Regge amplitudes from AdS/CFT

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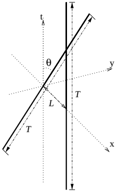
Zakopane Lectures (May 2011)

Initial work: R.Janik, R.P., hep-th//9907177,0003059,/0110024
1[rst] Lecture (1-4): M.Giordano, S.Seki, R.P., arXiv:1106.xxxx [hep-th]
2[nd] Lecture (5-8): M.Giordano, R.P., arXiv:1105.xxxx [hep-th]

Idea: Use the Gauge/Gravity duality in a generic confining framework

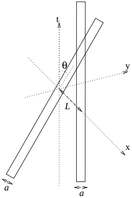
R.Janik, R.P., hep-th/9907177,0003059,0110024

Wilson Loop geometries: Wilson Loops for Strong (QCD) Interactions:



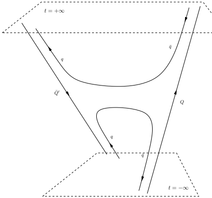
qq elastic

Infinite Wilson loop
Geometries: Single helicoid
Problems: Divergences



dipole elastic

Wilson loop Correlator
"Double helicoid"
Approximate geometry

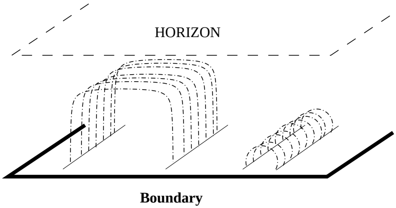


$q\bar{q}$ exchange

Finite "twisted" loo
Loop on helicoid
Floating Boundarie

Holography

G/G Tool: Wilson Loops $\Leftrightarrow$ Minimal Surfaces



$$\langle e^{iP\int_C \vec{A}\cdot\vec{d}l}\rangle = \int_\Sigma e^{-\frac{Area(\Sigma)}{\alpha'}} \approx e^{-\frac{Min.Area}{\alpha'}} \times Fluctuations$$

Confining theory $\sim$ QCD

$$ds^2_{AdS/BH} = \frac{16}{9}\frac{1}{z^{2/3}(1-(z/R_0)^4}\frac{dz^2}{z^2} + \frac{\eta_{\mu\nu}dx^\mu dx^\nu}{z^2} + \dots$$

Near-Horizon approximation

$\mathcal{N} = 4$ Conformal Theory

$$ds^2_{AdS} = \frac{dz^2}{z^2} + \frac{\eta_{\mu\nu}dx^\mu dx^\nu}{z^2}$$

Non-planar minimal surfaces

Outline of the lectures

1. **Introduction: Wilson loops and Confining Gauge/Gravity Duality**
2. **Reggeon Exchange: Euclidean Space**
3. **Reggeon Exchange: Minkowskian Space**
4. **Inelastic** *(and Elastic Rescattering)* **Reggeon amplitudes**

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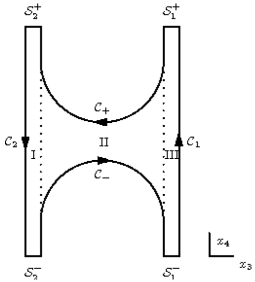
Euclidean Formalism

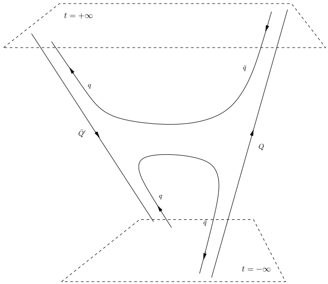


$$\tilde{a}(\vec{b}, \theta, T) = \mathcal{Z}^{-1} \int \mathcal{D}\mathcal{C}_+ \mathcal{D}\mathcal{C}_- \ \langle W[\mathcal{C}] \rangle \ e^{-mL[\mathcal{C}]} \ \mathcal{I}[\mathcal{C}] \Rightarrow \text{saddle}-\text{point}$$

