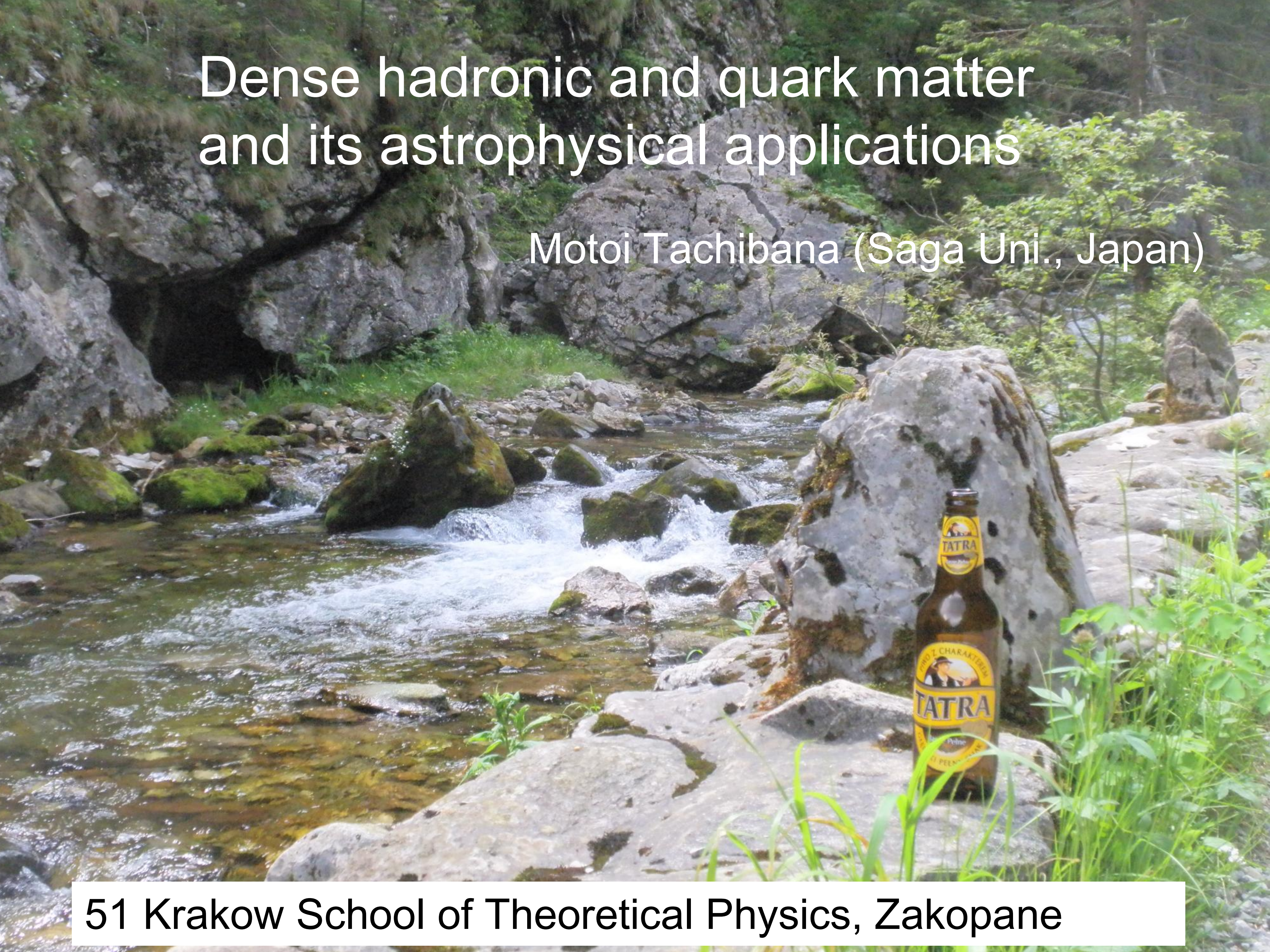
Dense hadronic and quark matter and its astrophysical applications

Motoi Tachibana (Saga Uni., Japan)



51 Krakow School of Theoretical Physics, Zakopane

Topics

1. QCD phase transition @ $\mu \neq 0$

   Ginzburg-Landau study for dense matter with axial anomaly, strange quark mass, and meson (Kaon) condensation

   [work w/ T. Hatsuda, N. Yamamoto, G. Baym: A. Schmitt and S. Stetina]

2. Astrophysical applications

   Collective modes in CFL quark matter and its application to superfluid-vortex dynamics

   [in progress w/ M. Ruggieri and M. Mannarelli]

Content of 1st topic

1. QCD phase transition and its phase diagram

1. Chiral-super interplay —anomaly-driven CP—

2. Excitation spectra —generalized pion—

4. Meson condensation —CFL K^0 condensate—

Content of 2nd topic

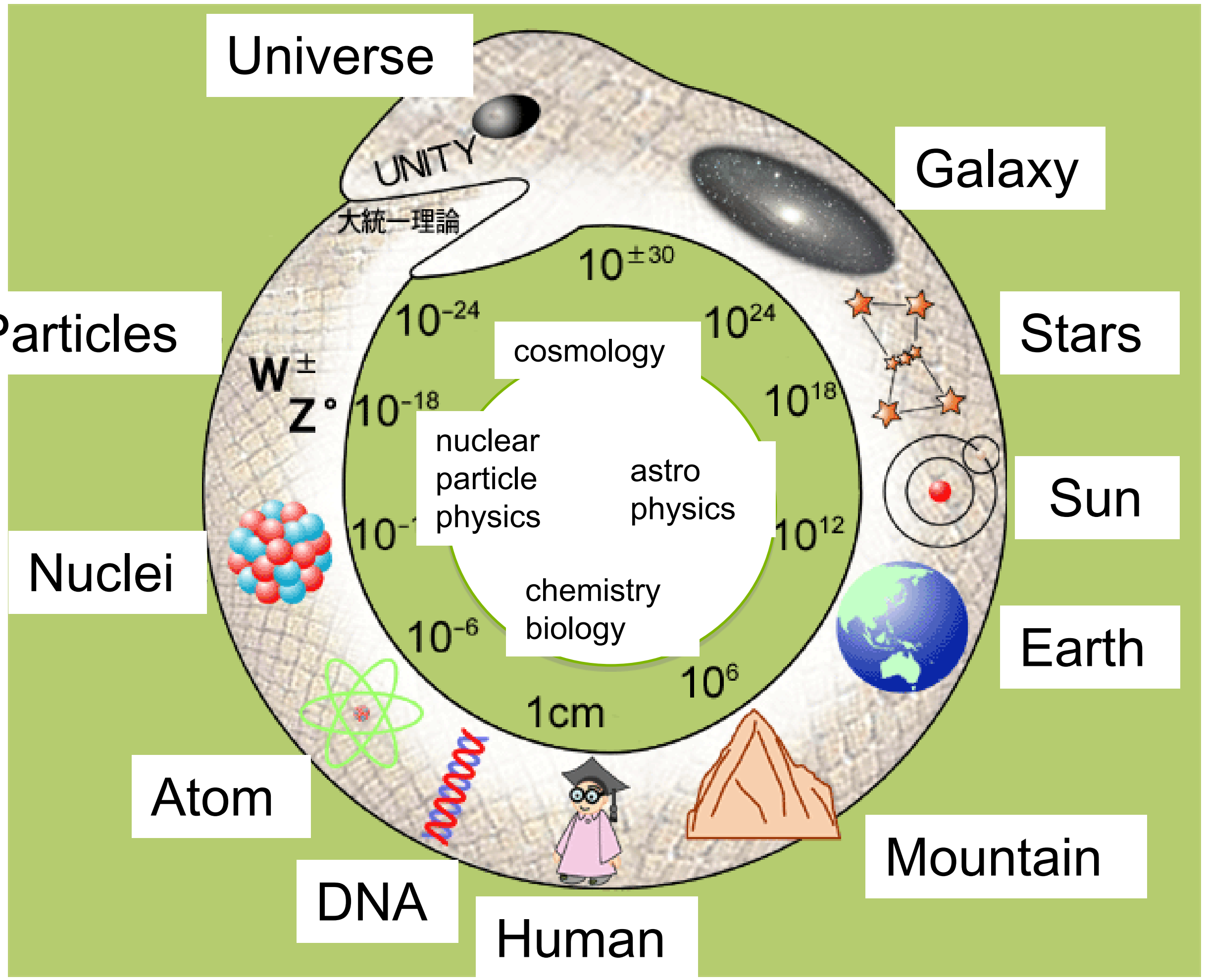
1. The r-mode instability of neutron star

1. Mutual friction ー vortex dynamics in superfluidity ー

2. Collective modes in CFL quark matter

3. Caroli-de Gennes-Matricon (CdGM) mode

Thanks to Gamow and Penzias-Wilson, who proposed and discovered the cosmic microwave background (CMB)



"UROBOROS" = unity of matters & universe

An interesting connection

between

matters and universe

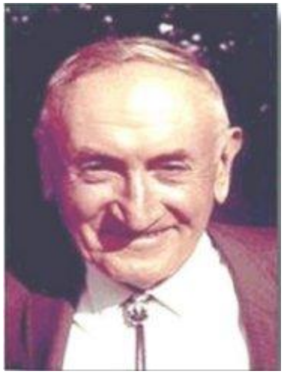
Compact Stars and Dense Matter

Discovery of Neutron Star

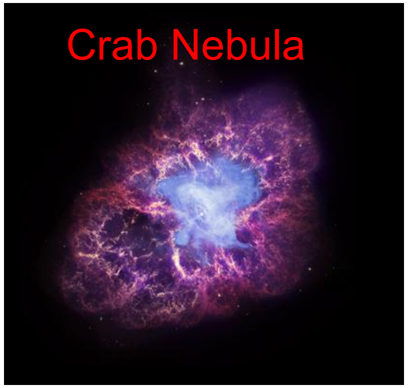
- A star made by neutron -

Baade and Zwicky (1934): 1st theoretical prediction of NS

W. Baade



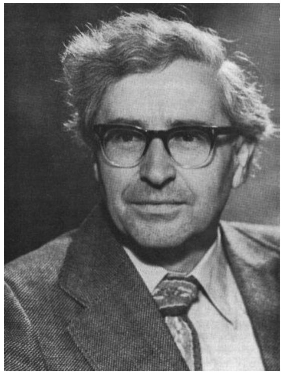
F. Zwicky



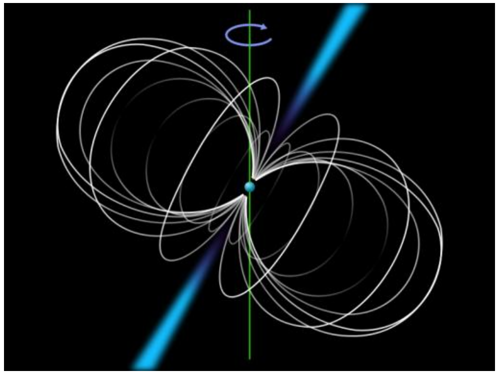
supernova

Bell and Hewish (1967): Discovery of Pulsar (1st pulsar!)

Jocelyn Bell Burnell



A. Hewish



pulsar image

Basic profiles of a neutron star

Macroscopic

(Typical) radius: $R \sim 10 km$

(Typical) mass: $M \sim 1.4 M_*$ M_* : solar mass

Temperature: $T < 10 MeV$ 1eV $\sim 10^4 K$

Magnetic field: $B \sim 10^{12} G$ Earth's *B* field $\sim 0.6G$

Microscopic

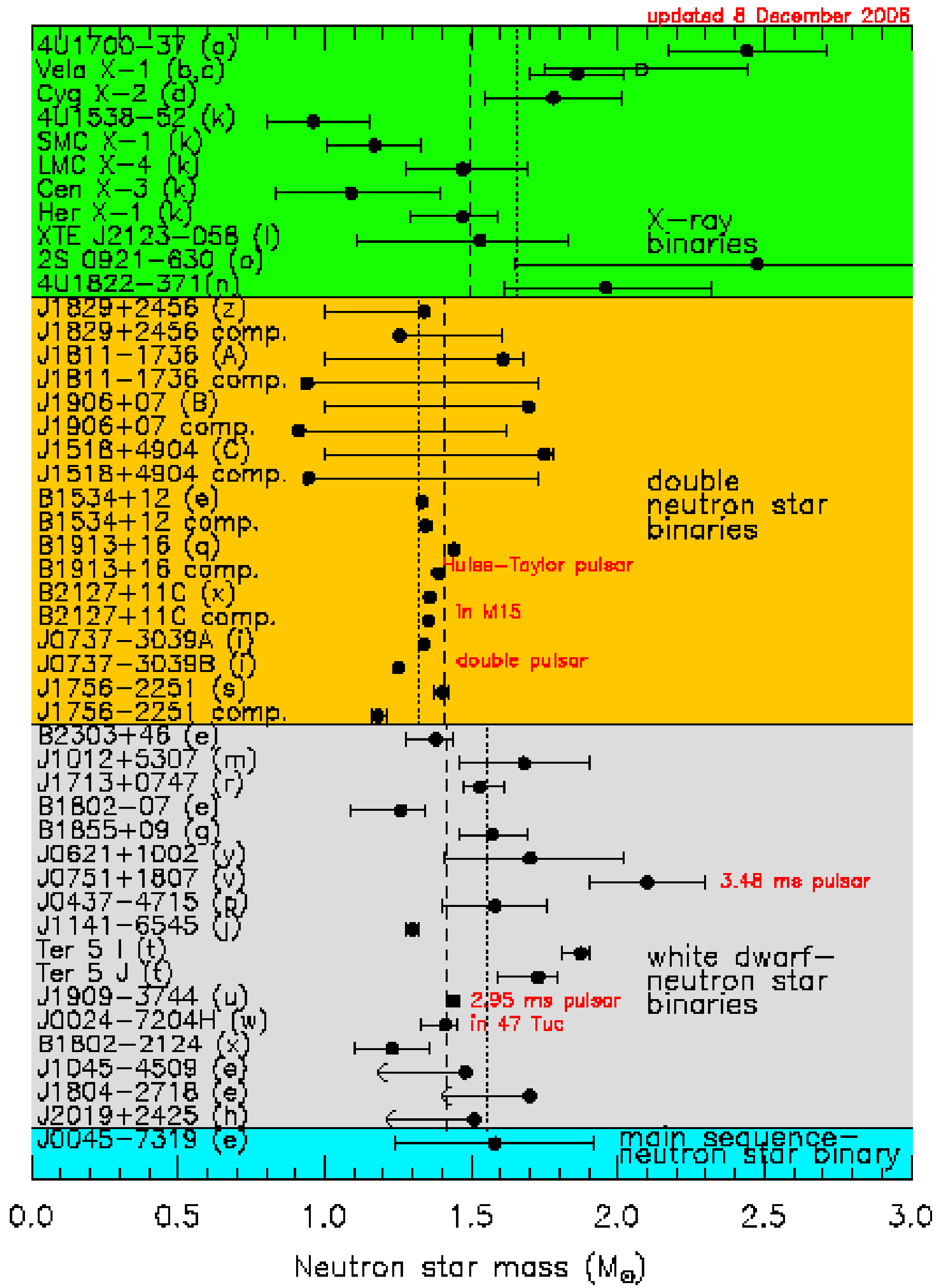
(Typical) density: $\rho_{nucl.} = 2.5 \times 10^{14} (g/cm^3)$ one teaspoon ~ 900×Giza's pyramid

Fermi momentum: $k_F \sim 300 - 600 MeV$

Fermi energy: $E_F = k_F^2 / 2m \sim 60 - 150 MeV$

Chemical potential: $\mu_B = 500 - 1000 MeV$

$$\left(\begin{array}{l} L_{\mathrm{int.}} \sim L_{diam.} \\ E_{\mathrm{int.}} >> E_{kin.} \end{array} \right)$$



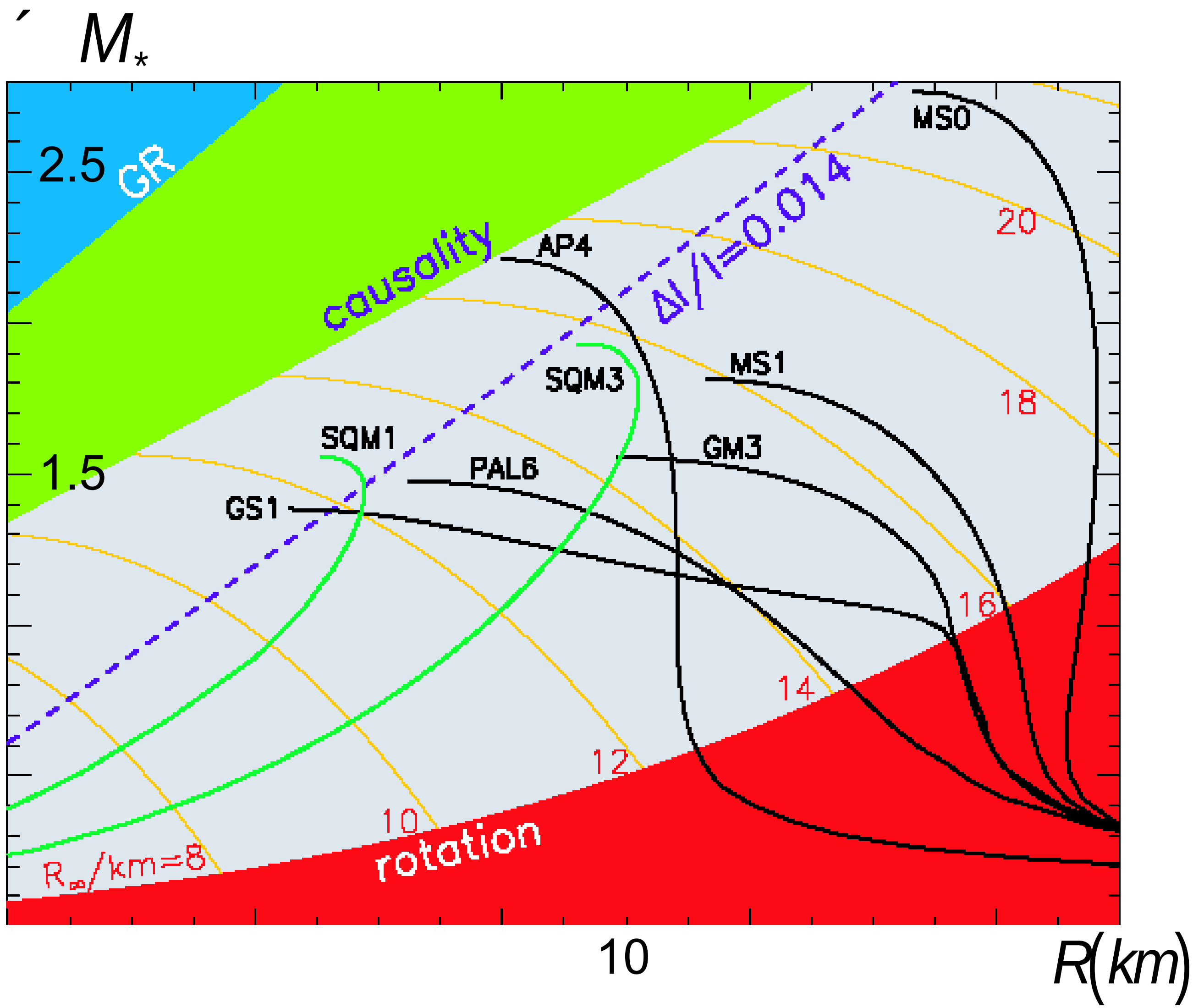
X-ray binaries

double NS binaries

WD-NS binaries

MS-NS binary

Mass-Radius relationship



[Blue region]

