Introduction to and Recent Progress in Lattice QCD

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lattice field theory talk

examples to reach the physical limit (physical mass & continuum)

Outline

User's guide to lattice QCD results

- Full lattice results have three main ingredients

1. (tech.) technically correct: control systematics (users can't prove)
2. (m_q) physical quark masses: $m_s/m_{ud} \approx 28$ (and $m_c/m_s \approx 12$)
3. (cont.) continuum extrapolated: at least 3 points with $c \cdot a^n$

only a few full results (spectrum, m_q, nature, T_c, EoS, curvature)

ad 1: obvious condition, otherwise forget it
ad 2: difficult (CPU demanding) to reach the physical u/d mass
BUT even with non-physical quark masses: meaningful questions
e.g. in a world with $M_\pi = M_\rho/2$ what would be M_N/M_π
these results are universal, do not depend on the action/technique
ad 3: non-continuum results contain lattice artefacts
(they are good for methodological studies, they just "inform" you)

User's guide to lattice QCD results

- troubleshooting

i. clarify if all three conditions were satisfied

ii. if yes: OK with any scale setting (error estimates can be tricky)

iii. if out of the above three ingredients one was missing:
if (1: tech.) was missing: forget it
if (2: m_q) was missing: reliable answer to a well defined case
if (3: cont.) was missing: ask to carry out the continuum limit
show the scaling $c \cdot a^n$ in the scaling regime
n is known from theory c is provided by the simulations
that is why we need at least 3 different lattice spacings

iv. if out of the above three ingredients two were missing: well ...

The origin of mass of the visible Universe

source of the mass for ordinary matter (not a dark matter talk)

basic goal of LHC (Large Hadron Collider, Geneva Switzerland):

"to clarify the origin of mass"

e.g. by finding the Higgs particle, or by alternative mechanisms
order of magnitudes: 27 km tunnel and O(10) billion dollars

The vast majority of the mass of ordinary matter

ultimate (Higgs or alternative) mechanism: responsible for the mass of the leptons and for the mass of the quarks

interestingly enough: just a tiny fraction of the visible mass (such as stars, the earth, the audience, atoms)
electron: almost massless, $\approx$1/2000 of the mass of a proton
quarks (in ordinary matter): also almost massless particles

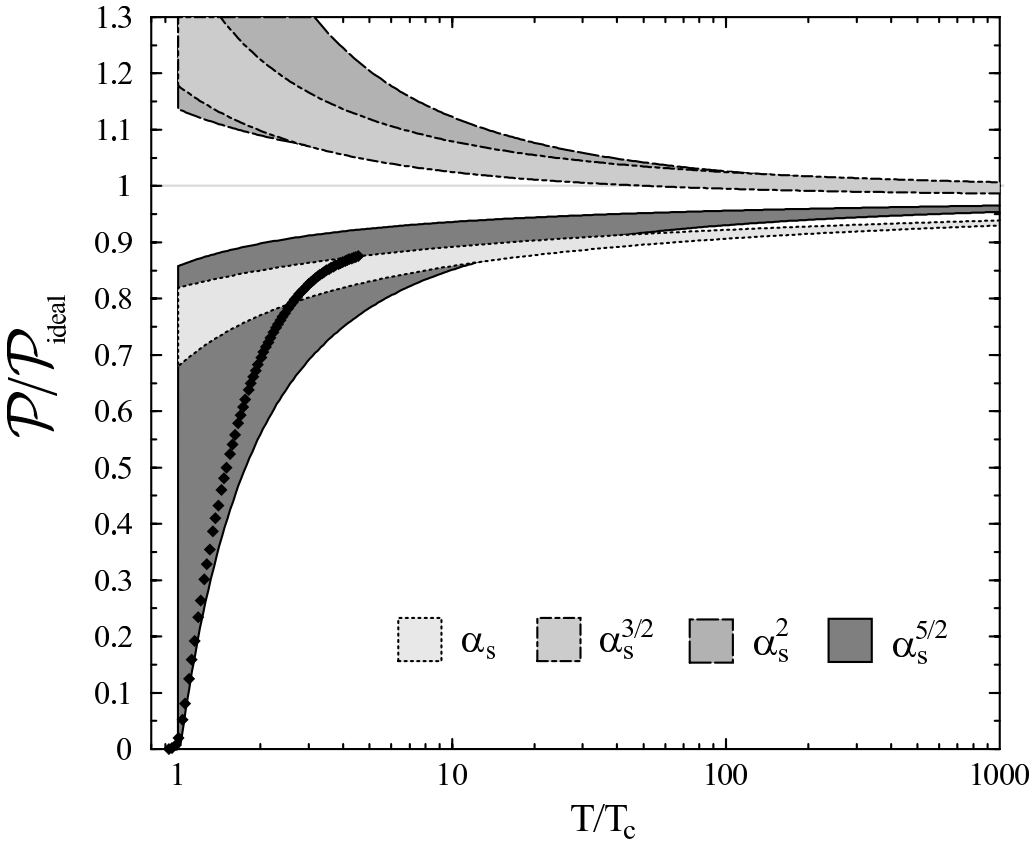
the vast majority (about 95%) comes through another mechanism
$\Longrightarrow$ this mechanism and this 95% will be the main topic of this talk

