PERTURBATIVE SATURATION AT WORK - TWO LECTURES.

Alex Kovner

University of Connecticut

with Misha Lublinsky (arXiv:1012.3398) and Tolga Altinoluk (arXiv:1102.5327)

PERTURBATIVE SATURATION (AKA CGC)

ITS A LONG STORY, BUT IN A NUTSHELL

GLUON DENSITY GROWS RAPIDLY AS ONE GOES TO LOW VALUES OF x IN HADRONIC WAVE FUNCTIONS.

THIS GENERATES "MOMENTUM DIVIDE" AT MOMENTUM SCALE EQUAL TO THE AVERAGE PARTON DENSITY IN THE TRANSVERSE PLANE, "THE SATURATION MOMENTUM" $Q_S \sim \rho$

AT SHORT DISTANCES $x < Q_S^{-1}$ USUAL PARTONIC PHYSICS REMAINS VALID, AS AT THIS SCALES BY DEFINITION EFFECTS OF DENSITY ARE NOT IMPORTANT.

HOWEVER WHEN PROBED ON TRANSVERSE DISTANCE SCALE $x > Q_S^{-1}$ THE HADRON THEN LOOKS LIKE A DENSE SYSTEM.

Q_S PLAYS A DUAL ROLE IN THIS PICTURE:

A. IT IS THE AVERAGE VALUE OF COLOR ELECTRIC FIELDS IN THE WAVE FUNCTION.

B. IT IS THE INVERSE OF THE LENGTH OVER WHICH THE COLOR ELECTRIC FIELDS ARE CORRELATED.

TO SEE THIS LET US USE THE STANDARD DIPOLE DIAGNOSTICS.

SUPPOSE WE HAVE A COLOR NEUTRAL DIPOLE THAT SCATTERS ON OUR SATURATED TARGET. FOR A GIVEN CONFIGURATION OF ELECTRIC FIELD IN THE TARGET, THE DIPOLE SCATTERING AMPLITUDE IS

$$N(r) = 1 - \mathrm{Tr}[S^\dagger(0)S(r)]$$

HERE S IS THE EIKONAL SCATTERING MATRIX $S(x) = e^{ig\int dx^+ A^-(x)}$.

THE POTENTIAL $A^-(x)$ (DISREGARDING COLOR FOR THE MOMENT) IS JUST THE USUAL $\partial_i A^- = F^{-i}$. LET'S DEFINE FOR CONVENIENCE INTEGRATED ELECTRIC FIELD $E_i = \int dx^+ F^{-i}$. THEN SINCE THE AVERAGE OF E_i OVER THE TARGET WAVE FUNCTION VANISHES

$$N(r) \sim 1 - e^{-(g\vec{r}\cdot\vec{E})^2}$$

FOR SMALL r WE HAVE PERTURBATIVE $N(r) \sim g^2 r^2 E^2$

REACHES UNITY FOR $r_s^2 = Q_S^{-2} \sim (gE)^{-2}$

WE ALSO KNOW THAT THE GLUON DISTRIBUTION IN THE TARGET IS CUTOFF BELOW MOMENTA $P_T \sim Q_S$. THUS COLOR ELECTRIC FIELDS ARE NOT LONG RANGE, BUT MUST BE DOMINATED BY WAVELENGTHS $\lambda \sim Q_S^{-1}$.

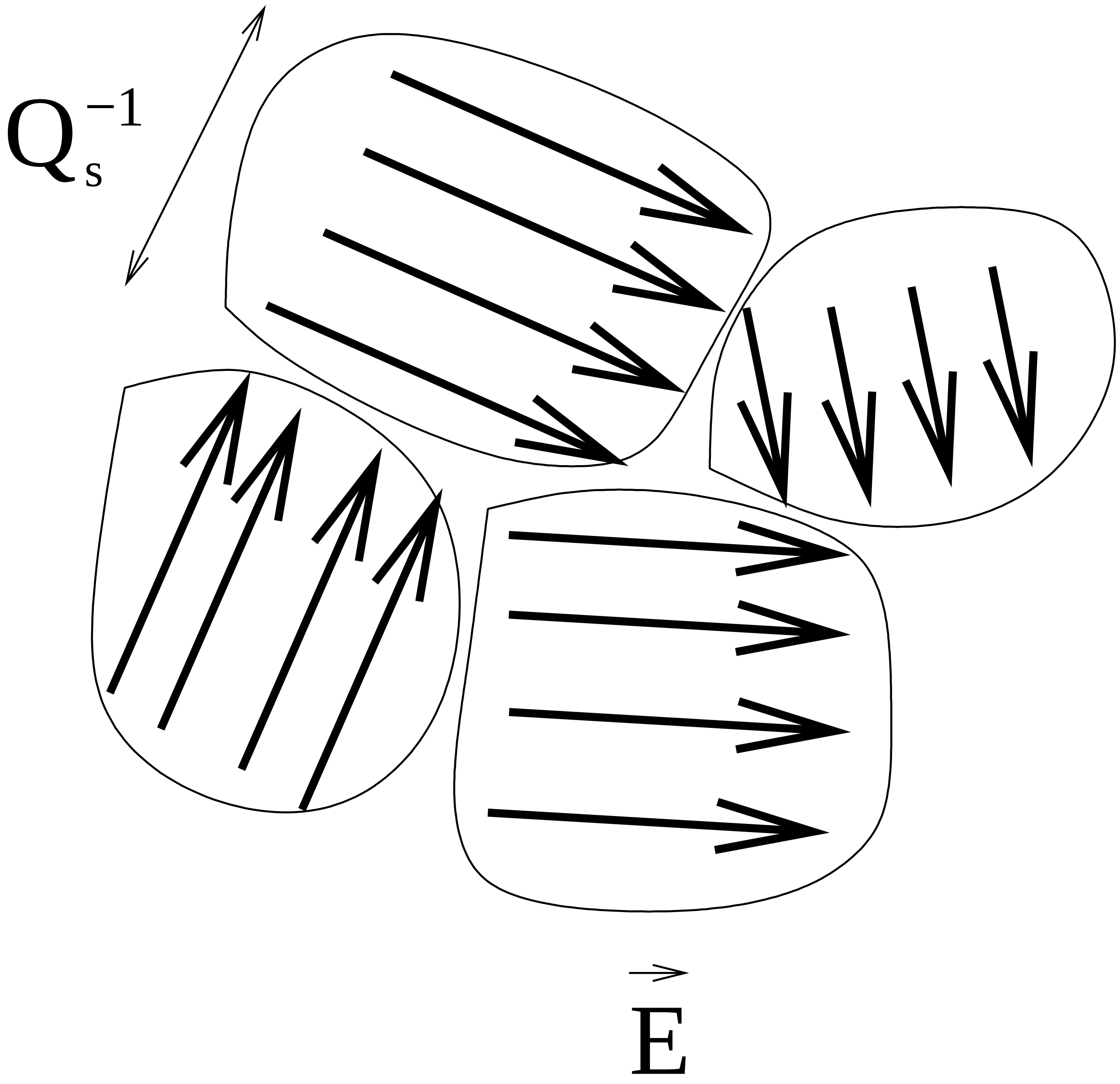


Figure 1: CARTOON OF A TYPICAL FIELD CONFIGURATION IN A SATURATED TARGET.

Q_S ALSO MANIFESTS ITSELF AS THE TYPICAL MOMENTUM FOR PARTICLES EMITTED INTO FINAL STATE IN COLLISIONS INVOLVING SUCH OBJECTS.

E.G. PERTURBATIVELY SINGLE INCLUSIVE SPECTRA ARE INFRARED DOMINATED $dN/dk^2 \propto 1/k^4$. BUT IN A SATURATED SYSTEM PRODUCTION IN THE INFRARED IS SUPPRESSED, SO THE SPECTRUM IS DOMINATED BY $k \sim Q_S$.

MULTIPLICITIES OF PRODUCED PARTICLES ALSO SHOULD SCALE WITH Q_S^2.

THERE IS A HOST OF OTHER EFFECTS THAT APPEAR WHEN PHYSICS IS SATURATION DOMINATED, BUT ALL DRIVEN BY TWO BASIC FEATURES: GENERALLY LESS PARTICLE PRODUCTION AND LESS CORRELATIONS BETWEEN PRODUCED PARTICLES

IF Q_S IS HIGH THERE IS A GOOD CHANCE TO DESCRIBE THIS PHYSICS PERTURBATIVELY, SINCE THE COUPLING CONSTANT AT THE RELEVANT SCALE IS SMALL...

THE BEST CHANCE FOR LARGE Q_S IS IN NUCLEI AT SMALL IMPACT PARAMETER, SO CENTRAL A-A COLLISIONS IS BEST.

BUT A-A IS VERY COMPLICATED - LOTS OF INTERACTION BETWEEN PRODUCED PARTICLES AFTER THE PRIMARY COLLISION, COLLECTIVE FLOW PHENOMENA, ETC.

ANOTHER FAVORED REGION OF PHASE SPACE IS HIGH ENERGY.

Q_S GROWS WITH ENERGY (RAPIDITY)

$$Q_S(\eta) \sim e^{\lambda\eta} \; (\; or \; e^{a\sqrt{\eta}})$$

LHC IS A NATURAL PLACE TO EXPLORE EFFECTS OF SATURATION MOMENTUM

ANOTHER PROMISING REGION - FORWARD REGION IN d-Au AT RHIC, WHERE ONE PROBES LOW x PART OF THE TARGET NUCLEUS WAVE FUNCTION, BUT NO COMPLICATED FINAL STATE EFFECTS.

RECENT COUPLE OF YEARS THERE HAS BEEN A LOT OF ACTIVITY IN THE CGC COMMUNITY INTERPRETING DATA IN THESE REGIMES THAT FAVOR MANIFESTATION OF SATURATION.

HEYDAY OF "CGC PHENOMENOLOGY"

FOUR (RECENT) PILLARS OF CGC PHENOMENOLOGY

FORWARD PARTICLE PRODUCTION IN d-Au -STRONG SUPPRESSION AT RHIC AT $\eta > 3$.

DIHADRON CORELATIONS IN d-Au - STRONG SUPPRESSION OF THE AWAY SIDE PEAK IN FORWARD PRODUCTION

"RIDGE"@LHC - LONG RANGE RAPIDITY AND ANGULAR CORRELATIONS IN p-p HIGH MULTIPLICITY EVENTS

"RIDGE"@RHIC- SAME IN Au-Au AND Cu-Cu

IN THESE LECTURES A LITTLE BIT ABOUT THE FORWARD SUPPRESSION AND THE RIDGE IN $p-p$.

FIRST LECTURE: A QUALITATIVE DISCUSSION OF NATURALNESS OF LONG RANGE RAPIDITY CORRELATIONS COUPLED WITH CORRELATIONS IN THE ANGLE OF PARTICLE EMISSION. THE ROLE OF SATURATION HERE, AS WE WILL SEE IS ACTUALLY TO MOVE THE EFFECT TO HIGHER TRANSVERSE MOMENTA AND ALSO TO MAKE IT EVENTUALLY WEAKER.