- $\log \langle W[\mathcal{C}] \rangle \sim -\frac{1}{2\pi\alpha'_{\text{eff}}} A[\mathcal{C}]$ = Area term
- $mL[\mathcal{C}] = mL[\mathcal{C}_+] + mL[\mathcal{C}_-]$ = Floating Length term
- $\mathcal{I}[\mathcal{C}] \equiv \otimes_{i=(1,2,+,-)} \mathcal{I}_i[\mathcal{C}_i]$ = Quark Spin Prefactor
- $\mathcal{Z}^{-1} \propto \langle W_1 \rangle \langle W_2 \rangle$ = (Infinite) Normalisation Constant

Wilson Loop

Gauge/Gravity Saddle-Point

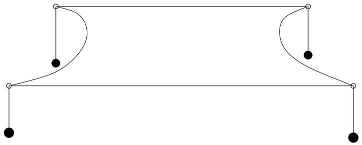


$$-\log \tilde{a} \approx S_{\mathrm{eff,\,E}}[\tau(\sigma)] \sim \frac{1}{2\pi\alpha'_{\mathrm{eff}}} A^{\mathrm{hel}}_{\mathrm{min}}[\tau(\sigma)] + 2m(L^{\mathrm{hel}}[\tau(\sigma)] - L_0[\tau(\sigma)])$$

$$\boxed{\delta S_{\mathrm{eff,\,E}}[\tau_{\mathrm{s.p.}}(\sigma)] = 0}$$

Warm-Up planar exercise

Soap Film with Floating Boundaries

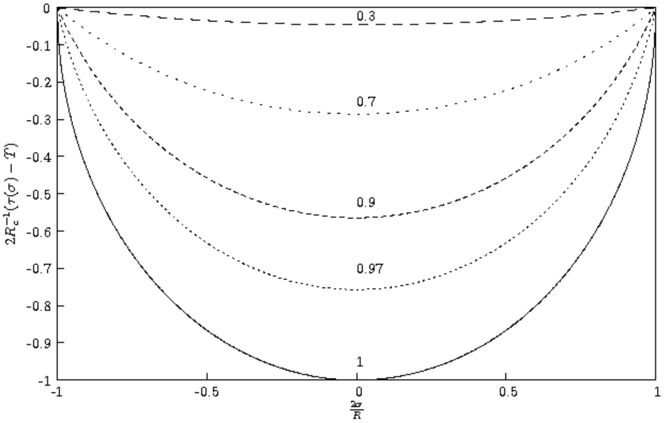


$$(\mathrm{m}) \times L = \int_{-\frac{R}{2}}^{+\frac{R}{2}} d\sigma \sqrt{1+(\dot{\tau}(\sigma))^2}\,, \qquad \left(\frac{1}{4\pi\alpha'}\right) \times \mathrm{A} = 2\int_{-\frac{\mathrm{R}}{2}}^{+\frac{\mathrm{R}}{2}} \mathrm{d}\sigma\ \tau(\sigma)\,.$$

- Euler-Lagrange Equation : $\ddot{\tau} - 2\left\{4\pi\alpha' m\right\}^{-1}\left(1+\dot{\tau}^2\right)^{\frac{3}{2}} = 0$
- Fixing the Floating boundary : $\tau(\sigma) - \tau_0 = \frac{R_c}{2}\left[1 - \sqrt{1 - \left(\frac{2\sigma}{R_c}\right)^2}\right]$
- Critical Size Before Splitting: $\sigma \in [-R/2, R/2]\,,\ R \leq R_c = 4\pi\alpha' m$

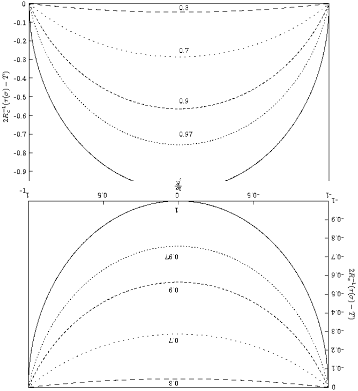
Warm-Up planar exercise

Soap Film with Floating Boundaries: Solution



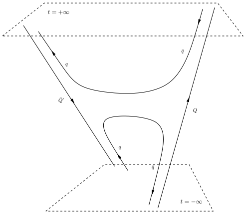
Warm-Up planar exercise

Soap Film with Floating Boundaries: Solution



Helicoidal Surface

Helicoidal Minimal surface with Floating boundaries $p_{\mathrm{helicoid}} \equiv \theta / b$

