

Thermalization of boost-invariant plasma from AdS/CFT

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M. Heller, RJ, P. Witaszczyk, 1103.3452

M. Heller, RJ, P. Witaszczyk, to appear

- 1 Motivation
- 2 Key physical questions
- 3 Boost-invariant flow
- 4 AdS/CFT methods for evolving plasma
- 5 Numerical relativity setup
- 6 Main results
 - Nonequilibrium vs. hydrodynamic behaviour
 - Entropy
 - Characteristics of thermalization
- 7 Conclusions

- There are strong indications that the quark-gluon plasma produced at RHIC is a strongly coupled system
- This poses numerous problems for the theoretical description (but also makes it very interesting)
- Static properties:
 - Thermodynamics — entropy/energy density etc.
 - Lattice QCD is an effective tool
 - Directly deals with QCD!
 - Quantitative results
- Real time properties:
 - Expansion of the plasma in heavy-ion collisions
 - *Derivation* of hydrodynamic expansion in the later stages of the collision
 - Dynamics far from equilibrium – fast thermalization of the plasma
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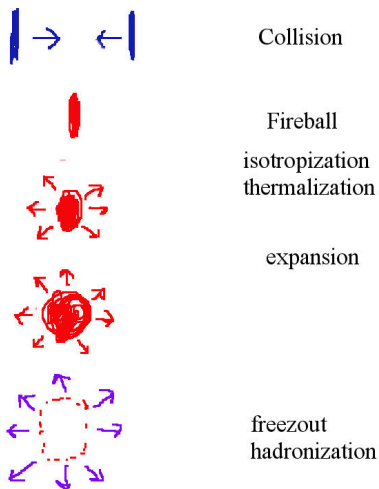
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Point of reference: heavy-ion collision at RHIC:

c.f. lectures by Hirano



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- To what extent would higher order (even all-order) viscous hydrodynamics explain plasma dynamics or do we need to incorporate genuine nonhydrodynamic degrees of freedom in the far from equilibrium regime
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Ways to proceed (apart from building phenomenological models):

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Similarities:

- Deconfined phase
- Strongly coupled
- No supersymmetry!

Differences:

- No running coupling \rightarrow Even at very high energy densities the coupling remains strong
- (Exactly) conformal equation of state \rightarrow Differences close to T_c , no bulk viscosity... (but not that different around $1.5 - 2.5 T_c$)
- No confinement/deconfinement phase transition \rightarrow Plasma fireball cools indefinitely

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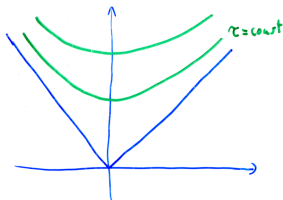
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Bjorken '83

Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.



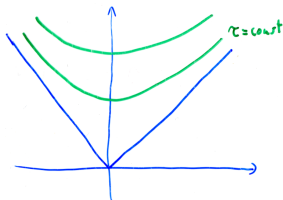
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- The longitudinal and transverse pressures are then given by

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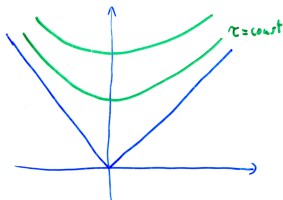
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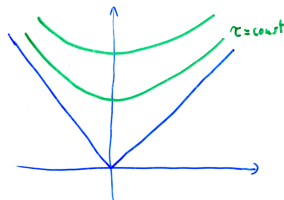
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- Current result for large τ :

RJ,Peschanski;RJ;RJ,Heller;Heller

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}}3^{\frac{3}{4}}}\frac{1}{\tau^2} + \frac{1+2\log 2}{12\sqrt{3}}\frac{1}{\tau^{\frac{8}{3}}} + \frac{-3+2\pi^2+24\log 2-24\log^2 2}{324\cdot 2^{\frac{1}{2}}3^{\frac{1}{4}}}\frac{1}{\tau^{\frac{10}{3}}} + \dots$$