SECOND IS THE DISCUSSION OF THE CGC BASED CALCULATION OF FORWARD PARTICLE SUPPRESSION. HERE I WILL DISCUSS ADDITIONAL CONTRIBUTIONS WHICH WERE NOT INCLUDED IN CALCULATIONS SO FAR AND WILL EXPLAIN THAT THOSE ARE THE ONES MORE DIRECTLY AFFECTED BY SATURATION EFFECTS.

LECTURE I - ON CMS ANGULAR "RIDGE" CORRELATIONS

CMS - TWO PARTICLE CORRELATIONS IN P-P, LONG RANGE IN RAPIDITY AND PEAKED IN FORWARD DIRECTION - "RIDGE" IN P-P COLLISIONS

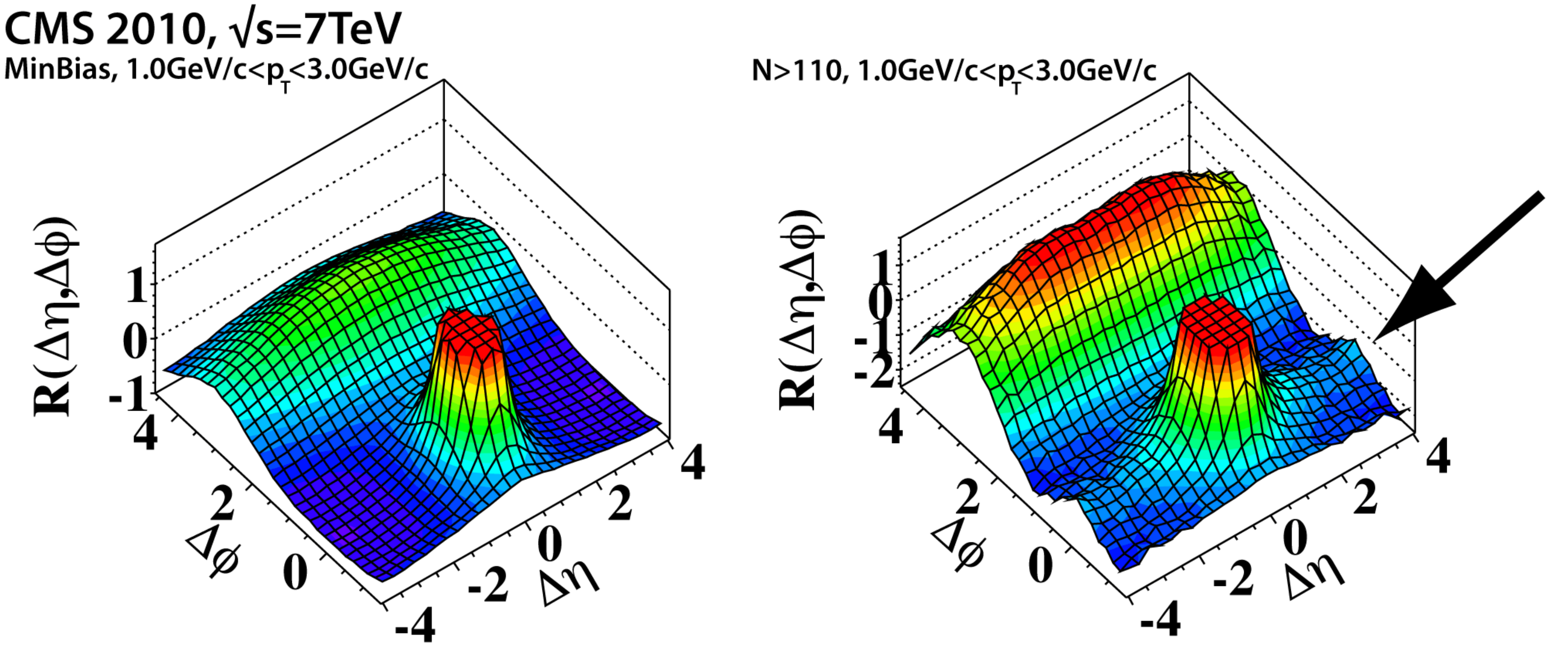


Figure 2: THE CMS RIDGE.

SIMILAR CORRELATIONS HAVE BEEN MEASURED ALSO AT RHIC BY STAR AND PHOBOS.

Correlated yield on near-side (|Δϕ| < 1):

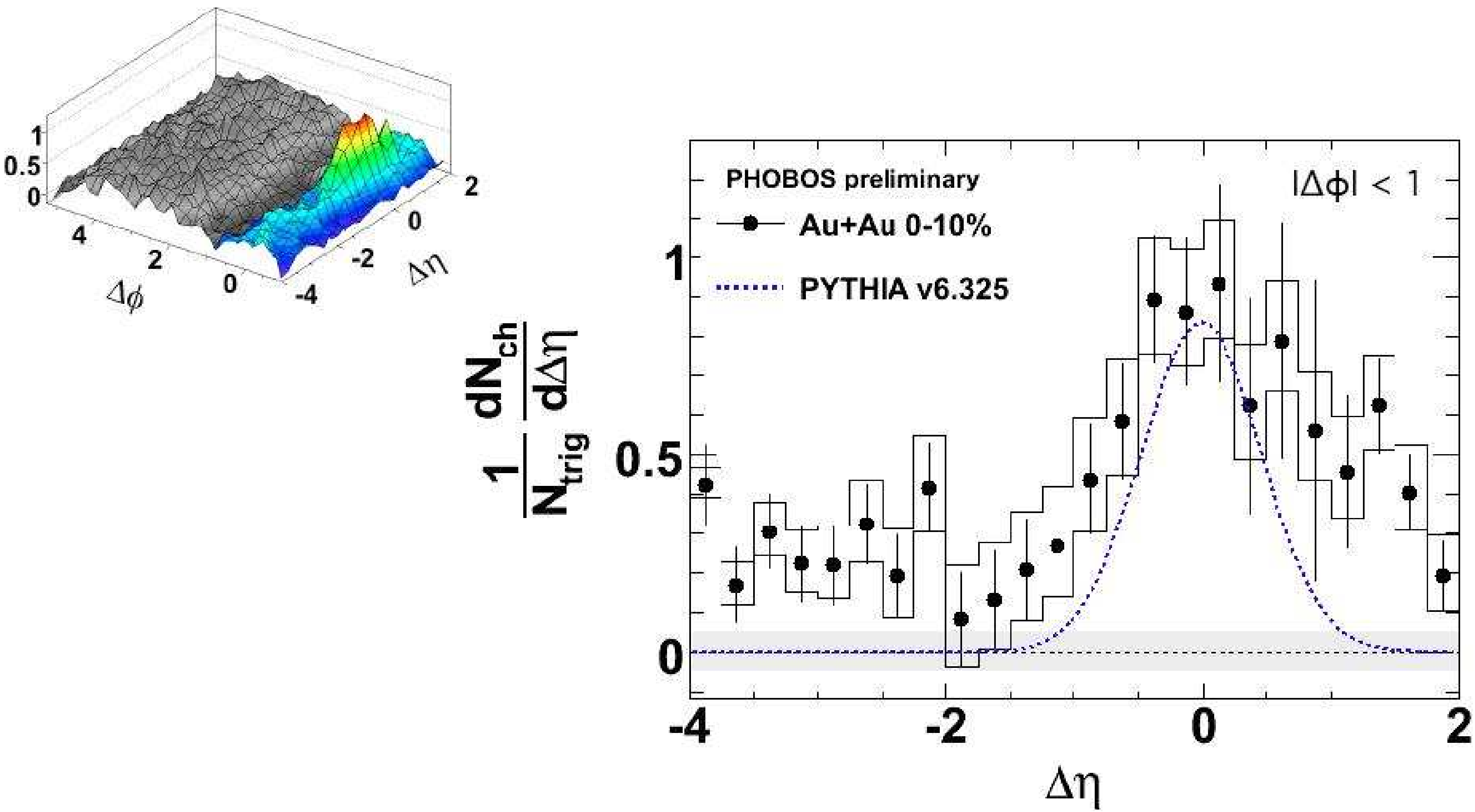


Figure 3: HARD RIDGE AT PHOBOS

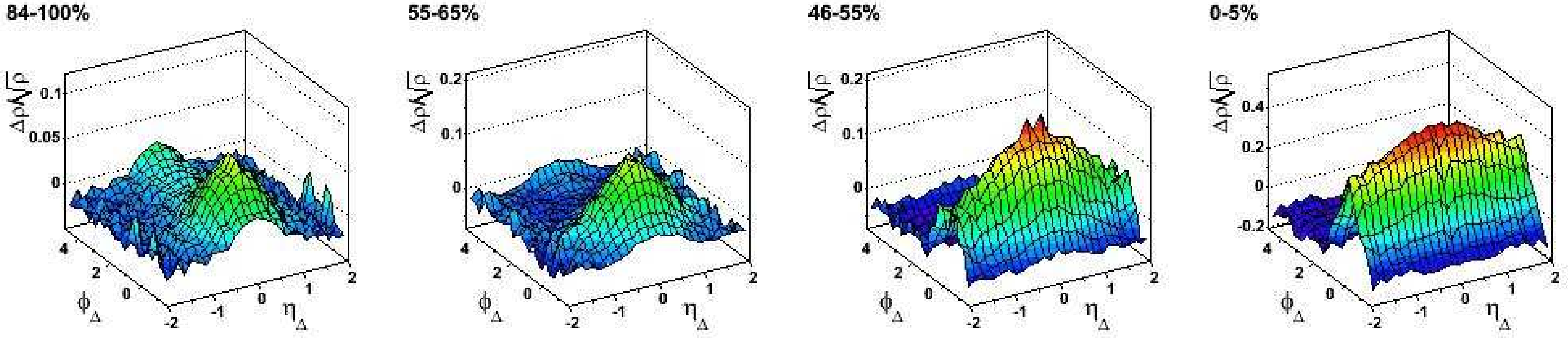


Figure 4: SOFT RIDGE AT STAR

THE RHIC RIDGE MAY HAVE A DIFFERENT NATURE, AS IT IS BELIEVED THAT FINAL STATE INTERACTIONS CAN GENERATE RADIAL FLOW WHICH PRODUCES ANGULAR CORRELATIONS INDEPENDENT FROM THOSE IN THE INITIAL STATE.

THE DISCUSSION HERE IS DECOUPLED FROM FINAL STATE, AND SO IS ONLY PERTINENT TO THE CMS MEASUREMENTS.

THERE IS AN ONGOING CALCULATIONAL EFFORT IN THE CGC COMMUNITY TO DESCRIBE THE CMS CORRELATION QUANTITATIVELY. HERE I ONLY GIVE A QUALITATIVE DISCUSSION, BUT ALSO POINT OUT TO SOME IMPORTANT PHYSICS WHICH NEEDS TO BE MUCH BETTER HANDLED QUANTITATIVELY IN THESE CALCULATIONS.