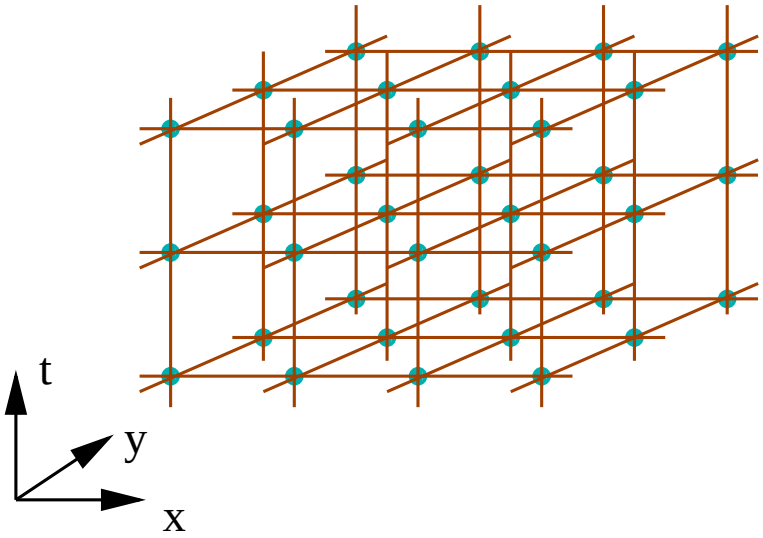
quantum chromodynamics (QCD, strong interaction) on the lattice

QCD: need for a systematic non-perturbative method

in some cases: good perturbative convergence; in other cases: bad

pressure at high temperatures converges at T=10^{300} MeV





fine lattice to resolve the structure of the proton ($\lesssim$0.1 fm)
few fm size is needed
50-100 points in 'xyz/t' directions
$a \Rightarrow a/2$ means 100-200$\times$CPU

mathematically
10^9 dimensional integrals

advanced techniques,
good balance and
several Tflops are needed

Importance sampling

$$Z = \int \prod_{n,\mu} [dU_\mu(n)] e^{-S_g} \det(M[U])$$

we do not take into account all possible gauge configuration

each of them is generated with a probability $\propto$ its weight

importance sampling, Metropolis algorithm:
(all other algorithms are based on importance sampling)

$$P(U \to U') = \min\left[1, \exp(-\Delta S_g) \det(M[U'])/\det(M[U])\right]$$

gauge part: trace of 3$\times$3 matrices (easy, without M: quenched)
fermionic part: determinant of $10^6 \times 10^6$ sparse matrices (hard)

more efficient ways than direct evaluation (Mx=a), but still hard

Hadron spectroscopy in lattice QCD

Determine the transition amplitude between:
having a "particle" at time 0 and the same "particle" at time t
$\Rightarrow$ Euclidean correlation function of a composite operator $\mathcal{O}$:

$$C(t) = \langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle$$

insert a complete set of eigenvectors $|i\rangle$

$$= \sum_i \langle 0|\mathrm{e}^{Ht}\,\mathcal{O}(0)\,\mathrm{e}^{-Ht}|i\rangle\langle i|\mathcal{O}^\dagger(0)|0\rangle = \sum_i |\langle 0|\mathcal{O}^\dagger(0)|i\rangle|^2\,\mathrm{e}^{-(E_i-E_0)t},$$

where $|i\rangle$: eigenvectors of the Hamiltonian with eigenvalue E_i.