Excluded by GR

[Green region]

Excluded by causality

[Red region]

Excluded by rotation

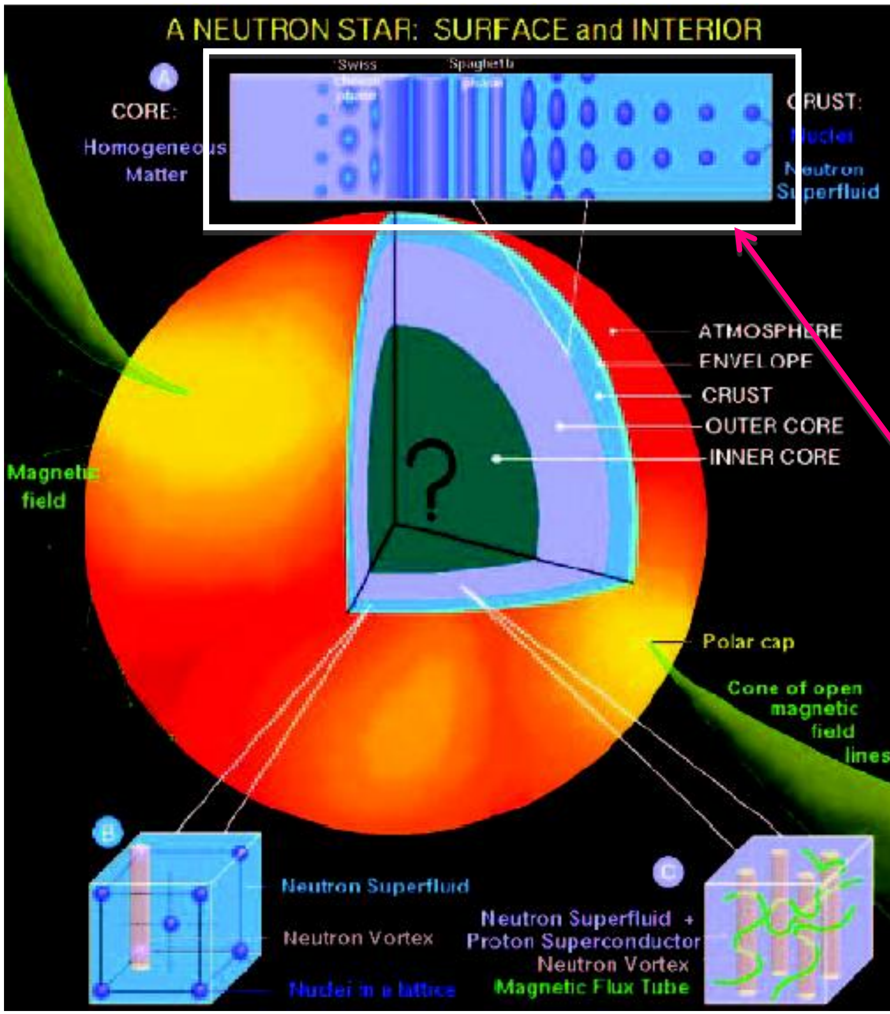
<Black curves>

Nuclear matter EOS

<Green curves>

Quark matter EOS

Neutron Star (NS) as "CIPOLLA" (Theorist's view)



① Atmosphere: hydrogen, a mix of heavy elements
(providing info. of temperature)

② Envelope: a few tens of meters
(acts as a thermal insulator)

③ Crust: 500-1000m thickness
(contains nuclei, forming a quantum liquid of superfluid neutron)

④ Nuclear Pasta: In-btw crust/core
(spaghetti, lasagnas, swiss cheese)

⑤ Outer Core: a few kilometers
(neutron superfluidity and proton superconductivity)

⑥ Inner Core: Big question mark!

Exotica

Several new states of matter may be favored in NS:

- Bose-Einstein condensate (BEC) of mesons
- Hyperons
- Deconfined (superconducting) quark matter

→ Qualitative/Quantitative differences among them?

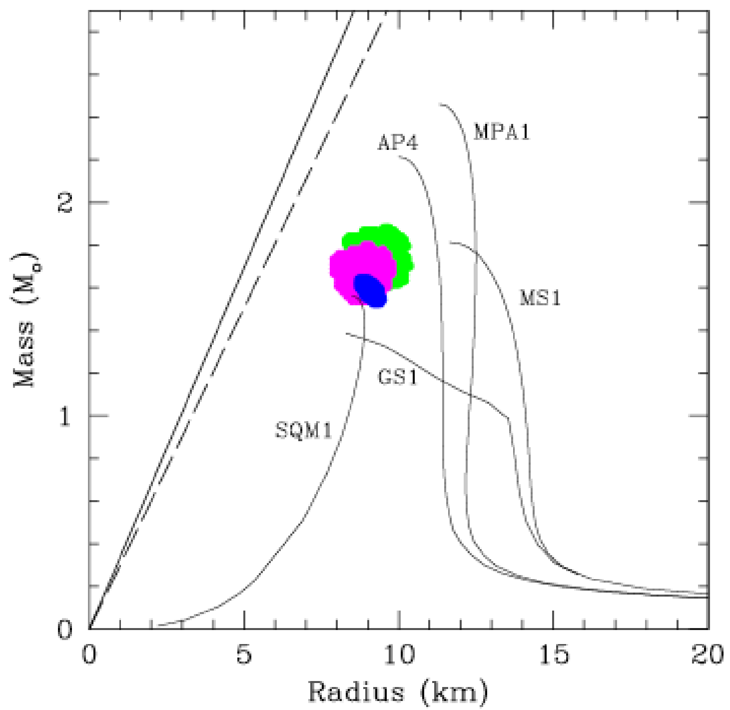
Neutron star as *laboratories* for the understanding of dense matter!

Tweet...the experiments in this labo. are less controlled than those on earth.

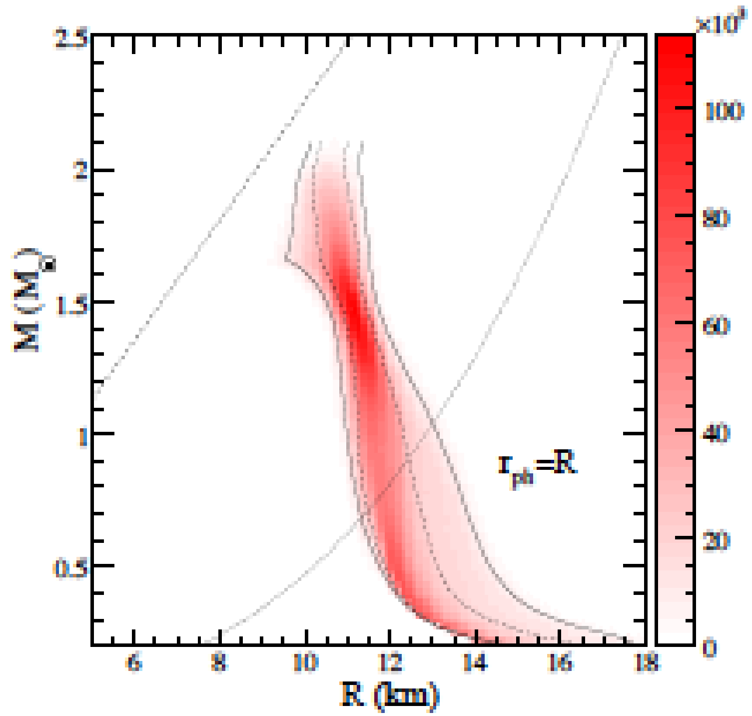
3 recent topics for NS observations

(Please don't ask me. Just smell the flavors)

From TOV to EOS



Özel-Baym-Guver (1002.3153)

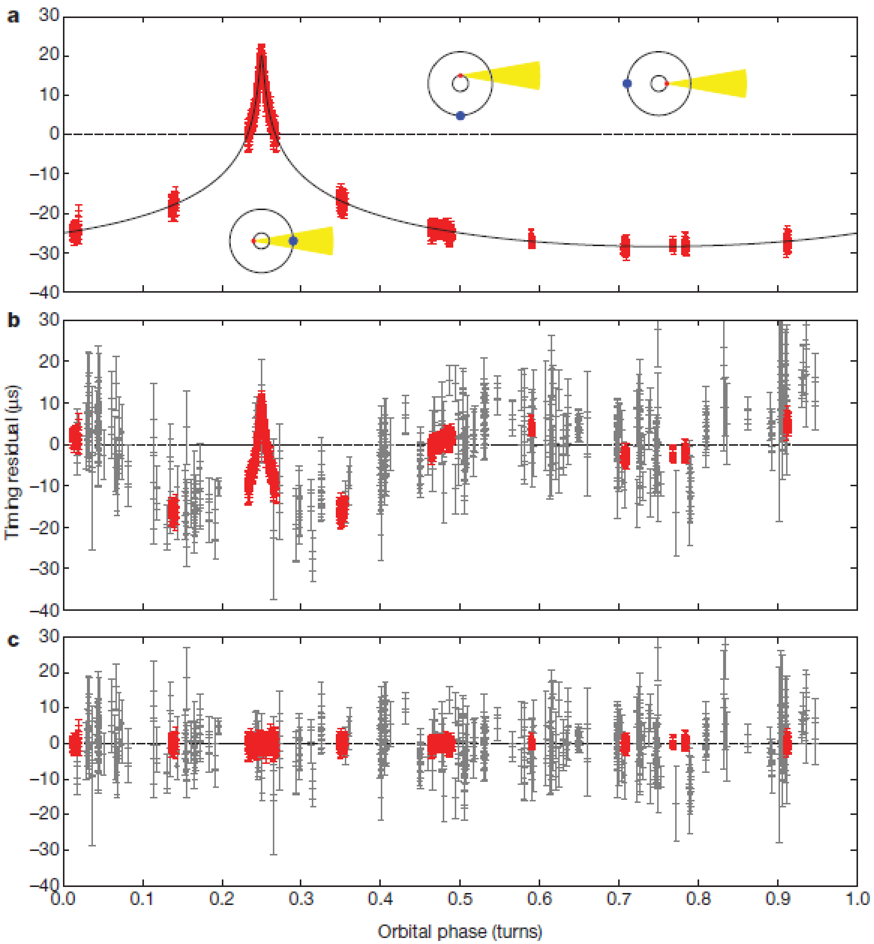


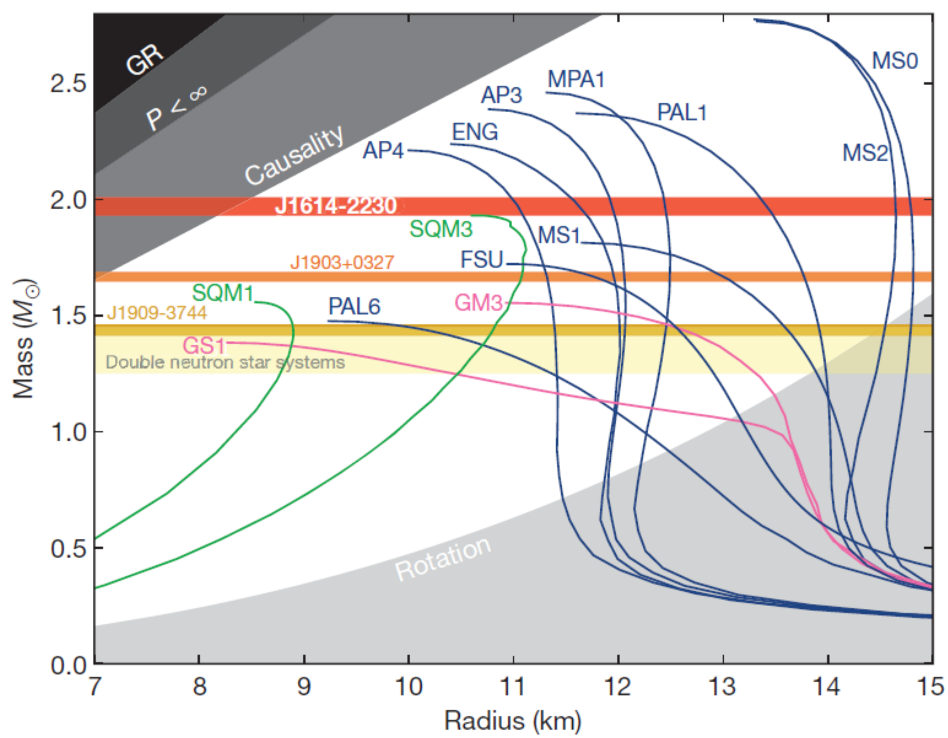
Steiner-Lattimer-Brown (1005.0811)

$$M = M(R) \quad \Longleftrightarrow \quad P = P(r)$$

A $2M_{SUN}$ NS measured using Shapiro-delay

Demorest et al., Nature 467:1081–1083(2010)





Evidence of $^3P_2$ superfluidity in NS ?

D. Page et al, Phys.Rev.Lett.106, 081101 (2011)

Selected for a Viewpoint in *Physics*

PRL **106,** 081101 (2011) PHYSICAL REVIEW LETTERS week ending 25 FEBRUARY 2011

Rapid Cooling of the Neutron Star in Cassiopeia A Triggered by Neutron Superfluidity in Dense Matter

Dany Page,[1] Madappa Prakash,[2] James M. Lattimer,[3] and Andrew W. Steiner[4]

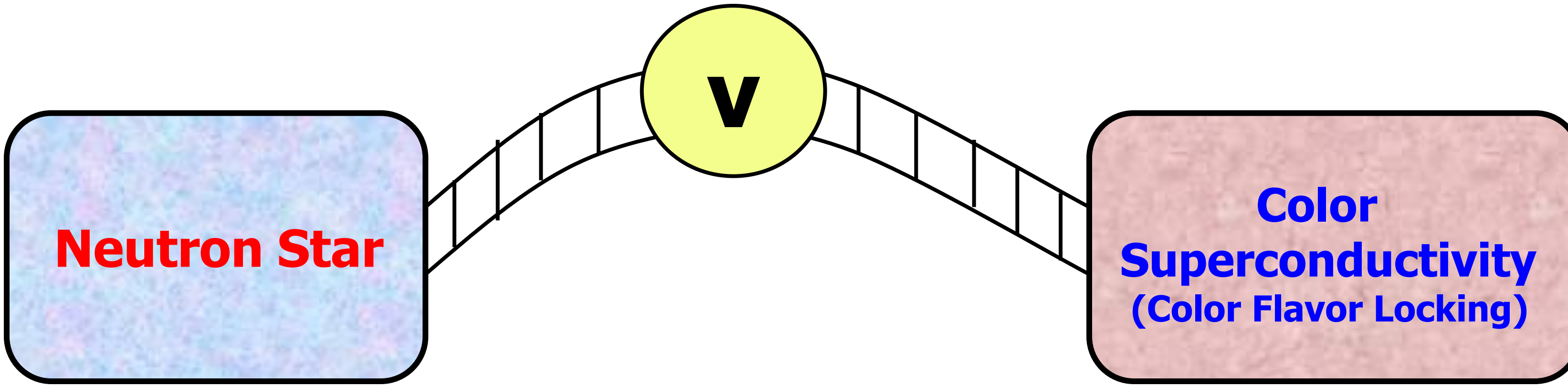
[1]*Instituto de Astronomía, Universidad Nacional Autónoma de México, Mexico D.F. 04510, Mexico*
[2]*Department of Physics and Astronomy, Ohio University, Athens, Ohio 45701-2979, USA*
[3]*Department of Physics and Astronomy, State University of New York at Stony Brook, Stony Brook, New York 11794-3800, USA*
[4]*Joint Institute for Nuclear Astrophysics, National Superconducting Cyclotron Laboratory and, Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA*

(Received 29 November 2010; published 22 February 2011)

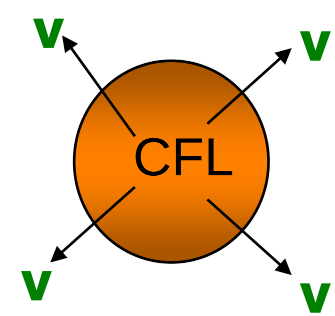
We propose that the observed cooling of the neutron star in Cassiopeia A is due to enhanced neutrino emission from the recent onset of the breaking and formation of neutron Cooper pairs in the $^3P_2$ channel. We find that the critical temperature for this superfluid transition is $\simeq 0.5 \times 10^9$ K. The observed rapidity of the cooling implies that protons were already in a superconducting state with a larger critical temperature. This is the first direct evidence that superfluidity and superconductivity occur at supranuclear densities within neutron stars. Our prediction that this cooling will continue for several decades at the present rate can be tested by continuous monitoring of this neutron star.

