Properties of The Light-Heavy and Heavy-Heavy Flavoured Mesons in NRQCD

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Outline

- ***Introduction***
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Thanks

Introduction

- *Recently, there have been renewed interest in the spectroscopy of the heavy flavoured hadrons due to number of experimental facilities (CLEO, DELPHI, Belle, BaBar, LHCb etc) which have been continuously providing and expected to provide more accurate and new information about the hadrons from low flavour to heavy flavour sector.*

- *The heavy flavour mesons are those in which at least one of the quark or antiquark or both the quark and antiquark belong to heavy flavour sector; particularly the charm or beauty. They are represented by $Q \bar Q$ mesonic systems which include the quarkonia* $(c\bar{c}$ and $b\bar{b})$ and B_c $(c\bar{b}$ or $b\bar{c})$ *mesons.*

- *The investigation of the properties of these mesons gives very important insight into heavy quark dynamics. Heavy quarkonia have a rich spectroscopy with many narrow States lying under the threshold of open flavour production.*

- *The success of theoretical model predictions with experiments can provide important information about the quark-antiquark interactions.*

Among many theoretical attempts or approaches to explain the hadron properties based on its quark structure very few were successful in predicting the hadronic properties starting from mass spectra to decay widths.

The new role of the heavy flavour studies as the testing ground for the non-perturbative aspects of QCD, demands extension of earlier phenomenological potential model studies on quarkonium masses to their predictions of decay widths with the non-perturbative approaches like NRQCD.

Our attempt here would be then to study the heavy-heavy flavour mesons in the charm and beauty sector in a general framework of the potential models. The model parameters used for the predictions of the masses and their radial wave functions would be used for the study of their decay properties using NRQCD formalism.

Non-relativistic Treatment for $Q \bar Q$ systems

- *In the center of mass frame of the heavy quark-antiquark system, the momenta of quark and antiquark are dominated by their rest mass* $m_{Q,\bar{Q}} \gg \Lambda_{QCD} \sim |\vec{p}\,|$ *which constitutes the basis of the non-relativistic treatment.*

- *Hence, for the study of heavy-heavy bound state systems such as* ($c\bar{c}$ and $b\bar{b}$) and B_c ($c\bar{b}$ or $b\bar{c}$) *we consider a non-relativistic Hamiltonian given by A K Rai (2002-05-06)*

$$H = M + \frac{p^2}{2m} + V(r)$$

Where

$$M = m_Q + m_{\bar{Q}}, \quad and \quad m = \frac{m_Q\, m_{\bar{Q}}}{m_Q + m_{\bar{Q}}}$$

m_Q and $m_{\bar{Q}}$ *are the mass parameters of quark and antiquark respectively, p is the relative momentum of each quark and V(r) is the quark antiquark potential.*

- *we have considered a general power potential with colour coulomb term of the form*

$$V(r) = \frac{-\alpha_c}{r} + Ar^{\nu}$$

as the static quark-antiquark interaction potential (CPP_{ν})

for the study of mesons, $\alpha_c = \frac{4}{3}\alpha_s$, α_s *being the strong running coupling constant, A is the potential parameter and v is a general power, such that the choice, v =1 corresponds to the coulomb plus linear potential.*

- *Hence for the present study of heavy-heavy flavour mesons, we employ the exponential trial wave function of the hydrogenic type to generate the mass spectra. Within the Ritz variational scheme using the trial radial wave function we obtain the expectation values of the Hamiltonian as* $(\langle H \rangle = E(\mu, \nu))$

- *The experimental spin average masses are computed from the experimental masses of the pseudoscalar and vector mesons using the relation,*

$$M_{SA} = M_P + \frac{3}{4}(M_V - M_P)$$

We account this correction to the value of R(0) by considering

$$R_{nJ}(0) = R(0)\left[1 + (SF)_J \frac{< \varepsilon_{SD} >_{nJ}}{M_{SA}}\right]$$

Where $(SF)_J$ and $< \varepsilon_{SD} >_{nJ}$ *is the spin factor and spin interaction energy of the meson in the nJ state, while R(0) and* M_{SA} *correspond to the radial wave function at the zero separation and spin average mass respectively of the $Q \bar Q$ system.*

The spin-spin and spin-orbit interactions are taken as

$$V_{S_Q \cdot S_{\bar{Q}}}(r) = \frac{8}{9}\frac{\alpha_s}{m_Q m_{\bar{Q}}} \vec{S}_Q \cdot \vec{S}_{\bar{Q}} \, 4\pi\delta(r)$$

$$V_{L \cdot S}(r) = \frac{4}{3}\frac{\alpha_s}{m_Q m_{\bar{Q}}} \frac{\vec{L} \cdot \vec{S}}{r^3}$$

- *The results are compared with known experimental values as well as with other the theoretical predictions. Mass predictions with v between 1.0 and 1.5 are found in accordance with the experimental results Particle Data Group.*

