# Regge amplitudes from AdS/CFT

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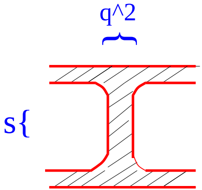
Zakopane Lectures (May 2011)

Initial work: R.Janik, R.P., hep-th//9907177,0003059,/0110024
$1^{rst}$ Lecture (1-4): M.Giordano, S.Seki, R.P., arXiv:1106.xxxx [hep-th]
$2^{nd}$ Lecture (5-8): M.Giordano, R.P., arXiv:1105.xxxx [hep-th]

# Motivation

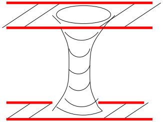
**Aim:** Use the AdS/CFT and Gauge/Gravity duality to study high-energy soft scattering amplitudes.

**String Theory Ancestor:** The “dual” Regge amplitudes for Strong Interactions:




Veneziano Amplitude

$A_R(s,q^2)$



Shapiro-Virasoro Amplitude

$A_P(s,q^2)$

$$A_{R,P}(s, t = -q^2) \xrightarrow{?} \sum_i \left(\frac{s}{s_0}\right)^{\alpha_i(t)} \beta^i(t)$$

## Questions

- **Old string scenario** (1968-1974): Problems

  Consistent (super) string: (10) 26 dimensions and Zero-mass on-shell states

  Reggeon exchange: $\rightarrow$ Gauge interactions

  Pomeron exchange: $\rightarrow$ Gravity

- **New string scenario** (1998-2011-): AdS/CFT correspondence and Gauge/Gravity duality at strong coupling

  Question # 1: Regge behaviour for the conformal $\mathcal{N} = 4$ gauge field theory from AdS/CFT?

  Question # 2: Regge behaviour for a non conformal gauge field theory $\sim$ QCD from Gauge/Gravity duality?

1. **Introduction: old/new Reggeon problem**
2. AdS/CFT and minimal surfaces
3. $gg$ amplitude in $\mathcal{N} = 4$ sYM theory
4. Regge Amplitudes in $\mathcal{N} = 4$ sYM theory

## Outline of the lectures

1. **Introduction: old/new Reggeon problem**
2. **AdS/CFT and minimal surfaces**
3. $gg$ **amplitude in $\mathcal{N} = 4$ sYM theory**
4. **Regge Amplitudes in $\mathcal{N} = 4$ sYM theory**

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2. **AdS/CFT and minimal surfaces**
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1. **Introduction: old/new Reggeon problem**
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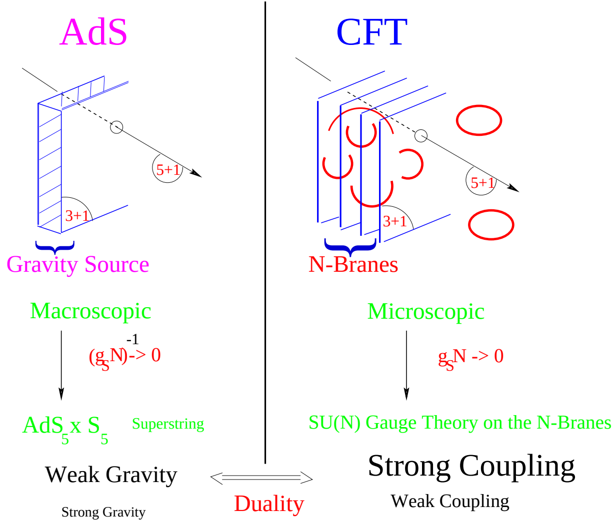
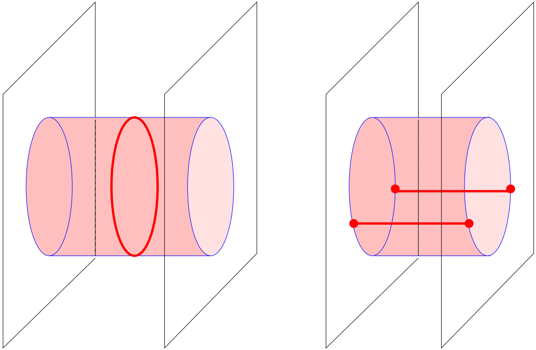
| $\mathcal{N} = 4$ Super Yang-Mills theory | $\equiv$ | Superstrings on $AdS_5 \times S^5$ |
|---|---|---|
| strong coupling | | (semi-)classical strings |
| nonperturbative physics | | or supergravity |
| very difficult | | 'easy' |
| weak coupling | | highly quantum regime |
| 'easy' | | very difficult |

- New ways of looking at nonperturbative gauge theory physics...
- Intricate links with General Relativity...
- This is an equivalence! Any state/phenomenon on the gauge theory side should have its dual counterpart...
- **Caveat:** the dual counterpart does not neccessarily have to be in the well understood (super)gravity sector...