NS Cooling via ν-emission in CFL Matter

[**D. K. Hong, H. K. Lee, M. A. Nowak and M. Rho hep-ph/0010156,**
Reddy-Sadzikowski-Tachibana('02), Jaikumar-Prakash-Schäfer('02)]



NS cooling via ν-emission through the interactions with the Nambu-Goldstone bosons



Neutrino mean free path (λ)
Neutrino emissivity (ε)

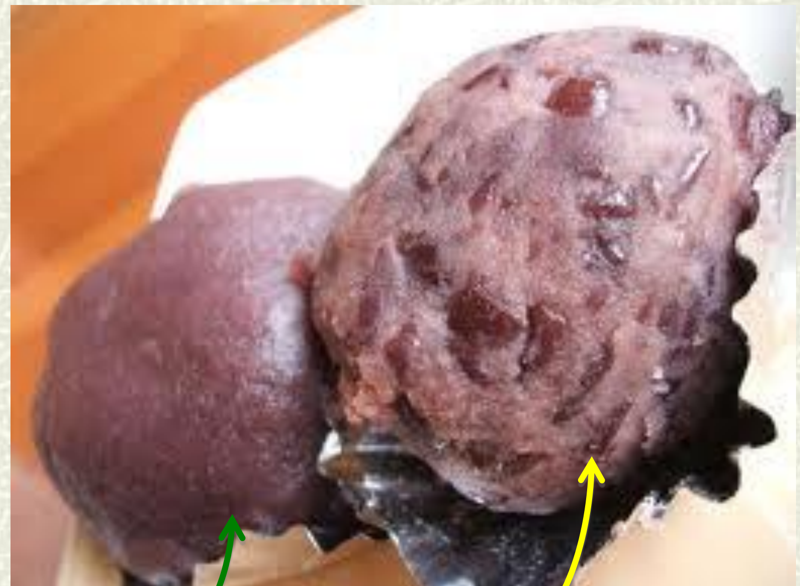
→

$$C_V \frac{dT}{dt} = - e$$

(Diffusion equation)

1. Ginzburg-Landau study in dense QCD: chiral-super interplay, axial anomaly, and meson condensation(s)

Hadrons
(AZUKI)



Quark matter
(KOSHI-AN)

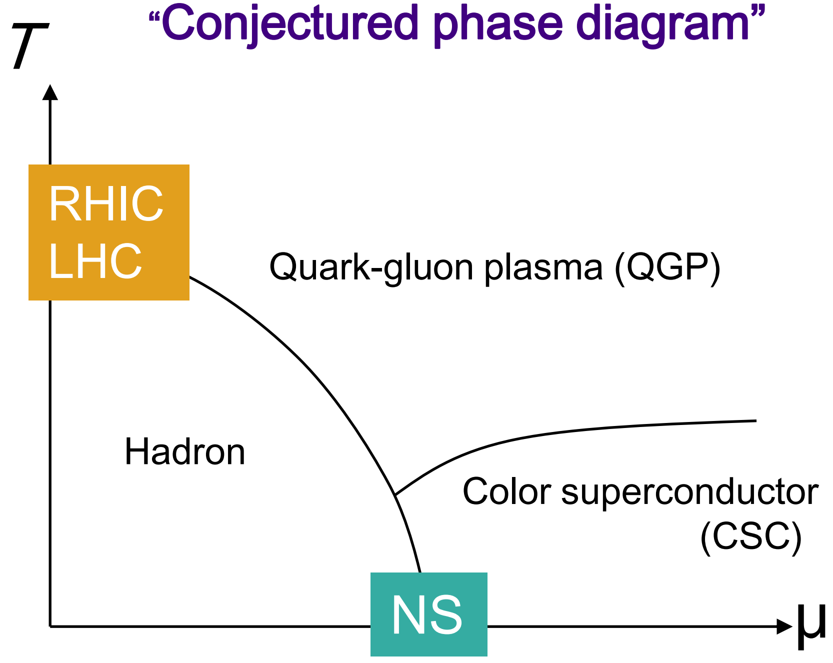
Hadronic matter
(TSUBU-AN)

QCD phase transition and its phase diagram

QCD @ **high** temperature(T) / density(ρ)   [Collins-Perry (1975)]

QCD vacuum undergoes **a phase change** at some values of T and ρ!

“Conjectured phase diagram”



Each phase characterized by

$\langle \bar{q}q \rangle$: chiral condensate

$\langle qq \rangle$: diquark condensate

”order parameters”

What we REALLY know is surprisingly less

- Large μ/T regime → sign problem Fodor's talk
- Entanglement among orders (eg. High-T_c SC)

↓

Ginzburg-Landau (GL) study in dense QCD

(Bailin-Love, Iida-Baym)

Ginzburg-Landau (GL) analysis

Ginzburug-Landau (GL) approach : model independent, analytic

1. Topological structure of the phase diagram
2. Order of the phase transition
3. Critical properties

Recipe

$$Z = \int [d\sigma] \exp\left(-\int dx L_{eff}(\sigma(x);K)\right)$$

$\sigma(x)$: Order parameter field

$$L_{eff} = \frac{1}{2}(\nabla\sigma)^2 + \sum_n a_n(K)\sigma^n$$

Same symmetry with underlying theory

$K = \{T, m, \mu, \ldots\}$: External parameters

Ginzburg-Landau = Saddle point approximation

Wilson = Fluctuations by renormalization group method

Chiral-super interplay in dense QCD

QCD symmetry: $SU(3)_C \times \left[SU(N_f)_L \times SU(N_f)_R\right] \times U(1)_B \times \cancel{U(1)}_A$

Order parameters

Chiral field:

$$\Phi_{ij} \sim \left(\bar{q}_R\right)_a^j \left(q_L\right)_a^i$$

$$\Phi \to e^{-2i\alpha_A} V_L \Phi V_R^+$$

Diquark field:

$$\left(d_L\right)_{ia} \sim \varepsilon_{ijk}\varepsilon_{abc}\left(q_L\right)_b^j \left(q_L\right)_c^k$$

$$d_L \to e^{2i\alpha_A} e^{2i\alpha_B} V_L d_L V_C^T$$

Underlying sym. (QCD)

GL potential

$$\mathcal{V}(\Phi, d) = \mathcal{V}_\chi(\Phi) + \mathcal{V}_d(d_L, d_R) + \mathcal{V}_{\chi d}(\Phi, d_L, d_R)$$

GL potential

$$\mathcal{V}_\chi = \frac{a_0}{2}\mathrm{tr}\,\Phi^\dagger\Phi + \frac{b_1}{4!}\left(\mathrm{tr}\,\Phi^\dagger\Phi\right)^2 + \frac{b_2}{4!}\mathrm{tr}\left(\Phi^\dagger\Phi\right)^2 \boxed{-\frac{c_0}{2}\left(\det\Phi + \det\Phi^\dagger\right)},$$

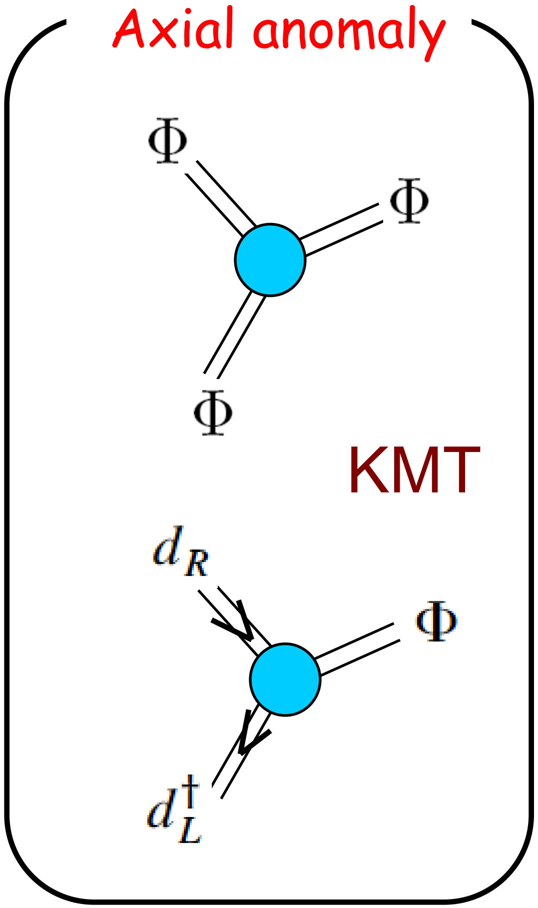
(Pisarski-Wilczek)

$$\begin{aligned}\mathcal{V}_d = {} & \alpha_0\ \mathrm{tr}[d_L d_L^\dagger + d_R d_R^\dagger] \\ & + \beta_1\left([\mathrm{tr}(d_L d_L^\dagger)]^2 + [\mathrm{tr}(d_R d_R^\dagger)]^2\right) \\ & + \beta_2\left(\mathrm{tr}[(d_L d_L^\dagger)^2] + \mathrm{tr}[(d_R d_R^\dagger)^2]\right) \\ & + \beta_3\ \mathrm{tr}[(d_R d_L^\dagger)(d_L d_R^\dagger)] + \beta_4\ \mathrm{tr}(d_L d_L^\dagger)\mathrm{tr}(d_R d_R^\dagger)\end{aligned}$$

(Iida-Baym)

$$\begin{aligned}\mathcal{V}_{\chi d} = {} & \boxed{\gamma_1\ \mathrm{tr}[(d_R d_L^\dagger)\Phi + (d_L d_R^\dagger)\Phi^\dagger]} \\ & + \lambda_1\ \mathrm{tr}[(d_L d_L^\dagger)\Phi\Phi^\dagger + (d_R d_R^\dagger)\Phi^\dagger\Phi] \\ & + \lambda_2\ \mathrm{tr}[d_L d_L^\dagger + d_R d_R^\dagger]\cdot\mathrm{tr}[\Phi^\dagger\Phi] \\ & + \lambda_3\left(\det\Phi\cdot\mathrm{tr}[(d_L d_R^\dagger)\Phi^{-1}] + h.c\right)\end{aligned}$$

(HTYB)



leading int. term come from anomaly

2 possible ways of contraction of 6-q KMT vertex in dense QCD !!

A simple ansatz for condensate fields

3-flavor massless quark matter $\left(m_u = m_d = m_s = 0\right)$

$$\Phi = \begin{pmatrix} \sigma & & \\ & \sigma & \\ & & \sigma \end{pmatrix} \qquad \delta_L = -\delta_R = \begin{pmatrix} \delta & & \\ & \delta & \\ & & \delta \end{pmatrix}$$

Color-Flavor Locking (CFL)

Alford-Rajagopal-Wilczek (1998)

$$\left\langle q_a^i C \gamma_5 q_b^j \right\rangle = \epsilon_{abc} \epsilon^{ijk} \left(\delta_L\right)_k^c \propto \epsilon_{abA} \epsilon^{ijA} = \delta_a^i \delta_b^j - \delta_a^j \delta_b^i$$

$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_{c+L+R}$$

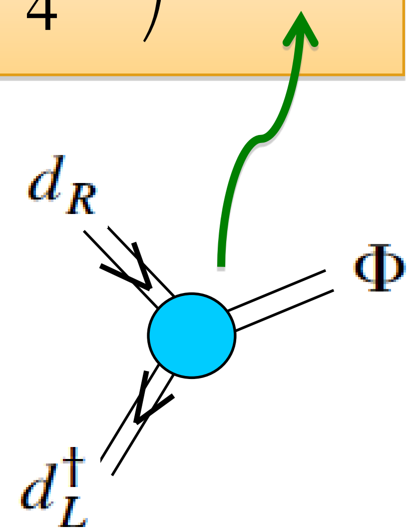
CFL breaks chiral symmetry!

Srednicki
-Susskind ('81)

For 3 flavor massless quarks (mean field level)

$$\Omega_{3F} = \left(\frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4\right) + \left(\frac{\alpha}{2}d^2 + \frac{\beta}{4}d^4\right) - \gamma d^2\sigma$$

- *s* : Chiral condensate
- *d* : Diquark condensate (CFL)
- *a, b, c, a, b, g*: GL parameters



Possible phases

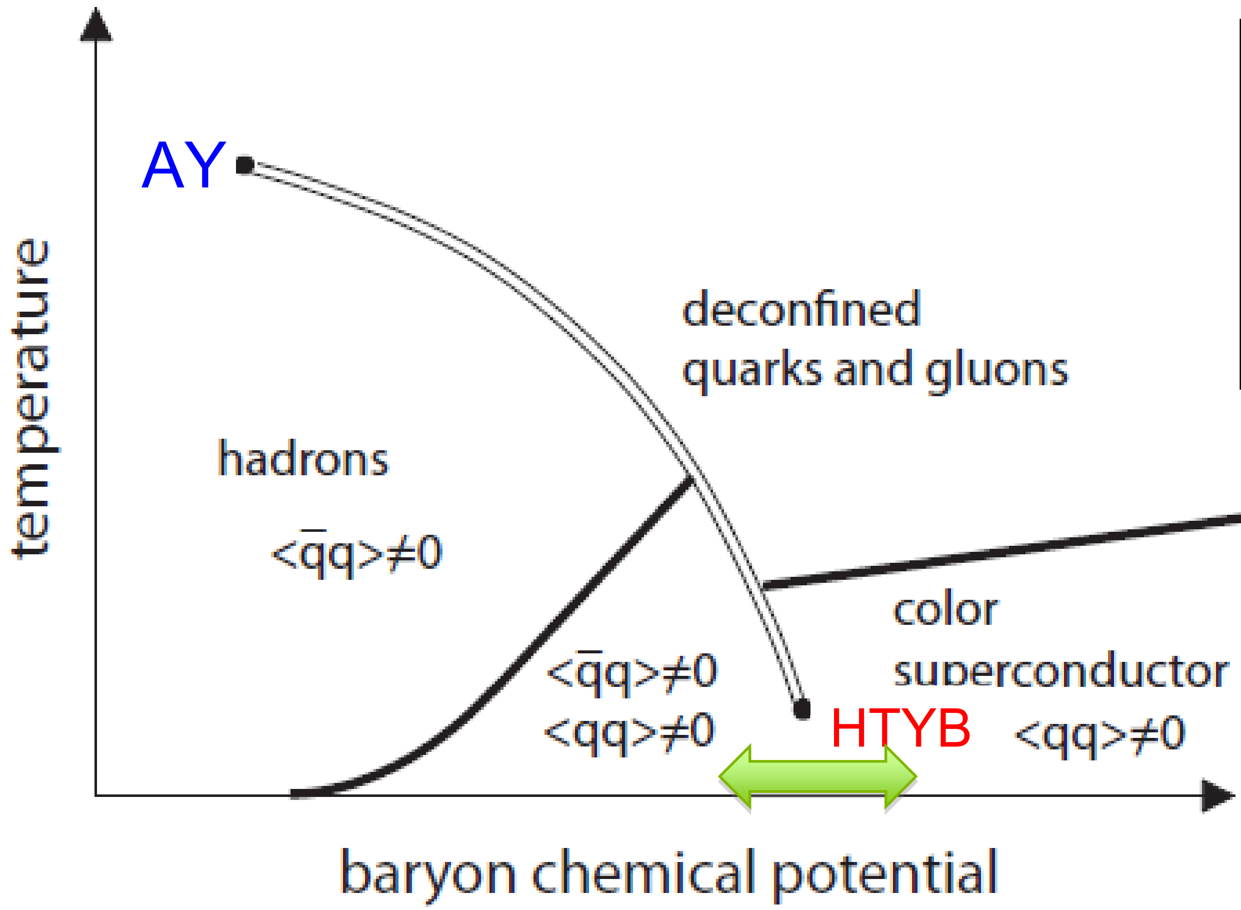
$\sigma = d = 0$	NOR
$\sigma \neq 0$	NG
$d \neq 0$	CSC
$\sigma \neq 0, d \neq 0$	COE

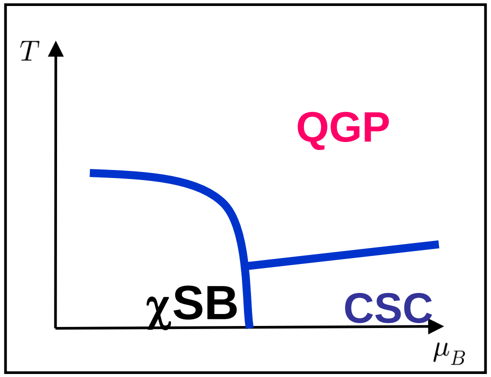
- $d, \sigma \neq 0$ *is favored*
- ext. source for σ

Equivalent to Ising-ferro !

→ Critical point !

Possible phase diagram in QCD





AY=Asakawa•
Yazaki

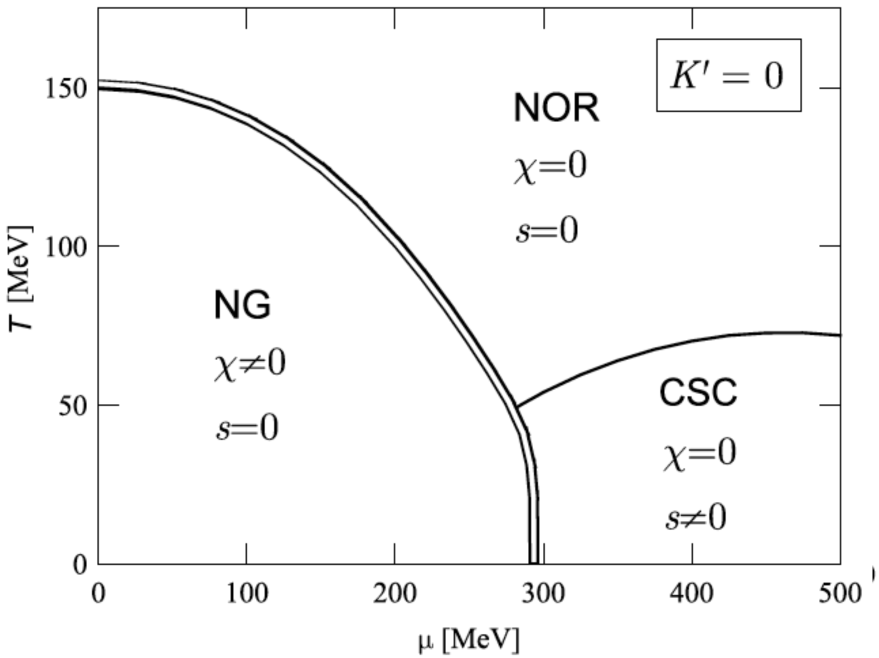
"Anomaly-induced critical point in dense QCD"

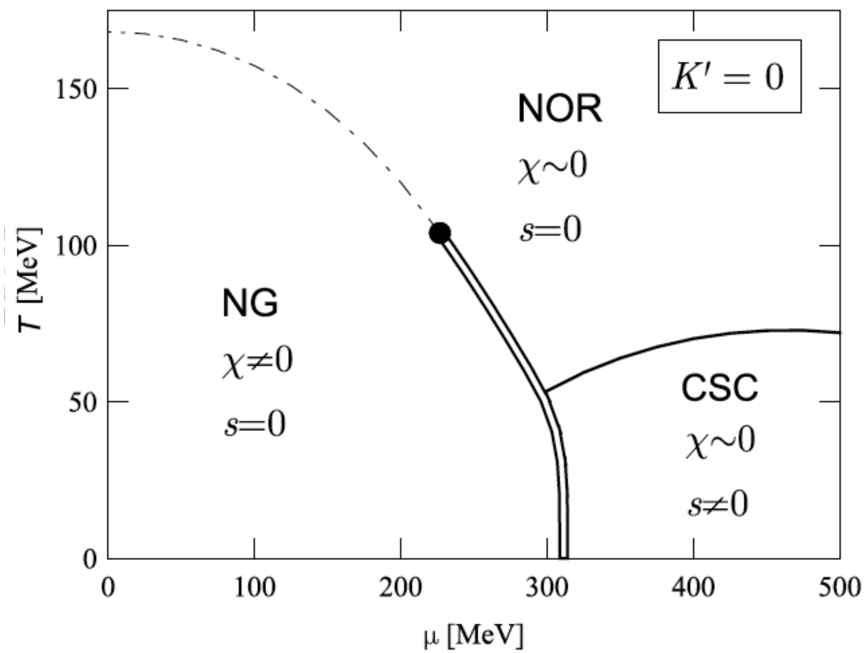
Hatsuda, Tachibana, Yamamoto & Baym, PRL('06)

NJL model calculation

Abuki et al., Phys.Rev.D81:125010 (2010)

K': strength of KMT int.