and $$\mathcal{O}(t) = \mathrm{e}^{Ht}\,\mathcal{O}(0)\,\mathrm{e}^{-Ht}.$$

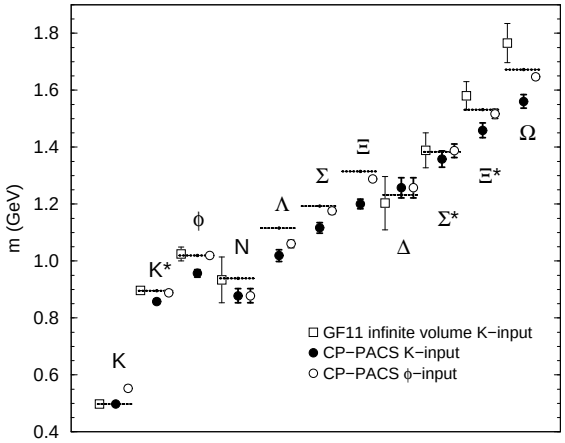
t large $\Rightarrow$ lightest states (created by $\mathcal{O}$) dominate: $C(t) \propto e^{-M\cdot t}$
t large $\Rightarrow$ exponential fits or mass plateaus M_t=log[C(t)/C(t+1)]

Quenched results

QCD is 35 years old $\Rightarrow$ properties of hadrons (Rosenfeld table)

non-perturbative lattice formulation (Wilson) immediately appeared
needed 20 years even for quenched result of the spectrum (cheap)
instead of det(M) of a $10^6 \times 10^6$ matrix trace of 3×3 matrices

always at the frontiers of computer technology:
GF11: IBM "to verify quantum chromodynamics" (10 Gflops, '92)
CP-PACS Japanese purpose made machine (Hitachi 614 Gflops, '96)

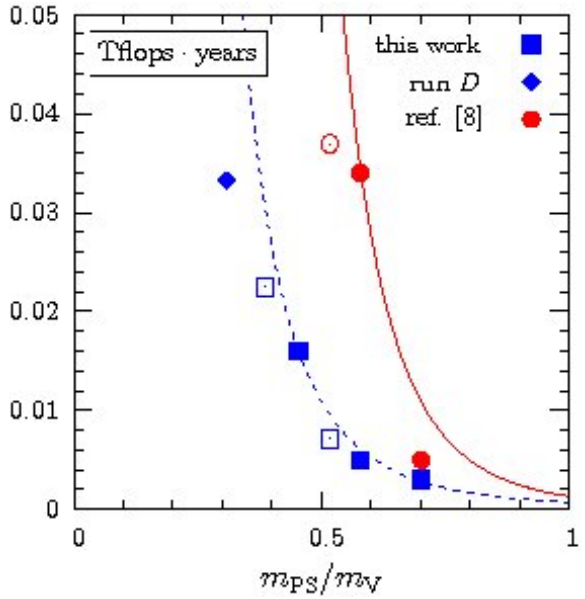


the $\approx$10% discrepancy was believed to be a quenching effect

Difficulties of full dynamical calculations

though the quenched result can be qualitatively correct
uncontrolled systematics $\Rightarrow$ full "dynamical" studies
by two-three orders of magnitude more expensive (balance)
present day machines offer several hundreds of Tflops