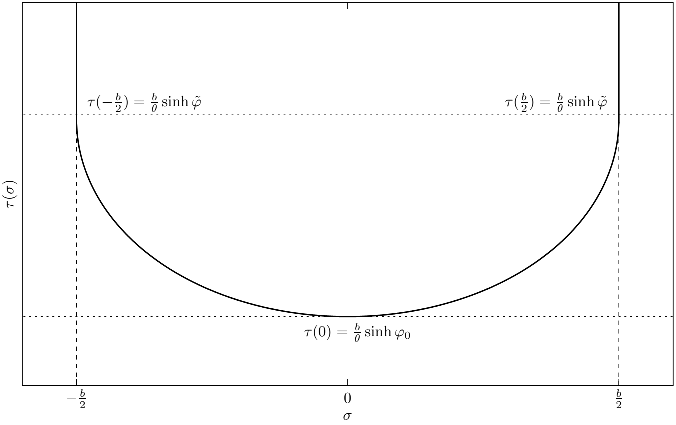


$$L = \frac{1}{p} \int_{-\frac{\theta}{2}}^{+\frac{\theta}{2}} ds \sqrt{1 + [t(s)]^2 + [\dot{t}(s)]^2}\,, \qquad A = \frac{1}{p^2} \int_{-\frac{\theta}{2}}^{+\frac{\theta}{2}} ds \int_{-t(s)}^{+t(s)} dy \sqrt{1 + y^2}$$

- Euler-Lagrange Equation : $\frac{2m}{(1+t^2+\dot{t}^2)^{\frac{3}{2}}}[(\ddot{t}-t)(1+t^2)-2t\dot{t}^2]-\frac{1}{\pi\alpha'_{\mathrm{eff}}p}\sqrt{1+t^2}=0$
- Euclidean Solution : $S_{\mathrm{eff,\,E}} = \left\{ \frac{b^2}{2\pi\alpha'_{\mathrm{eff}}\theta}\, f(\tilde{\varphi}) \right\}_A + \left\{ \frac{4mb}{\theta} \left(B(\varphi_0, \tilde{\varphi}) - \sinh\tilde{\varphi}\right) \right\}_L$
- Critical Impact Parameter: $b \le b_c = 4\pi\alpha'_{\mathrm{eff}} m$

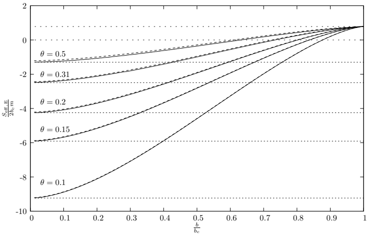
Helicoidal Surface

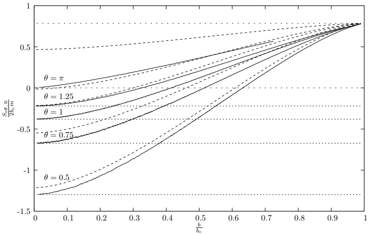
Helicoidal Minimal surface with Floating boundaries $\tau_{\text{helicoid}} \equiv \frac{\sinh \varphi}{p_{\text{helicoid}}}$