Maldacena (1997):



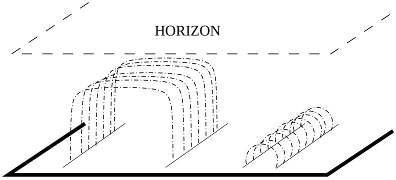
Gauge/Gravity Duality: a wider concept



Schomerus

- Open ⇔ Closed String duality
- Tree-level Closed String ⇔ 1-loop Open String
- Classical Gravity ⇔ Quantum Gauge field Theory
- Small/Large Distance ⇒ Weak/Strong Gauge coupling ?

**G/G Tool:** Wilson Loops $\Leftrightarrow$ Minimal Surfaces

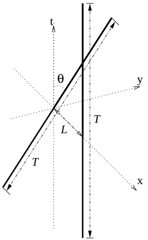



$$\langle e^{iP\int_C \vec{A}\cdot\vec{d}l}\rangle = \int_\Sigma e^{-\frac{Area(\Sigma)}{\alpha'}} \approx e^{-\frac{Min.Area}{\alpha'}} \times Fluctuations$$

| Confining theory $\sim$ QCD | $\mathcal{N}=4$ Conformal Theory |
|---|---|
| Linear potential $\mathcal{A} \propto T \times L$ | Coulombic potential $\mathcal{A} \propto T/L$ |
| Quasi-planar Approximation | Non-planar minimal surfaces |
| $\sim$ *QCD* Regge amplitude ? | $\mathcal{N}=4$ sYM Regge amplitude? |

**G/G Tool:** Eikonal approximation $\Leftrightarrow$ Helicoidal Geometry

Janik, R.P. 99-01




$$a(L = |\vec{b}|, \chi) \equiv \frac{i}{2s} \int \frac{d^2\vec{q}}{(2\pi)^2} e^{-i\vec{q}\cdot\vec{b}} \; A_{\mathcal{R}}(s,t)$$

$$\Leftrightarrow \;\; \tilde{a}(L, \theta \to -i\chi, T) = \mathcal{Z}^{-1} \int \mathcal{DC} \, \langle W[\mathcal{C}] \rangle \sim e^{-\frac{1}{\alpha'} \mathcal{A}_{Helicoid}}$$

Confining theory $\sim$ QCD

$\sim \mathbb{R}_4$ Helicoid known

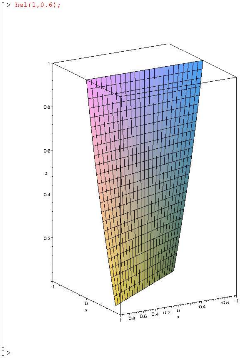
Analytic Continuation $\to$ Minkowski ?

$\mathcal{N} = 4$ Conformal Theory

$AdS_5$ "Helicoid" unknown

$\mathcal{N} = 4$ sYM Regge amplitude?

**Flat space example:** Eikonal approximation ⇔ Helicoidal Geometry

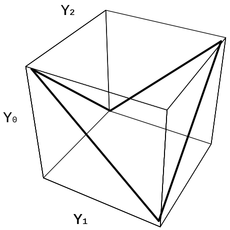



$$Area = LT\sqrt{1+\frac{T^2\theta^2}{L^2}} + \frac{L^2}{2\theta}\log\frac{\sqrt{1+\frac{T^2\theta^2}{L^2}}+\theta\frac{T}{L}}{\sqrt{1+\frac{T^2\theta^2}{L^2}}-\theta\frac{T}{L}}$$

## Alday-Maldacena Amplitude

$4-$ *gluon* amplitude: $\Leftrightarrow$ Minkowskian Minimal surface ($z$ near $\infty$)

Alday, Maldacena (2007)