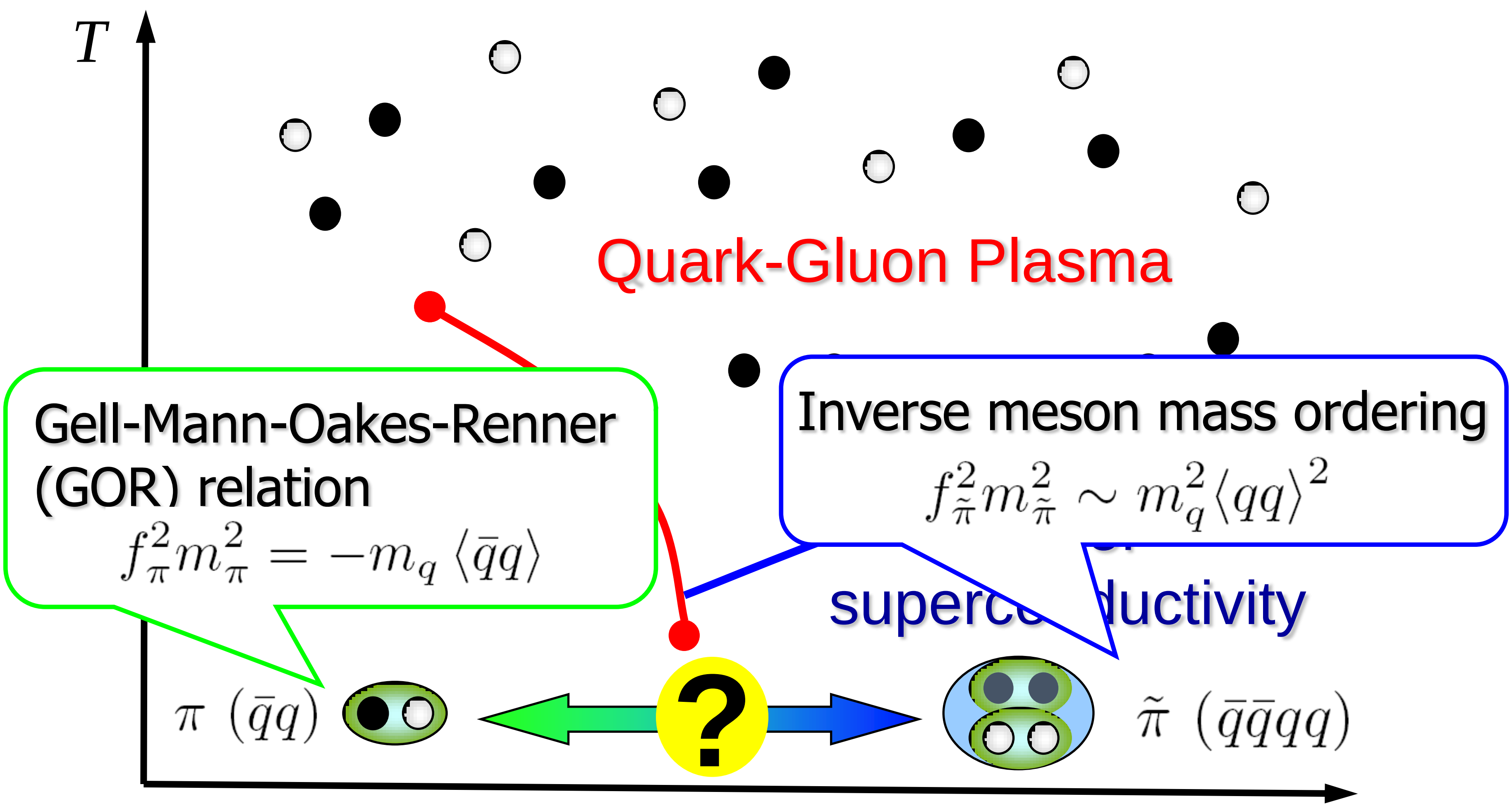
$m_u = m_d = m_s = 0$

$m_u = m_d = m_s = 5.5$

Excitation spectra
----pions at intermediate density----

A concrete realization of quark-hadron continuity?

Pion at intermediate density



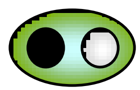
What is the form & mass spectrum of the pion at intermediate density?

Ginzburg-Landau effective Lagrangian

"Pion" on the hadron side

$$\Phi = \sigma \Sigma e^{-2i\theta}$$

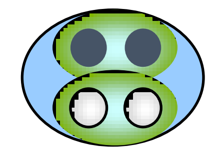
$$\Sigma = \exp\left(i \frac{\lambda^I \pi^I}{f_\pi} \right)$$



"Pion" on the CSC side

$$d_L = dU_L e^{2i\tilde{\theta}+2i\phi}, \quad d_R = -dU_R e^{-2i\tilde{\theta}+2i\phi}$$

$$\tilde{\Delta} = U_L U_R^\dagger = \exp\left(i \frac{\lambda^I \tilde{\pi}^I}{f_{\tilde{\pi}}} \right)$$



Effective Lagrangian:

- Kinetic term $\mathcal{L}^{\mathrm{kin}} = f_\pi^2 \mathrm{Tr}\left(g_\pi^{\mu\nu} \partial_\mu \Sigma \partial_\nu \Sigma^\dagger \right) + f_{\tilde{\pi}}^2 \mathrm{Tr}\left(g_{\tilde{\pi}}^{\mu\nu} \partial_\mu \tilde{\Delta} \partial_\nu \tilde{\Delta}^\dagger \right)$

$$g_\pi^{\mu\nu} = \mathrm{diag}\left(1, \vec{v}_\pi^2\right), \quad g_{\tilde{\pi}}^{\mu\nu} = \mathrm{diag}\left(1, \vec{v}_{\tilde{\pi}}^2\right)$$

- Mixing term: $\mathcal{L}^{\mathrm{mix}} = -\gamma d^2 \sigma \left[\mathrm{Tr}\left(\tilde{\Delta}^\dagger \Sigma \right) + \mathrm{h.c.} \right]$

- Mass term: (up to $\mathcal{O}(M)$) $\mathcal{L}^{\mathrm{mass}} = A_0 \left[\mathrm{Tr}\left(M \Sigma^\dagger \right) + \mathrm{h.c.} \right] + \Gamma_1 \left[\mathrm{Tr}\left(M \tilde{\Delta}^\dagger \right) + \mathrm{h.c.} \right]$

Mass spectrum of the generalized pion

Mass eigenstate : $\begin{pmatrix} \Pi \\ \tilde{\Pi} \end{pmatrix} = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} \pi \\ \tilde{\pi} \end{pmatrix}$

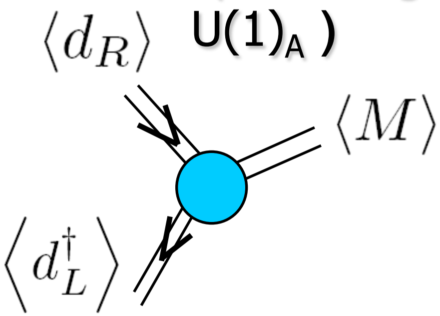
Π is a mixed state of $\pi\ (\bar{q}q)$ & $\tilde{\pi}\ (\bar{q}\bar{q}qq)$ with mixing angle ϑ

Generalized Gell-Mann-Oakes-Renner (GOR) relation

$$\left(f_\pi^2 + f_{\tilde{\pi}}^2\right) m_\Pi^2 = -m_q \left(\langle \bar{q}q \rangle + \Gamma \langle qq \rangle^2\right)$$

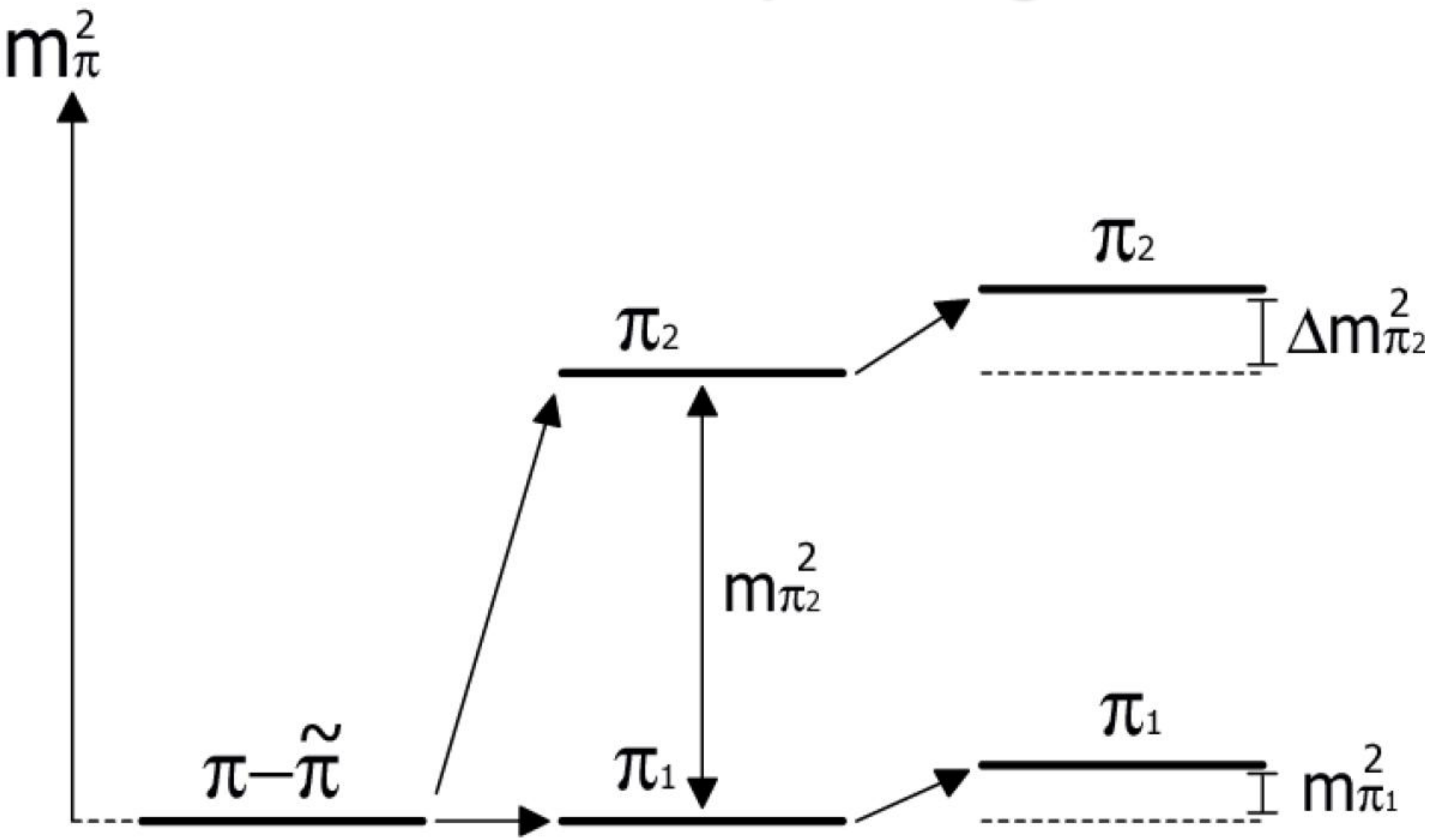
[red box] = **Axial anomaly (breaking U(1)$_A$)**

$$\Gamma \sim \mu (\Lambda_{\mathrm{QCD}}/\mu)^9 (1/g)^{14}$$



- Hadron-quark continuity is also realized for excited states.
- Axial anomaly plays a crucial role on pion mass spectrum.

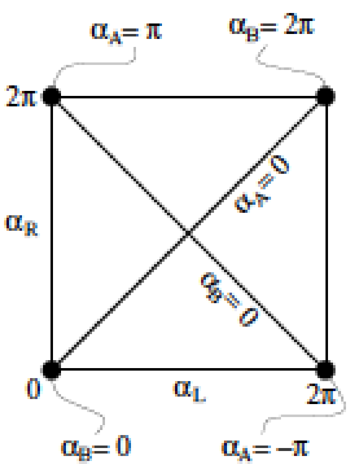
Mass splitting

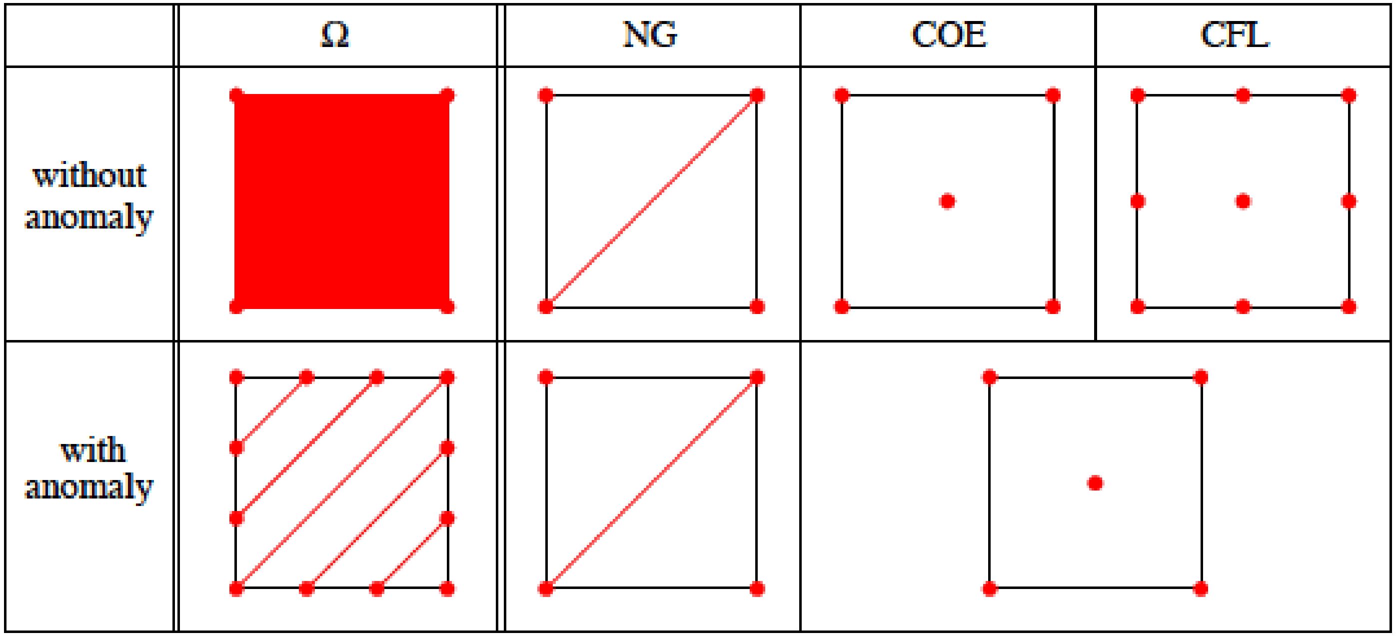


$$\mathcal{L}^{\rm kin} \qquad \mathcal{L}^{\rm kin}+\mathcal{L}^{\rm mix} \qquad \mathcal{L}^{\rm kin}+\mathcal{L}^{\rm mix}+\mathcal{L}^{\rm mass}$$

★ Physics of external stress★

—Meson (Kaon) condensation in CFL—





A.Schmitt, S.Stephan, M.T., PRD (2011)

Question

If finite strange quark mass is introduced from asymptotically high density side, what happens?

『Physics of Stress』

1. Direct effect from M_s
2. Indirect effect from M_s

→ Meson condensation in CFL

(Bedaque-Schafer, 2002)

CFL meson condensation in GL

$$\delta_L = \delta_R^{+} = \delta \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi/2) & ie^{i\mu t}\sin(\phi/2) \\ 0 & ie^{-i\mu t}\sin(\phi/2) & \cos(\phi/2) \end{pmatrix}$$

ϕ : K^0 field μ: Kaon chemical potential

Muto-Tatsumi, PLB(92)

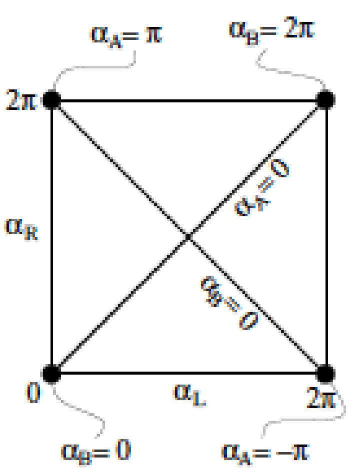
$$\Phi = \begin{pmatrix} \sigma_u & 0 & 0 \\ 0 & \sigma_d & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}$$

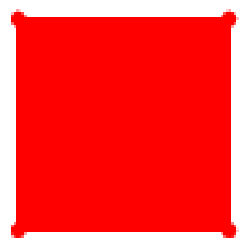
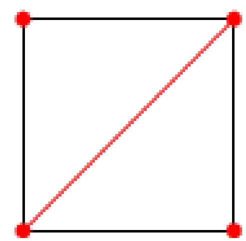
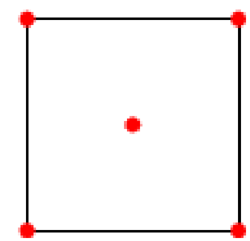
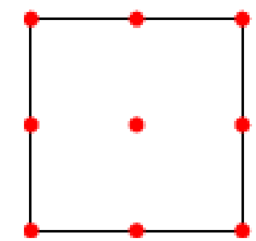
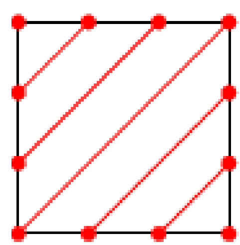
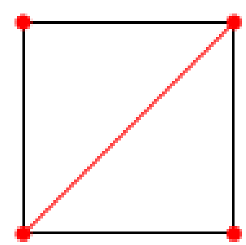
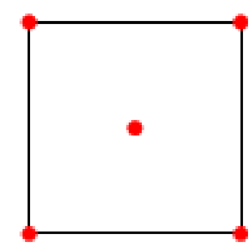
Chiral condensate

Comment on gauge invariance

Elitzur's theorem = "Local gauge invariance cannot be broken"

The use of diquark operators is just conventional. Alternatively, one can utilize the operators of the form $(\bar{q}q)^2$, $(qqq)^2$ for chiral and baryon symmetry breaking, respectively.