G. T. Bodwin E. Braaten and G. P. Lepage, PRD 51 (1995)1125, Erratum 55(1997)5853

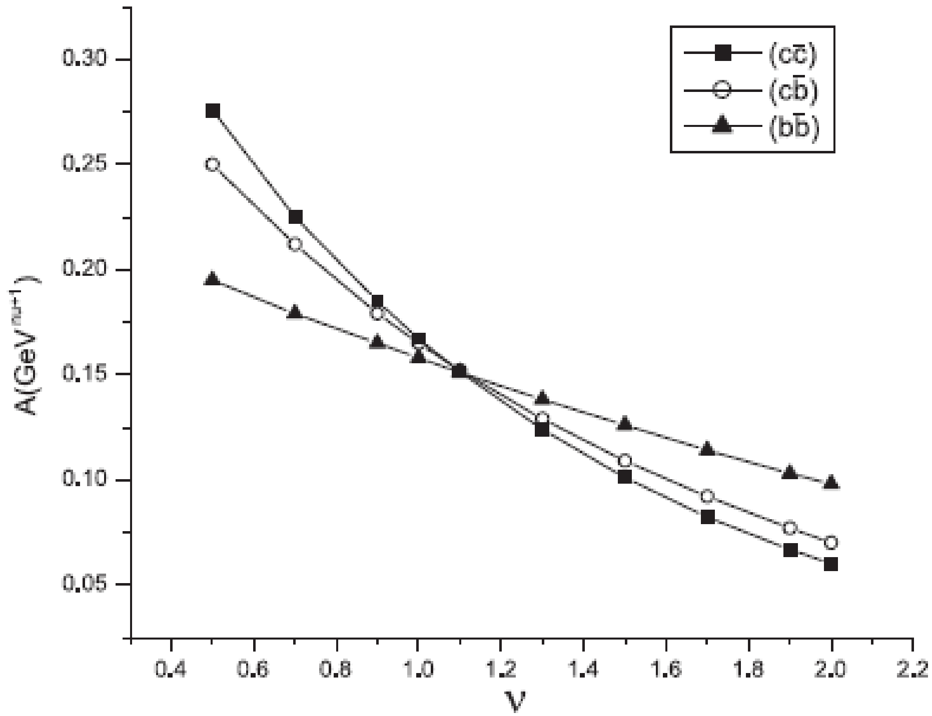


FIG. 1: Fitted potential parameter A against potential index ν

S-Wave and P-Wave Masses (in GeV) of $c\bar{c}$ meson

ν	1^1S_0	$1\,^3S_1$	1^1P_1	$1\,^3P_0$	$1\,^3P_1$	$1\,^3P_2$	2^1S_0	$2\,^3S_1$	2^1P_1	2^3P_0	$2\,^3P_1$	$2\,^3P_2$	3^1S_0	$3\,^3S_1$
0.5	3.000	3.092	3.313	3.292	3.302	3.323	3.352	3.375	3.519	3.494	3.507	3.531	3.541	3.553
0.7	2.980	3.100	3.373	3.341	3.357	3.389	3.427	3.464	3.666	3.623	3.644	3.687	3.697	3.717
0.9	2.960	3.109	3.429	3.383	3.406	3.451	3.495	3.547	3.808	3.742	3.775	3.842	3.846	3.878
1.0	2.950	3.112	3.450	3.398	3.424	3.477	3.522	3.583	3.872	3.792	3.832	3.911	3.912	3.950
1.1	2.942	3.116	3.473	3.414	3.444	3.503	3.549	3.619	3.936	3.843	3.889	3.983	3.979	4.024
1.3	2.962	3.123	3.513	3.414	3.477	3.550	3.597	3.683	4.055	3.933	3.994	4.116	4.102	4.161
1.5	2.912	3.129	3.547	3.461	3.504	3.590	3.636	3.739	4.162	4.009	4.085	4.239	4.212	4.285
1.7	2.899	3.134	3.576	3.477	3.526	3.625	3.668	3.788	4.257	4.073	4.165	4.394	4.309	4.395
1.9	2.887	3.141	3.603	3.491	3.547	3.658	3.696	3.832	4.345	4.129	4.237	4.453	4.396	4.500
2.0	2.882	3.144	3.615	3.497	3.556	3.673	3.708	3.852	4.385	4.153	4.269	4.501	4.436	4.547
[1, 2]	2.980	3.097	3.511	3.415		3.556		3.686				3.929		4.040
[12]	2.979	3.096	3.526	3.424	3.511	3.556	3.588	3.686	3.945	3.854	3.929	3.972	3.991	4.088
[47]	2.980	3.097	3.527	3.416	3.508	3.558	3.597	3.686	3.960	3.844	3.894	3.994	4.014	4.095

(1,2) PDG(2006)(2002). (12) D Ebert et al, PRD 67 (2003)014027.
(47) S F Redford et al. PRD 75(2007) 074031

S-Wave and P-Wave Masses (in GeV) of $b\bar{b}$ meson

ν	1^1S_0	$1\,^3S_1$	1^1P_1	$1\,^3P_0$	$1\,^3P_1$	$1\,^3P_2$	2^1S_0	$2\,^3S_1$	2^1P_1	2^3P_0	$2\,^3P_1$	$2\,^3P_2$	3^1S_0	$3\,^3S_1$
0.5	9.426	9.463	9.670	9.664	9.683	9.672	9.696	9.702	9.808	9.803	9.806	9.811	9.824	9.827
0.7	9.419	9.465	9.712	9.703	9.731	9.716	9.751	9.760	9.908	9.898	9.903	9.913	9.931	9.936
0.9	9.414	9.467	9.751	9.740	9.774	9.757	9.803	9.816	10.006	9.991	9.999	10.014	10.038	10.045
1.0	9.411	9.468	9.768	9.755	9.792	9.775	9.826	9.841	10.053	10.035	10.044	10.062	10.088	10.097
1.1	9.408	9.468	9.784	9.769	9.809	9.791	9.846	9.865	10.097	10.076	10.086	10.108	10.136	10.147
1.3	9.403	9.470	9.815	9.797	9.840	9.824	9.888	9.910	10.184	10.155	10.170	10.198	10.230	10.244
1.5	9.399	9.472	9.842	9.820	9.866	9.852	9.924	9.951	10.264	10.228	10.246	10.282	10.317	10.334
1.7	9.394	9.473	9.864	9.840	9.887	9.877	9.955	9.985	10.335	10.291	10.313	10.357	10.394	10.416
1.9	9.390	9.474	9.885	9.857	9.905	9.900	9.982	10.017	10.401	10.350	10.376	10.428	10.466	10.492
2.0	9.389	9.475	9.896	9.866	9.913	9.911	9.995	10.032	10.434	10.379	10.406	10.462	10.501	10.529
[1, 2]		9.460		9.860	9.893	9.913		10.023		10.232	10.255	10.268		10.355
[12]	9.400	9.460	9.901	9.863	9.892	9.913	9.993	10.023	10.261	10.234	10.255	10.268	10.328	10.355
[47]	9.414	9.461	9.900	9.861	9.891	9.912	9.999	10.023	10.262	10.231	10.255	10.272	10.345	10.364

(1,2) PDG(2006)(2002). (12) D Ebert et al, PRD 67 (2003)014027.
(47) S F Redford et al. PRD 75(2007) 074031

Decay constants ($f_{P/V}$) of the heavy flavoured mesons

- *The decay constants of mesons are important parameters in the study of leptonic or non-leptonic weak decay processes.*

- ***The Van-Royen-Weisskopf formula***

$$f^2_{P/V} = \frac{12\left|\psi_{P/V}(0)\right|^2}{M_{P/V}}\bar{C}^2(\alpha_s)$$

Where $\bar{C}^2(\alpha_s)$ *is the QCD correction factor given by E Braaten (1995)*

Gershtein (1998)

$$\bar{C}^2(\alpha_s) = 1 - \frac{\alpha_s}{\pi}\left[\delta^{P,V} - \frac{m_Q - m_{\bar{Q}}}{m_Q + m_{\bar{Q}}}\, ln\frac{m_Q}{m_{\bar{Q}}}\right]$$

Here $\delta^P = 2$ and $\delta^V = 8/3$.

Decay constants(f_P & f_V) (in MeV) of 1S and $c\bar{c}$ mesons states.

Models	$R_p(0)$ $GeV^{3/2}$	$R_v(0)$ $GeV^{3/2}$	f_P MeV	$f_P(cor.)$ MeV	f_V MeV	$f_V(cor.)$ MeV
ERHM	0.726	0.752	410	317	418	323
BT	0.874	0.909	499	382	505	389
PL	0.971	1.009	550	399	560	407
LOG	0.877	0.914	496	379	506	387
Cornell	1.171	1.217	663	532	676	543
ν =0.5	0.763	0.787	430	348	437	326
0.7	0.875	0.912	495	401	506	377
0.9	0.967	1.018	549	444	564	421
1.0	1.005	1.063	572	463	589	439
1.1	1.041	1.107	593	480	613	457
1.3	1.104	1.184	627	507	655	488
1.5	1.158	1.252	663	536	692	516
1.7	1.204	1.311	691	559	724	539
1.9	1.245	1.366	716	579	753	561
2.0	1.264	1.391	728	589	767	571
K W Edwards et al. (CLEO) PRL 86 (2001) 30				335± 75	459± 28	416± 6

Decay constants(f_P & f_V) (in MeV) of $1S$ $b\bar{b}$ meson state.