- Leading term — perfect fluid behaviour
- second term — 1st order viscous hydrodynamics
- third term — 2nd order viscous hydrodynamics
- fourth term — 3rd order viscous hydrodynamics...
- As we decrease τ more and more dissipation will start to be important
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$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}}3^{\frac{3}{4}}} \frac{1}{\tau^2} + \frac{1 + 2 \log 2}{12\sqrt{3}} \frac{1}{\tau^{\frac{8}{3}}} + \frac{-3 + 2\pi^2 + 24 \log 2 - 24 \log^2 2}{324 \cdot 2^{\frac{1}{2}}3^{\frac{1}{4}}} \frac{1}{\tau^{\frac{10}{3}}} + \dots$$

- Leading term — perfect fluid behaviour
- second term — 1st order viscous hydrodynamics
- third term — 2nd order viscous hydrodynamics
- fourth term — 3rd order viscous hydrodynamics...
- As we decrease τ more and more dissipation will start to be important
- The large τ expansion is completely determined in terms of a single overall scale
- For small τ , in contrast, initial conditions will be very important leading to a dependence on a multitude of scales/parameters... **(this talk)**

Aim: Study the evolution of $\varepsilon(\tau)$ all the way from $\tau = 0$ to large τ starting from various initial conditions...

Questions:

- When and how does the transition to hydrodynamics (\equiv thermalization/isotropization) occur? (p_L and p_T are determined in terms of $\varepsilon(\tau)$)
- To what extent would higher order (even all-order) viscous hydrodynamics explain plasma dynamics or do we need to incorporate genuine nonhydrodynamic degrees of freedom in the far from equilibrium regime
- Does there exist some physical characterization of the initial state which determines the main features of thermalization and subsequent evolution?
- What is the produced entropy from $\tau = 0$ to $\tau = \infty$ (asymptotically perfect fluid regime)

It is convenient to eliminate explicit dependence on the number of degrees of freedom and use an *effective* temperature T_{eff} instead of $\varepsilon(\tau)$

$$\langle T_{\tau\tau} \rangle \equiv \varepsilon(\tau) \equiv N_c^2 \cdot \frac{3}{8} \pi^2 \cdot T_{eff}^4$$

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Describe it in terms of lightest degrees of freedom on the AdS side
which are relevant at strong coupling



$$ds^2 = \frac{g_{\mu\nu}(x^\rho, z) dx^\mu dx^\nu + dz^2}{z^2} \equiv g_{\alpha\beta}^{5D} dx^\alpha dx^\beta$$

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- The above choice ensures that we will control the boundary conditions even though they may be in a strongly curved part of the spacetime
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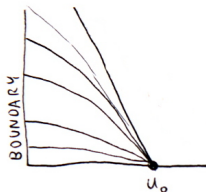
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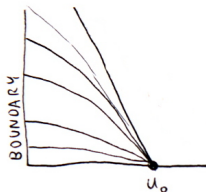
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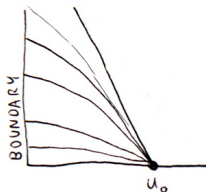
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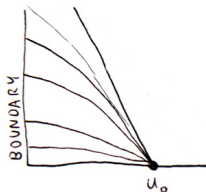
- In the ADM formulation we are free to choose how to foliate spacetime into 'equal time' hypersurfaces
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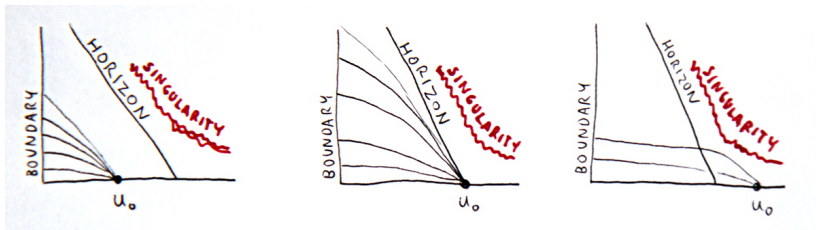