DUMITRU, DUSLING, GELIS, JALILIAN-MARIAN, LAPPI, VENUGOPALAN - arXiv:1009.5295 AND ONGOING - THE SAME MECHANISM ALBEIT IN A LITTLE DIFFERENT GUISE OF "GLASMA FLUX TUBES". BUT REALLY THE SAME!

LEVIN, REZAEIAN - arXiv:1105.3275 - AGAIN THE SAME EXACT MECHANISM, BUT DRESSED IN ROBES OF POMERON CALCULUS.

CHOICE OF FRAME

CENTER OF MASS COMPLICATED FOR CENTRAL RAPIDITY

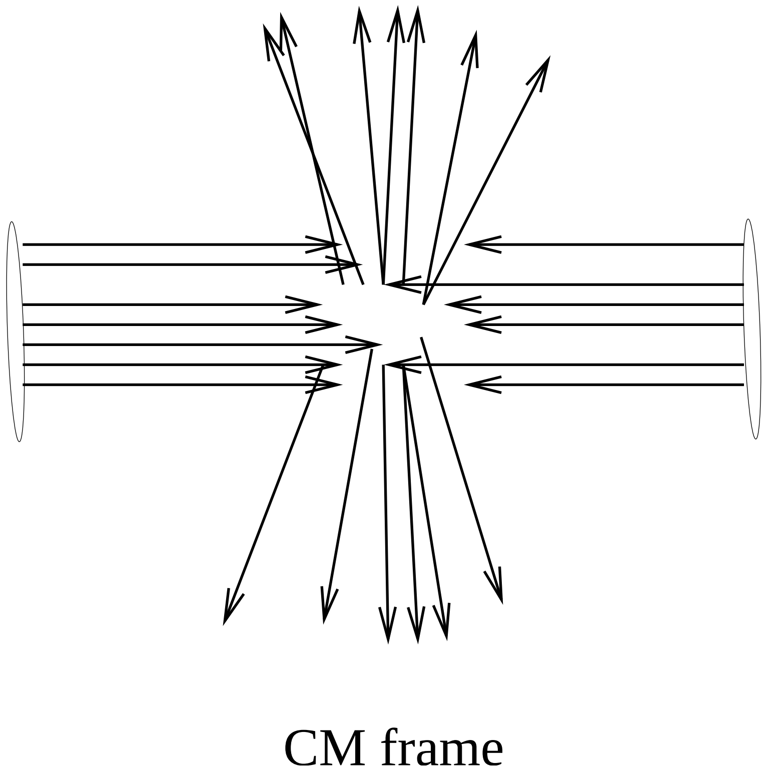


Figure 5: COLLISION IN CM - NOT EIKONAL AT Y=0.

RECOIL CEARLY NONNEGLIGIBLE - EIKONAL APPROXIMATION NOT APPLICABLE - FOR SCATTERED PARTONS $P_T \sim P^+$

AFTER COLLISION YOU HAVE THE "GLASMA" MESS - LONGITUDINAL FIELDS, "FLUX TUBES"...

BUT IT IS MUCH SIMPLER IN THE LAB FRAME

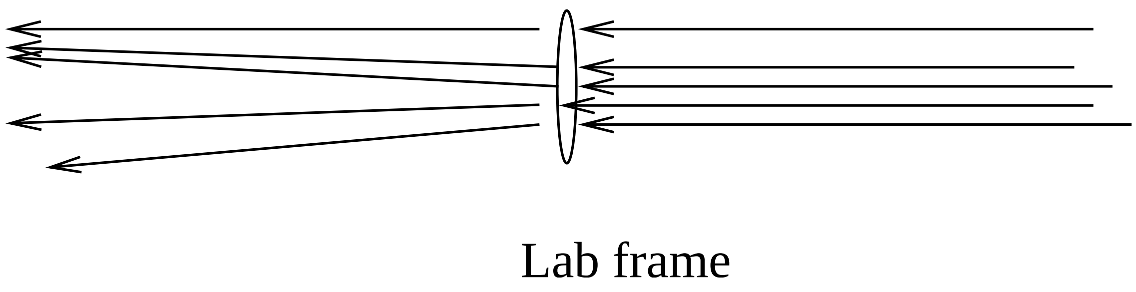


Figure 6: COLLISION IN LAB FRAME - EIKONAL OK.

PARTONS SCATTER EVER SO SLIGHTLY - $P^+ \gg P_T$ - EIKONAL APPROXIMATION SHOULD BE OK

LONGITUDINAL FIELDS STILL EXIST, BUT THE PICTURE THANKFULLY IS RATHER MORE MUNDANE

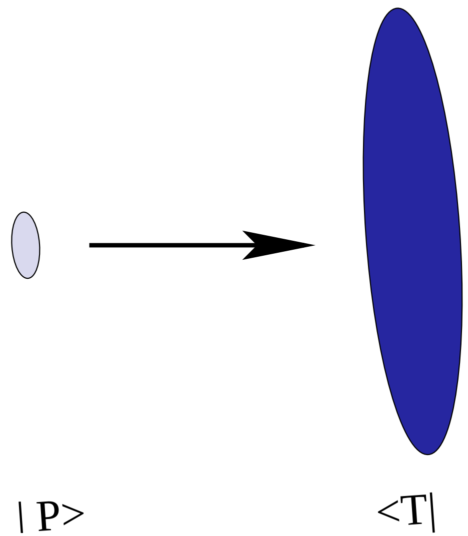


Figure 7: PARTONIC EIKONAL SCATTERING

PARTONS OF THE PROJECTILE SCATTER OFF THE FIELDS OF THE TARGET.

PROJECTILE CARRIES COLOR CHARGE DENSITY $\rho^a(x)$.

THIS CHARGE DENSITY DRAGS WITH IT COULOMB (OR WEIZSACKER-WILLIAMS) FIELD SQUEEZED BY LORENTZ BOOST

$$F^a_{-i} = b^a_i(x)\delta(x^-)$$

DETERMINED BY THE MAXWELL (YANG-MILLS) EQUATIONS:

$$\partial_i b^a_i(x) = \rho^a(x); \qquad \partial_i b^a_j - \partial_j b^a_i - g f^{abd} b^b_i(x) b^c_j(x) = 0$$

AFTER PASSING THROUGH THE TARGET FIELDS, THE SOURCES SCATTER

$$\rho^a(x) \to S^{ab}(x)\rho^b(x)$$

AND ALSO THE SOFT GLUONS SCATTER:

$$b_i^a(x) \to S^{ab}(x)b_i^b(x)$$

RESULT: SOFT GLUONS DECOHERE FROM THE SOURCES AND FLY AWAY AS FINAL STATE GLUONS. THEY FLY THEIR OWN WAY NOT ENTIRELY PARALLEL TO THE BEAM, AND THEIR OWN WEIZSACKER-WILLIAMS FIELDS NOW HAVE A COMPONENT PARALLEL TO THE INITIAL BEAM DIRECTION - "THE LONGITUDINAL FIELDS OF GLASMA".

THUS THE LONGITUDINAL FIELDS ARE SIMPLY THE ATTRIBUTES OF THE SCATTERED GLUONS.

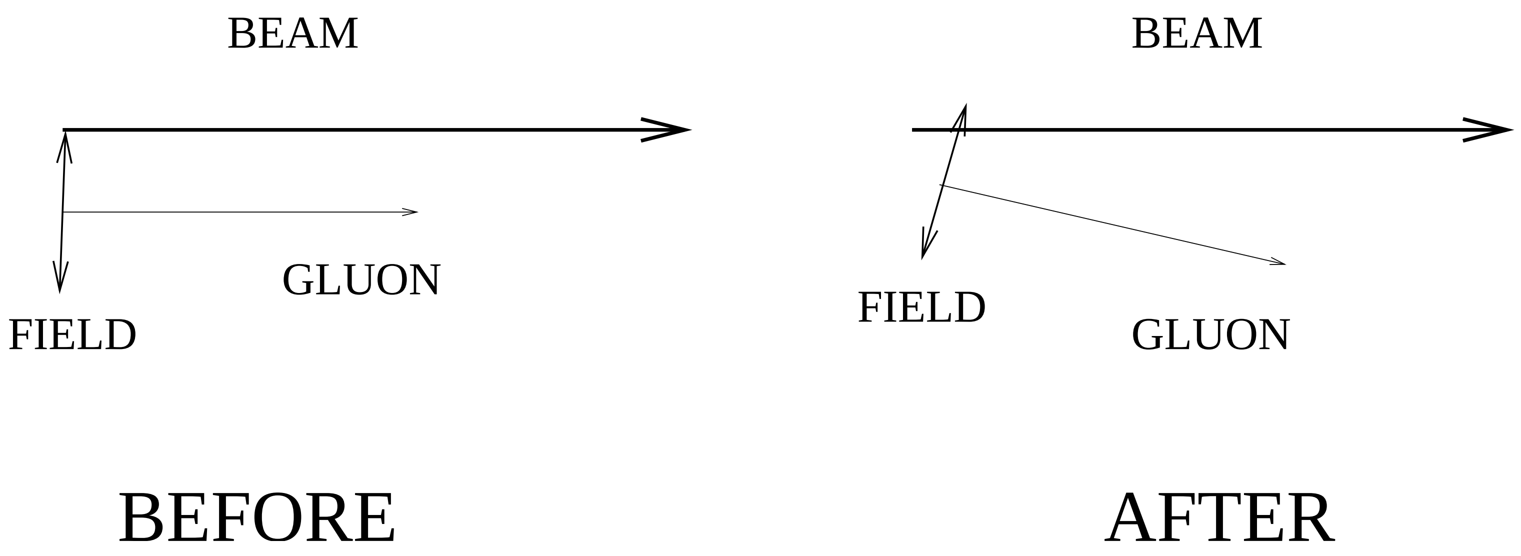


Figure 8: GLUONS SCATTER OUT - LONGITUDINAL FIELDS "TRIVIAL".

SO LETS JUST KEEP IN MIND THAT IN THE LAB FRAME EVERYTHING IS DESCRIBED BY A BUNCH OF INCOMING GLUONS THAT SCATTER ON THE TARGET FIELDS

NAIVE PICTURE OF EIKONAL GLUON PRODUCTION

LONG RANGE RAPIDITY CORRELATIONS COME FOR FREE WITH BOOST INVARIANCE

INCOMING $|P>$ IS BOOST INVARIANT: EXACTLY THE SAME GLUON DISTRIBUTIONS AT η_1 AND η_2. AND THEY SCATTER ON EXACTLY THE SAME TARGET

WHAT HAPPENS AT η_1, HAPPENS ALSO AT η_2

TRUE **CONFIGURATION BY CONFIGURATION** IF THERE IS A "'CLASSICAL"' AVERAGE FIELD IN THE PROJECTILE - FLUCTUATIONS ARE SMALL. BUT EVEN OTHERWISE ONE CERTAINLY EXPECTS SOME LONG RANGE CORRELATIONS IN RAPIDITY.