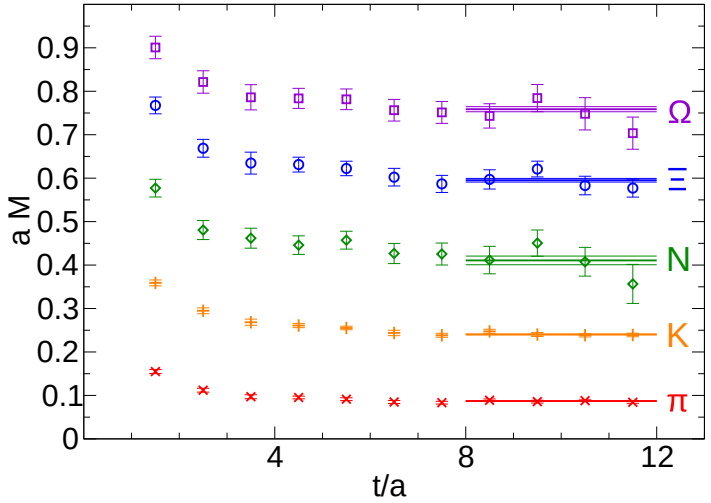
no revolution but evolution in the algorithmic developments
Berlin Wall '01: it is extremely difficult to reach small quark masses:



Scale setting and masses in lattice QCD

in meteorology, aircraft industry etc. grid spacing is set by hand
in lattice QCD we use g,m_{ud} and m_s in the Lagrangian ('a' not)
measure e.g. the vacuum mass of a hadron in lattice units: M_Ωa
since we know that M_Ω=1672 MeV we obtain 'a'

masses are obtained by correlated fits (choice of fitting ranges)
illustration: mass plateaus at the smallest $M_\pi \approx$190 MeV (noisiest)



volumes and masses for unstable particles: avoided level crossing
decay phenomena included: in finite V shifts of the energy levels

Parameters of the Lagrangian

three parameters of the Lagrangian: coupling strength g, m_{ud} and m_s

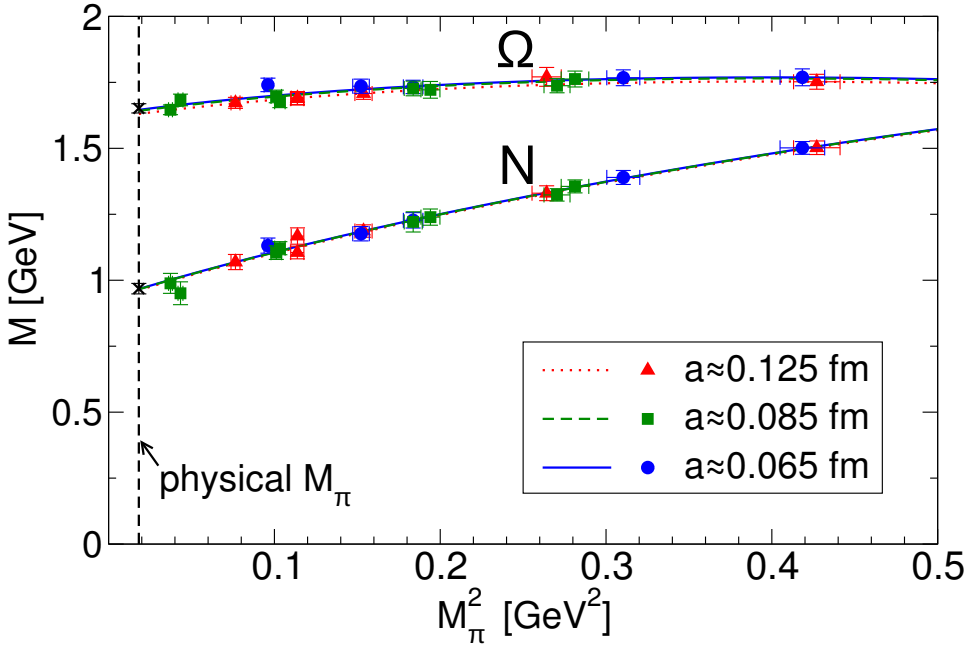
asymptotic freedom: for large cutoff (small lattice spacing) g is small
in this region the results are already independent of g (scaling)

QCD predicts only dimensionless combinations (e.g. mass ratios)
$\Rightarrow$ we can eliminate g as an input parameter by taking ratios

the pion mass M_π is particularly sensitive to m_{ud}
the kaon mass M_K is particularly sensitive to m_s

relatively easy to set the strange quark mass m_s to its physical value
it is very CPU demanding to approach the physical m_{ud}