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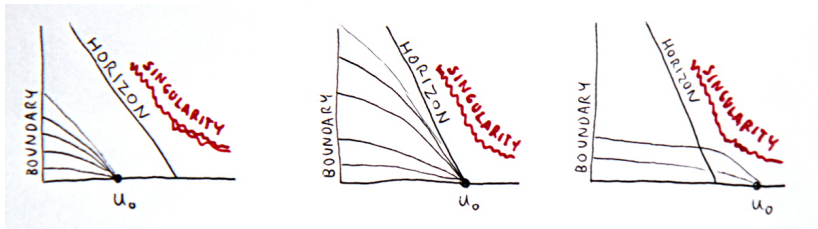
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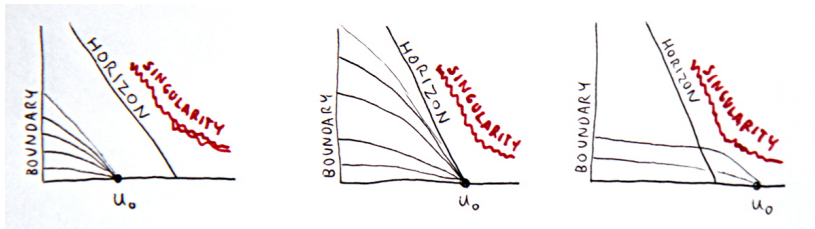
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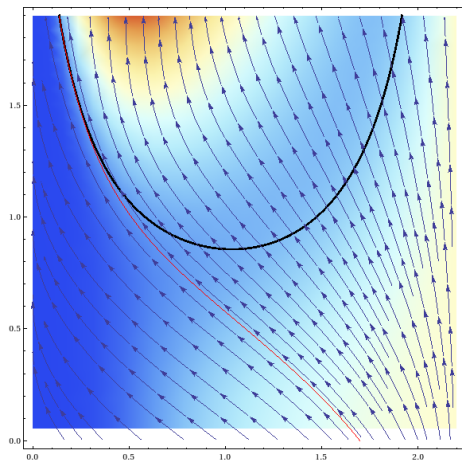
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black line – dynamical horizon, arrows – null geodesics, colors represent curvature

- We have considered 20 initial conditions, each given by a choice of the metric coefficient $c(\tau = 0, u)$
- We have chosen quite different looking profiles e.g.

$$c_1(u) = \cosh u$$

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Nonequilibrium vs. hydrodynamic behaviour

- Introduce the dimensionless quantity $w(\tau) \equiv T_{eff}(\tau) \cdot \tau$
- Viscous hydrodynamics (up to any order in the gradient expansion) leads to equations of motion of the form

$$\frac{\tau}{w} \frac{d}{d\tau} w = \frac{F_{hydro}(w)}{w}$$

where $F_{hydro}(w)$ is a *universal function* completely determined in terms of the hydrodynamic transport coefficients (shear viscosity, relaxation time and higher order ones) e.g.

$$\frac{F_{hydro}(w)}{w} = \frac{2}{3} + \frac{1}{9\pi w} + \frac{1 - \log 2}{27\pi^2 w^2} + \frac{15 - 2\pi^2 - 45 \log 2 + 24 \log^2 2}{972\pi^3 w^3} + \dots$$

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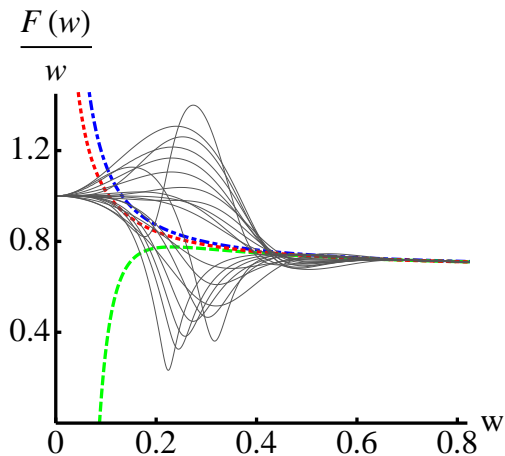
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A plot of $F(w)/w$ versus w for various initial data

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$$\Delta p_L \equiv 1 - \frac{p_L}{\varepsilon/3} = 12F(w) - 8$$