	Ω	NG	COE	CFL
without anomaly				
with anomaly				

GL free energy with Ms and meson condensation

After taking $s_u = s_d = s_s \equiv s$ for simplicity, we obtain

$$\Omega_{GL}(\sigma, d, \phi) = a_0\sigma + \frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4$$
$$+\frac{1}{2}\left[\alpha_0 + (\alpha - \alpha_0)\cos\phi\right]d^2 + \frac{1}{4}\left[\beta_0 + (\beta - \beta_0)\cos\phi - \underline{\mu^2 \sin^2\phi}\right]d^4$$
$$-\left[\gamma_0 + (\gamma - \gamma_0)\cos\phi\right]d^2\sigma$$

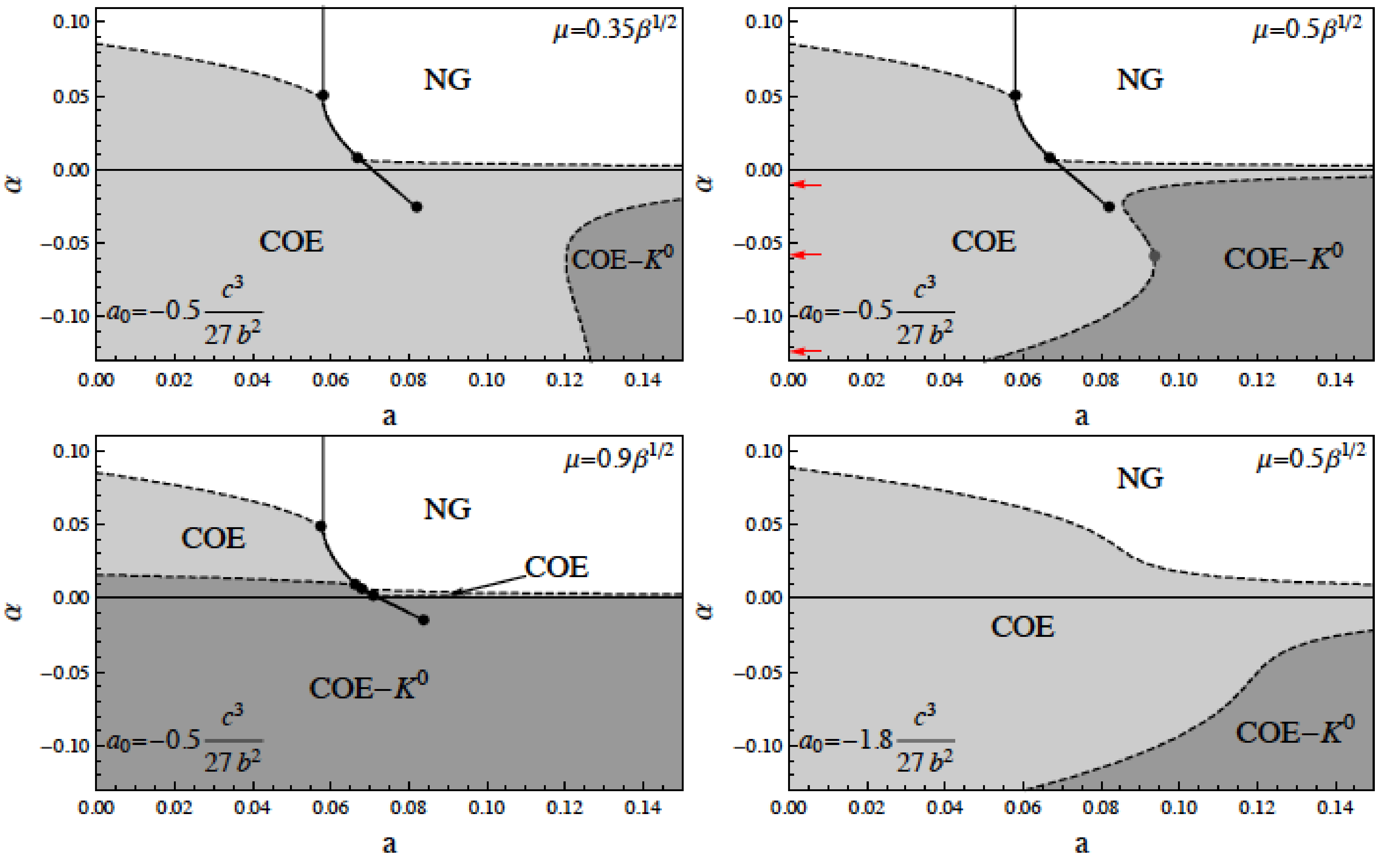
a_0 : GL coefficient via *direct* Ms

BS term

Direct effect → location of critical point

Indirect effect → Rearrangement of ground state

GL phase diagrams



Collective modes in CFL quark matter and its application to vortex dynamics in superfluidity

Key ingredients

- The r(otational)-mode instability of NS
- Mutual friction in superfluid vortex system
- Collective modes in CFL quark matter

  - A chance to pin-down NS matter content -

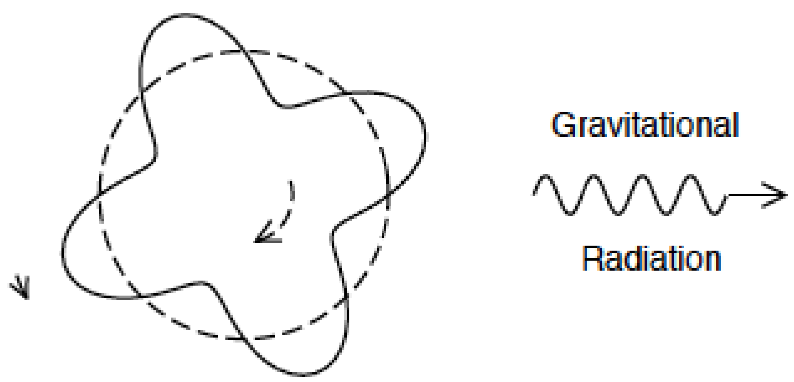
Work in progress w/ M. Ruggieri & M. Mannarelli

The r(otational) mode instability of NS

(eg. N. Andersson)

Non-radial oscillations of the star with
the Coriolis force acting as the restoring force

If dissipative phenomena are
not strong enough,
the oscillations will grow
exponentially, and the star
will keep slowing down until
some dissipation mechanism
can damp the r-modes.



Therefore, the study of the r-modes is
useful in constraining the stellar structure.

Various mechanisms been proposed

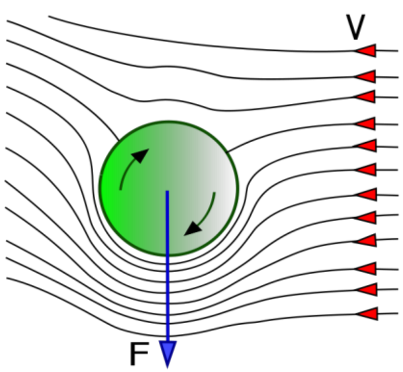
Mutual friction in superfluid vortex system

[Hall-Vinen (1956)]

An interaction btw normal & superfluid components provided by vortices. It manifests in experiment as a dissipation present in rotating superfluid state.

- Magnus force btw superfluid comp. and vortex

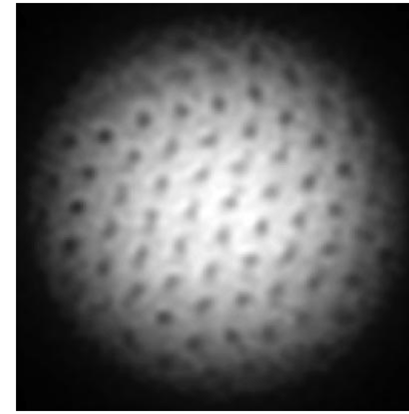
$$\vec{F}_M = \kappa \rho_s (\vec{v}_s - \vec{v}_L) \times \hat{\vec{z}}$$



- Force produced by the normal excitations

$$\vec{F}_N = -D(\vec{v}_n - \vec{v}_L) - D' \hat{\vec{z}} \times (\vec{v}_n - \vec{v}_L)$$

- Force balance condition for a vortex



$$\vec{F}_M + \vec{F}_N = 0$$

- The vortex velocity is given by

$$\vec{v}_L = \vec{v}_s + \alpha'(\vec{v}_n - \vec{v}_s) + \alpha \hat{\vec{z}} \times (\vec{v}_n - \vec{v}_s)$$

where,

$$\alpha = \frac{\delta_{\parallel}}{\delta_{\parallel}^2 + (1 - \delta_{\perp})^2}, \quad \alpha' = \frac{1 - \delta_{\perp}}{\delta_{\parallel}^2 + (1 - \delta_{\perp})^2}$$

$$\delta_{\parallel} = D / \kappa \rho_s, \quad \delta_{\perp} = D' / \kappa \rho_s$$

depend on microscopic physics and related to scattering cross sections

(Hall-Vinen parameters)

- Mutual friction is the force on the superfluid

$$\vec{F}_{MF} = -\vec{F}_M$$

If a perturbation of the superfluid velocity $\delta\vec{v}_L$ is introduced, there is no guarantee that two forces are balanced ($\delta\vec{F}_v \equiv \delta\vec{F}_N + \delta\vec{F}_M \neq 0$

Energy dissip: $\left(\frac{dE}{dt}\right)_{MF} = \delta\vec{F}_v \cdot \delta\vec{v}_s$ ⇨ $\frac{1}{\tau_{MF}} = -\frac{1}{2E}\left(\frac{dE}{dt}\right)_{MF}$

Application to color-flavor locked quark matter

In CFL phase, baryon number symmetry $U(1)_B$ is broken.
So CFL quark matter is a superfluid. If such a state exists in NS, then it is worth to consider the mutual friction.

Caroli-de Gennes-Matricon (CdGM) mode

In the system of fermionic superfluidity with vortex, there are fermionic excitations trapped in a vortex core, called the Caroli-de Gennes-Matricon (CdGM) modes. Roughly speaking, the excitation energy is given by binding energy associated with radial direction of the vortex.

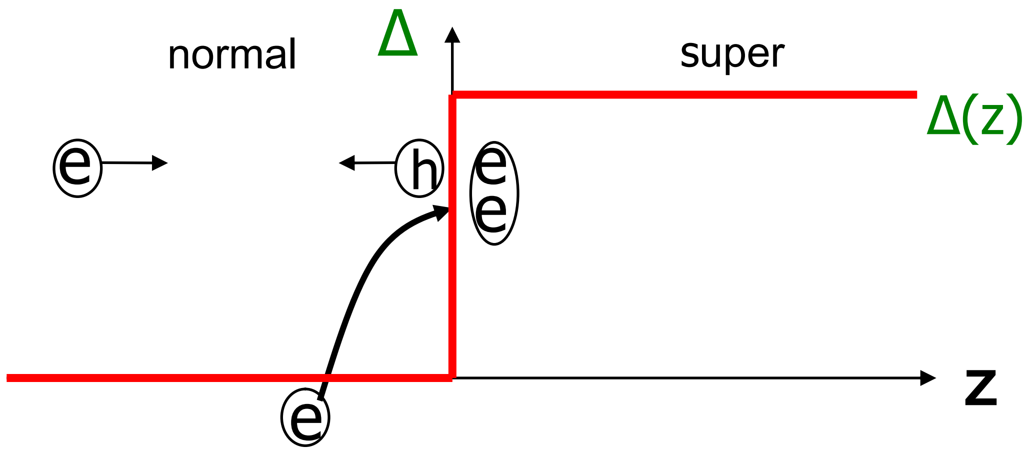
$$E_g \sim O(1) \times \frac{\Delta^2}{\varepsilon_F}, \quad \delta E = \frac{\Delta^2}{\varepsilon_F}$$

(mini-gap)

In ordinary superconductor, since the value of Δ / ε_F is going to be around $10^{-4} \sim 10^{-2}$, the spectrum is almost continuous. So they dominate the low energy dynamics. While, in color superconductor, according to some model calculations, $\Delta / \varepsilon_F \approx 10^{-2} \sim 10^{-1}$. Instead of those modes, one can consider fluctuations around superfluid gap.

Andreev reflection in color-super.

Sadzikowski, M.T. (2002, 2003)
Partyka, Sadzikowski, M.T. (2009)



"Andreev reflection"
An incident electron from normal metal hits the interface of metal/super, then reflected as a hole.

If there exist various phases in NS, a similar phenomena might happen inside there. And it may affect the transport such as energy and charge flows, etc.

Andreev reflection in color supercond.!

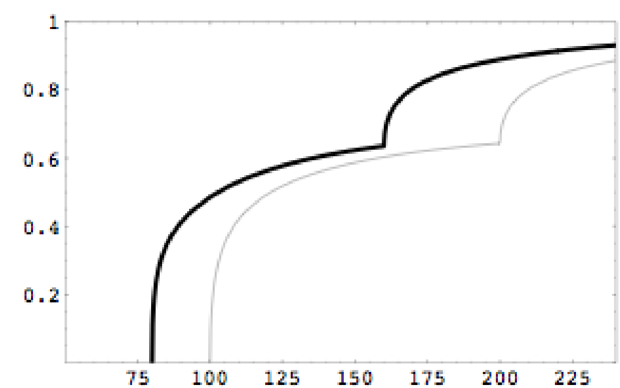
Bogoliubov—de Gennes eqs.

$$i\dot{f} = \left(-\frac{\partial_z^2}{2m} - E_F\right) f + \Delta g$$

$$i\dot{g} = \left(-\frac{\partial_z^2}{2m} + E_F\right) g + \Delta^* f$$

f : particle

g : hole



Incident energy vs. Probability

We are on the way of the calculations. . .

Summary

QCD phase diagram and collective dynamics

1. Chiral-super interplay
2. Generalized pion
   (cf. generalized vector meson in QSR)
3. CFLK ^{0} condensate

Vortex dynamics in rotating superfluidity

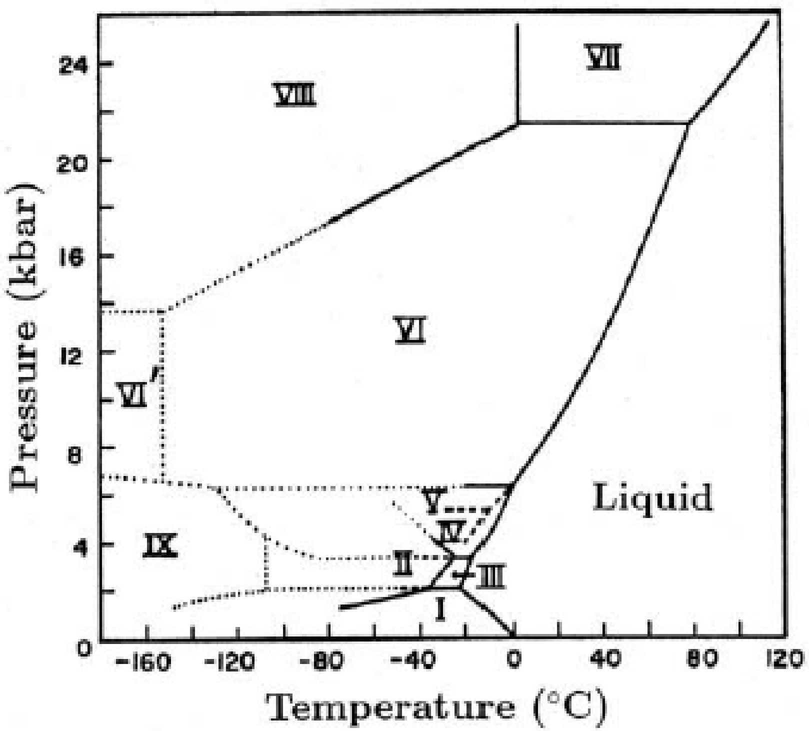
1. Mutual friction
2. Application to CFL quark matter


SUKIENNICE - „KONIK MUZEALNY"
Weekendowe warsztaty w formie gry rodzinnej
Weekend fun! Activities for all the family
YOUR VISIT STARTS
MUZEUM
NARODOWE
W KRAKOWIE
GANBARE NIPPON!!
RATUJMY NIPPON!!
Uczniowie Szkoły Języka Japońskiego
Dziękuję!