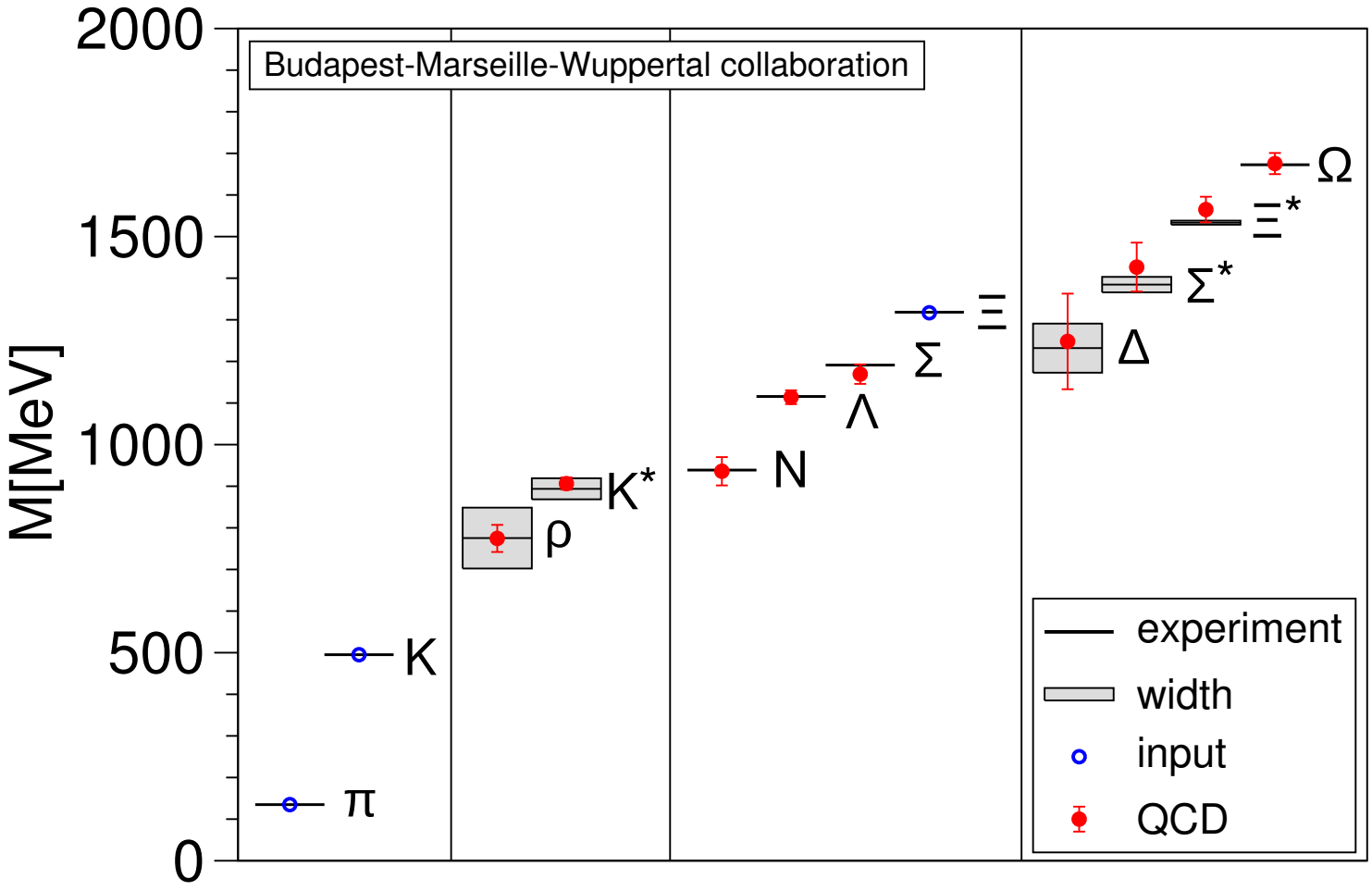
altogether 15 points for each hadrons



smooth extrapolation to the physical pion mass (or m_{ud})
small discretization effects (three lines barely distinguishable)

continuum extrapolation goes as $c \cdot a^n$ and it depends on the action
in principle many ways to discretize (derivative by 2,3... points)
goal: have large n and small c (in this case $n = 2$ and c is small)