- Minkowskian Boundary: Polygon of light-like gluons (+boosts)
- Exact minimal surface in T-dual $AdS_5$ metric: $ds^2 = \frac{R^2}{r^2}\left(\eta_{\mu\nu}dy^\mu dy^\nu + dr^2\right)$
- **Amplitude from minimal Area:**

$$A(s,t) = e^{-i\mathcal{A}} = \exp\left[2iS_{\rm div}(s) + 2iS_{\rm div}(t) + \frac{\sqrt{\lambda}}{8\pi}\left(\log\frac{s}{t}\right)^2 + \tilde{C}\right]$$

$$iS_{\rm div}(p) = -\frac{1}{\epsilon^2}\frac{1}{2\pi}\sqrt{\frac{\lambda\mu^{2\epsilon}}{(-p)^\epsilon}} - \frac{1}{\epsilon}\frac{1}{4\pi}(1-\log 2)\sqrt{\frac{\lambda\mu^{2\epsilon}}{(-p)^\epsilon}}, \quad (p = s,t)$$

Regge limit: $(-s) \to \infty, (-t)$ fixed

- Divergent part

$$iS_{\rm div}(p) = -\frac{1}{\epsilon^2}\frac{\sqrt{\lambda}}{2\pi} + \frac{1}{\epsilon}\frac{\sqrt{\lambda}}{4\pi}\log\frac{-2p}{e\mu^2} - \frac{\sqrt{\lambda}}{16\pi}\log^2\frac{-p}{\mu^2} + \frac{\sqrt{\lambda}}{8\pi}(1-\log 2)\log\frac{-p}{\mu^2}$$

- Regge Amplitude:

$$A(s,t) = A_{\rm div}\cdot A_{\mathcal{R}}\cdot e^{\mathcal{O}(\epsilon)}$$

$$A_{\rm div}(s,t) = \exp\left[-\frac{1}{\epsilon^2}\frac{2\sqrt{\lambda}}{\pi} + \frac{1}{\epsilon}\frac{\sqrt{\lambda}}{2\pi}\left(\log\frac{-s}{\mu^2} + \log\frac{-t}{\mu^2} - 2(1-\log 2)\right)\right]$$

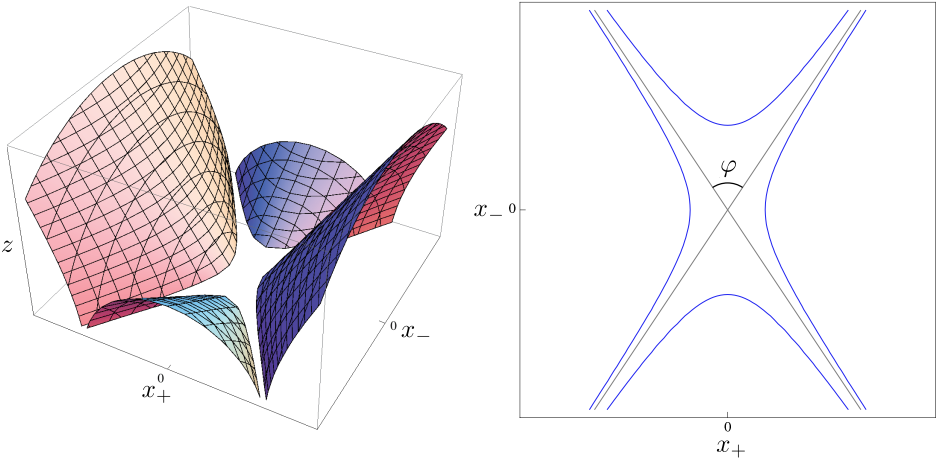
$$A_{\mathcal{R}}(s,t) = \left(\frac{-s}{\mu^2}\right)^{-\frac{1}{4}f(\lambda)\log\frac{-t}{\mu^2}+g(\lambda)}\exp\left(\frac{g(\lambda)}{4}\log\frac{-t}{\mu^2} + \tilde{C}\right)$$

- Regge Trajectory:

$$\boxed{\alpha_{\mathcal{R}}(t) = -\frac{f(\lambda)}{4}\log\frac{-t}{\mu^2} + \frac{g(\lambda)}{4}} \quad f(\lambda) = \frac{\sqrt{\lambda}}{\pi};\ g(\lambda) = \frac{\sqrt{\lambda}}{\pi}(1-\log 2)$$