Back-up

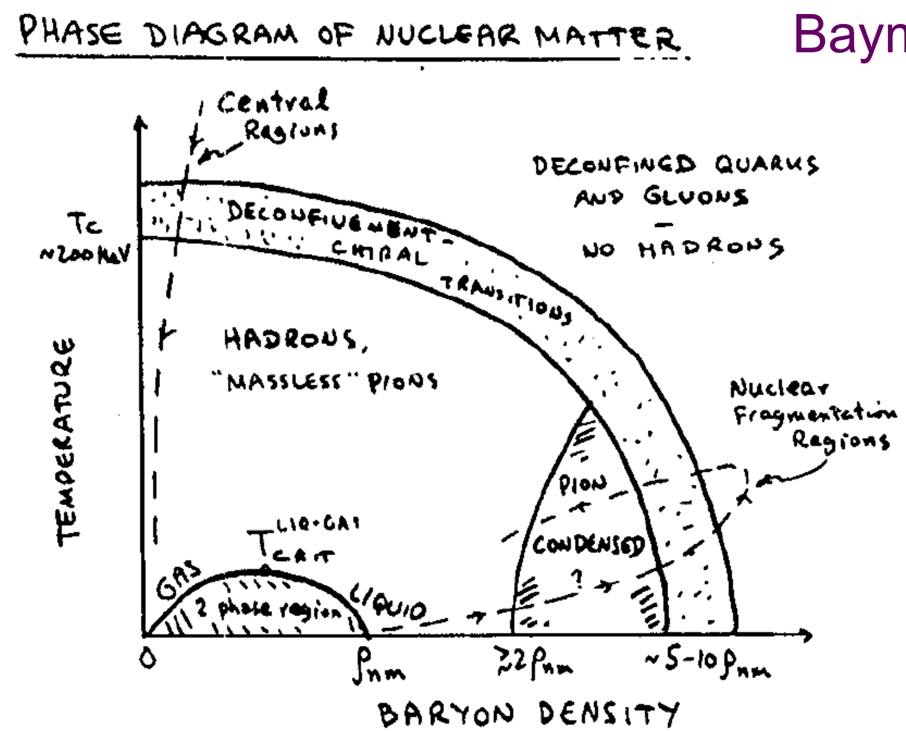
★ Physics of Interplay ★

—competition/interplay among orders—



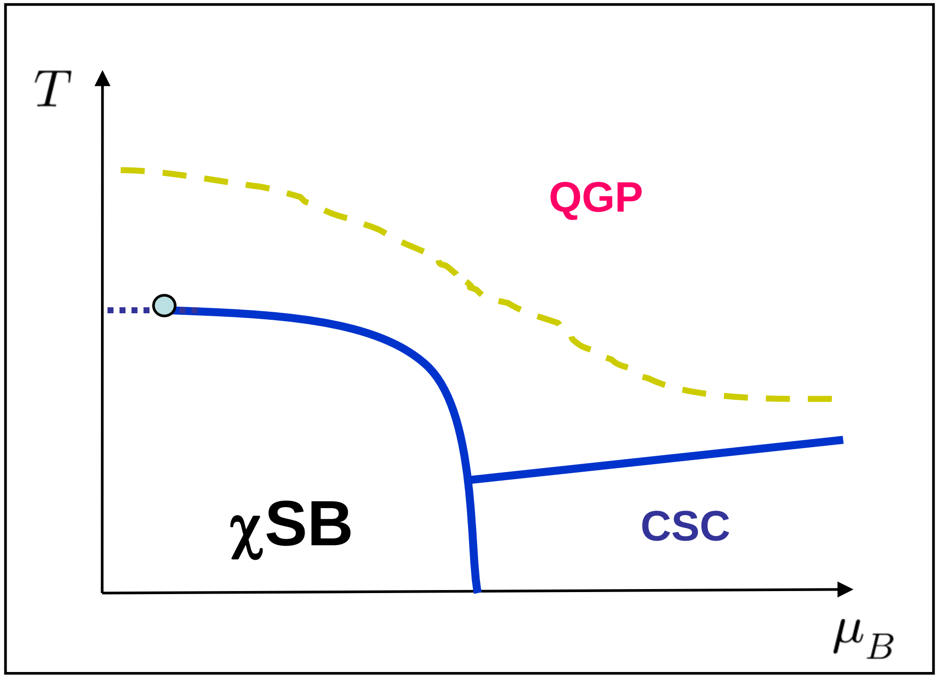
H_2O

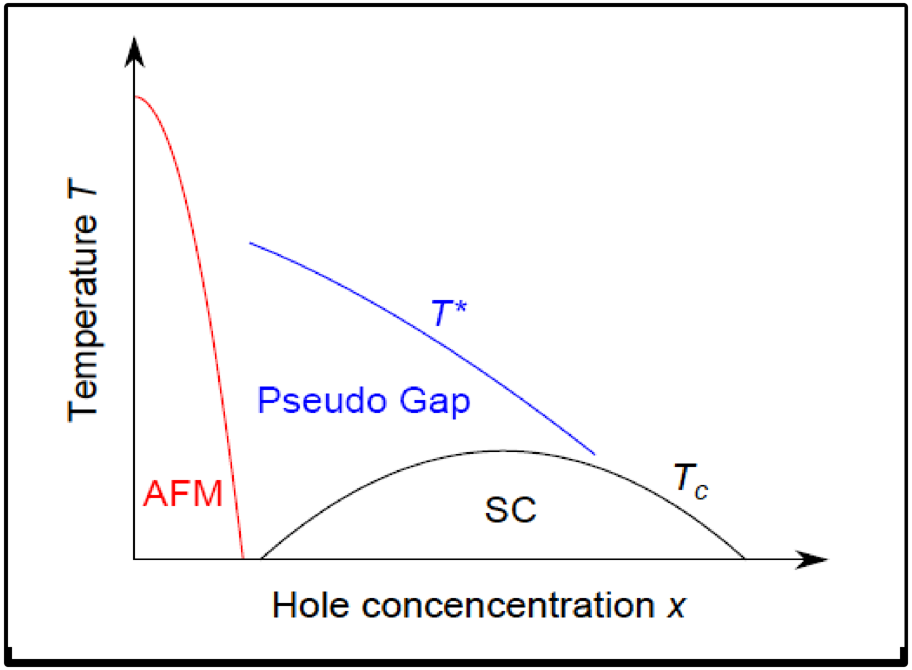
Baym



Nucl. matter?

Similarity between QCD and High Tc Superconductor





<u>Common features in QCD, HTS, and ultracold atoms</u>

1. Competition between different orders
2. Strong coupling
   - Sigrist and Ueda, ('91)
   - Babaev, Int. J. Mod. Phys. A16 ('01)
   - Kitazawa, Nemoto, Kunihiro, PTP ('02)
   - Abuki, Itakura & Hatsuda, PRD ('02)
   - Chen, Stajic, Tan & Levin, Phys. Rep. ('05)

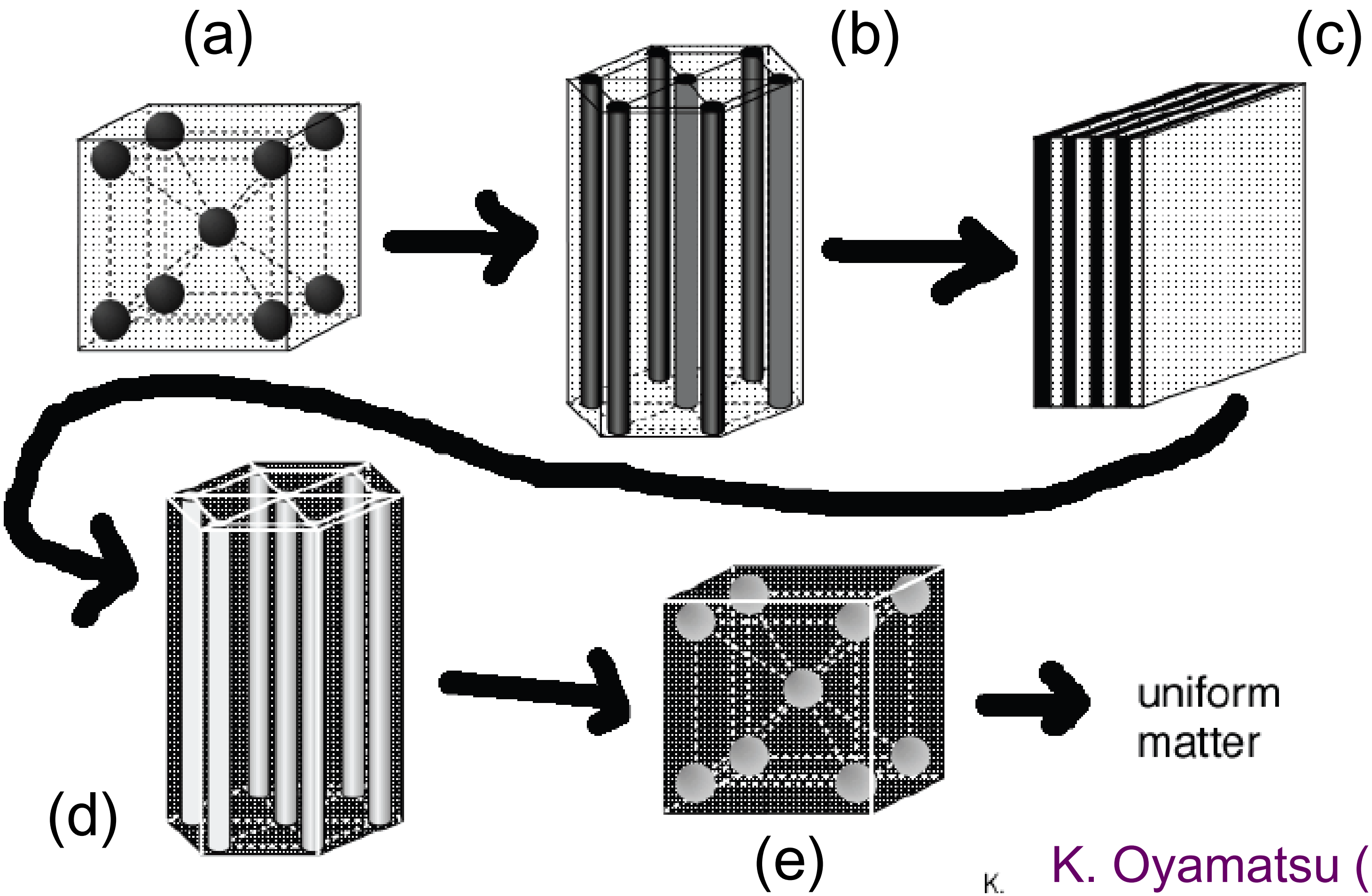
Hadron-quark continuity (Schäfer & Wilczek, 99)

Continuity between hyper nuclear matter & CFL phase

$$G = SU(3)_L \times SU(3)_R \times U(1)_B \times \cancel{U(1)_A} \times SU(3)_C$$

Phase	Hyper nuclear matter	CFL phase
Symmetry breaking Pattern	$SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{L+R}$	$SU(3)_L \times SU(3)_R \times SU(3)_C \times U(1)_B \rightarrow SU(3)_{L+R+C}$
Order parameter	chiral condensate	diquak condensate
$U(1)_B$	broken in the H-dibaryon channel	broken by d
Elementary excitations	Pseudo-scalar mesons (**π** etc)	NG bosons
	vector mesons (**ρ** etc)	massive gluons
	baryons	massive quarks (CFL gap)

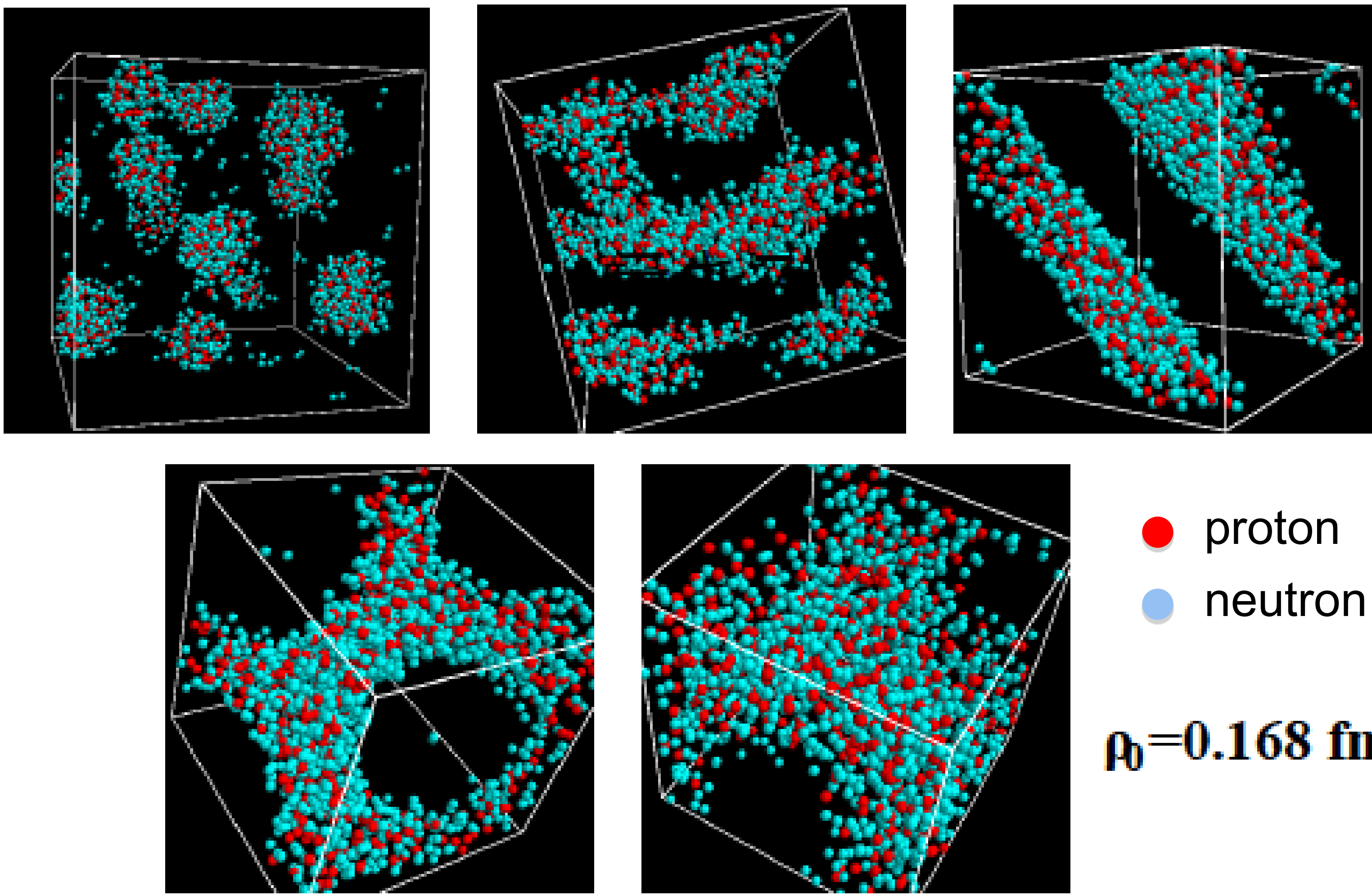
Nuclear "Pasta"



K. Oyamatsu (1993)

(a) meatball (b) spaghetti (c) lasagna (d) anti-spaghetti (e) swiss cheese

Quantum Molecular Dynamics (QMD)



proton

neutron

$\rho_0 = 0.168\ \mathrm{fm}^{-3}$

G. Watanabe et al. (2002)

Proven uncertainties in high-mass NS in NS-WD

Pulsar J0751+1807

2.1 ± 0.2 solar mass

Nice et al., ApJ 634 (2005) 1242

Nice, talk@40 Years of Pulsar, McGill,
Aug 12-17, 2007

$1.26^{+0.14}_{-0.12}$ solar mass

difficulties in Bayesian analysis for WD mass

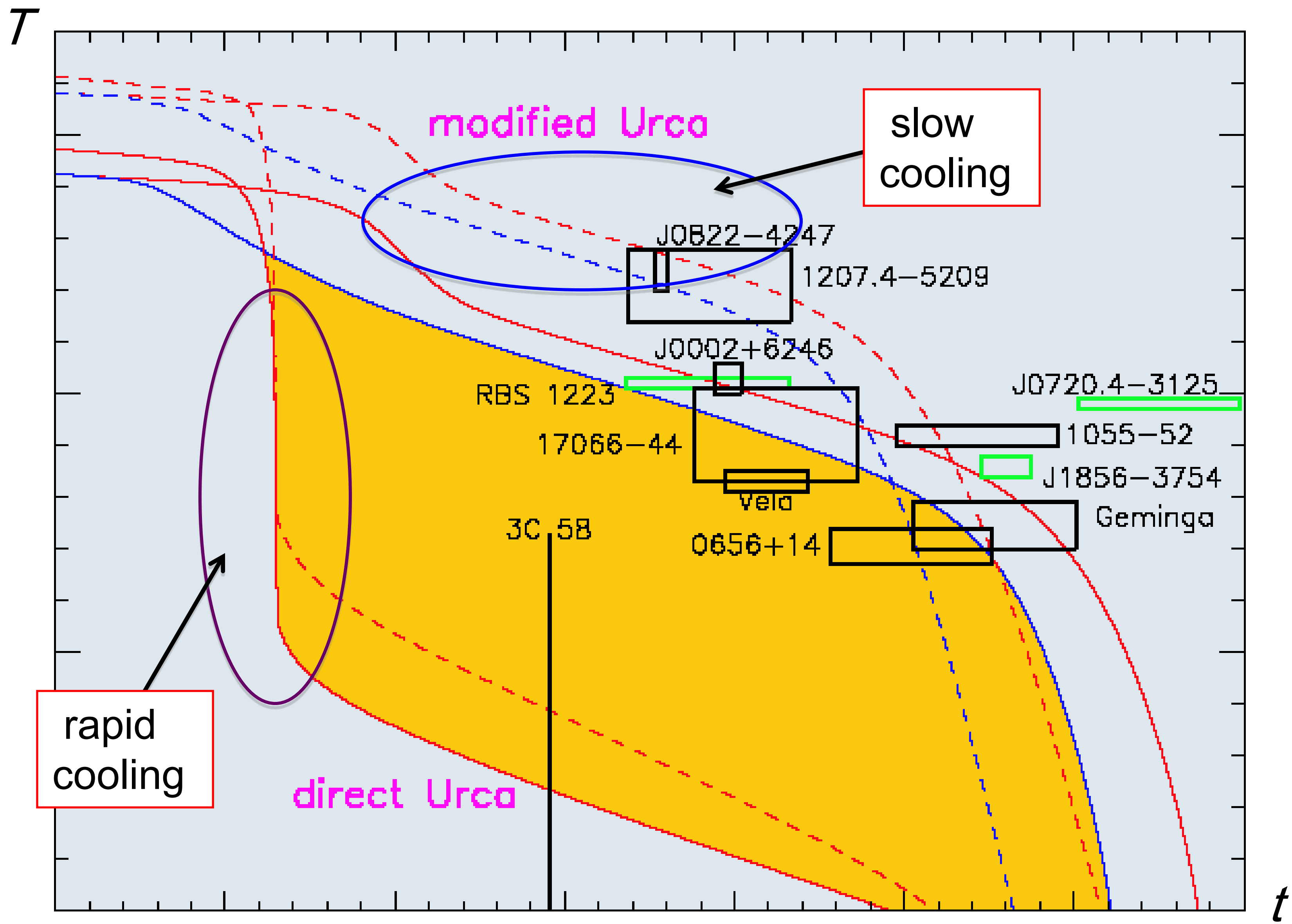
From the slide of C.H.Lee, 2011, Kyoto

Table 1. Neutron star mass measurements (1σ uncertainties).