Models	$R_p(0)$	$R_v(0)$	f_P	$f_P(cor.)$	f_V	$f_V(cor.)$
	$GeV^{3/2}$	$GeV^{3/2}$	MeV	MeV	MeV	MeV
ERHM	2.232	2.235	709	601	710	601
BT	2.527	2.551	807	683	810	686
PL	2.132	2.146	680	563	682	565
LOG	2.206	2.221	703	594	706	596
Cornell	3.706	3.762	1185	1022	1194	1029
ν =0.5	1.971	1.979	627	537	629	509
0.7	2.178	2.189	693	594	695	563
0.9	2.358	2.371	751	643	753	609
1.0	2.436	2.451	776	665	778	630
1.1	2.513	2.529	801	686	803	650
1.3	2.648	2.667	844	723	847	685
1.5	2.764	2.785	881	755	884	715
1.7	2.867	2.891	914	783	918	743
1.9	2.962	2.989	945	809	949	768
2.0	3.009	3.037	960	822	964	780
				711[19]		708

C W Hwang et al. JPG 34 (2007) 687

Heavy-light Systems

- Hamiltonian for the heavy-light system

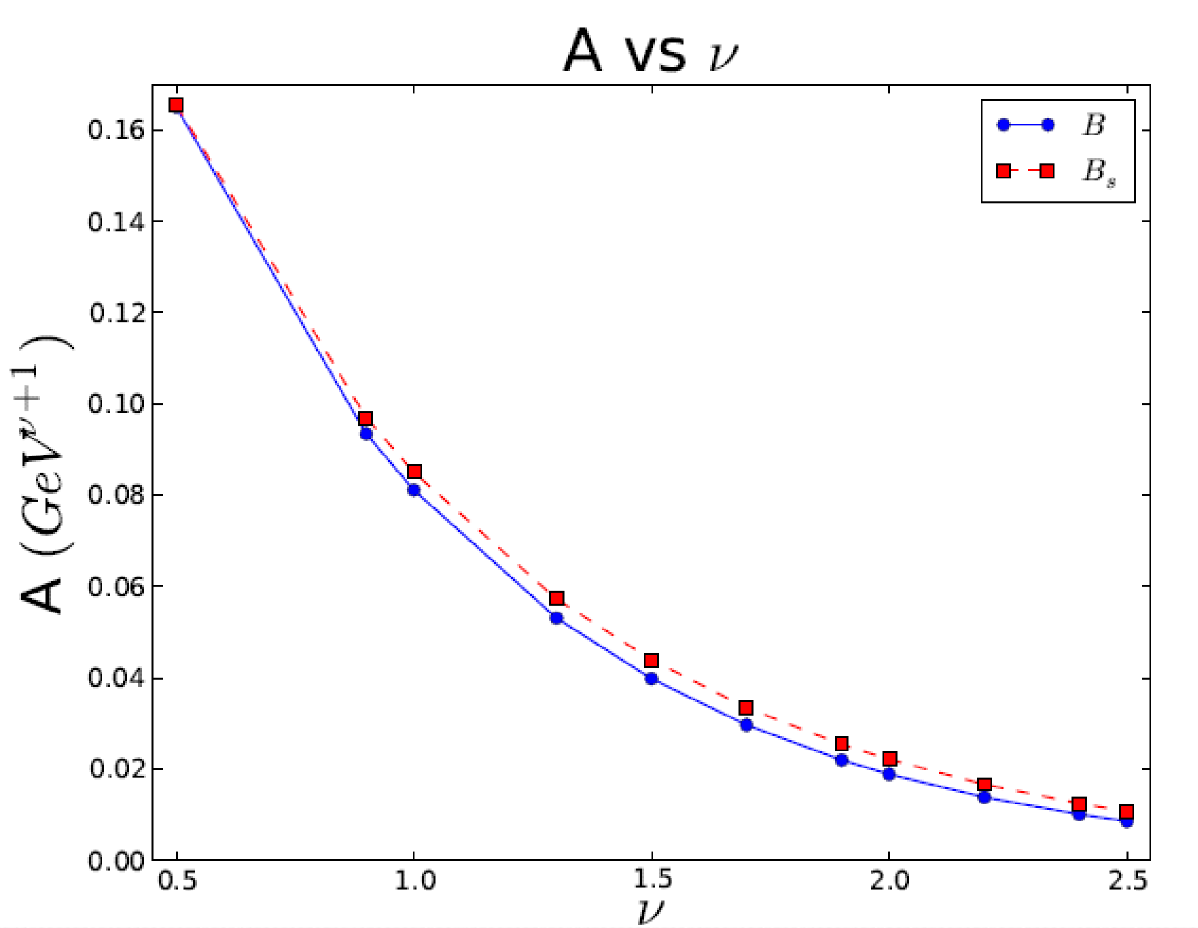
$$H = \sqrt{\mathbf{p}^2 + m_Q^2} + \sqrt{\mathbf{p}^2 + m_{\bar{q}}^2} + V(\mathbf{r})$$

- Quark-antiquark potential of the form

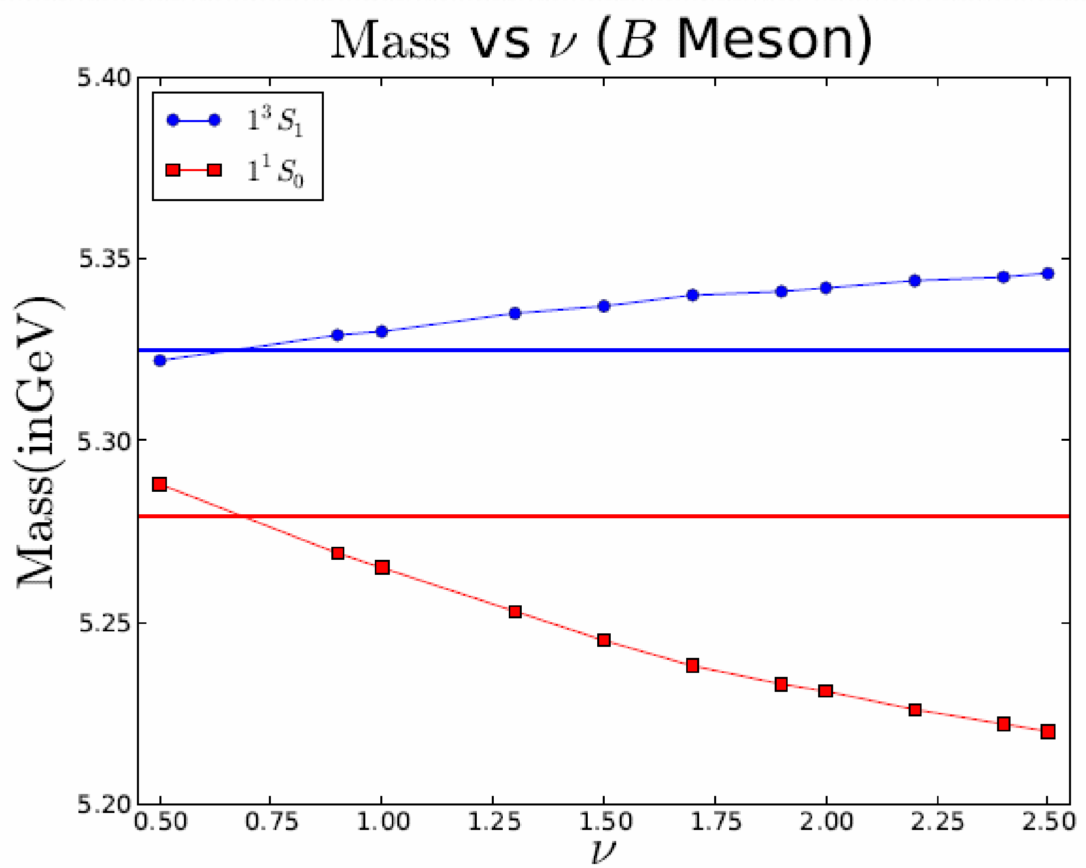
$$V(r) = -\frac{\alpha_c}{r} + Ar^{\nu}$$