- For a perfect fluid $\Delta p_L \equiv 0$. For a sample initial profile we get

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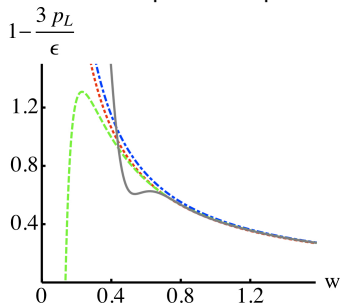
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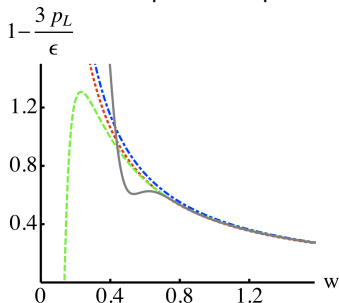
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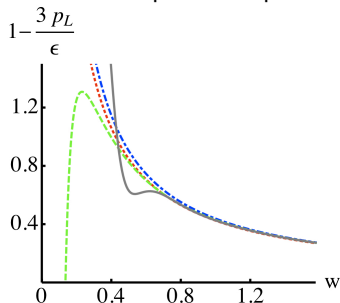
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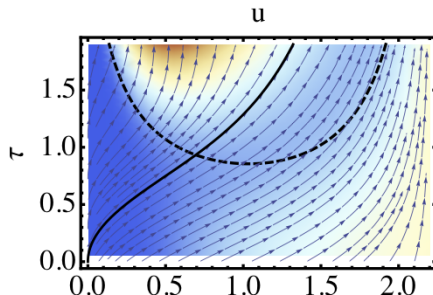
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Recall the complicated nonequilibrium dynamics...

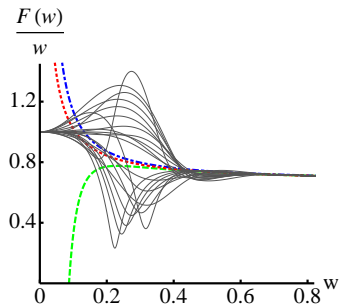
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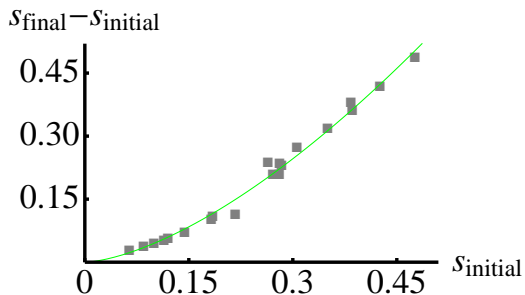
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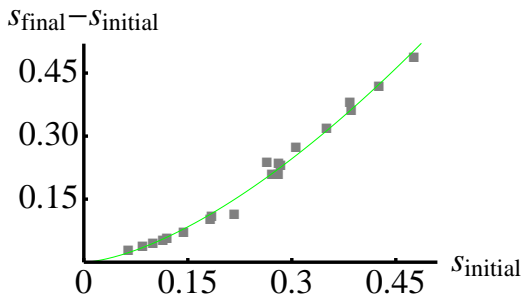
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- The initial entropy turns out to be a key characterization of the initial state
- There seems to be a lot of hidden regularity in the nonequilibrium dynamics
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A numerical criterion for thermalization

- We want to study systematically the properties of the plasma at the point when the dynamics becomes describable by viscous hydrodynamics...
- We adopted a numerical criterion for thermalization

$$\left\| \frac{\tau \frac{d}{d\tau} w}{F_{hydro}^{3^{rd} order}(w)} - 1 \right\| < 0.005$$