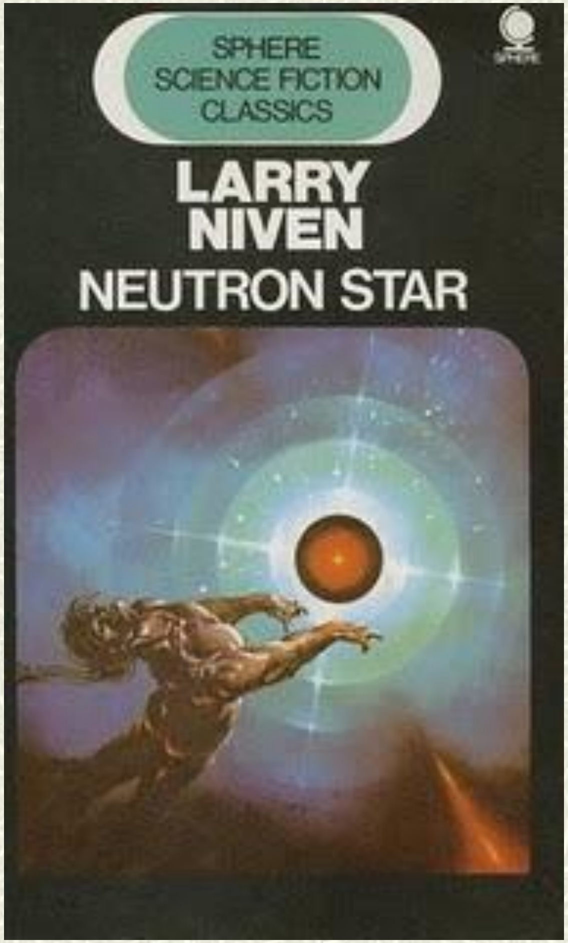
Object	Mass ($M_\odot$)	Ref.	Object	Mass ($M_\odot$)	Ref.
X-ray binaries					
4U1700-37*	$2.44^{+0.27}_{-0.27}$	(*80*)	Vela X-1†	$1.86^{+0.16}_{-0.16}$	(*81*, *82*)
Cyg X-2	$1.78^{+0.23}_{-0.23}$	(*83*)	4U1538-52	$0.96^{+0.19}_{-0.16}$	(*84*)
SMC X-1	$1.17^{+0.16}_{-0.16}$	(*84*)	LMC X-4	$1.47^{+0.22}_{-0.19}$	(*84*)
Cen X-3	$1.09^{+0.30}_{-0.26}$	(*84*)	Her X-1	$1.47^{+0.12}_{-0.18}$	(*84*)
XTE J2123-058	$1.53^{+0.30}_{-0.42}$	(*85*, *86*)	2A 1822-371	>0.73	(*87*)
Mean = 1.53 $M_\odot$, weighted mean = 1.48 $M_\odot$					
Neutron star–neutron star binaries					
1518+49	$1.56^{+0.13}_{-0.44}$	(*88*)	1518+49 companion	$1.05^{+0.45}_{-0.11}$	(*88*)
1534+12	$1.3332^{+0.0010}_{-0.0010}$	(*88*)	1534+12 companion	$1.3452^{+0.0010}_{-0.0010}$	(*88*)
1913+16	$1.4408^{+0.0003}_{-0.0003}$	(*88*)	1913+16 companion	$1.3873^{+0.0003}_{-0.0003}$	(*88*)
2127+11C	$1.349^{+0.040}_{-0.040}$	(*88*)	2127+11C companion	$1.363^{+0.040}_{-0.040}$	(*88*)
J0737-3039A	$1.337^{+0.005}_{-0.005}$	(*46*)	J0737-3039B	$1.250^{+0.005}_{-0.005}$	(*46*)
Mean = 1.34 $M_\odot$, weighted mean = 1.41 $M_\odot$					
Neutron star–white dwarf binaries					
B2303+46	$1.38^{+0.06}_{-0.10}$	(*88*)	J1012+5307	$1.68^{+0.22}_{-0.22}$	(*89*)
J1713+0747‡	$1.54^{+0.07}_{-0.08}$	(*90*)	B1802-07	$1.26^{+0.08}_{-0.17}$	(*88*)
B1855+09‡	$1.57^{+0.12}_{-0.11}$	(*90*)	J0621+1002	$1.70^{+0.32}_{-0.29}$	(*91*)
J0751+1807	$2.20^{+0.20}_{-0.20}$	(*92*, *93*)	J0437-4715	$1.58^{+0.18}_{-0.18}$	(*94*)
J1141-6545	$1.30^{+0.02}_{-0.02}$	(*95*)	J1045-4509	<1.48	(*88*)
J1804-2718	<1.70	(*88*)	J2019+2425	<1.51	(*96*)
Mean = 1.58 $M_\odot$, weighted mean = 1.34 $M_\odot$					
Neutron star–main sequence binary					
J0045-7319	$1.58^{+0.34}_{-0.34}$	(*88*)			

*Could possibly be a black hole, due to lack of pulsations. †Data from (*81*) used. ‡Reflects binary period–white dwarf mass constraint from (*97*).

Thermal history of a Neutron Star



1. High Density Star as “CIPOLLA”



(1966)

1. Limiting spin frequency

r-mode instability (gravity wave radiation reaction-driven)

bulk and shear viscosities

2. Galactic supernova neutrinos

~20 ν from SN1987A by Kamiokande

Now ~10000ν by SuperKamiokande
~1000ν by SNO

} from the center of our galaxy

3. Gravity waves

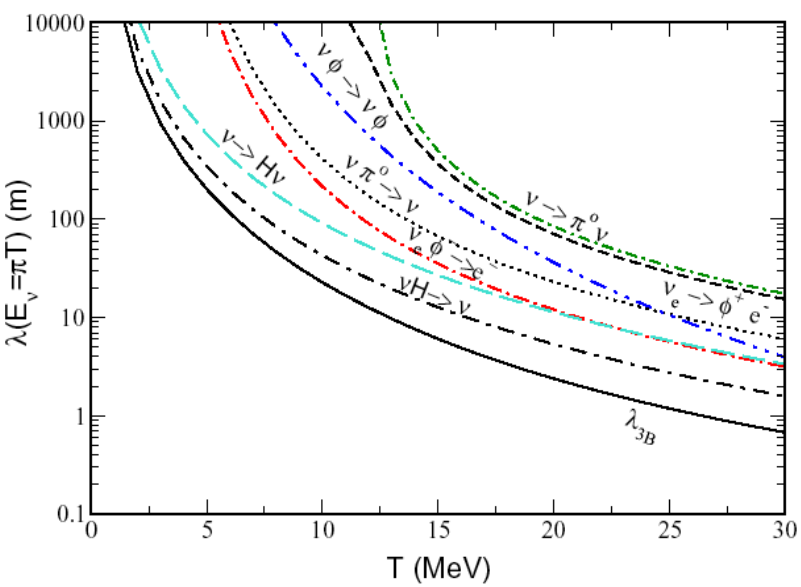
The detection of gravity waves from a *binary system* in Laser Interferometer Gravitational-Wave Observatory (LIGO)

Microscopic計算への重要なInput

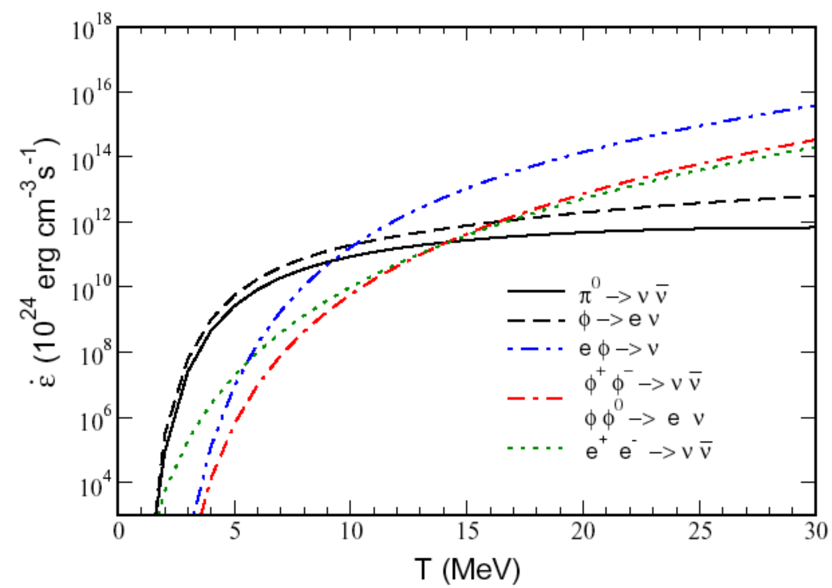
例) ν散乱振幅 = CFLカイラル有効理論を用いて評価

Reddy-Sadzikowski-M.T. (2002,2003)

$$L_{CFL} = \left(f_p^t\right)^2 (\partial_0 S)^2 - \left(f_p^s\right)^2 (\partial_i S)^2 + cTr[MS] + h.c.$$



ν平均自由行程



νエネルギー損失

秩序間の共存/競合のアイデア

Iida, Matsuura, Hatsuda, M.T. PRL(2004)

GL ポテンシャル

$$W = a(D_1^2 + D_2^2 + D_3^2) - eD_3^2 - hD_1^2$$

$$+ b_1(D_1^2 + D_2^2 + D_3^2)^2 + b_2(D_1^4 + D_2^4 + D_3^4)$$

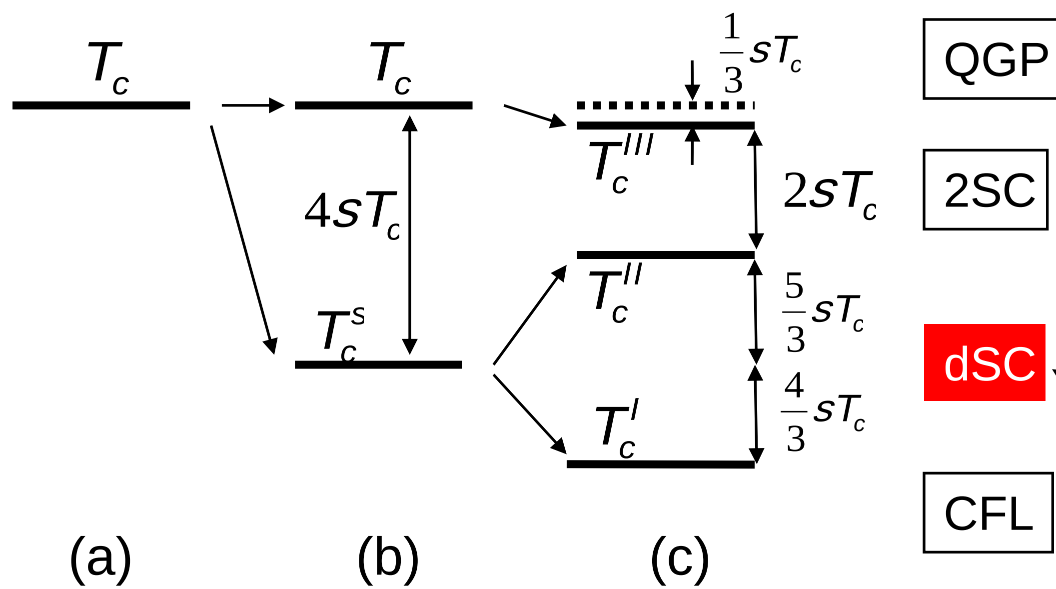
Ms及び電気的中性条件の寄与

有限温度相転移

$D_1 \sim (ds)$

$D_2 \sim (us)$

$D_3 \sim (ud)$



我々が見出した新しい相！

(a) masslessの場合 (b)有限のMs を考慮した場合 (c) 電気的中性条件を加味した場合

Some simple ansatz for condensate fields

(I) **3-flavor massless quark matter** $\left(m_u = m_d = m_s = 0\right)$

Ansatz

$$\Phi = \begin{pmatrix} s & & \\ & s & \\ & & s \end{pmatrix} \qquad d_L = -d_R = \begin{pmatrix} d & & \\ & d & \\ & & d \end{pmatrix}$$

Color-Flavor Locking (CFL)

Alford-Rajagopal -Wilczek (1998)

(※)Srednicki-Susskind proposed a similar idea in the context of χSB (1981)

(II) **2-flavor massless quark matter** $\left(m_u = m_d = 0,\ m_s = \infty\right)$

Ansatz

$$\Phi = \begin{pmatrix} s & & \\ & s & \\ & & 0 \end{pmatrix} \qquad d_L = -d_R = \begin{pmatrix} 0 & & \\ & 0 & \\ & & d \end{pmatrix}$$

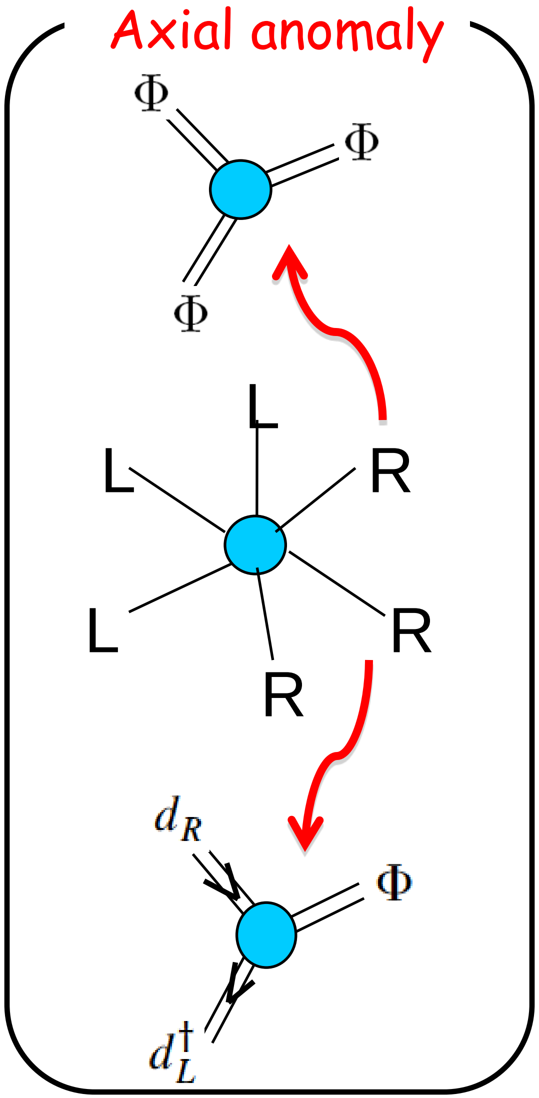
2-flavor color superconductivity (2SC)

Complete classification of the GL potential (m=0)

$$\mathcal{V}_\chi = \frac{a_0}{2}\mathrm{tr}\ \Phi^\dagger\Phi + \frac{b_1}{4!}\left(\mathrm{tr}\ \Phi^\dagger\Phi\right)^2 + \frac{b_2}{4!}\mathrm{tr}\left(\Phi^\dagger\Phi\right)^2 - \frac{c_0}{2}\left(\det\Phi + \det\Phi^\dagger\right),$$

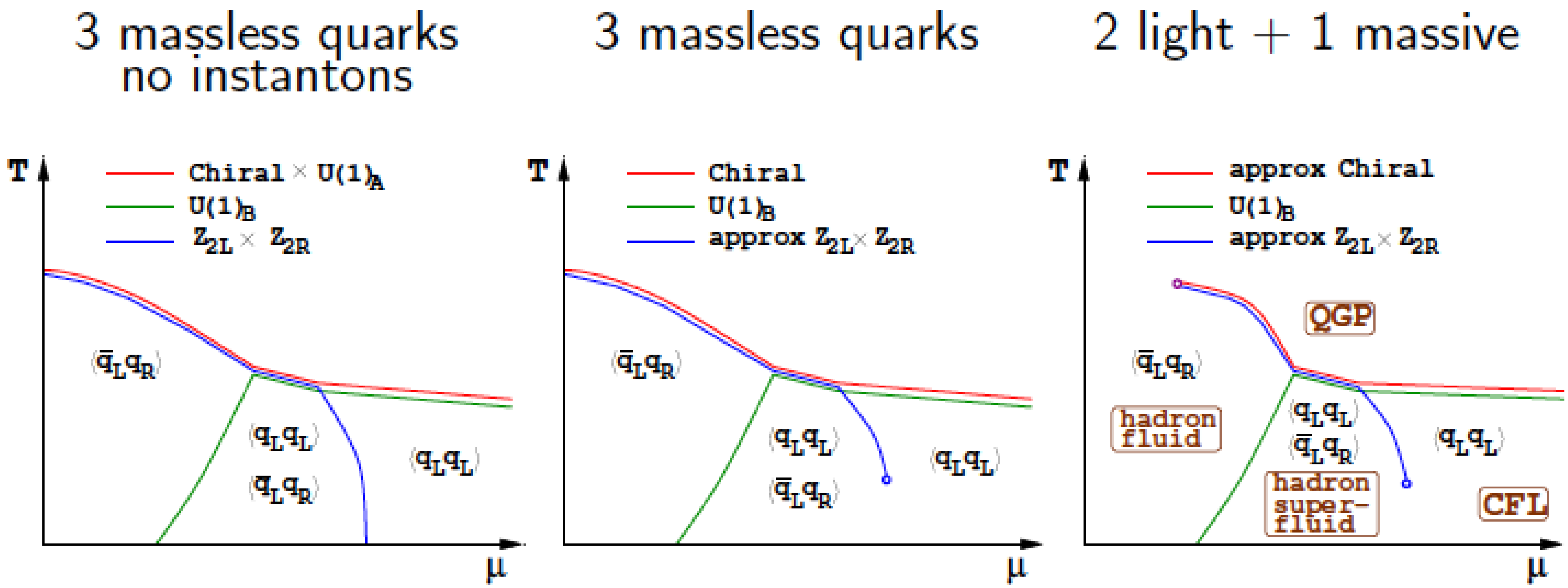
$$\begin{aligned}\mathcal{V}_d = \ & \alpha_0\ \mathrm{tr}[d_L d_L^\dagger + d_R d_R^\dagger] \\ + \ & \beta_1\left([\mathrm{tr}(d_L d_L^\dagger)]^2 + [\mathrm{tr}(d_R d_R^\dagger)]^2\right) \\ + \ & \beta_2\left(\mathrm{tr}[(d_L d_L^\dagger)^2] + \mathrm{tr}[(d_R d_R^\dagger)^2]\right) \\ + \ & \beta_3\ \mathrm{tr}[(d_R d_L^\dagger)(d_L d_R^\dagger)] + \beta_4\ \mathrm{tr}(d_L d_L^\dagger)\mathrm{tr}(d_R d_R^\dagger)\end{aligned}$$

$$\begin{aligned}\mathcal{V}_{\chi d} = \ & \gamma_1\ \mathrm{tr}[(d_R d_L^\dagger)\Phi + (d_L d_R^\dagger)\Phi^\dagger] \\ & +\lambda_1\ \mathrm{tr}[(d_L d_L^\dagger)\Phi\Phi^\dagger + (d_R d_R^\dagger)\Phi^\dagger\Phi] \\ & +\lambda_2\ \mathrm{tr}[d_L d_L^\dagger + d_R d_R^\dagger]\cdot\mathrm{tr}[\Phi^\dagger\Phi] \\ & +\lambda_3\left(\det\Phi\cdot\mathrm{tr}[(d_L d_R^\dagger)\Phi^{-1}] + h.c\right)\end{aligned}$$



An explanation by Hatsuda-Alford

Phase transition lines associated with spontaneous breaking of *exact* symmetries *cannot end*.
If the symmetry is explicitly broken, the line can end at a critical point.