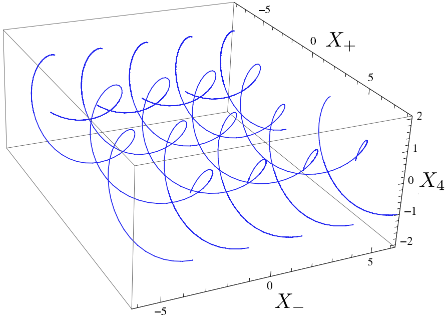
**A show of $AdS_5$ Minimal surfaces**

Regge limit: Minimal Surface at the IR $z \to \infty$

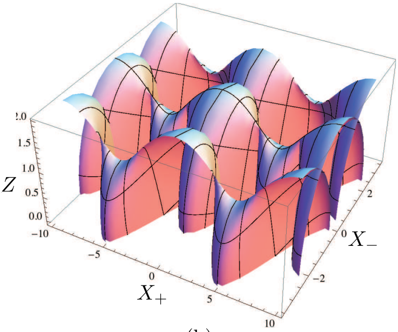



(a) (b)

Regge limit: Minimal Surface at the UV $z \to 0$

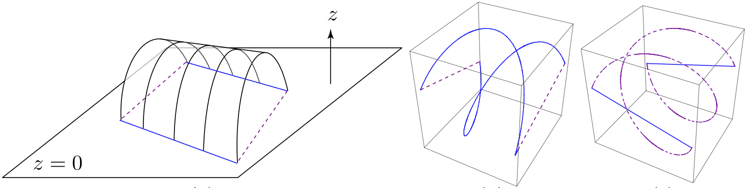



(a)




(b)

Regge limit: Wilson Lines Boundary $z = 0$



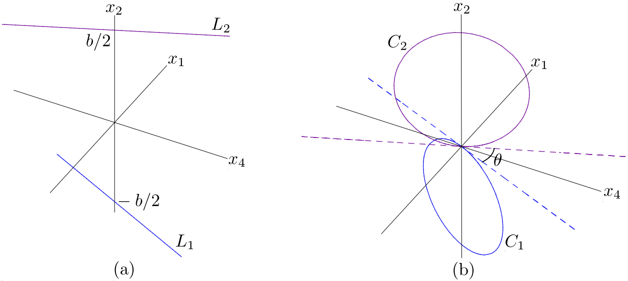

(a) (b) (c)

Parallels | $gg$ double helix | $qq$ double helix

Different "helicoids" in $AdS_5$ for $gg$ and $qq$ !!! So what to do?

Minimal Surface for $qq$ amplitude: Conformal Transform of Helicoid

Giordano,R.P.,Seki (2011)




- Invariance by 5-d Inversion

$$x_\mu \to \frac{x_\mu}{|x_\mu|^2 + z^2}\,, \quad z \to \frac{z}{|x_\mu|^2 + z^2}$$

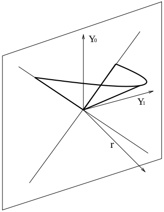
- Euclidean Two-cusp Dominance

$$\boxed{A^{\rm quark}_{\rm minimal} = 2\Gamma^{E}_{\rm cusp}(\theta)\log(M_q b) + \cdots}$$

Analytic Continuation Euclid $\rightarrow$ Minkowski

- Cusp Minimal Surface in $AdS_5$

Kruczenski (2002)




- Cusp: $\theta = \pi + i\chi,\ \Gamma_{\text{cusp}}(\chi) \rightarrow -\frac{f(\lambda)}{4}\chi \quad \text{for} \quad \chi \gg 1$

$$A^{\text{quark}}_{\text{minimal}}(Mb,\chi) = -\frac{f(\lambda)}{2}\log\frac{Mb}{\chi} + \Psi(\chi)$$

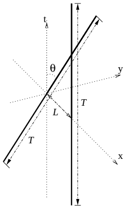
- Back to Momentum space:

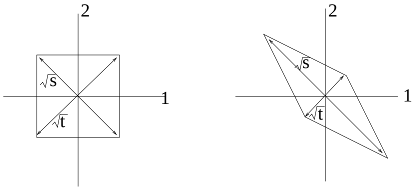
$$\mathrm{A}^{\text{quark}}_{\text{Regge}}(s,t) \approx 2(-s/M^2)^{-\frac{f}{4}\log\frac{-t}{\mu^2}+\frac{f}{2}\log 2}$$

**Results:** Eikonal approximation *vs.* Alday-Maldacena

qq 2−cusp amplitude

gg 4 → 2−cusp amplitude




(a) (b)

$$\alpha_{\mathcal{R}}(t) = -\frac{f(\lambda)}{4} \log \frac{-t}{{\mu_{qq}}^2}$$

$$\alpha_{\mathcal{R}}(t) = -\frac{f(\lambda)}{4} \log \frac{-t}{{\mu_{gg}}^2}$$