Final result for the hadron spectrum



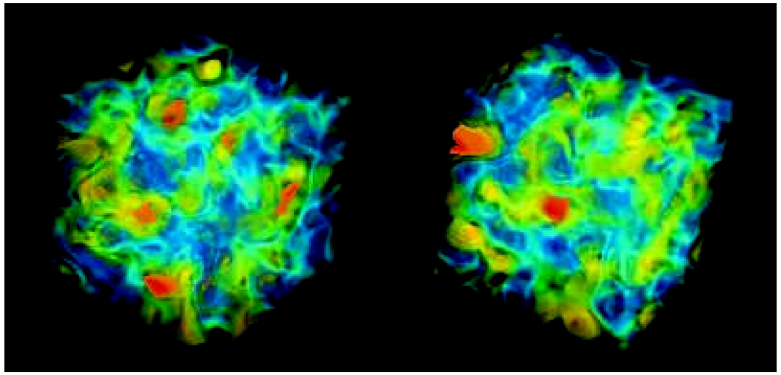
Breakthrough of the Year

Proton's Mass 'Predicted'

STARTING FROM A THEORETICAL DESCRIPTION OF ITS INNARDS, physicists precisely calculated the mass of the proton and other particles made of quarks and gluons. The numbers aren't new; experimenters have been able to weigh the proton for nearly a century. But the new results show that physicists can at last make accurate calculations of the ultracomplex strong force that binds quarks.

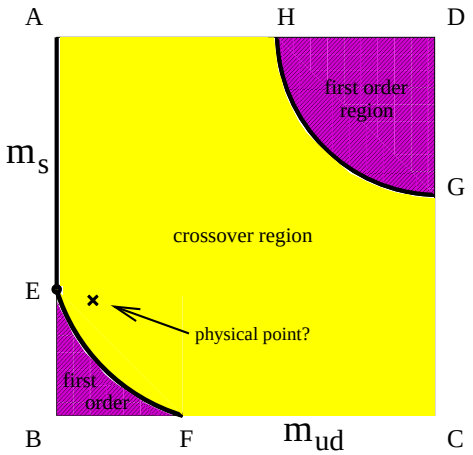
In simplest terms, the proton comprises three quarks with gluons zipping between them to convey the strong force. Thanks to the uncertainties of quantum mechanics, however, myriad gluons and quark-antiquark pairs flit into and out of existence within a proton in a frenzy that's nearly impossible to analyze but that produces 95% of the particle's mass.

To simplify matters, theorists from France, Germany, and Hungary took an approach known as "lattice quantum chromodynamics." They modeled continuous space and time as a four-dimensional array of points—the lattice—and confined the quarks to the points and the gluons to the links between them. Using supercomputers, they reckoned the masses of the proton and other particles to a precision of about 2%—a tenth of the uncertainties a decade ago—as they reported in November.



In 2003, others reported equally precise calculations of more-esoteric quantities. But by calculating the familiar proton mass, the new work signals more broadly that physicists finally have a handle on the strong force.

Phase diagram and its uncertainties



physical quark masses: important for the nature of the transition

n_f=2+1 theory with m_q=0 or ∞ gives a first order transition

intermediate quark masses: we have an analytic cross over (no χPT)

F.Karsch et al., Nucl.Phys.Proc. 129 ('04) 614; G.Endrodi et al. PoS Lat'07 182('07);

de Forcrand, S. Kim, O. Philipsen, Lat'07 178('07)

continuum limit is important for the order of the transition:

n_f=3 case (standard action, N_t=4): critical $m_{ps}\approx$300 MeV

different discretization error (p4 action, N_t=4): critical $m_{ps}\approx$70 MeV

the physical pseudoscalar mass is just between these two values

Lattice formulation

$$Z = \int dU d\Psi d\bar{\Psi} e^{-S_E} \tag{1}$$

S_E is the Euclidean action

Parameters (the lattice spacing does not appear explicitely):
gauge coupling g
quark masses m_i ($i = 1..N_f$)
(Chemical potentials μ_i)
Volume (V) and temperature (T)

