Hydrodynamic Approach to Relativistic Heavy Ion Collisions

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Chapter 4

Hydrodynamics and Flow

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1 Introduction and Disclaimer

The main purpose of the lecture was to lead students and young postdocs to the frontier of the hydrodynamic description of relativistic heavy-ion collisions (H.I.C.) in order for them to understand talks and posters presented in the Quark Matter 2008 (QM08) conference in Jaipur, India [1]. So the most recent studies were not addressed in this lecture as they would be presented during the QM08 conference itself. Also, we try to give a very pedagogical lecture here. For the readers who may want to study relativistic hydrodynamics and its application to H.I.C. as an advanced course, we strongly recommend them to consult the references.
Two Lectures

1. Basic aspects of relativistic heavy ion collisions and latest results at LHC
2. Hydrodynamic analysis of relativistic heavy ion collisions at RHIC and LHC
Physics of Relativistic Heavy Ion Collisions
Fate of Smashing Two Nuclei

Multiplicity of charged hadrons $\sim 700$ per unit rapidity in a head-on collision at $\sqrt{s_{NN}} = 200$ GeV
New Era Just Started!

Multiplicity of charged hadrons $\sim 1600$ per unit rapidity in a head-on collision at $\sqrt{s_{NN}}=2.76$ TeV
Primary Goals of Heavy Ion Collisions at Ultrarelativistic Energies

- Understanding of QCD matter under extreme conditions, quark gluon plasma (high T and low $n_B$)
  - Confinement, chiral symmetry breaking
  - Relevant to early universe
  - Properties of matter under extreme conditions governed by strong interaction
  - Unique opportunity
Big Bang vs. Little Bang

3D Hubble expansion

Nearly 1D Hubble expansion* + 2D transverse expansion

Figure adapted from http://www-utap.phys.s.u-tokyo.ac.jp/~sato/index-j.htm

*BJorken (’83)
Big Bang vs. Little Bang (contd.)

<table>
<thead>
<tr>
<th></th>
<th>Big Bang</th>
<th>Little Bang</th>
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</thead>
<tbody>
<tr>
<td><strong>Time Scale</strong></td>
<td>$10^{-5}$ sec $\gg$ m.f.p./c</td>
<td>$10^{-23}$ sec $\sim$ m.f.p./c</td>
</tr>
<tr>
<td><strong>Expansion Rate</strong></td>
<td>$10^{5-6}$/sec</td>
<td>$10^{22-23}$/sec</td>
</tr>
<tr>
<td><strong>Spectrum</strong></td>
<td>Red-shifted (CMB)</td>
<td>Blue-shifted (hadrons)</td>
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Local thermalization is not trivial in heavy ion collisions.

Collective flow is a key to see whether local thermalization is achieved.
Jargon: Centrality

“Centrality” characterizes a collision and categorizes events.

- central event
- peripheral event

Participant-Spectator picture is valid
How to Quantify Centrality

\( N_{\text{part}} \): The number of participants
\( N_{\text{coll}} \): The number of binary collisions

\( N_{\text{part}} \) and \( N_{\text{coll}} \) as a function of impact parameter

197\(^{\text{Au}}\)+197\(^{\text{Au}}\)

PHENIX: Correlation btw. BBC and ZDC signals
Basic Checks of Exp. Data at RHIC
Sufficient Energy Density?

Bjorken energy density

\[ \epsilon_{\text{Bj}}(\tau) = \frac{\langle m_T \rangle \, dN}{\tau \pi R^2 \, dy} \]

- \( \tau \): proper time
- \( y \): rapidity
- \( R \): effective transverse radius
- \( m_T \): transverse mass

Bjorken ('83)

total energy (observables)
Estimated Energy Density at RHIC

Well above $\varepsilon_c$ from lattice simulations in central collision at RHIC

$$\varepsilon_B(\tau) = \frac{\langle m_T \rangle \, dN}{\tau \pi R^2 \, dy}$$

PHENIX('05)
STAR('08)
Matter in (Chemical) Equilibrium?

\[ n_i(T, \mu) = \frac{g}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T]} \pm 1 \]

\[ \langle N_i \rangle = V \left[ n_i^{th}(T, \mu) + \sum_R \Gamma_{R \rightarrow i} n_R(T, \mu) \right] \]

Two fitting parameters: \( T_{ch}, \mu_B \)
Chemical Freezeout Temperature

\[ T = 177 \text{ MeV}, \quad \mu_B = 29 \text{ MeV} \]

Close to \( T_c \) from lattice
Matter in (Kinetic) Equilibrium?

Kinetically equilibrated matter at rest

Isotropic distribution

Kinetically equilibrated matter at finite velocity

Lorentz-boosted distribution
Radial Flow

Kinetic equilibrium inside matter

Pressure gradient
→ Driving force of flow
→ Flow vector points to radial direction

Blast wave model (thermal+boost)
e.g. Sollfrank et al. (’93)
Spectral change is seen in AA!

Power law in pp & dAu

Convex to Power law in Au+Au

• “Consistent” with thermal + boost picture
• Large pressure could be built up in AA collisions

Adapted from O. Barannikova, (QM05)
Thermalized?

Excess of photon radiation

Consistent with inverse slope

$T \sim 220 \pm 20$ MeV
Basic checks cleared

- Energy density can be well above $e_c$.
- “Temperature” can be extracted.
  - Chemically frozen $T \sim T_c$
  - Space-time averaged $T > T_c$
- High pressure could be built up.