- We looked at the following features of thermalization:
 - 1 the dimensionless quantity $w = T_{eff} \cdot \tau$
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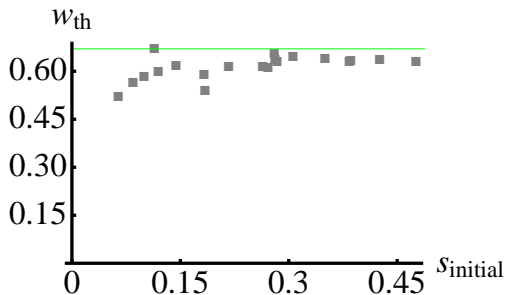
- We want to study systematically the properties of the plasma at the point when the dynamics becomes describable by viscous hydrodynamics...
- We adopted a numerical criterion for thermalization

$$\left\| \frac{\tau \frac{d}{d\tau} w}{F_{hydro}^{3^{rd} order}(w)} - 1 \right\| < 0.005$$

- We looked at the following features of thermalization:
 - 1 the dimensionless quantity $w = T_{eff} \cdot \tau$
 - 2 The thermalization time in units of initial temperature $\tau_{th} \cdot T_{eff}(0)$
 - 3 The temperature at thermalization relative to the initial temperature $T_{th}/T_{eff}(0)$

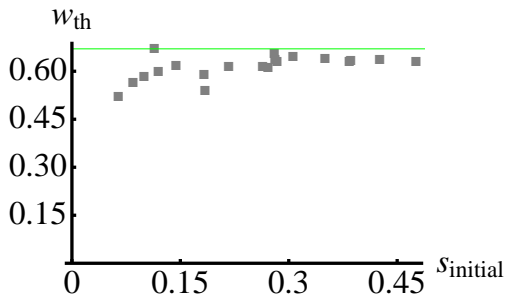
- w at thermalization is approximately constant and for the initial profiles considered does not exceed $w = 0.67$. It seems to decrease for profiles with smaller initial entropy
- N.B. sample initial conditions for hydrodynamics at RHIC ($\tau_0 = 0.25 \text{ fm}$, $T_0 = 500 \text{ MeV}$) assumed in [Broniowski, Chojnacki, Florkowski, Kisiel] correspond to $w = 0.63$
- The pressure anisotropy at thermalization is still sizable

$$\Delta p_L \equiv 1 - \frac{p_L}{\epsilon/3} = 12F(w) - 8 \simeq 12F_{hydro}(w) - 8 \sim 0.72 - 0.73$$



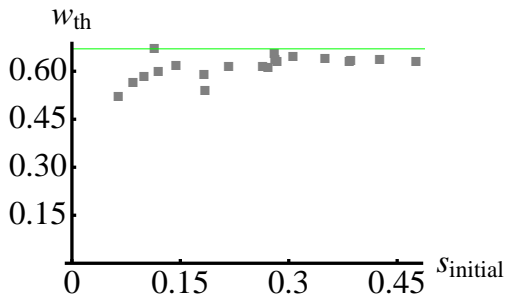
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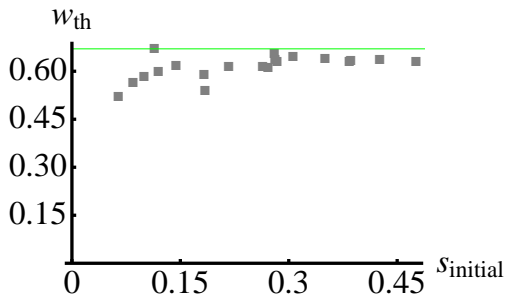
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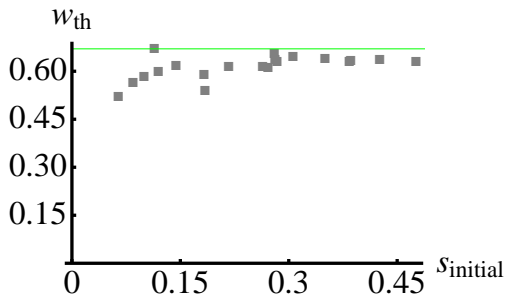
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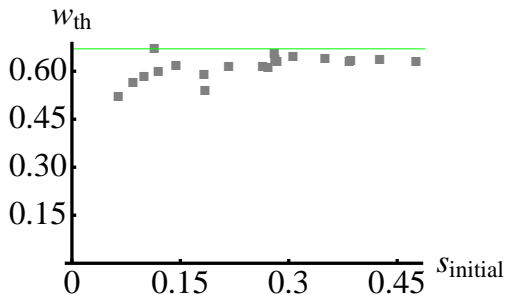
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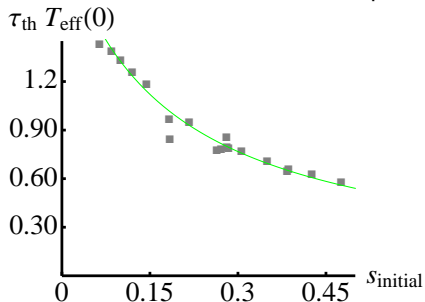
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- Thermalization time in units of the initial *effective* temperature $T_{eff}(0)$

