $B = 0, 1/2, 1, 2, 5$ (top curve to bottom curve).

Properties of Modified Potential

- For $k < 1$ the potential is globally stable.
- For $k > 1$, $W \underset{h\to\infty}{\longrightarrow} 0$.
- For $(k-1)B > 1$ the critical point at $h = 0$ becomes a local minimum with Higgs mass

$$\tilde{m}_h^2 = m_h^2 \frac{(k-1)B - 1}{2(1+B)^{k+1}}.$$

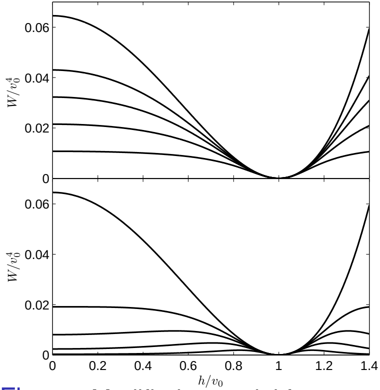


Figure: Modified potential for $k = 1$ (top) and $k = 3$ (bottom) and for $B = 0, 1/2, 1, 2, 5$ (top curve to bottom curve).

Effective Potential at Finite Temperature

- Each particle species contributes a free energy density term

$$U(h,T) = W(h) + \sum_j F_{\pm,j}(h,T),$$

$$F_\pm(h,T) = \mp\frac{g_s T^4}{2\pi^2}\int_0^\infty \ln\left(1 \pm e^{-E/T}\right) z^2 dz,$$

$$E/T = \sqrt{z^2 + g^2h^2/T^2}, \qquad z = p/T.$$

Effective Potential at Finite Temperature

- Particles with $gh \ll T$ don't contribute to the difference in effective potential.
- For our analysis to remain valid down to the scale $T \simeq 1$ GeV we include Higgs, gauge bosons $W^{\pm}$, Z^0 and the top, bottom, charm quarks, and the tau lepton. The other particles are approximately massless at this scale.

Critical Temperatures

- T_{c_1}: Massless phase is restored.
- T_{c_2}: Critical points become degenerate. Pressures of two phases are equal.
- T_{c_3}: Massive phase is eliminated.

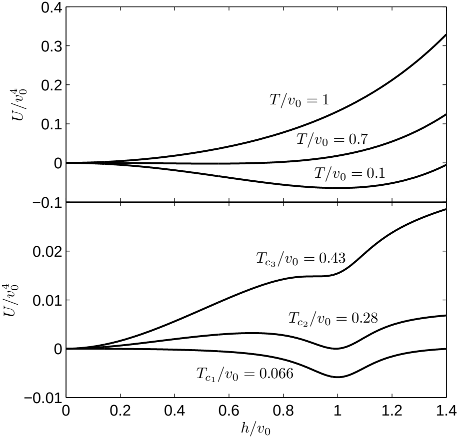


Figure: Higgs Effective Potential for $B = 0$, $k = 1$ (top) and $B = 10$, $k = 1$ (bottom) for Higgs-top quark system at finite temperature.

Computing Critical Temperatures

- Critical points are zeros of

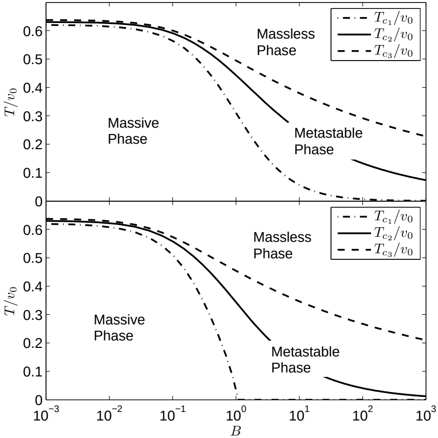
$$G(h,T) = \frac{\lambda_h h(h^2 - v_0^2)(1 - B(k-1)(h^2/v_0^2 - 1)^2)}{(1 + B(h^2/v_0^2 - 1)^2)^{k+1}} + \sum_j \frac{g_s g^2 T^3 h}{2\pi^2} \int_0^\infty \frac{z^2/E}{(e^{E/T} \pm 1)} dz \quad (1)$$

- To track the location of the critical point for $T > 0$ we solve the ode

$$\frac{dh}{dT} = -\frac{\partial G}{\partial T}\left(\frac{\partial G}{\partial h}\right)^{-1}. \quad (2)$$

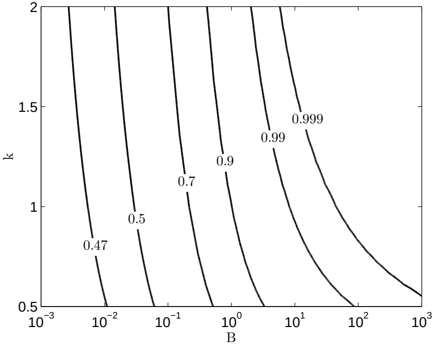
Phase Diagram

Phase domains as a function of the Higgs potential parameter B for $k = 1$ (top) and $k = 2$ (bottom).



Phase Diagram

$\Delta h/v_0$ at critical temperature T_{c_2} in the k, B plane.



Summary

- Maintains properties of the Higgs in the present day massive phase.
- Creates strong first order phase transition.
- Significantly lowers phase transition temperature.
- Possible exploration of EW phase transition at laboratory energies.
- Work is ongoing to provide a theoretical basis for effective potentials of this type.

References and Acknowledgments

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