Importance of systematic study based on dynamical framework
Discovery of nearly perfect fluidity at RHIC
Hydrodynamics for QGP at Work

Universe May Have Begun as Liquid, Not Gas

Associated Press
Tuesday, April 19, 2005; Page A05

New results from a particle collider suggest that the universe behaved like a liquid in its earliest moments, not the fiery gas that was thought to have pervaded the first microseconds of existence.

Early Universe was a liquid

Quark-gluon blob surprises particle physicists.

by Mark Peplow
news@nature.com

The Universe consisted of a perfect liquid in its first moments, according to results from an atom-smashing experiment.

Scientists at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory on Long Island, New York, have spent five years searching for the quark-gluon plasma that is thought to have filled our Universe in the first microseconds of its existence. Most of them are now convinced they have found it. But, strangely, it seems to be a liquid rather than the expected hot gas.
What is Elliptic Flow?

How does the system respond to spatial anisotropy?

No secondary interaction

Hydro behavior

Spatial Anisotropy

Interaction among produced particles

Momentum Anisotropy
Elliptic Flow in Kinetic Theory

$\lambda = \frac{1}{\sigma \rho} \propto \eta$

$\lambda \rightarrow 0$ : Ideal hydro

$\sigma \rightarrow \infty$ : strongly interacting system

$v_2$ is

- generated through secondary collisions
- saturated in the early stage
- sensitive to cross section ($\sim 1/m.f.p. \sim 1/\text{viscosity}$)

Zhang-Gyulassy-Ko ('99)
Arrival at Hydrodynamic Limit

Experimental data reach hydrodynamic limit curve for the first time at RHIC.

\[ \varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle x^2 + y^2 \rangle} \]

\[ \frac{v_2}{\varepsilon} = \frac{\text{momentum anisotropy}}{\text{spatial anisotropy}} = \frac{\text{output}}{\text{input}} = \text{response} \]

Experimental data reach hydrodynamic limit curve for the first time at RHIC.
Contact: Karen McNulty Walsh, (631) 344-8350 or Mona S. Rowe, (631) 344-5056

RHIC Scientists Serve Up “Perfect” Liquid

New state of matter more remarkable than predicted -- raising many new questions

April 18, 2005

TAMPA, FL -- The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) -- a giant atom “smasher” located at the U.S. Department of Energy’s Brookhaven National Laboratory -- say they’ve created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC’s heavy ion collisions...
New Results at LHC
Transverse Energy at LHC

~2.5 times larger than at RHIC

$\varepsilon \sim 16 \text{ GeV/fm}^3 \ @ \text{LHC}$
$\varepsilon \sim 5.4 \text{ GeV/fm}^3 \ @ \text{RHIC}$
assuming $\tau = 1 \text{ fm}/c$

Adapted from talk by A. Toia (ALICE) at QM2011
Almost identical to RHIC!?

~30% increase from RHIC to LHC
Initial Fluctuation

Ideal situation:
Initial eccentricity drives elliptic flow.

Real situation:
Initial spatial fluctuation causes higher order harmonics.
Profile itself determines “event” planes to respond.

Adapted from talk by J.Jia at QM2011
Higher Harmonics in Wide $p_T$ Range

Higher harmonics are measured up to 10 GeV/c

Adapted from talk by J. Jia (ATLAS) at QM2011
Two Particle Correlation

Adapted from talk by J.Jia(ATLAS) at QM2011
Response to Eccentricity

Linear increase with multiplicity!?

Adapted from talk by J. Velkovska (CMS) at QM2011
Summary of Flow Phenomena

- An almost perfect fluid is created at RHIC for the first time.
  - Concept of strongly interacting quark-gluon many body system is established.
  - How perfect?
- First LHC results just appear!
- Fluctuation and dissipation would be a key to understand the dynamics and, in turn, transport properties of the QGP.
Remarks on this Estimate

- **Even e^+e^- or pp data can be fitted well!**
  
  See, e.g., Becattini&Heinz('97)

- **What is the meaning of fitting parameters?**
  
  See, e.g., Rischke('02), Koch('03)

- **Why so close to T_c?**
  
  → No chemical eq. in hadron phase!?
  
  → Essentially dynamical problem!

\[ \partial \cdot u(x) \quad \leftrightarrow \quad \sum_j \langle \sigma_{ij} v_{ij} \rangle \rho_j \]

- **expansion rate**
- **reaction rate**
Remarks on this Estimate

- Not necessary to be thermalized completely
  - Results from hadronic cascade models.
- How is radial flow generated dynamically?
- Finite radial flow even in pp collisions?
  - \((T, v_T) \sim (140\text{MeV}, 0.2)\)
  - Is blast wave reliable quantitatively?
- Consistency?
  - Chi square minimum located a different point for \(f\) and \(W\)
- Flow profile? Freezeout hypersurface? Sudden freezeout?
One data point kills most of models
As \sim 2.1 \text{ times many as at RHIC}

Higher Order Harmonics

- $v_n$ (n=2,3,4 and 5) are observed at LHC
- $v_n$ (n>3) mainly from initial fluctuation effects
- $v_2 \sim v_3$ in central events

ALICE Collaboration
arXiv:1105.3865