IF IT IS PROBABLE TO PRODUCE A GLUON AT η_1, IT IS ALSO PROBABLE TO PRODUCE GLUON AT η_2

BUT EXACTLY BY THE SAME LOGIC THERE MUST BE ANGULAR CORRELATIONS: IF THE FIRST GLUON IS MOST LIKELY TO BE SCATTERED TO THE RIGHT, THE SECOND GLUON **AT THE SAME IMPACT PARAMETER** WILL BE ALSO SCATTERED TO THE RIGHT

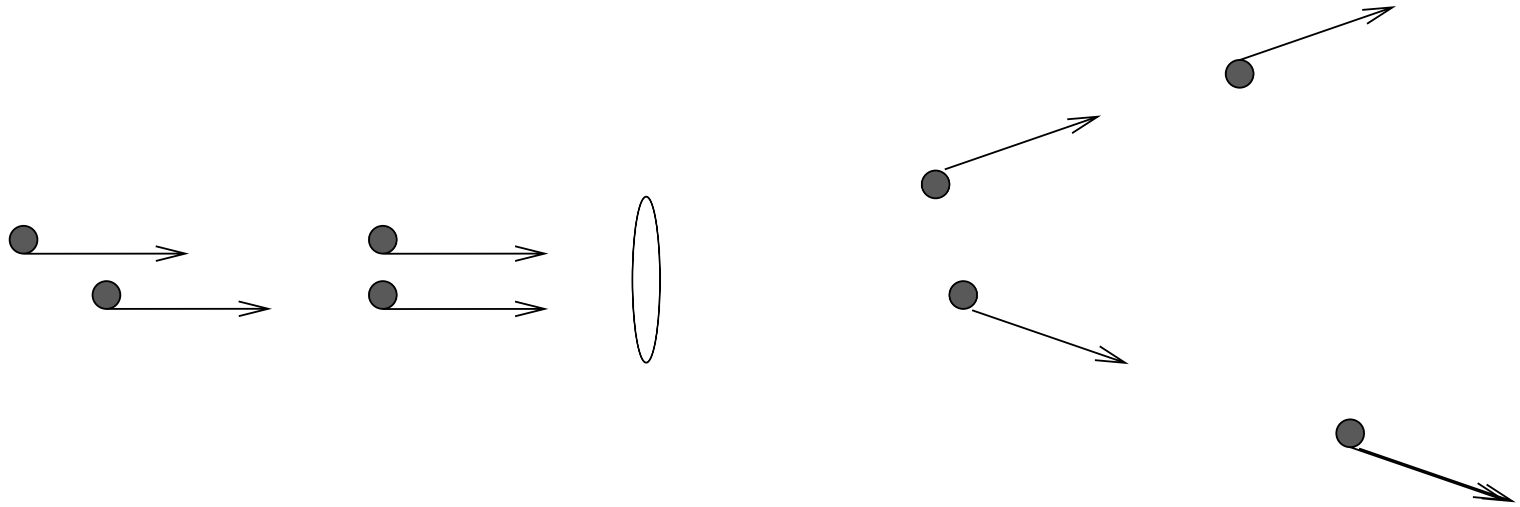


Figure 9: SAME IMPACT PARAMETER - SAME KICK

IN TERMS OF OUR DOMAIN TYPE CARTOON OF THE TARGET, THE PARTON WITH CHARGE q THAT HITS AT AN IMPACT PARAMETER x PICKS UP A MOMENTUM

$$\Delta \vec{P}_T = gq \int dx^+ \vec{F}^- = gq\vec{E}$$

AND THE NEXT PARTON (AT ANOTHER RAPIDITY) PICKS UP EXACTLY THE SAME MOMENTUM, IF IT HAS THE SAME CHARGE q. BUT SINCE THE INCOMING WAVE FUNCTION IS BOOST INVARIANT, THE TWO PARTONS VERY LIKELY WILL HAVE THE SAME CHARGE q.

CAN WE EASILY SEE IT IN THE ACTUAL GLUON PRODUCTION FORMULAE?

TWO GLUON INCLUSIVE PRODUCTION

WE NEGLECT THE EVOLUTION BETWEEN THE TWO PRODUCED GLUONS

AND ALSO ASSUME DILUTE PROJECTILE

(almost Bayer, A.K, Nardi, Wiedemann 2005)

$$\frac{dN}{d^2p\,d^2k\,d\eta\,d\xi} = < A^{ab}(k,p)A^{*ab}(k,p) >_{P,T}$$

WITH

$$A^{ab}(k,p) = \int_{u,z} e^{ikz+ipu}$$

$$\int_{x_1,x_2} \Big\{ g f_i(z-x_1)\,[S(x_1)-S(z)]^{ac}\,\rho^c(x_1) \Big\} \Big\{ g f_j(u-x_2)\,[S(u)-S(x_2)]^{bd}\,\rho^d(x_2) \Big\}$$
$$-\frac{g}{2}\int_{x_1} f_i(z-x_1) f_j(u-x_1) \Big\{ [S(x_1)-S(z)]\,\bar\rho(x_1)\,\Big[S^\dagger(u)+S^\dagger(x_1)\Big] \Big\}^{ab}$$
$$+g\int_{x_1} f_i(z-u) f_j(u-x_1)\,\Big\{ (S(z)-S(u))\,\bar\rho(x_1) S^\dagger(u) \Big\}^{ab}.$$

HERE

$$\bar\rho \equiv T^a\rho^a, \qquad f_i(x-y) = \frac{(x-y)_i}{(x-y)^2}$$

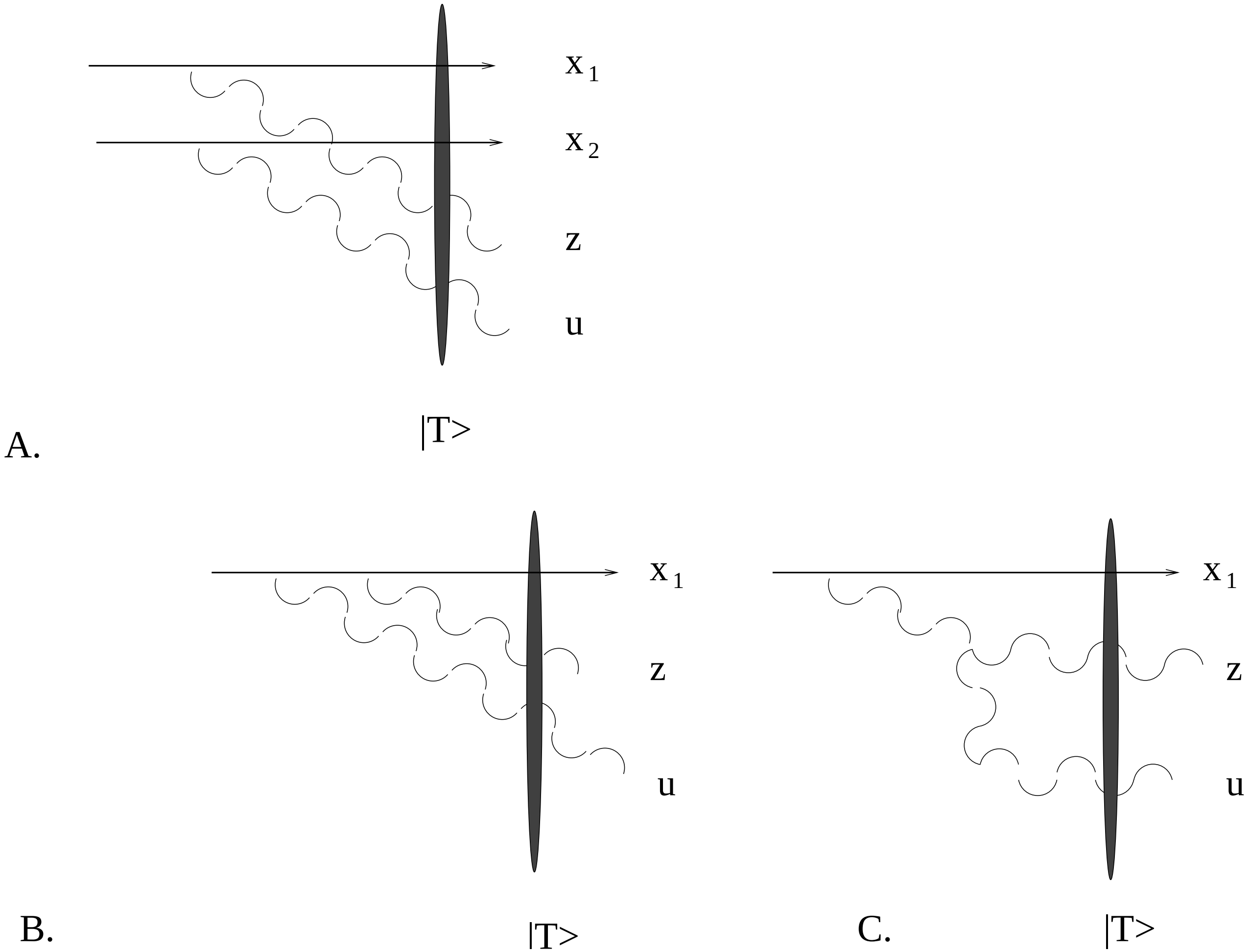


Figure 10: THE THREE CONTRIBUTION TO PRODUCTION AMPLITUDE.

A. IS LEADING IN THE LARGE FIELD LIMIT $\rho \propto \frac{1}{g}$. IT IS INDEPENDENT EMISSION OF THE TWO GLUONS BY TWO COLOR CHARGES TWO POMERONS

B. IS THE EMISSION OF THE TWO GLUONS FROM THE SAME VALENCE

SOURCE

C. IS EMISSION OF THE GLUON ATu WHICH SUBSEQUENTLY EMITS THE GLUON AT z

THESE CORRESPOND TO TWO GLUON PRODUCTION FROM A SINGLE POMERON AND ARE NOT RELEVANT TO THE PRESENT DISCUSSION.