- The wave function

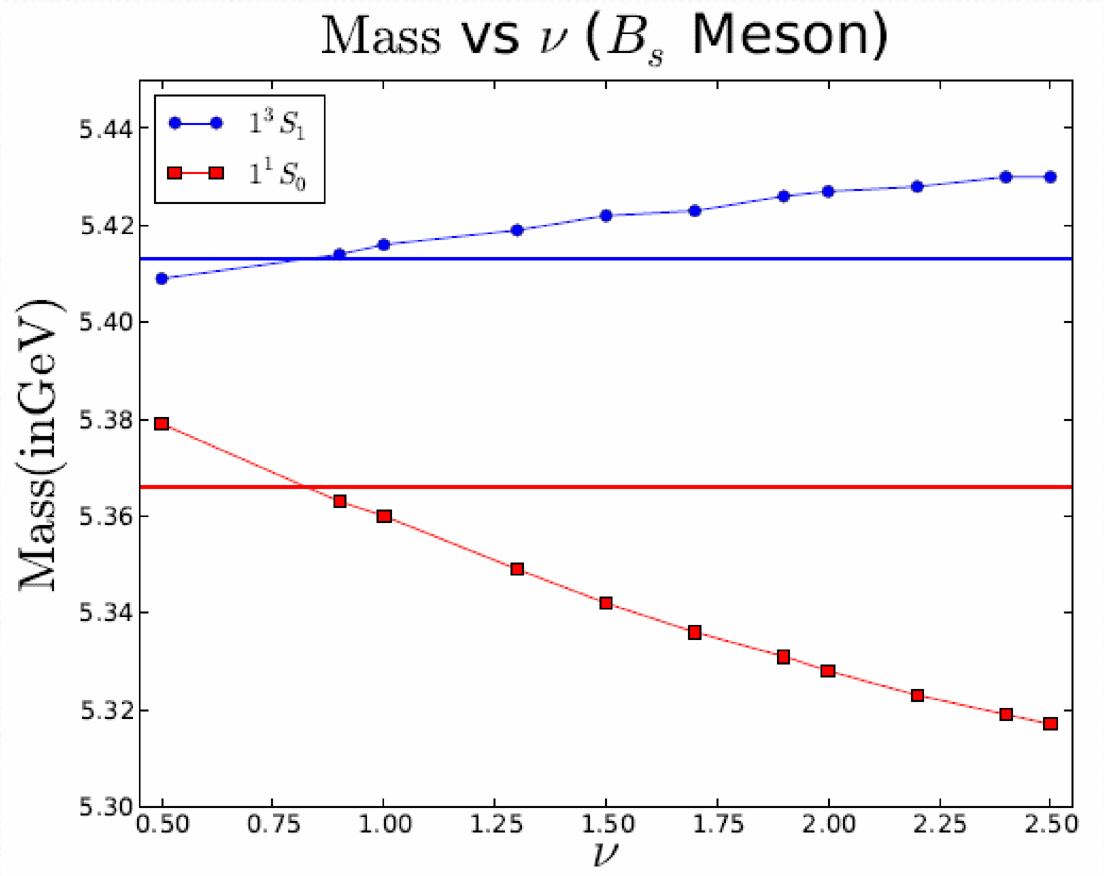
$$R_{nl}(r) = \left(\frac{\mu^3 (n-l-1)!}{2n(n+l)!} \right)^{1/2} (\mu\, r)^l \, e^{-\mu r/2} \, L_{n-l-1}^{2l+1}(\mu r)$$



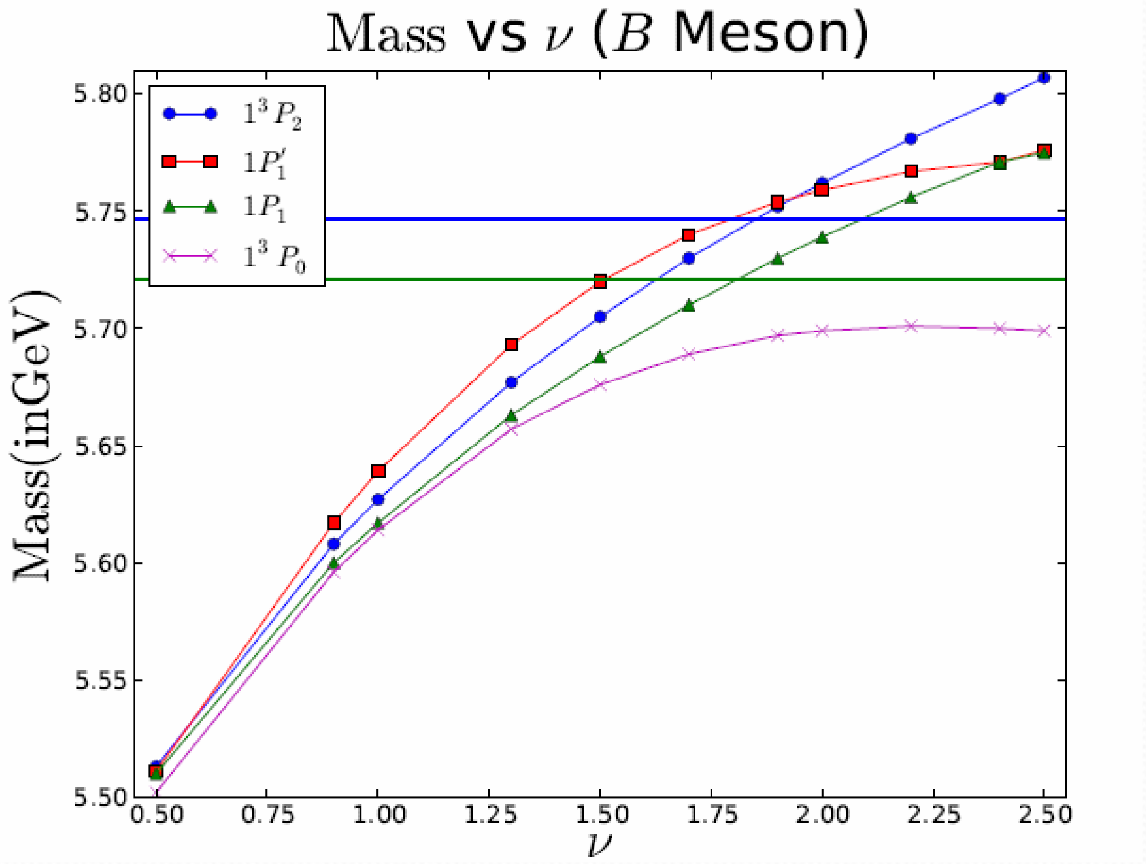
A vs ν
B
B_s
A $(GeV^{\nu+1})$
ν
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.5
1.0
1.5
2.0
2.5