- Again we see a clean dependence on the initial entropy $S_{initial}$
- The data can be fitted by

$$\tau_{th} \cdot T_{eff}(0) \sim \frac{1}{0.48 + 2.74 \cdot S_{initial}}$$

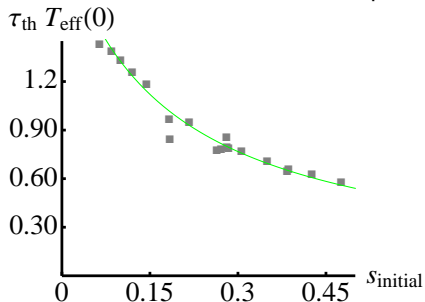
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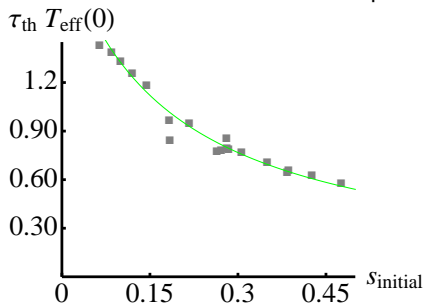
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Temperature at thermalization

- It is interesting to consider the ratio of the temperature at thermalization to the initial effective temperature
- This gives information on which part of the cooling process occurs in the far from equilibrium regime and which part occurs during the hydrodynamic evolution

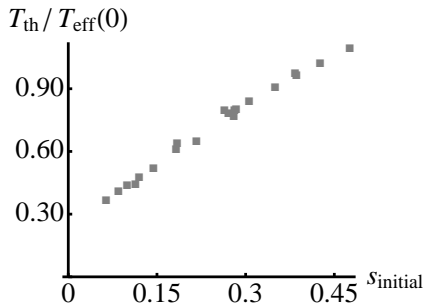
- Note: for initial profiles with large $s_{initial}$, the energy density initially rises and only then falls \rightarrow even for $T_{th}/T_{eff}(0) \sim 1$ there is still sizable nonequilibrium evolution
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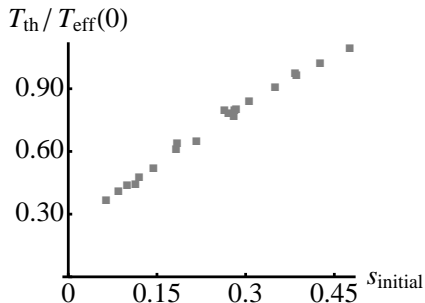
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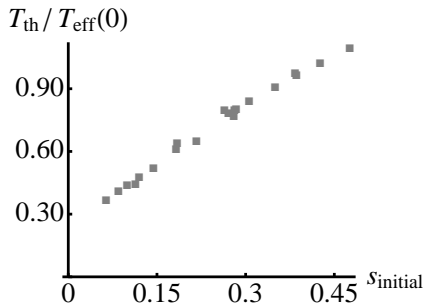
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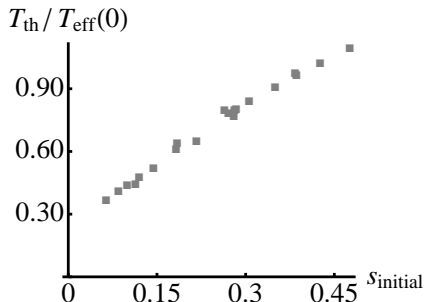
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