SQUARING THE AMPLITUDE OF COURSE LEADS TO ZILLIONS OF TERMS - BUT WE WILL ONLY LOOK EXPLICITLY AT ONE OF THEM

$$\sigma^4 = \int_{z,\bar{z},u,\bar{u},x_1,\bar{x}_1,x_2\bar{x}_2} e^{ik(z-\bar{z})+ip(u-\bar{u})}\alpha_s^2 \vec{f}(\bar{z}-\bar{x}_1)\cdot\vec{f}(x_1-z)\,\vec{f}(\bar{u}-\bar{x}_2)\cdot\vec{f}(x_2-u)$$

$$\times\left\{\rho(x_1)[S^\dagger(x_1)-S^\dagger(z)][S(\bar{x}_1)-S(z)]\rho(\bar{x}_1)\right\}\left\{\rho(x_2)[S^\dagger(u)-S^\dagger(x_2)][S(\bar{u})-S(\bar{x}_2)\rho(\bar{x}_2)\right\}$$

ROBUST CORRELATION

$$\sigma^4 = \langle \sigma_1(k)\sigma_1(p) >$$

CONFIGURATION BY CONFIGURATION (FOR FIXED CONFIGURATION OF PROJECTILE CHARGES ρ AND FIXED TARGET FIELDS S)

$$\sigma_1(k) = \int_{z,\bar{z},x_1,\bar{x}_1} e^{ik(z-\bar{z})} \alpha_s \vec{f}(\bar{z}-\bar{x}_1)\cdot\vec{f}(x_1-z) \left\{ \rho(x_1)[S^\dagger(x_1) - S^\dagger(z)][S(\bar{x}_1) - S(z)]\rho(\bar{x}_1) \right\}$$

$\sigma_1(k)$ IS A SINGLE GLUON EMISSION PROBABILITY FOR A **GIVEN** CONFIGURATION OF COLOR CHARGES IN THE PROJECTILE AND A **GIVEN** CONFIGURATION OF TARGET FIELDS

$\sigma_1(k)$ IS A NONTRIVIAL REAL FUNCTION OF k, WHICH HAS A MAXIMUM AT SOME VALUE $k = q_0$. CLEARLY THEN THE TWO GLUON PRODUCTION PROBABILITY CONFIGURATION BY CONFIGURATION HAS A MAXIMUM AT

$$k = p = q_0$$

THE VALUE OF q_0 DEPENDS ON CONFIGURATION, BUT THE FACT THAT k AND p ARE THE SAME DOES NOT.

IS THE MAXIMUM OF σ_1 UNIQUE?

$$\sigma_1(k) = a(k)a^*(k) = a(k)a(-k)$$

$$a(k) = \int_{z,x_1,} e^{ikz} g\vec{f}(x_1 - z)\,[S(x_1) - S(z)]\rho(x_1)$$

THUS σ_1 IS SYMMETRIC UNDER $k \to -k$ AND IS DOUBLY DEGENERATE - WITH MAXIMA AT q_0 AND $-q_0$

THIS MEANS THAT σ^4 HAS A SYMMETRY $k, p \to -k, p$ AND THEREFORE **HAS MAXIMA AT TWO RELATIVE ANGLES** $\phi = 0$ **AND** $\phi = \pi$

THE MAXIMUM AT $\phi = \pi$ IS OF COURSE VERY DIFFICULT TO DISTINGUISH EXPERIMENTALLY

ALL THESE FEATURES REMAIN TRUE FOR THE LEADING TERM BEYOND THE WEAK SOURCE APPROXIMATION, SINCE THE TWO GLUON INCLUSIVE PRODUCTION PROBABILITY IS STILL CONFIGURATION BY CONFIGURATION A SQUARE OF A SINGLE GLUON INCLUSIVE PRODUCTION PROBABLITY, WHICH IS A SQUARE OF A **REAL** CLASSICAL FIELD (AMPLITUDE).

DEGENERACY IS EASY TO UNDERSTAND IN OUR SIMPLE PICTURE.

THE FIELDS ARE COLORED AND THE PARTONS ARE GLUONS - ALSO COLORED.

SUPPOSE THE TARGET ELECTRIC FIELD IS IN THE THIRD DIRECTION IN COLOR SPACE, E_i^3; AND INCOMING GLUON FIELD HAS INDEX 1, b_i^1.

WITH RESPECT TO THE THIRD DIRECTION SUCH A GLUON FIELD HAS EQUAL NUMBER OF POSITIVELY AND NEGATIVELY CHARGED PARTONS $W_1 = W^+ + W^-$.

THUS PROBABILITY TO BE SCATTERED PARALLEL AND ANTIPARALLEL TO THE FIELD ARE EQUAL, DUR TO REALITY OF THE ADJOINT REPRESENTATION.

THE DEGENERACY THUS DOES NOT HOLD FOR QUARKS, AND ONE EXPECTS SHARPER CORRELATION AT VANISHING AZYMUTHAL ANGLE.

WHAT ABOUT "NONCLASSICAL" TERMS?

FIRST OFF, THERE IS NO ANGULAR DEGENERACY

THE AMPLITUDE DOES NOT FACTORIZE, SO ITS REALITY MEANS ONLY PARITY SYMMETRY $k, p \to -k, -p$

IS THERE POSITIVE CORRELATION AT $\phi = 0$?

$$A_{u \text{ emits } z} = g \int_{x_1} f_i(z-u) f_j(u-x_1) \left\{ (S(z) - S(u)) \, \bar{\rho}(x_1) S^\dagger(u) \right\}^{ab}$$

FOR z TO DECOHERE FROM u, AND THEREFORE BE EMITTED, THE TWO GLUONS MUST PREFERRABLY HIT AT DIFFERENT IMPACT PARAMETERS. WHEN EMITTED AT THE SAME IMPACT PARAMETER THE TWO GLUONS WILL HAVE OPPOSITE TRANSVERSE MOMENTA DUE TO CORRELATIONS IN THE INITITAL STATE - LARGE AWAY SIDE RAPIDITY INDEPENDENT MAXIMUM AT $\Delta\phi = \pi$

$$A_{x \text{ emits } u \text{ and } z} = -\frac{g}{2} \int_{x_1} f_i(z-x_1) f_j(u-x_1) \Big\{ [S(x_1) - S(z)] \, \bar{\rho}(x_1) \left[S^\dagger(u) + S^\dagger(x_1) \right] \Big\}^{ab}$$

HERE z HAS TO HIT FAR FROM x, BUT u LIKES TO BE CLOSE TO x IN FACT THIS TERM PROBABLY PRODUCES ONE GLUON AT RELATIVELY LARGE p_T - GREATER THAN q_s WITH THE BALANCING MOMENTUM APPEARING AT MORE FORWARD RAPIDITY

HOW BIG IS THE EFFECT?

TRANSVERSE CORRELATION LENGTH IN THE HADRON $L = \frac{1}{Q_s}$

TO BE CORRELATED THE TWO GLUONS HAVE TO BE IN THE SAME INCOMING STATE AND HAVE TO SCATTER OF THE SAME TARGET FIELD HAVE TO SIT WITHIN $\Delta X < L_{min}$ OF EACH OTHER.

THE CORRELATED PRODUCTION $\propto S/Q_s^2$,

WHILE THE TOTAL MULTIPLICITY $\propto S$

$$\left[\frac{d^2N}{d^2pd^2k} - \frac{dN}{d^2k}\frac{dN}{d^2p}\right] / \frac{dN}{d^2k}\frac{dN}{d^2p} \sim \frac{1}{(Q_s^{max})^2 S_{min}} .$$

IS IT N_c SUPPRESSED?

THE CALCULATIONS OF THE BNL GROUP ARE BASED ON FACTORIZATION.

AT LARGE N_c THE LEADING CONTRIBUTION IS WHEN THE CHARGE DENSITIES ARE PAIRWISE IN COLOR SINGLETS. HAVE TO AVERAGE OVER THE PROJECTILE AND TARGET WAVE FUNCTIONS

$$\langle\rho^a(x_1)\rho^a(\bar{x}_1)\rho^b(x_2)\rho^b(\bar{x}_2)\rangle_P$$

$$\times\langle\mathrm{Tr}\left\{[S^\dagger(x_1)-S^\dagger(z)][S(\bar{x}_1)-S(\bar{z})]\right\}\mathrm{Tr}\left\{[S^\dagger(x_2)-S^\dagger(u)][S(\bar{x}_2)-S(\bar{u})]\right\}\rangle_T\,.$$

THE SIMPLEST APPROACH (BNL "GLASMA FLUX TUBES")

EXPAND ALL $S=1+\alpha$; KEEP ONLY LEADING TERM

$$N\propto\{\rho\alpha\alpha\rho\}(k)\{\rho\alpha\alpha\rho\}(p)$$

NOW AVERAGE WITH GAUSSIAN WEIGHTS

$$<\rho\rho\rho\rho>=3<\rho\rho><\rho\rho>;\qquad <\alpha\alpha\alpha\alpha>=3<\alpha\alpha><\alpha\alpha>$$

TAKE $< \rho\rho >= \Phi_{BK}$ AND THE SAME FOR $< \alpha\alpha >$ GIVES

$$N \propto 9\Phi^P_{BK}\Phi^P_{BK}\Phi^T_{BK}\Phi^T_{BK}$$

WITH GAUSSIAN AVERAGING

$$\langle\rho^a(x_1)\rho^a(\bar{x}_1)\rho^b(x_2)\rho^b(\bar{x}_2)\rangle_{Gauss\ and\ leading\ N_c} = \langle\rho^a(x_1)\rho^a(\bar{x}_1)\rangle_{Gauss}\langle\rho^b(x_2)\rho^b(\bar{x}_2)\rangle_{Gauss}\,.$$

AND THE SAME FACTORIZATION FOR THE TARGET AVERAGES OF S's

AND SO

$$\frac{d^2N}{d^2p\,d^2k} = \frac{dN}{d^2k}\frac{dN}{d^2p}$$

WITHIN GAUSSIAN (FACTORIZABLE) APPROXIMATION CORRELATIONS ARE SUBLEADING IN $1/N_c$

BUT IT DOES NOT HAVE TO BE LIKE THIS!

WHEN IS FACTORIZABLE AVERAGING GOOD? WHEN THE POINTS ARE FAR AWAY IN SPACE

$$\langle \rho^a(x_1)\rho^a(\bar{x}_1)\rho^b(x_2)\rho^b(\bar{x}_2)\rangle$$

IF $(x_1, \bar{x}_1)$ IS FAR FROM $(x_2, \bar{x}_2)$ THEY DON'T KNOW ABOUT EACH OTHER AND THE AVERAGE FACTORIZES.

BUT WE ARE INTERESTED PRECISELY IN THE OPPOSITE SITUATION - WHEN ALL FOUR POINTS ARE WITHIN THE CORRELATION LENGTH, AND THEREFORE WE ARE SAMPLING CONFIGURATIONS WHICH AT ALL POINTS ARE SIMILAR

FACTORIZABILITY IS NOT AN INHERENT PROPERTY OF THE LARGE N LIMIT. E.G. FOR "DIPOLE DENSITY"

$$n(x_1, \bar{x}_1) = \Big(\rho^a(x_1) - \rho^a(\bar{x}_1)\Big)^2 .$$

IN BFKL EVOLVED WAVE FUNCTION OF A SINGLE DIPOLE (PARENT DIPOLE LARGER THAN DAUGHTERS)

$$\langle n(x_1,\bar{x}_1)n(x_2,\bar{x}_2)\rangle - \langle n(x_1,\bar{x}_1)\rangle\langle n(x_2,\bar{x}_2)\rangle \sim \langle n(x_1,\bar{x}_1)\rangle\langle n(x_2,\bar{x}_2)\rangle \left(\frac{b}{x}\right)^{-\lambda}$$

THERE IS NO REASON AT ALL TO BELIEVE THAT THE AVERAGES FACTORIZE. THUS VERY LIKELY THERE IS A CONTRIBUTION TO THE CORRELATED PRODUCTION ALREADY IN THE LEADING ORDER IN LARGE N_C

CONCLUSIONS

GLUON PRODUCTION AT HIGH ENERGY LEADS NATURALLY TO RAPIDITY CORRELATIONS (TRIVIALLY) AND ANGULAR CORRELATIONS (A LITTLE LESS TRIVIALLY). THERE JUST HAVE TO BE MANY GLUONS SO THAT MORE THAN ONE IS PRODUCED AT FIXED IMPACT PARAMETER (WITHIN $\Delta b \sim \frac{1}{Q_s}$ (- HOT SPOTS, HIGH MILTIPLICITY EVENTS?))