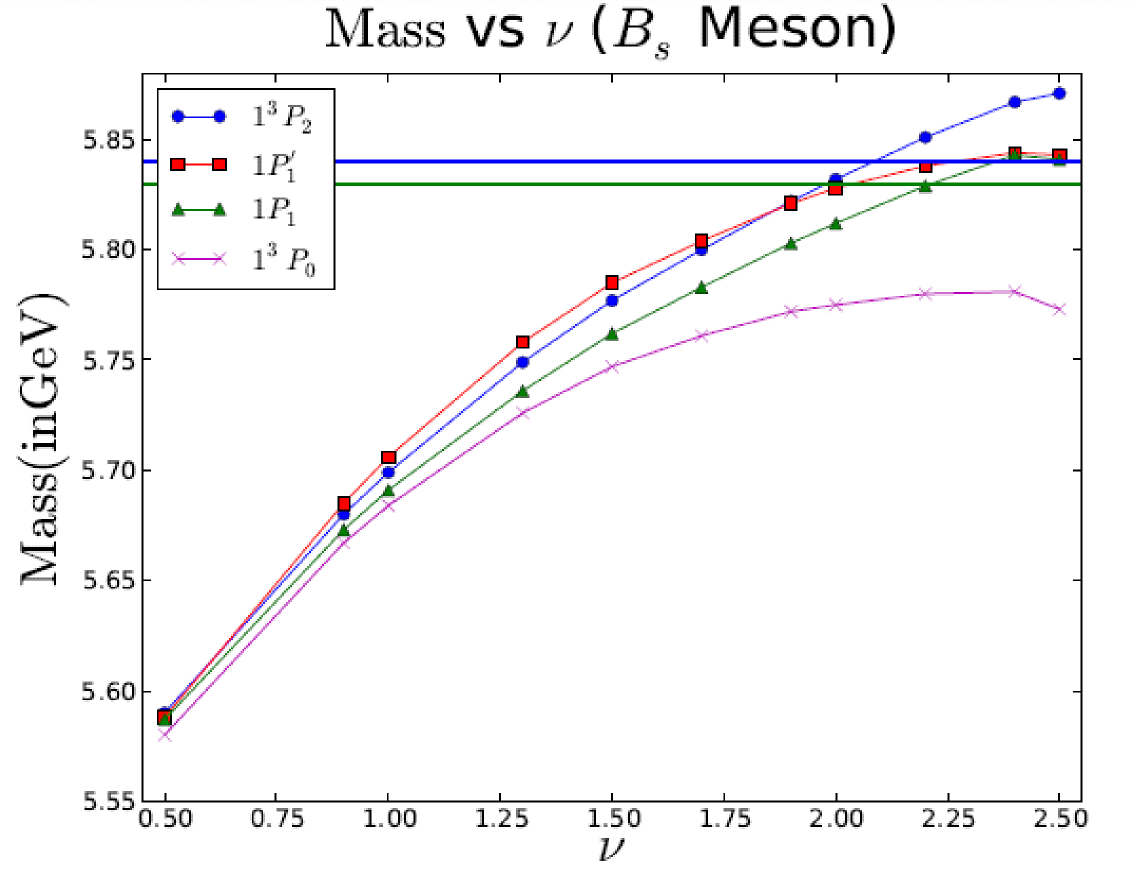
Mass vs ν (B Meson)
1^3S_1
1^1S_0
Mass(inGeV)
5.40
5.35
5.30
5.25
5.20
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.25
2.50
ν



Mass vs ν (B_s Meson)
1^3S_1
1^1S_0
Mass(inGeV)
5.44
5.42
5.40
5.38
5.36
5.34
5.32
5.30
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.25
2.50
ν

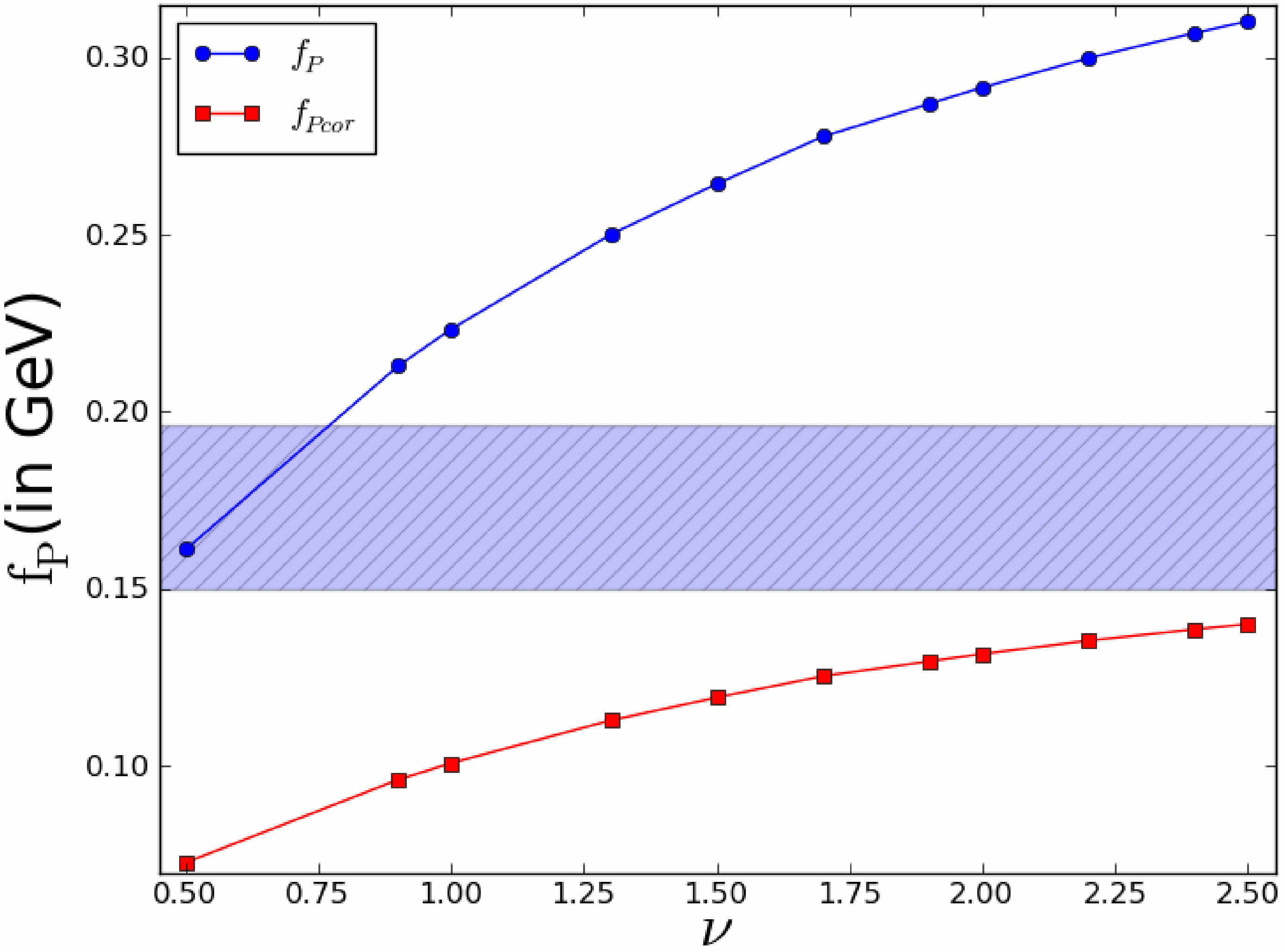


Mass vs ν (B Meson)
1^3P_2
$1P_1'$
$1P_1$
1^3P_0
Mass(inGeV)
5.80
5.75
5.70
5.65
5.60
5.55
5.50
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.25
2.50
ν