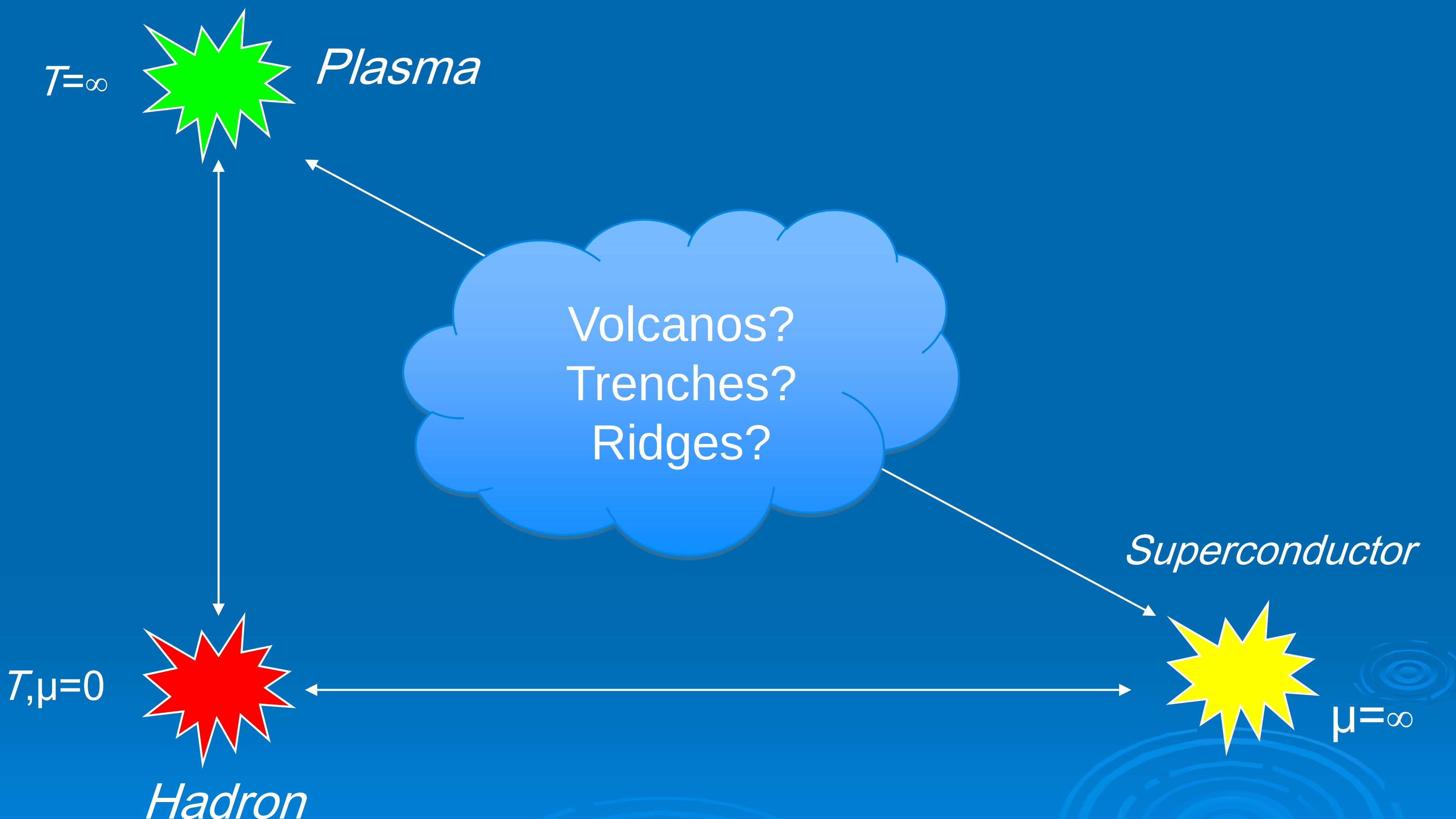


$Z_{2L}: q_L \to -q_L, q_R \to q_R$

$Z_{2R}: q_L \to q_L, q_R \to -q_R$

Slide from NFQCD10@Kyoto

Three major "islands" in the ends of hot/dense QCD world



How these islands connected under the QCD sea?

Effective Lagrangian for NG bosons

(Casalbuoni-Gatto PLB464('99)111, Hong-Rho-Zahed PLB468('99)261
Son-Stephanov PRD61('00)074012, Bedaque-Schafer NPA697('02)802)

$$\mathcal{L}_{\mathrm{eff}} = \frac{f_\pi^2}{4}\left[Tr\nabla_0\Sigma\nabla_0\Sigma^* - v^2 Tr\vec{\nabla}\Sigma\cdot\vec{\nabla}\Sigma^* \right]$$

$$+ f_\pi^2\left[\frac{a}{2} Tr\tilde{M}(\Sigma+\Sigma^*) + \frac{\chi}{2} TrM(\Sigma+\Sigma^*) \right]$$

$\mathrm{S}\ ^{\mathrm{o}}\ \exp(2i\mathrm{P}/f_p)$

(Π : NG bosons)

poorly known at low densities

$$\nabla_0\Sigma = \partial_0\Sigma - i[(\mu_Q Q - X_L)\Sigma - \Sigma(\mu_Q Q - X_R)]$$

χ: instanton contribution
(T. Schäfer, PRD65('02)094033)

$$X_L = MM^*/2n, \qquad X_R = M^*M/2n$$

$$\tilde{M}\ ^{\mathrm{o}}\ (\det M)M^{-1},\ \ M\ ^{\mathrm{o}}\ diag(m_u, m_d, m_s)$$

$$Q = diag(2/3, -1/3, -1/3)$$

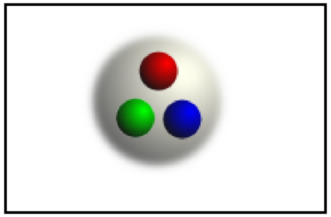
$$v^2 = \frac{1}{3} \qquad a = \frac{3\Delta^2}{\pi^2 f_\pi^2}$$

$$f_p = 0.21n$$

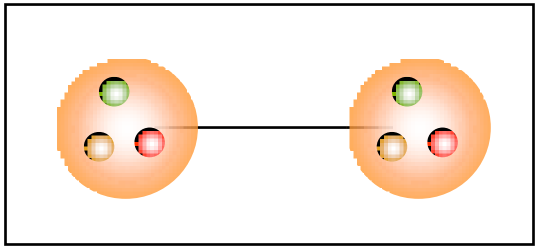
perturbative results

Grand Challenges

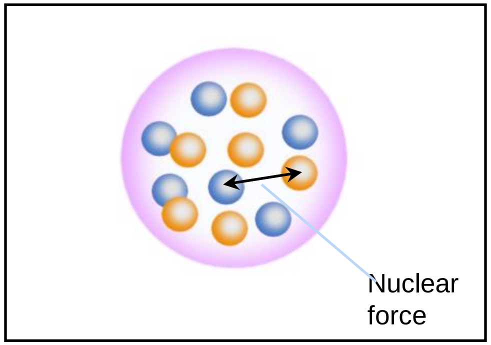
- Space-time evolution of QCD matter -



Hadrons

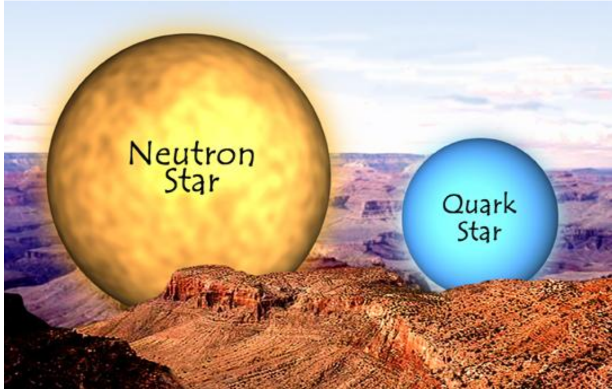


Nucleons

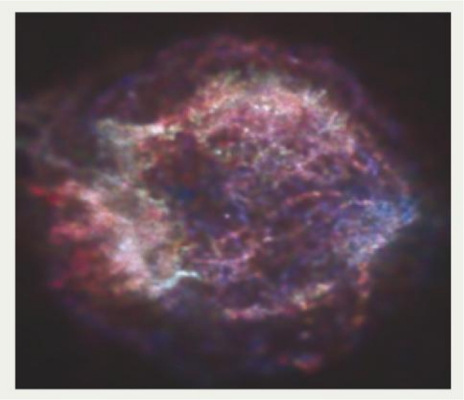


Nuclei

Phase transition in
the early universe



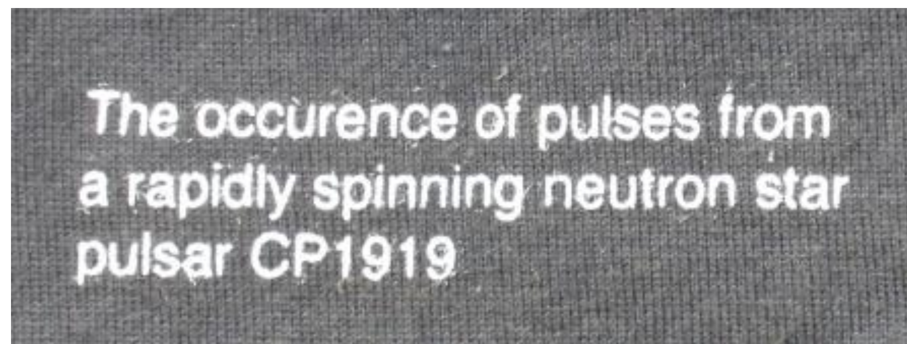
Neutron/quark star



Supernovae

The answer to the ultimate question

"Why the matter of our universe can be stable?"

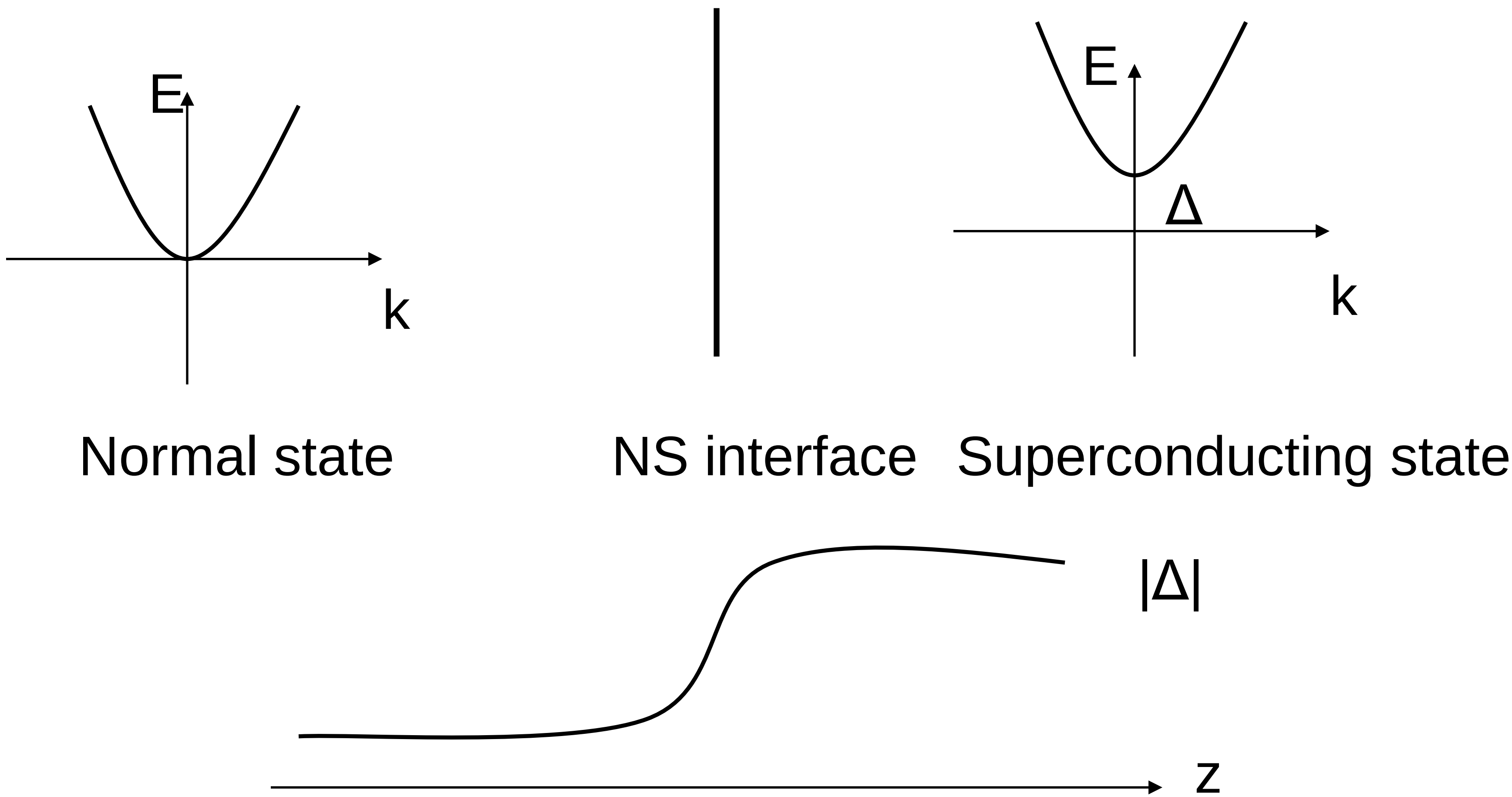


Neutron Star Tシャツ！

★インターフェイスの物理★

ーAndreev散乱ー

(Andreev, Zh. Eksp. Teor. Fiz. 46 (1964) 1823)



超対称ゲージ理論による有限密度QCDの解析

Maru-M.T. MPL(2005)1495

Central idea

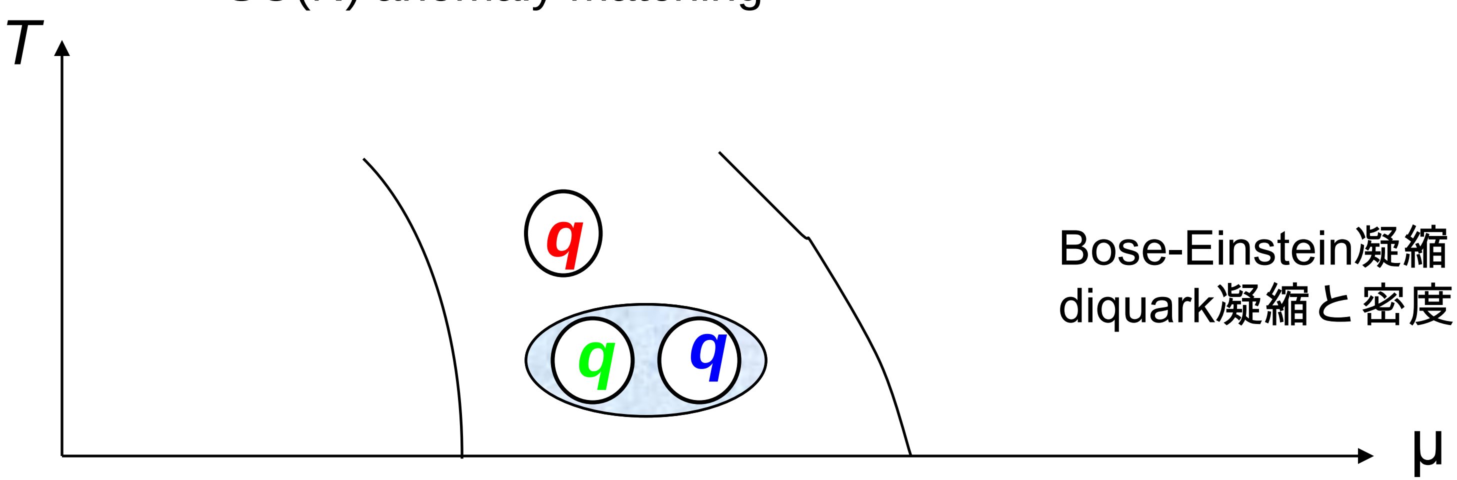
Exact results of SUSY + chemical potential effects ~~(SUSY)~~

N=1 SO(N)×SU(3)c SUSY composite model

$$\left(Q, \overline{Q}, X\right) \Rightarrow \left((Q^2), (\overline{Q}^2), X^2, QX, \overline{Q}X, Q\overline{Q}\right)$$

“diquarks” “quarks”

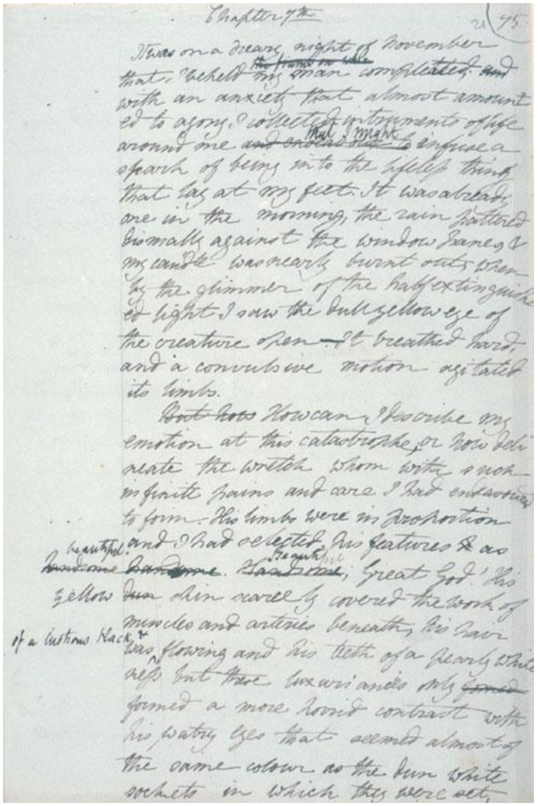
SO(N) anomaly matching



Bose-Einstein凝縮としての
diquark凝縮と密度相転移

「発明は、何もないところから生まれるのでは
ありません。混沌とした中から生まれるのです」
(メアリー・シェリー)

メアリー・シェリー



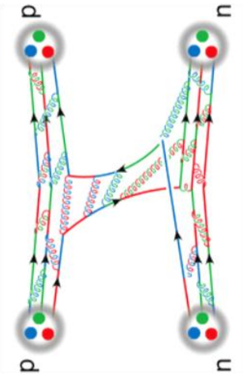
Chapter 7th
It was on a dreary night of November
that I beheld my man completed and
with an anxiety that almost amount
ed to agony I collected instruments of life
around me that I might infuse a
spark of being into the lifeless thing
that lay at my feet. It was already
one in the morning, the rain pattered
dismally against the window panes &
my candle was nearly burnt out, when
by the glimmer of the half extinguish
ed light I saw the dull yellow eye of
the creature open — It breathed hard,
and a convulsive motion agitated
its limbs.
How can I describe my
emotion at this catastrophe or how deli
neate the wretch whom with such
infinite pains and care I had endeavoured
to form. His limbs were in proportion
and I had selected his features as
beautiful. Beautiful; Great God! His
yellow skin scarcely covered the work of
muscles and arteries beneath, his hair
of a lustrous black & was flowing and his teeth of a pearly white
ness but these luxuriances only
formed a more horrid contrast with
his watery eyes that seemed almost of
the same colour as the dun white
sockets in which they were set

「フランケンシュタイン」草稿

「物理学の法則は単純です。
でもこの世界は決してつまらないものではない。
理想的にできているのだと思います。」
（南部陽一郎）

南部陽一郎

「クォーク」



QCD Lagrangian (Nambu, 1965)

$$L_{QCD} = \bar{q}^a \left(i\gamma_\mu D^\mu - m_q \right)_{ab} q^b - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

Just one line, but very fertile in physics and mathematics.

Providing testing field for any kind of new ideas.

Still lots of unsolved problems (inexhaustible spring).

http://www.pbase.com/daveb/image