Hagedorn spectrum in large N_c QCD

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based on T.D. Cohen, V. Krejčiřík, arXiv:1104.4783 [hep-th]

introduction

- hadrons – particles feeling strong interaction
  - in the 60s and 70s, a LOT of hadrons discovered
  - various types, properties
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- the properties of all hadrons (masses, widths) from first principles remain unresolved

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- Hagedorn spectrum – number of hadrons grows exponentially with mass

$$\rho(m) \sim e^{\frac{m}{T_H}}$$

  - phenomenological evidence
  - "proof" of its existence in large N_c QCD

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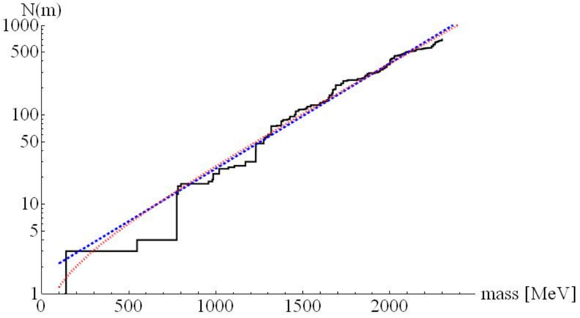

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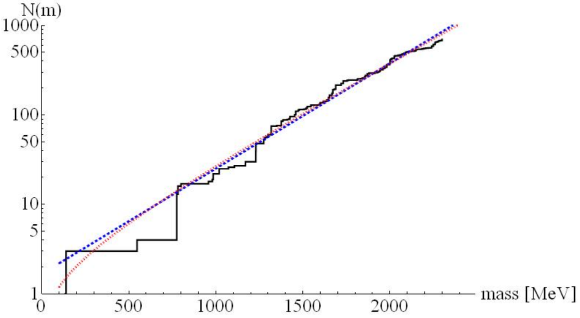

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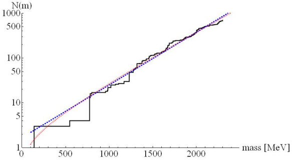

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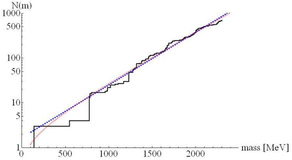

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- looks like exponential over 2 orders of magnitude
- naive fit leads to Hagedorn temperature
  - $T_H = 369\ \mathrm{MeV}$ (for e^{m/T_H})
  - $T_H = 426\ \mathrm{MeV}$ (for $m^a e^{m/T_H}$)
- better agreement for high lying states
  - expected from string-like behavior

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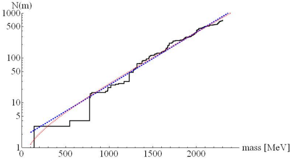

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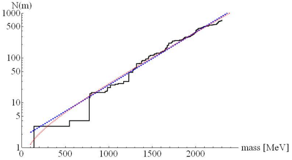

- arguments against

- Hagedorn temperature much bigger than critical temperature from lattice
- exponential look maybe a coincidence
  - opening of new spin/parity channels (Regge trajectories $J^2 \sim m$)
- not MASSES but properties of RESONANCES

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  - theoretical inputs
  - outline of procedure
  - individual details

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  - theoretical inputs
  - outline of procedure
  - individual details
- not rigorous proof, merely an argument provided certain assumptions are met

theoretical inputs

- large N_c world
  - mesons are stable $\Gamma \sim 1/N_c$
    masses well defined
  - mesons are noninteracting $\sigma \sim 1/N_c$
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- asymptotic freedom
- standard assumptions regarding the validity of perturbation theory

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- Hagedorn spectrum

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- building blocks

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- sets

$$\mathcal{S}_1 = \{J_1, J_2\} = \{\overline{q} O_1 q, \overline{q} O_2 q\}$$
$$\mathcal{S}_2 = \{J_{11}, J_{12}, J_{21}, J_{22}\} = \{\overline{q} O_1 O_1 q, \overline{q} O_1 O_2 q, \cdots\}$$
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$$\cdots$$