CORRELATIONS EXIST CONFIGURATION BY CONFIGURATION AND THEREFORE GAUSSIAN AVERAGING VERY LIKELY UNDERESTIMATES THEM. **THERE IS NO REASON NOT TO HAVE CORRELATIONS AT LEADING ORDER IN $1/N_C$.**

THIS MEANS WE HAVE TO UNDERSTAND HOW TO EVOLVE IN RAPIDITY OBJECTS MORE COMPLICATED THAN "DIPOLES" - BUT IT IS ESSENTIAL TO UNDERSTAND THE ANGULAR CORRELATIONS.

"'CLASSICAL"' TERM LEADS TO THE STRONGEST CORRELATIONS - THUS THE CORRELATIONS SHOULD BE STRONGEST FOR NUCLEUS PROJECTILE WHERE IT DOMINATES. ON THE OTHER HAND EFFECT BECOMES WEAKER WITH INCREASING Q_S. SO MAYBE ACTUALLY THE OTHER WAY ROUND - IT IS STRONGEST FOR $p-p$ IN A LIMITED RANGE OF ENERGIES?

LECTURE II - ON FORWARD SUPPRESSION

DATA IS FIT REASONABLY WELL.

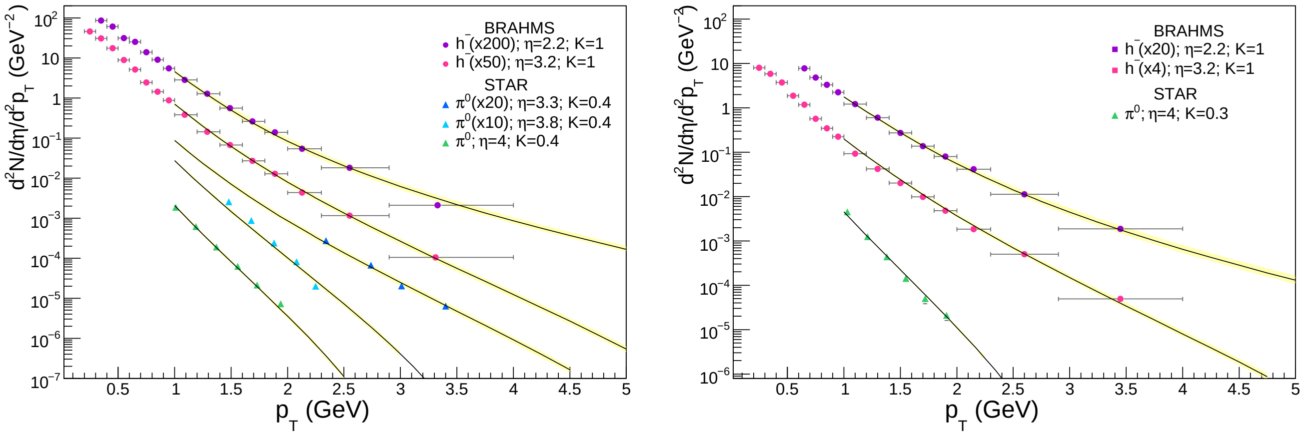


Figure 11: Negatively charged hadron and π^0 yields in proton-proton (at pseudo-rapidities (2.2, 3.2) and (3.3, 3.8 and 4)) and deuteron-gold (at pseudo-rapidities (2.2, 3.2) and 4) collisions at $\sqrt{s_{NN}} = 200$ GeV. Data by the BRAHMS and STAR collaborations. (From Albacete and Marquet)

BUT A CURIOUSLY SLOW APPROACH TO PERTURBATIVE REGIME (OR RATHER THE LACK THEREOF)

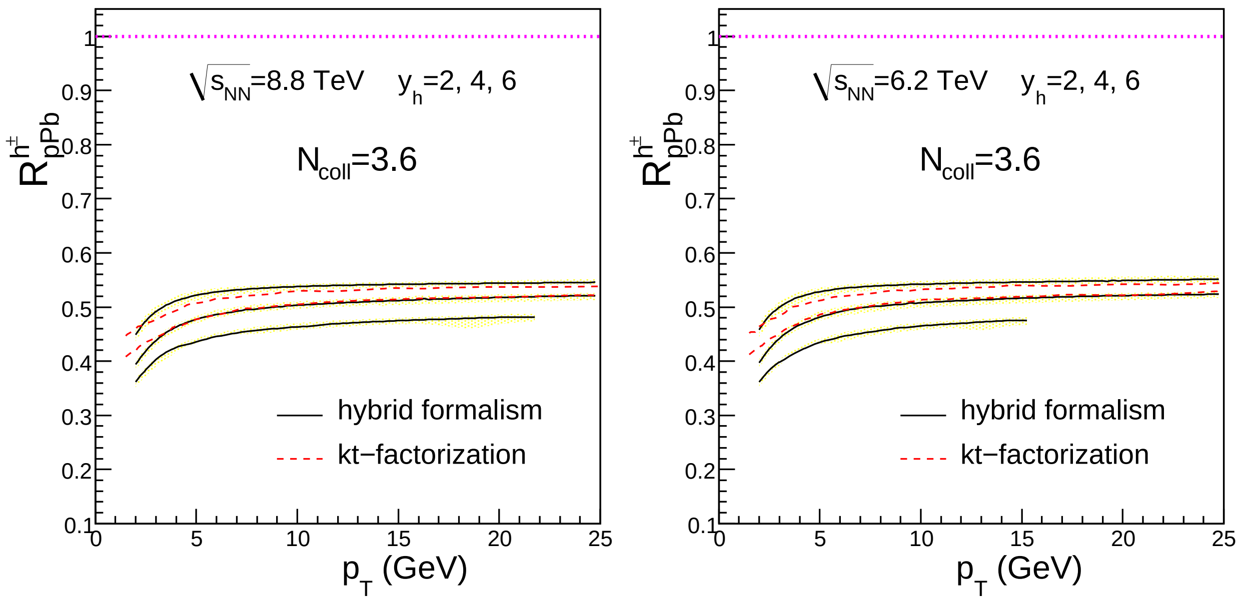


Figure 12: Nuclear modification factors for $h^\pm$ production in p+Pb collisions, $R^{h^\pm}_{pPb}$, for collision energies $\sqrt{s_{NN}} = 8.8$ (left) and 6.2 TeV (right) and for rapidities $y_h = 2$, 4, and 6. For comparison, the red dashed line corresponds to the same quantity calculated in the k$_t$-factorization scheme. (From Albacete and Marquet)

WHAT GOES INTO THIS CALCULATION?

"HYBRID FORMALISM" OF DUMITRU, HAYASHIGAKI, JALILIAN-MARIAN

VERY INTUITIVE

$$\frac{dN}{d^2k\,d\eta} = \frac{1}{(2\pi)^2}\int_{x_F}^{1}\frac{dz}{z^2}$$

$$\left[x_1 f_g(x_1,Q^2)N_A(x_2,\frac{k}{z})D_{h/g}(z,Q) + \Sigma_q x_1 f_q(x_1,Q^2)N_F(x_2,\frac{k}{z})D_{h/q}(z,Q)\right]$$

WITH

$$x_F = \frac{k}{\sqrt{s_{NN}}}e^{\eta}; \qquad x_1 = \frac{x_F}{z}; \qquad x_2 = x_1 e^{-2\eta}$$

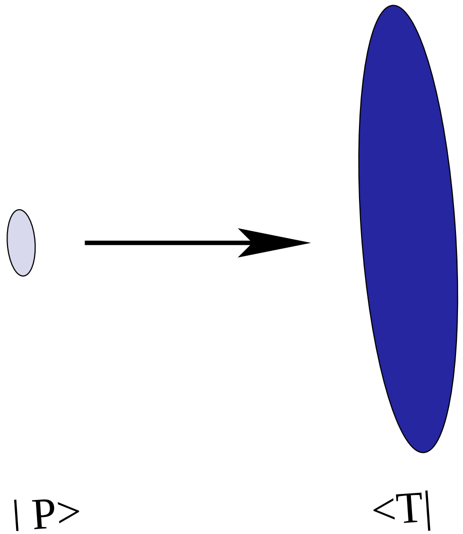
SOME QUESTIONS:

DOES IT TAKE INTO ACCOUNT ALL LEADING TWIST CONTRIBUTIONS IMPORTANT AT HIGH k_T?

HOW IS IT RELATED TO k_T FACTORIZED APPROACH?

LETS ACTUALLY DERIVE IT.

SMAILL PERTURBATIVE PROJECTILE $|P\rangle$ SCATTERS ON A LARGE DENSE TARGET $\langle T|$.



$|P\rangle$ IS CALCULATED PERTURBATIVELY KEEPING EXACT KINEMATICS (NO SOFT APPROXIMATION).

$|T\rangle$ IS MODELLED AS DISTRIBUTION OF CLASSICAL COLOR FIELDS

THE SCATTERING IS APPROXIMATED BY EIKONAL (NO RECOIL) SCATTERING OF THE PROJECTILE PARTONS ON THE FIELDS OF THE TARGET.

WE START BY CONSIDERING GLUONS ONLY - INCLUDING QUARKS IS STRAIGHTFORWARD AND WILL BE DONE LATER.

THE INCOMING PROJECTILE WAVE FUNCTION

$$|\Psi\rangle_{in} = \Omega|v\rangle$$

$|v\rangle$ - ZERO ORDER PERTURBATIVE WAVE FUNCTION

Ω DIAGONALIZES QCD HAMILTONIAN

$$\Omega^{\dagger} H_{QCD} \Omega = H_{diag}$$

OUTGOING WAVE FUNCTION

$$|\Psi\rangle_{out} = S|\Psi\rangle_{in}$$

THE NUMBER OF PRODUCED GLUONS:

$$\frac{dN}{d^2k dk^+} = \frac{1}{(2\pi)^3}\langle v|\Omega^\dagger S^\dagger \Omega a^\dagger(k,k^+)a(k,k^+)\Omega^\dagger S\Omega|v\rangle$$

WHY? BECAUSE:

$$H\Omega|v\rangle = E_v\Omega|v\rangle \quad \rightarrow \quad H[\Omega a^\dagger(k)\Omega^\dagger]\Omega|v\rangle = [E_v + \omega(k)][\Omega a^\dagger(k)\Omega^\dagger]\Omega|v\rangle$$

THUS THE OPERATOR $\Omega a^\dagger(k)\Omega^\dagger$ CREATES A DRESSED GLUON - AN EIGENSTATE OF INTERACTING HAMILTONIAN!