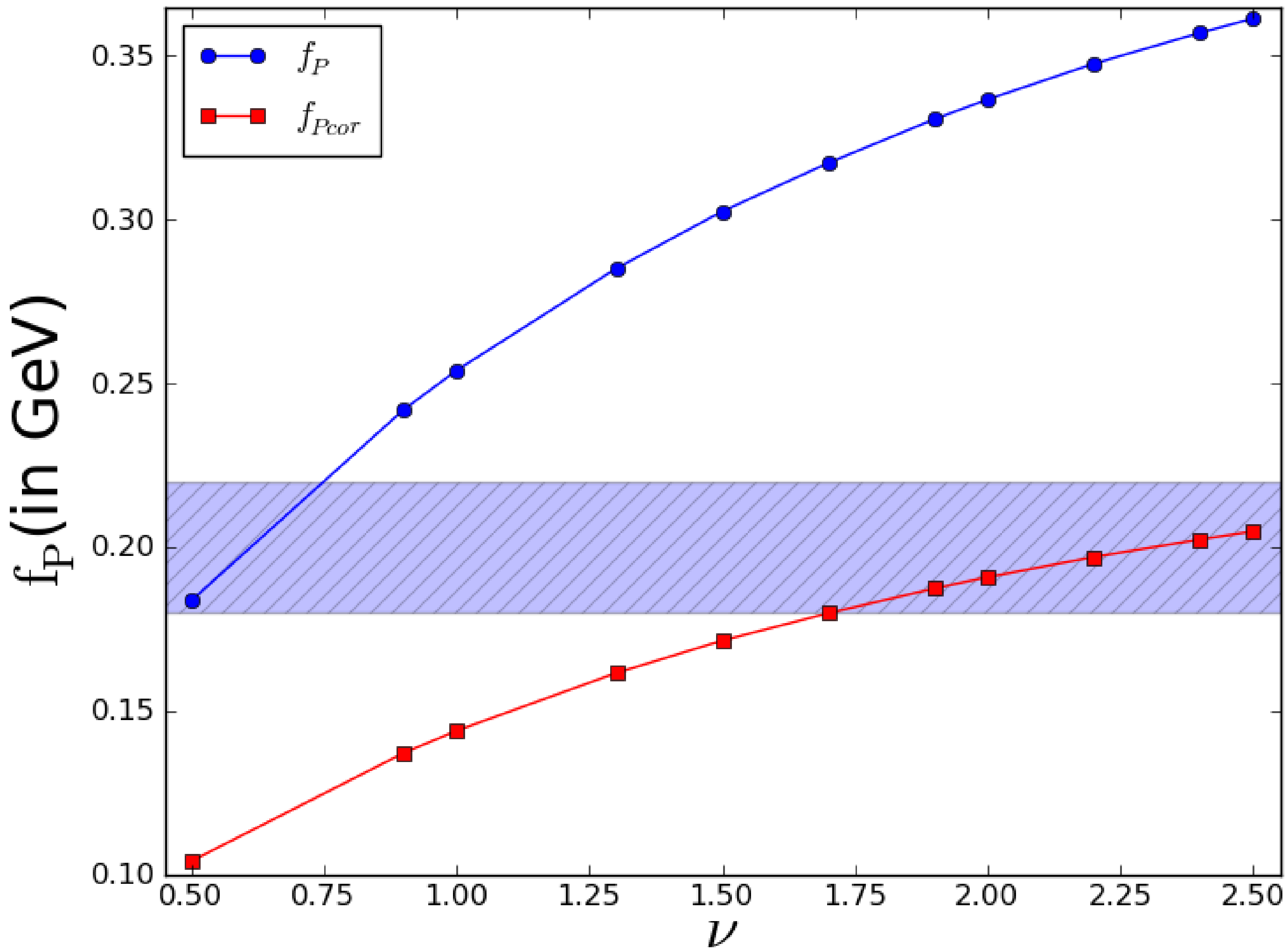


Mass vs ν (B_s Meson)
1^3P_2
$1P_1'$
$1P_1$
1^3P_0
Mass(inGeV)
5.55
5.60
5.65
5.70
5.75
5.80
5.85
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.25
2.50
ν

$f_P(1S)$ vs ν for B meson



$f_P(1S)$ vs ν for B_s meson



Decay Rates of Quarkonia

The two photon decay width of the pseudoscalar meson is given by (A K Rai 2005)

$$\Gamma(0^{-+} \rightarrow 2\gamma) = \Gamma_0 + \Gamma_{0R}$$

Here Γ_0 is the conventional Van Royen-Weisskopf term for the $0^{-+} \rightarrow \gamma\gamma$ decays [Van Royen-Weisskopf 1967], where Γ_R is due to the radiative corrections for this decay which is given by

$$\Gamma_0 = \frac{12\alpha_e^2 e_Q^4}{M_P^2} R_P^2(0)$$

and

$$\Gamma_{0R} = \frac{\alpha_s}{\pi}\left(\frac{\pi^2 - 20}{3}\right)\Gamma_0$$

Similarly, the leptonic decay width of the vector meson is computed as

$$\Gamma(1^{--} \rightarrow l^+ l^-) = \Gamma_{VW} + \Gamma_R$$

where

$$\Gamma_{VW} = \frac{4\alpha_e^2 e_Q^2}{M_V^2} R_V^2(0)$$

Γ_R, the radiative correction is given by

$$\Gamma_R = \frac{-16}{3\pi}\alpha_s \Gamma_{VW}$$

NRQCD formalism

- *The decay rates of the heavy-quarkonium states into photons and pairs of leptons are among the earliest applications of perturbative quantum chromodynamics (QCD) (T Appelquist and H D Politzer 1975, R Barbieri and Gatto 1976). In these analysis, it was assumed that the decay rates of the meson factored into a short-distance part that is related to the annihilation rate of the heavy quark and antiquark, and long-distance factor containing all non perturbative effects of the QCD.*

- *The short-distance factor calculated in terms of the running coupling constant $\alpha_s(m_Q)$ of QCD, evaluated at the scale of the heavy-quark mass m_Q, while the long-distance factor was expressed in terms of the meson's non relativistic wave function, or its derivatives, evaluated at origin.*

- *In case of S-wave decays and in case of P-wave decays into Photons, the factorization assumption was supported by explicit calculations at next-to-leading order in α_s.*

- *An elegant effort was provided by the NRQCD formalism (Bodwin and Lepage 1995-97). It consists of a non-relativistic Schrodinger field theory for the heavy quark and antiquark that is coupled to the usual relativistic field theory for light quarks and gluons. NRQCD not only organize calculation of all orders in* α_s, *but also elaborate systematically the relativistic corrections to the conventional formula.*