- number of elements $= 2^n$
- mass dimension $= 4n + 3$
- spin 0 scalars or pseudoscalars

current current correlator

- sequence of correlator (in Euclidean time) matrices

$$\Pi_{ab}^{(n)}(\tau) = \left\langle J_a^\dagger(\tau) J_b(0) \right\rangle \quad , \quad J_{a,b} \in \mathcal{S}_n$$

- spectral decomposition

$$\Pi_{ab}^{(n)}(\tau) = \sum_k \int \frac{\mathrm{d}^3 \vec{p}}{(2\pi)^3} \; C_{ab,k} \;\; \Delta(\tau; m_k)$$

  - consequence of large N_c + confinement
  - Js creates only single particle states

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- useful tool for proof of inequality

$$N(an+b) \geq \; -\frac{\mathrm{d}}{\mathrm{d}\tau} \mathrm{Tr} \log \Pi^{(n)} \;\; \geq W(m_N)$$

running recap

- done
  - sets of local operators growing exponentially $N = 2^n$
  - tool for the proof of the key inequality $-\frac{\mathrm{d}}{\mathrm{d}\tau}\mathrm{Tr}\log\Pi^{(n)}$

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- to do
  - two inequalities to show

right	$-\frac{\mathrm{d}}{\mathrm{d}\tau}\mathrm{Tr}\log\Pi^{(n)} \geq W(m_N)$
left	$N(an+b) \geq -\frac{\mathrm{d}}{\mathrm{d}\tau}\mathrm{Tr}\log\Pi^{(n)}$

right inequality

- well known properties of correlation functions

$$\lim_{\tau\to\infty} -\frac{\mathrm{d}}{\mathrm{d}\tau}\log\langle J(\tau)J(0)\rangle = m_0$$
$$-\frac{\mathrm{d}}{\mathrm{d}\tau}\log\langle J(\tau)J(0)\rangle > m_0$$

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- straightforward[1] generalization to matrix of correlators

$$-\frac{\mathrm{d}}{\mathrm{d}\tau}\mathrm{Tr}\log\Pi^{(n)} \geq \sum_{k}^{2^n} m_k = W(m_{2^n})$$

  - currents create single particle states
  - properties of scalar propagator
  - logarithmic and derivative structure

[1] after a few months of work

left inequality

- for $\tau \to 0$, asymptotic freedom allow us to treat fields as free
- large N_c guarantees the matrix is diagonal

$$\Pi_{ab}^{(n)} = \delta_{ab} \, \mathrm{const} \; \tau^{-8n+6}$$

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$$-\frac{\mathrm{d}}{\mathrm{d}\tau} \mathrm{Tr} \log \Pi^{(n)} = (2^n) \frac{8n+6}{\tau}$$

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- asymptotic value is reached for $\tau_0 < \tau$, with τ_0 independent of n
- Hagedorn spectrum plus estimate of $T_H \leq \frac{8\log_2(e)}{\tau_0}$

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  - sets of local operators growing exponentially
  - right inequality $-\frac{\mathrm{d}}{\mathrm{d}\tau}\mathrm{Tr}\log\Pi^{(n)} \quad \geq \quad W(m_N)$
  - left inequality $N(an+b) \quad \geq \quad -\frac{\mathrm{d}}{\mathrm{d}\tau}\mathrm{Tr}\log\Pi^{(n)}$
    - provided that correlators are at their asymptotically free values
  - we obtained Hagedorn spectrum

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- to do
  - corrections to asymptotic values – perturbation theory

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  - standard assumption that perturbative expansion is valid as long as the correction are small

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perturbative corrections

- corrections included via perturbation theory
  - standard assumption that perturbative expansion is valid as long as the correction are small
- key thing to show – scaling with n
  - at fixed τ and fixed order, corrections scales at most lineraly with n
- involved, technical and boring procedure[2]
  - planarity of diagrams at large N_C
  - logarithmic structure
  - $n \gg$ order of perturbation theory

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- no string dynamics assumed
- confinement assumed only in a sense that all physical states are color singlets
    - no center symmetry breaking or Wilson loop area law
- argument can be extended to 2+1 dimensions and used for other large N gauge theories too

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