DIAGRAMMATICALLY THIS IS EQUIVALENT TO TAKING INTO ACCOUNT THE FINAL STATE EMISSION...

ITS JUST THE USUAL PERTURBATIVE DEFINITION OF OBSERVABLE, JUST IN DIFFERENT NOTATIONS...

PERTURBATIVELY WE FIND

$$\Omega = e^{-iG} = 1 - iG + \ldots$$

WITH

$$G = -gf^{abc} \int_{k,p,k^+,p^+>0} \frac{1}{\sqrt{2k^+p^+(k^++p^+)}} \frac{1}{\omega_{p+k}-\omega_p-\omega_k}$$

$$\left\{ - \left[\frac{p^+}{k^+}k_i - p_i\right] a_i^b(k^+,k)a_j^c(p^+,p)a_j^{a\dagger}(k^++p^+,k+p)\right.$$

$$\left. +\frac{p^+}{p^++k^+}k_j a_i^b(k^+,k)a_i^c(p^+,p)a_j^{a\dagger}(k^++p^+,k+p)\right\} + h.c.$$

AND THE SINGLE INCLUSIVE GLUON SPECTRUM

$$\frac{dN}{d^2k dk^+} = \frac{1}{(2\pi)^3}\langle v| \left[\hat{S}^\dagger G - G\hat{S}^\dagger\right] a_k^{a\dagger}(k^+,k)a_k^a(k^+,k) \left[G\hat{S} - \hat{S}G\right] |v\rangle$$

FOR EIKONAL SCATTERING $\hat{S}^\dagger a_i^a(q^+,v)\hat{S} = S^{ab}(v)a_i^b(q^+,v)$

AT THIS POINT WE ARE SORT OF AT A CROSSROADS. WE HAVE TERMS OF THE TYPE:

$$\langle v|a^\dagger(p^+ + k^+)a(p^+)a^\dagger(q^+)a(q^+ + k^+)|v\rangle$$

NEGLECTING k^+ IS THE SOFT LIMIT. IT LEADS TO CORRELATOR OF THE COLOR CHARGE DENSITIES

$$\langle v|\rho^a(x)\rho^b(y)|v\rangle$$

ON THE OTHER HAND "PARTONIC LIMIT" (EXPANSION IN NUMBER OF PARTONS IN THE PROJECTILE) IS WRITING THIS AS

$$\delta(p^+ - q^+)\langle v|a^\dagger(p^+ + k^+)a(p^+ + k^+)|v\rangle + \langle v|a^\dagger(p^+ + k^+)a^\dagger(q^+)a(p^+)a(q^+ + k^+)|v\rangle$$

AND NEGLECTING THE SECOND TERM

THE "HYBRID" FORMALISM OF DHJ IS PARTONIC IN NATURE AND CORRESPONDS TO THE SECOND OPTION

THEN

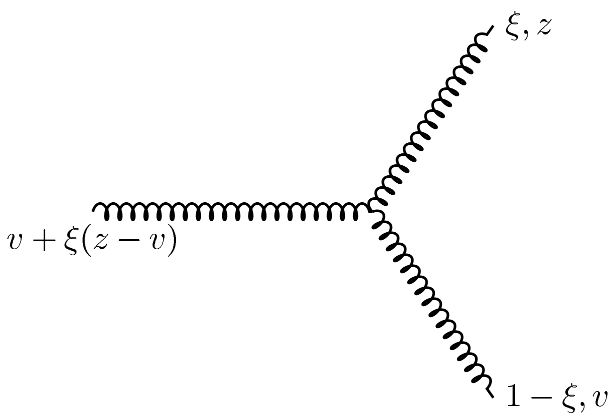
$$\frac{dN}{d^2k\,dk^+} = \frac{\alpha_s}{2\pi^2}\frac{1}{(2\pi)^2}\frac{1}{N_c^2-1}\int_x^1 \frac{d\xi}{\xi}\frac{1}{k^+}e^{ik(z-\bar z)}\frac{2}{(1-\xi)}\left[(1-\xi)^2+\xi^2+(1-\xi)^2\xi^2\right]\frac{(v-\bar z)_i}{(v-\bar z)^2}\frac{(v-z)_i}{(v-z)^2}$$

$$\times tr\left\{\left[S^\dagger((1-\xi)v+\xi\bar z)T^a S((1-\xi)v+\xi\bar z)-S^\dagger_v T^a S_{\bar z}\right]\right.$$

$$\left.\times\left[S^\dagger((1-\xi)v+\xi z)T^a S((1-\xi)v+\xi z)-S^\dagger_z T^a S_v\right]\right\}$$

$$\times\frac{k^+}{2\pi\xi}\langle a_j^{b\dagger}(\frac{k^+}{\xi},(1-\xi)v+\xi\bar z)a_j^b(\frac{k^+}{\xi},(1-\xi)v+\xi z)\rangle$$

ROUGHLY: NUMBER OF GLUONS IN THE LEADING ORDER STATE $\times$ PROBABILITY TO EMIT THE GLUON WITH LONGITUDINAL MOMENTUM FRACTION ξ $\times$ SCATTERING PROBABILITY OF THE PARTONIC SYSTEM



THE SOFT LIMIT.

TO GET SOME INTUITION CONSIDER THE SOFT LIMIT FIRST $\xi \ll 1$

$$\frac{dN}{d^2k dk^+} = \frac{\alpha_s}{\pi^2}\frac{1}{(2\pi)^2}\frac{N_c}{N_c^2-1}\int \frac{1}{k^+}e^{ik(z-\bar{z})}\frac{(v-\bar{z})_i}{(v-\bar{z})^2}\frac{(z-v)_i}{(z-v)^2}tr\left\{S_z S^\dagger_{\bar{z}} + 1 - S^\dagger_v S_z - S^\dagger_{\bar{z}} S_v\right\}$$

$$\times \langle a_j^{a\dagger}(\frac{k^+}{\xi}, v) a_j^a(\frac{k^+}{\xi}, v)\rangle$$

C.F. STANDARD k_T FACTORISED EXPRESSION:

$$\frac{dN}{d^2k dk^+} = \frac{\alpha_s}{\pi^2}\frac{1}{(2\pi)^2}\frac{1}{N_c^2-1}\int \frac{1}{k^+}e^{ik(z-\bar{z})}\frac{(v-\bar{z})_i}{(v-\bar{z})^2}\frac{(z-v)_i}{(z-v)^2}tr\left\{S_z S^\dagger_{\bar{z}} + 1 - S^\dagger_v S_z - S^\dagger_{\bar{z}} S_v\right\}$$

$$\times \langle \rho^a_v \rho^a_{\bar{v}}\rangle$$

IN THE PARTONIC APPROXIMATION, INDEED

$$\langle \rho^a(v)\rho^a(\bar{v})\rangle = \delta^2(v-\bar{v})N_c\langle\int \frac{dp^+}{2\pi} a_i^{\dagger a}(p^+, v) a_i^a(p^+, v)\rangle$$

SOFT LIMIT OF "HYBRID" IS THE SAME AS PARTONIC LIMIT (INDEPENDENT COLOR CHARGES IN THE TRANSVERSE PLANE) OF k_T FACTORISED

CUSTOMARY DEFINITION OF TMD:

$$x f_g\left(x, Q = \frac{1}{|u-v|}\right) \equiv \frac{p^+}{2\pi}\int d^2b \langle a_i^{a\dagger}(p^+, u) a_i^a(p^+, v)\rangle) = \int d^2b \int \frac{d^2p}{\pi} e^{ip\cdot(u-v)}\phi(p, b; x$$

$$\approx \int d^2b \int_0^{\frac{1}{|u-v|^2}} dp^2 \phi(p, b; x)$$

IN THE SOFT LIMIT

$$\langle \rho^a(v)\rho^a(\bar{v})\rangle = \frac{1}{8\pi\alpha_s}\int d^2p e^{ip\cdot(v-\bar{v})} p^2 \phi_P(p, b)$$

$$tr[1 - S^\dagger(v)S(\bar{v})] = 2\pi\alpha_s N_c \int d^2p e^{ip\cdot(v-\bar{v})}\frac{1}{p^2}\phi_T(p, b)$$

AND THE SINGLE GLUON SPECTRUM:

$$\frac{dN}{d^2b d^2k d\eta} = \frac{\alpha_s N_c}{N_c^2-1}\frac{1}{k^2}\int_l \phi_T(l+k, Y-\eta)\phi_P(l,\eta)$$

$$= \frac{\alpha_s N_c}{N_c^2-1}\int_l \Big[\frac{1}{(l+k)^2} + \frac{1}{(l+k)^2}\frac{l^2}{k^2} + 2\frac{1}{(l+k)^2}\frac{l\cdot k}{k^2}\Big]\phi_T(l+k)\phi_P(l)$$

IN THE LARGE PRODUCED MOMENTUM LIMIT $k \gg Q_s, \Lambda_{QCD}$ THIS IS DOMINATED BY TWO INTEGRATION REGIONS:

A. $|l| \ll |k|$: $N_E = \frac{\alpha_s N_c}{N_c^2-1}\frac{1}{k^2}\phi_T(k)\int_{l<Q\sim k}\phi_P(l)$

THIS IS "'ELASTIC"' SCATTERING OF A LOW p_t GLUON WITH LARGE MOMENTUM TRANSFER.

B. $|l+k| \ll |k|$: $N_I \frac{\alpha_s N_c}{N_c^2-1}\frac{1}{k^2}\phi_P(k)\int_{q<Q\sim k}\phi_T(q)$

THIS IS " INELASTIC " SCATTERING: HIGH p_t GLUON SCATTERS WITH SMALL MOMENTUM TRANSFER.

INELASTIC - BECAUSE HIGH p_T GLUONS LIVE IN THE WAVEFUNCTION ONLY AS A PART OF THE "UNRESOLVED STRUCTURE" OF THE NAIVE PARTON MODEL PARTONS!

THE INELASTIC PARTON SCATTERING

CONSIDER A STATE:

$$|in>= \left[|v,a> + gT^a_{bc} f(v,z)|v,b;\ z,c> \right]$$

AFTER SCATTERING:

$$|out>= \left[S^{ab}(v)|v,b> + gT^a_{bc} f(v,z) S^{bd}(v) S^{ce}(z)|v,d;\ z,e> \right]$$

THE INELASTIC COMPONENT:

$$|out>_{inelastic}= gT^a_{bc} f(v,z) S^{bd}(v) \left[S^{ce}(z)|v,d;\ z,e> - S^{ce}(v)|v,d;\ z,e> \right]$$

IF $S(v)$ IS SLOWLY VARYING, THE HIGH MOMENTUM IN THE FINAL STATE CAN ONLY ARISE FROM HIGH MOMENTUM COMPONENT OF THE AMPLITUDE $f(v,z)$

$$gT^a_{bc} \int d^2 z e^{ik\cdot(z-v)} f(v,z) S^{bd}(v) \left[S^{ce}(z) - S^{ce}(v) \right]$$

SQUARING AND INTEGRATING OVER v WITH THE GLUON DENSITY REPRODUCES THE INELASTIC CONTRIBUTION TO THE GLUON PRODUCTION

HOW IMPORTANT IS THE INELASTIC CONTRIBUTION?