Helicoidal Surface

Euclidean Effective Action: *Numerics vs. Analytics*



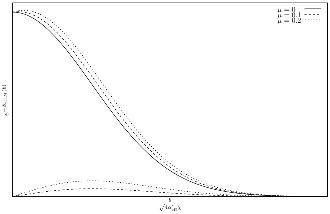


Analytic *vs.* numerics *small* θ , Analytic *vs.* numerics *large* θ

- Analytic Approximation : $\cosh\tilde\varphi = \frac{4\pi\alpha'_{\rm eff}m}{b} = \frac{b_c}{b}$; $\varphi_0 = {\rm arccosh}\,\frac{b_c}{b} - \frac{\theta}{2}$
- Effective Action : $S_{\rm eff,\,E} = \frac{b^2}{2\pi\alpha'_{\rm eff}\theta}{\rm arccosh}\,\frac{b_c}{b} + 2\pi^2\alpha'_{\rm eff}m^2 - \frac{2bm}{\theta}\sqrt{\left(\frac{b_c}{b}\right)^2 - 1}$
- Euclid $\rightarrow$ Minkovski: $b \leq b_c = 4\pi\alpha'_{\rm eff}m \rightarrow b \sim \sqrt{s} \gg b_c$

Euclid → Minkowski

Analytic continuation $b > b_c$

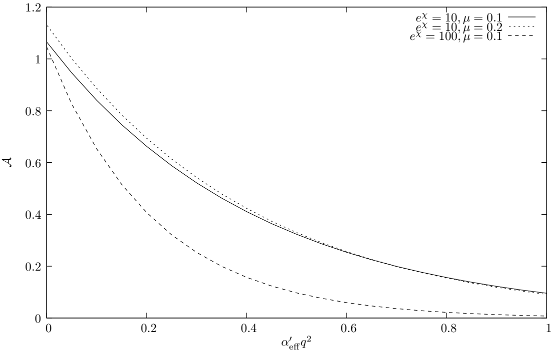


$$S_{\mathrm{eff,\,E}} \to S_{\mathrm{eff,\,M}} = \frac{b^2}{2\pi\alpha'_{\mathrm{eff}}\chi}\arccos\frac{b_c}{b} - \frac{2bm}{\chi}\sqrt{1-\left(\frac{b_c}{b}\right)^2} + 2\pi^2\alpha'_{\mathrm{eff}}m^2$$

- Almost Gaussian profile of b-amplitude:
  $a(s,b) \sim \exp -S_{\mathrm{eff,\,M}} \simeq \exp\left(-\frac{b^2}{4\alpha'_{\mathrm{eff}}\chi} + \frac{4bm}{\chi} + \cdots\right)$
- Mass effects : (small) Increase of mean radius with energy

The Reggeon amplitude

Reggeon in Momentum space



$$\mathcal{A}_{\mathcal{R}}(s, t = -q^2) \equiv -2is \int d^2\vec{q}\ e^{i\vec{q}\cdot\vec{b}}\ a(\vec{b}, \chi) = -4i\pi s \int_0^\infty db\, b\, J_0(qb)\ a(b, \chi)$$

- Increase of Slope: $\frac{\partial \mathcal{A}_{\mathcal{R}}}{\partial t}(s, t = 0) = \alpha'_{\text{eff}} \left(\chi + 6m\sqrt{\pi \alpha'_{\text{eff}} \chi}\right)$

The Reggeon amplitude

Regge Singularity Structure

$$\tilde{\mathcal{A}_{\mathcal{R}}}(\omega, t) = \int_0^\infty d\chi e^{-\omega\chi} \mathcal{A}_{\mathcal{R}}(s, t) \approx \int_0^\infty d\chi e^{-\omega\chi} \left\{ \mathcal{T}_0(\chi, t) + m\mathcal{T}_1(\chi, t) + \cdots \right\}$$

- Mass effects : Regge Pole + Cut

$$\mathcal{A}^{(M)}(\omega, t) \underset{\omega \to \alpha'_{\text{eff}} t}{\simeq} \frac{1}{\omega - \alpha'_{\text{eff}} t} + m \left\{ \frac{8\alpha'_{\text{eff}} t^{\frac{1}{2}}}{\omega - \alpha'_{\text{eff}} t} + 4t^{-\frac{1}{2}} \log \frac{16\alpha'_{\text{eff}} t}{\omega - \alpha'_{\text{eff}} t} \right\}$$

- Linear Regge Trajectory: (small-)m independent

$$\boxed{\alpha_{\mathcal{R}}(t = 0) = \alpha_{\mathcal{R}}(t = 0) + \alpha'_{\text{eff}} t}$$