- *It also provides non-perturbative definitions of the long-distance factors in terms of matrix elements of NRQCD, making it possible to evaluate them in the numerical lattice calculations.*

- *Analyzing S-wave decays within this frame work, it recover, at leading order in* v^2, *standard factorization formulae, which contain a single non-perturbative parameter. At next to leading order in* v^2, *the decay rates satisfy a more general factorization formula, which contain two additional independent non-perturbative matrix elements related to their radial wave functions.*

- *It is expected that the NRQCD formalism has all the corrective contributions for the right predictions of the decay rates. NRQCD factorization expressions for the decay rates of quarkonium and decay are given by (Bodwin and Petrelli PRD 2002).*

$$\Gamma(^1S_0 \to \gamma\gamma) = \frac{F_{\gamma\gamma}(^1S_0)}{m_Q^2}\left|< 0|\chi^\dagger\psi|^1S_0 >\right|^2 + \frac{G_{\gamma\gamma}(^1S_0)}{m_Q^4} Re\left[<^1 S_0|\psi^\dagger\chi|0 >< 0|\chi^\dagger(-\frac{i}{2}\overrightarrow{D})^2\psi|^1S_0 >\right]$$

$$+\frac{H^1_{\gamma\gamma}(^1S_0)}{m_Q^6} <^1 S_0|\psi^\dagger(-\frac{i}{2}\overrightarrow{D})^2\chi|0 >< 0|\chi^\dagger(-\frac{i}{2}\overrightarrow{D})^2\psi|^1S_0 >$$

$$+\frac{H^2_{\gamma\gamma}(^1S_0)}{m_Q^6}\ Re\left[<^1 S_0|\psi^\dagger\chi|0 >< 0|\chi^\dagger(-\frac{i}{2}\overrightarrow{D})^4\psi|^1S_0 >\right]$$

$$\Gamma(^3S_1 \to e^+e^-) = \frac{F_{ee}(^3S_1)}{m_Q^2}\left|< 0|\chi^\dagger\sigma\psi|^3S_1 >\right|^2 + \frac{G_{ee}(^3S_1)}{m_Q^4} Re\Big[<^3 S_1|\psi^\dagger\sigma\chi|0 >< 0|\chi^\dagger\sigma(-\frac{i}{2}\overrightarrow{D})^2\psi|^3S_1 >$$

$$+\frac{H^1_{ee}(^1S_0)}{m_Q^6} <^3 S_1|\psi^\dagger\sigma(-\frac{i}{2}\overrightarrow{D})^2\chi|0 >< 0|\chi^\dagger\sigma(-\frac{i}{2}\overrightarrow{D})^2\psi|^3S_1 >$$

$$+\frac{H^2_{ee}(^1S_0)}{m_Q^6}\ Re\left[<^3 S_1|\psi^\dagger\sigma\chi|0 >< 0|\chi^\dagger\sigma(-\frac{i}{2}\overrightarrow{D})^4\psi|^3S_1 >\right]$$

The short distance coefficients F's and G's of the order of α_s^2 and α_s^3 are given by

$$F_{\gamma\gamma}(^1S_0) = 2\pi Q^4\alpha^2\left[1 + \left(\frac{\pi^2}{4} - 5\right)C_F\frac{\alpha_s}{\pi}\right]$$

$$G_{\gamma\gamma}(^1S_0) = -\frac{8\pi Q^4}{3}\alpha^2$$

$$H^1_{\gamma\gamma}(^1S_0) + H^2_{\gamma\gamma}(^1S_0) = \frac{136\pi}{45}Q^4\alpha^2$$

$$\begin{aligned} F_{ee}(^3S_1) \;=\; & \frac{2\pi Q^2\alpha^2}{3}\{1 - 4C_F\frac{\alpha_s(m)}{\pi} \\ & + \left[-117.46 + 0.82n_f + \frac{140\pi^2}{27}ln(\frac{2m}{\mu_A})\right] \\ & (\frac{\alpha_s}{\pi})^2\} \end{aligned}$$

$$G_{ee}(^3S_1) = -\frac{8\pi Q^2}{9}\alpha^2$$

$$H^1_{ee}(^3S_1) + H^2_{ee}(^3S_1) = \frac{58\pi}{54}Q^2\alpha^2$$

Results

Decay rates of $0^{-+} \rightarrow \gamma\ \gamma$ and the relevant correction terms of η_c meson

state	$\nu =$	0.5	0.7	0.9	1.0	1.5	2.0	Expt.[13]	[14]
1S	$\Gamma_0(0)$	8.173	10.918	13.465	14.649	19.971	24.297		
	$\Gamma_{0R}(0)$	5.538	7.397	9.123	9.925	13.531	16.462	7.2±0.7	8.500
	Γ_{NRQCD}	6.078	7.810	9.391	10.077	13.142	15.596		
2S	$\Gamma_0(0)$	1.642	2.534	3.488	3.962	6.285	8.346		
	$\Gamma_{0R}(0)$	1.112	1.717	2.363	2.685	4.258	5.655		2.400
	Γ_{NRQCD}	2.721	4.633	6.979	8.223	15.192	22.238		
3S	$\Gamma_0(0)$	0.735	1.014	1.727	1.993	3.323	4.540		
	$\Gamma_{0R}(0)$	0.498	0.687	1.170	1.350	2.251	3.076		0.880
	Γ_{NRQCD}	1.144	1.817	3.542	4.339	9.527	16.019		
4S	$\Gamma_0(0)$	0.446	0.727	0.843	1.236	2.131	2.909		
	$\Gamma_{0R}(0)$	0.302	0.493	0.838	1.248	1.440	1.971		
	Γ_{NRQCD}	0.695	1.339	1.905	2.899	7.170	13.158		
5S	$\Gamma_0(0)$	0.286	0.493	0.728	0.850	1.456	1.999		
	$\Gamma_{0R}(0)$	0.194	0.334	0.493	0.576	0.987	1.354		
	Γ_{NRQCD}	0.454	0.946	1.687	2.153	5.650	11.157		
6S	$\Gamma_0(0)$	0.205	0.359	0.534	0.624	1.065	1.449		
	$\Gamma_{0R}(0)$	0.139	0.243	0.362	0.423	0.721	0.982		
	Γ_{NRQCD}	0.335	0.722	1.323	1.709	4.731	9.806		

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Quarkonia decay into Light Hadrons

The annihilation rate of the heavy quarkonium state (η_c, η_b, J/ψ and Υ) into light hadrons (G. T. Bodwin et al 1995)

$$\Gamma(^1S_0 \to LH) = \frac{N_c}{\pi}\frac{Im\ f_1(^1S_0)}{m_Q^2}\left|\overline{R_{\eta_c}}\right|^2 - \frac{N_c}{\pi}\frac{Im\ g_1(^1S_0)}{m^4} Re\left(\overline{R_s}\overline{\nabla^2 R_s}\right) + O(v^4\Gamma)$$

$$\Gamma(^3S_1 \to LH) = \frac{N_c}{\pi}\frac{Im\ f_1(^3S_1)}{m_Q^2}\left|\overline{R_{\psi}}\right|^2 - \frac{N_c}{\pi}\frac{Im\ g_1(^3S_1)}{m^4} Re\left(\overline{R_s}\overline{\nabla^2 R_s}\right) + O(v^4\Gamma)$$