BOTH CONTRIBUTIONS ARE THE SAME ORDER IN α_s (ELASTIC DENSITY IS $O(1)$, BUT SCATTERING PROBABILITY $O(\alpha_s)$; INELASTIC DENISTY IS $O(\alpha_s)$, BUT SCATTERING PROBABILITY $O(1)$)

ASSUME PERTURBATIVE BEHAVIOR: $\phi = \frac{\mu^2}{p^2}$. THEN

$$\left[\frac{dN}{d^2k d\eta}\right]_{elastic} = \alpha_s \frac{\mu_P^2 \mu_T^2 \ln}{k^4} \frac{k^2}{\Lambda_{QCD}^2}$$

$$\left[\frac{dN}{d^2k d\eta}\right]_{inelastic} = \alpha_s \frac{\mu_P^2 \mu_T^2}{k^4} \ln \frac{k^2}{Q_s^2}$$

AT $p \gg Q_S$ IT IS NOT SUPPRESSED EVEN LOGARITHMICALLY!

BACK TO THE HYBRID FORMALISM.

$$\frac{dN}{d^2k\,dk^+} = \frac{\alpha_s}{2\pi^2}\frac{1}{(2\pi)^2}\frac{1}{N_c^2-1}\int_x^1 \frac{d\xi}{\xi}\frac{1}{k^+}e^{ik(z-\bar{z})}\frac{2}{(1-\xi)}\left[(1-\xi)^2+\xi^2+(1-\xi)^2\xi^2\right]\frac{(v-\bar{z})_i}{(v-\bar{z})^2}\frac{(v-z)_i}{(v-z)^2}$$

$$\times tr\left\{\left[S^\dagger((1-\xi)v+\xi\bar{z})T^aS((1-\xi)v+\xi\bar{z})-S_v^\dagger T^aS_{\bar{z}}\right]\right.$$

$$\left.\times\left[S^\dagger((1-\xi)v+\xi z)T^aS((1-\xi)v+\xi z)-S_z^\dagger T^aS_v\right]\right\}$$

$$\times\frac{k^+}{2\pi\xi}\langle a_j^{b\dagger}(\frac{k^+}{\xi},(1-\xi)v+\xi\bar{z})a_j^b(\frac{k^+}{\xi},(1-\xi)v+\xi z)\rangle$$

IT IS EASY TO IDENTIFY THE SAME PHYSICS IN THE HYBRID APPROACH.

ELASTIC CONTRIBUTION - ALL VARIATION IN $z-\bar{z}$ IS IN THE PHASE FACTORS

INELASTIC CONTRIBUTION: THE SEPARATION $|v-z|$ IS SMALL.

ALSO: $N_F(k)=\int d^2x\,e^{ikx}\frac{1}{N_c}\mathrm{Tr}[1-S_F^\dagger(0)S_F(x)]$

$$\left[\frac{dN}{d^2k\,d\eta}\right]_{elastic} \simeq \frac{\alpha_s}{\pi}\frac{1}{(2\pi)^2}\int_{p^2<Q^2}\frac{dp^2}{2p^2}\int_{x_F}^{1}\frac{d\xi}{\xi}P_{g/g}(\xi)x_F f_g(\frac{x_F}{\xi},p^2)N_A(k) + \text{"}valence\text{"}$$

$$= \frac{1}{(2\pi)^2}x_F f_g(x_F,Q^2)N_A(k)$$

$$\left[\frac{dN}{d^2k\,d\eta}\right]_{inelastic} = \frac{\alpha_s}{\pi^2}\frac{N_c^2}{N_c^2-1}\frac{1}{k^4}\int_{x_F}^{1}\frac{d\xi}{\xi}\left\{1-\xi+\xi^2\right\}P_{g/g}(\xi)x_F f_g(\frac{x_F}{\xi},Q)$$

$$\times \int_{p^2<Q^2}\frac{d^2p}{(2\pi)^2}p^2 N_F(p)$$

COMPLETE LEADING TWIST EXPRESSION (INCLUDING FRAGMENTATION CONTRIBUTION):

$$\frac{dN}{d^2k\,d\eta} = \int_{x_F}^{1}\frac{dz}{z^2}D_{h/g}(z,Q)\Bigg[x_1 f_g(x_1,Q^2)N_A(x_2,\frac{k}{z})$$

$$+\frac{\alpha_s}{\pi^2}\frac{N_c^2}{N_c^2-1}\frac{z^4}{k^4}\int_{x_1}^{1}\frac{d\xi}{\xi}\left\{1-\xi+\xi^2\right\}P_{g/g}(\xi)x_1 f_g(\frac{x_1}{\xi},Q)\int_{p^2<Q^2}\frac{d^2p}{(2\pi)^2}p^2 N_F(x_2,p)\Bigg]$$

QUARKS INCLUDED.

INCLUDING QUARK AND ANTIQUARK PRODUCTION IS STRAIGHTFORWARD IF A BIT TEDIOUS.

THE FINAL RESULT FOR THE EXTRA "INELASTIC" CONTRIBUTION IS

$$\left[\frac{dN_i}{d^2k d\eta}\right]_{inelastic} = \frac{\alpha_s}{2\pi^2}\frac{1}{k^4}\int_{p^2<Q^2}\frac{d^2p}{(2\pi)^2}p^2 N_F(p)x_F\int_{x_F}^{1}\frac{d\xi}{\xi}\Sigma_{j=q,\bar{q},g}w_{i/j}(\xi)P_{i/j}(\xi)f_j(\frac{x_F}{\xi},Q)$$

WITH "INELASTIC WEIGHTS"

$$w_{g/g}(\xi) = 2\frac{N_c^2}{N_c^2-1}(1-\xi+\xi^2)$$

$$w_{g/q}(\xi) = w_{g/\bar{q}}(\xi) = \frac{N_c^2}{N_c^2-1}\left[1+(1-\xi)^2-\frac{\xi^2}{N_c^2}\right]$$

$$w_{q/q}(\xi) = w_{\bar{q}/\bar{q}}(\xi) = \frac{N_c^2}{N_c^2-1}\left[1+\xi^2-\frac{(1-\xi)^2}{N_c^2}\right]$$

$$w_{q/g}(\xi) = w_{\bar{q}/g}(\xi) = \frac{1}{2}\left[(1-\xi)^2+\xi^2-\frac{2\xi(1-\xi)}{N_c^2-1}\right]$$

DISCUSSION.

SOME FEATURES OF THE RESULT:

FINAL STATES THAT CORRESPOND TO INELASTIC CONTRIBUTION ARE PAIRS OF PARTONS (PRESUMABLY DIHADRONS) WITH LARGE BALANCING TRANSVERSE MOMENTUM. AT FORWARD KINEMATICS THIS MUST BE DOMINATED BY ASYMMETRIC IN RAPIDITY CONFIGURATIONS, OTHERWISE BOTH PARTONS WOULD NEED TO CARRY LARGE FRACTIONS OF LONGITUDINAL MOMENTUM.

HOW DOES SATURATION EXHIBIT ITSELF?

ELASTIC CONTRIBUTION DEPENDS ONLY ON $N(k)$ AT LARGE MOMENTUM k AND THUS SHOULD NOT BE TOO SENSITIVE TO SATURATION!

INELASTIC CONTRIBUTION IS PROPORTIONAL TO THE TARGET GLUON DISTRIBUTION BELOW THE MEASURED MOMENTUM

$$\int_{p^2<Q^2} \frac{d^2p}{(2\pi)^2} p^2 N_F(p,x_2) \propto f_{target}(Q,x_2)$$

THIS IS STRONGLY SUPPRESSED BY THE EVOLUTION AT SMALL x_2 (FORWARD)!

THUS ONE EXPECTS THAT IT IS THE INELASTIC CONTRIBUTION THAT ARE MOSTLY SENSITIVE TO EFFECTS OF SATURATION IN $p-A$ ($d-Au$) VERSUS $p-p$.

SO HOW COME ALBACETE & MARQUET OBTAIN $R_{dA} < 1$, EVEN THOUGH THEY DO NOT INCLUDE THIS PIECE?

$$\frac{dN}{d^2k d\eta} \sim \frac{1}{(2\pi)^2} x_1 f_g(x_1, Q^2) N_A(x_2, k)$$

NAIVELY IF $N_A(k) = AN_p(k)$ AT INITIAL RAPIDITY, THEN **THE RATIO STAYS THE SAME AT ANY RAPIDITY IF THE RAPIDITY EVOLUTION IS LINEAR!**.

NEVERTHELESS BFKL EVOLUTION AFFECTS ELASTIC CONTRIBUTION AS WELL THROUGH THE "SHADOWING" INTRODUCED AT THE INITIAL RAPIDITY!

BFKL EQUATION EVOLVES ANY INITIAL CONDITION INTO THE "GEOMETRIC SCALING" FORM $\phi_{BFKL}(k, Y) \propto [Q_s(Y)/k]^{2-2\gamma}$

ANOMALOUS DIMENSION γ IS A SLOWLY VARYING FUNCTION OF TRANSVERSE MOMENTUM AND RAPIDITY, AND VARIES BETWEEN 0 IN THE ULTRAVIOLET AND $1/2$ IN THE INFRARED.

WITHIN BFKL Q_s IS NOT A SATURATION MOMENTUM AS SUCH, BUT RATHER HAS THE MEANING OF MOMENTUM AT WHICH THE DIPOLE SCATTERING PROBABILITY IS CLOSE TO UNITY. WITHIN THE BFKL EVOLUTION IT DEPENDS EXPONENTIALLY ON RAPIDITY $Q_s^2(Y) = Q_0^2 \exp\{\lambda Y\}$

HERE Q_0^2 IS THE SCALE IN THE INITIAL CONDITION. **THE INITIAL SCALES ARE DIFFERENT FOR THE PROTON (Λ^2_{QCD}) AND THE NUCLEUS ($A^{1/3}\Lambda^2_{QCD}$)** AND THE DISCREPANCY IN SCALES IS ENHANCED BY BFKL EVOLUTION.

THUS EVEN EXCLUDING THE SATURATION EFFECTS IN THE EVOLUTION ONE WOULD GET A NONTRIVIAL R_{dA} SOLELY DUE TO THE BFKL ANOMALOUS DIMENSION

$$R_{pA}(Y) = \frac{1}{N_{coll}} \left[\frac{Q_{sA}(Y)}{Q_{sp}(Y)}\right]^{2-2\gamma(Y)} = \left[\frac{Q_{0p}}{Q_{0A}}\right]^{2\gamma(Y)} < 1$$

RECALL THAT INELASTIC CONTRIBUTION IS NOT INCLUDED IN THE FITS AT THE MOMENT

SO WHAT WE SEE IN THE FITS IS LIKELY NOT EFFECTS OF THE SATURATION AT ALL, BUT RATHER EFFECTS OF THE LINEAR BFKL EVOLUTION!