The Reggeon amplitude

Companion Regge Contributions : “Rescattering” corrections

$$\arccos\frac{b_c}{b} \to \arccos\frac{b_c}{b} + 2n\pi \Rightarrow a_{\mathcal{R}}^{(n)}(b,\chi) = a_{\mathcal{R}}(b,\chi) \times \left[\exp\left\{-\frac{b^2}{\alpha'_{\text{eff}}\chi}\right\}\right]^{\mathbf{n}}$$

- Convolutions : Regge $\otimes$ multi-Elastic

$$\mathcal{A}_{\mathcal{R}}(s,t) = \int d^2b\, e^{i\vec{q}\cdot\vec{b}}\, a_{\mathcal{R}}(b,\chi) \left[\exp\left\{-\frac{b^2}{\alpha'_{\text{eff}}\chi}\right\}\right]^{\mathbf{n}} = i^n \mathcal{A}_{\mathcal{R}} \otimes \mathcal{A}_{el}^{\otimes \mathbf{n}}(s,t) \quad ation$$

- Elastic (re)scattering: consistent with Gauge/Gravity “Pomeron”

Janik,R.P. (2000)

$$\mathcal{A}_{el}(s,t) = -i2s \int d^2b\, e^{i\vec{q}\cdot\vec{b}} \exp\left\{-\frac{b^2}{\alpha'_{\text{eff}}\chi}\right\} = -2i\pi\alpha'_{\text{eff}}\chi\ s^{\left[1+\frac{\alpha'_{\text{eff}}t}{4}\right]\chi}$$

Main Results

Reggeons from Gauge/Gravity: m-dependent analysis

- Euclidean Minimal surface with floating boundaries $b < b_c = 4\pi\alpha' m$: $S_{\mathrm{eff,\,E}}(\theta, b) = \frac{b^2}{2\pi\alpha'_{\mathrm{eff}}\theta} f(\tilde{\varphi}) + \frac{4mb}{\theta}\left(B(\varphi_0, \tilde{\varphi}) - \sinh\tilde{\varphi}\right)$
- Analytic continuation to Minkowski space $b > b_c$: $S_{\mathrm{eff,\,M}}(s, b) \sim \frac{b^2}{2\pi\alpha'_{\mathrm{eff}}\chi}\arccos\frac{b_c}{b} - \frac{2bm}{\chi}\sqrt{1 - \left(\frac{b_c}{b}\right)^2} + 2\pi^2\alpha'_{\mathrm{eff}} m^2$
- Gaussian–like Reggeon–exchange amplitude in impact–parameter $a(\vec{b}, \chi) \equiv \frac{i}{2s}\int \frac{d^2\vec{q}}{(2\pi)^2} e^{-i\vec{q}\cdot\vec{b}}\, \mathcal{A}_{\mathcal{R}}(s,t) \propto \exp\left\{-\frac{b^2}{4\alpha'_{\mathrm{eff}}\chi} + \frac{4bm}{\chi} + \cdots\right\}$
- Linear Reggeon trajectory (pole(m=0)+cut(m)): $a_{\mathcal{R}(}t) \equiv \alpha_0 + \alpha'_{\mathrm{eff}}\, t\ , \quad \frac{\partial\mathcal{A}_{\mathcal{R}}}{\partial t}(s, t=0) = \alpha'_{\mathrm{eff}}\left(\chi + 6m\sqrt{\pi\alpha'_{\mathrm{eff}}\chi}\right)$
- "Rescattering corrections" to the "bare" $q\bar{q}$–Reggeon exchange $\mathcal{A}_{\mathcal{R}}(s,t) \to \sum_n i^n \mathcal{A}_{\mathcal{R}} \otimes \mathcal{A}_{el}^{\otimes n}\ , \quad \mathcal{A}_{el}(s,t) = -2i\pi s\ \alpha'_{\mathrm{eff}}\chi \exp\left\{\frac{\alpha'_{\mathrm{eff}}\, t}{4}\chi\right\}$