The short distance coefficients

$$Im\ f_1(^1S_0) = \frac{\pi C_F}{2N_c}\alpha_s^2$$

$$Im\ g_1(^1S_0) = -\frac{2\pi C_F}{3N_c}\alpha_s^2$$

$$Im\ f_1(^3S_1) = \frac{(\pi^2-9)(Nc^2-4)C_F}{54Nc)}\alpha_s^2\left[1 + (-9.46\ C_F + 4.13\ C_A - 1.161\ n_f)\alpha_s/\pi\right] + \pi Q^2\left(\sum_i Q_i^2\right)\alpha^2\left(1 - \frac{13}{4}C_F\frac{\alpha_s}{\pi}\right)$$

The imaginary part of the $Im\ g_1(^3S_1)$ vanish at order α_s

The quarkonia decay into light hadrons (relative order v^4)

$$\Gamma({}^1S_0 \to LH) = \frac{F_1({}^1S_0)}{m_Q^2}\left\langle {}^1S_0|O_1({}^1S_0)|{}^1S_0\right\rangle + \frac{G_1({}^1S_0)}{m_Q^4}\left\langle {}^1S_0|P_1({}^1S_0)|{}^1S_0\right\rangle$$
$$+\frac{F_8({}^3S_1)}{m_Q^2}\left\langle {}^1S_0|O_8({}^3S_1)|{}^1S_0\right\rangle + \frac{F_8({}^1S_0)}{m_Q^2}\left\langle {}^1S_0|O_8({}^1S_0)|{}^1S_0\right\rangle$$
$$+\frac{F_8({}^1P_1)}{m_Q^4}\left\langle {}^1S_0|O_8({}^1P_1)|{}^1S_0\right\rangle + \frac{H_1^1({}^1S_0)}{m_Q^6}\left\langle {}^1S_0|Q_1^1({}^1S_0)|{}^1S_0\right\rangle$$
$$+\frac{H_1^2({}^1S_0)}{m_Q^6}\left\langle {}^1S_0|Q_1^2({}^1S_0)|{}^1S_0\right\rangle$$

$$\Gamma({}^3S_1 \to LH) = \frac{F_1({}^3S_1)}{m_Q^2}\left\langle {}^3S_1|O_1({}^3S_1)|{}^3S_1\right\rangle + \frac{G_1({}^3S_1)}{m_Q^4}\left\langle {}^3S_1|P_1({}^3S_1)|{}^3S_1\right\rangle$$
$$+\frac{F_8({}^1S_0)}{m_Q^2}\left\langle {}^1S_0|O_8({}^1S_0)|{}^1S_0\right\rangle + \frac{F_8({}^3S_1)}{m_Q^2}\left\langle {}^1S_0|O_8({}^3S_1)|{}^1S_0\right\rangle$$
$$+\sum_{j=0,1,2}\frac{F_8({}^3P_J)}{m_Q^4}\left\langle {}^3S_1|O_8({}^3P_J)|{}^3S_1\right\rangle + \frac{H_1^1({}^3S_1)}{m_Q^6}\left\langle {}^3S_1|Q_1^1({}^3S_1)|{}^3S_1\right\rangle$$
$$+\frac{H_1^2({}^3S_1)}{m_Q^6}\left\langle {}^3S_1|Q_1^2({}^3S_1)|{}^3S_1\right\rangle$$

G.T. Bodwin et al PRD, 66 (2002)

The short distance coefficients

$$F_1(^1S_0) \;=\; \frac{\pi\ C_F}{Nc}\alpha_s^2\left[1+(\frac{\pi^2}{4}-5)C_F+(\frac{199}{18}-\frac{13\pi^2}{24})\ C_A-\frac{8}{9}\ n_f)\frac{\alpha_s}{\pi}\right]$$

Where $Nc = 3$ is the no. of colour, $C_F = 4/3$

$$G_1(^1S_0) = -\frac{4\pi C_F}{3N_c}\alpha_s^2$$

$$F_8(^3S_1) \;=\; \frac{\pi n_f}{3}\alpha_s^2\left[1+\frac{\alpha_s}{\pi}\left(-\frac{13}{4}\right)C_F+\left(\frac{133}{18}-\frac{2}{3}log2-\frac{\pi^2}{4}\right)\ C_A-\frac{10}{9}\ n_f T_F+2b_o Log\frac{\mu}{2m_Q}\right]$$
$$+5\alpha_s^3\left(-\frac{73}{4}+\frac{67}{36}\pi^2)\right)$$

$$F_8(^1S_0) \;=\; 2\pi B_F\alpha_s^2\left[1+\frac{\alpha_s}{\pi}\left(-5+\frac{\pi^2}{4}\right)C_F+\left(\frac{122}{9}-\frac{17}{24}\pi^2\right)\ C_A-\frac{16}{9}\ n_f T_F+2b_o Log\frac{\mu}{2m_Q}\right]$$

$$F_8(^1P_1) = \frac{\pi Nc}{6}\alpha_s^2$$

$$H_1^1(^1S_0) + H_1^2(^1S_0) = \frac{68\pi C_F}{45N_c}\alpha_s^2$$

$$H_1^1(^3S_1) + H_1^2(^3S_1) = \frac{(N_c^2-1)(N_c^2-4)}{N_c^2}\left(-\frac{833}{972} + \frac{(1609\pi^2)}{12960} + \frac{7}{81}log\frac{2m}{\mu_\Lambda}\right)\alpha_s^3$$

$\eta_c \rightarrow Light\ Hadrons$(in MeV)

ν	Expansion up to v^2		Expansion up to v^4		
	Γ_{NRQCD}	Γ_{wcr}	Γ_{NRQCD}	Γ_{wcr}	Γ_{Others}
0.5	11.34	5.82	77.01	41.93	
0.7	15.61	6.71	99.60	45.65	14.38
0.9	19.91	9.15	120.48	58.20	±
1.0	21.96	9.89	129.78	61.33	1.07
1.1	23.95	10.06	138.94	64.46	±
1.3	27.79	11.91	155.90	69.77	1.43
1.5	31.40	13.09	171.33	74.43	N. Faboano
1.7	34.76	14.07	185.22	78.48	(2002)
1.9	37.97	15.16	198.16	82.09	
2.0	39.47	15.61	204.31	83.77	

Conclusions

- *we have made a comprehensive study of the heavy-heavy flavour mesonic systems in the general frame work of potential models.*

- *The potential model parameters and the masses of the charmed and beauty quark obtained from the respective quarkonia mass predictions have been employed to study their decay properties in the frame work of NRQCD formalism as well as using the conventional Van-Royen-Weisskopf non-relativistic formula.*

- *It is interesting to note that the predictions of the di-gamma decay widths of η_c and leptonic decay widths of J/ψ and Upsilon are in good agreement with the respective experimental results.*

- *The present study of the decay rates of quarkonia clearly indicates the relative importance of QCD related corrections on the phenomenological potential models.*

- *We are also studying the heavy-light flavour mesons masses, decay constants, branching ratios and electromagnetic transitions in the same scheme.*

THANKS