Finite $T \leftrightarrow$ finite temporal lattice extension

$$T = \frac{1}{N_t a} \tag{2}$$

Continuum limit: $a \to 0 \Longleftrightarrow N_t \to \infty$; CPU demand scales as N_t^{8-12}

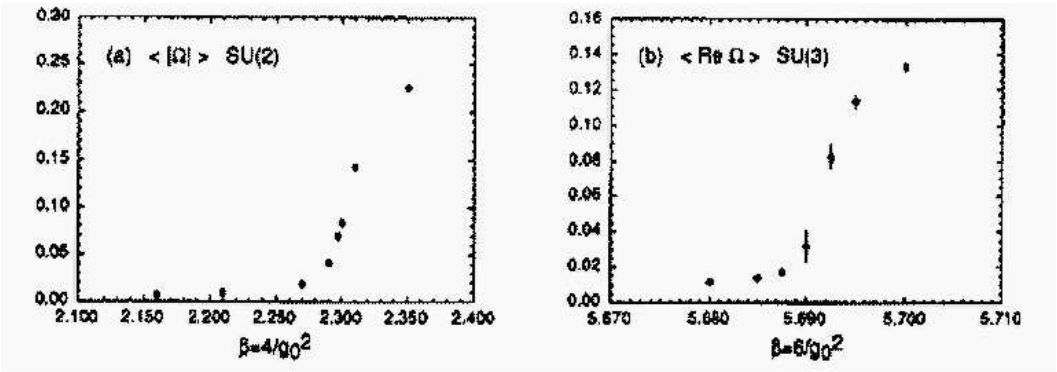
Finite-size scaling theory

problem with phase transitions in Monte-Carlo studies

Monte-Carlo applications for pure gauge theories ($V = 24^3 \cdot 4$)

existence of a transition between confining and deconfining phases:

Polyakov loop exhibits rapid variation in a narrow range of β



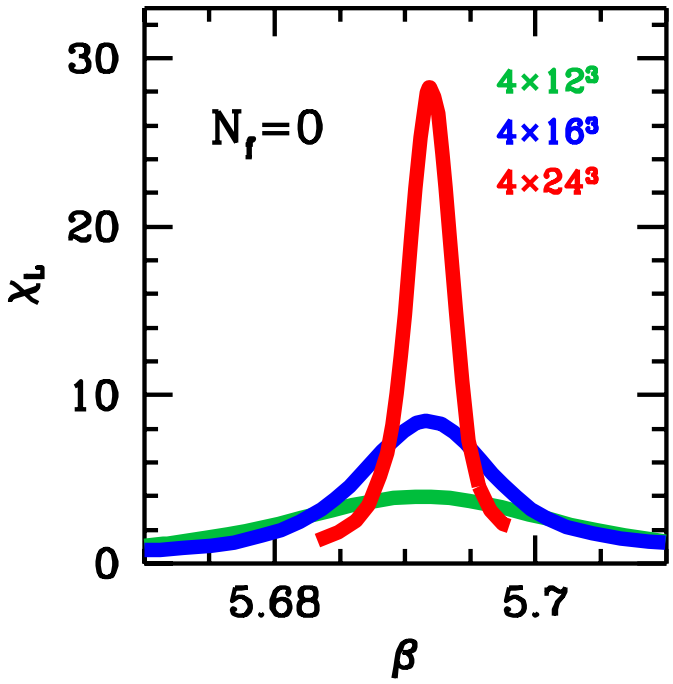
- theoretical prediction: SU(2) second order, SU(3) first order

$\Longrightarrow$ Polyakov loop behavior: SU(2) singular power, SU(3) jump

data do not show such characteristics!

Finite size scaling in the quenched theory

look at the susceptibility of the Polyakov-line
first order transition (Binder) $\Longrightarrow$ peak width $\propto$ 1/V, peak height $\propto$ V



finite size scaling shows: the transition is of first order

The nature of the QCD transition

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675

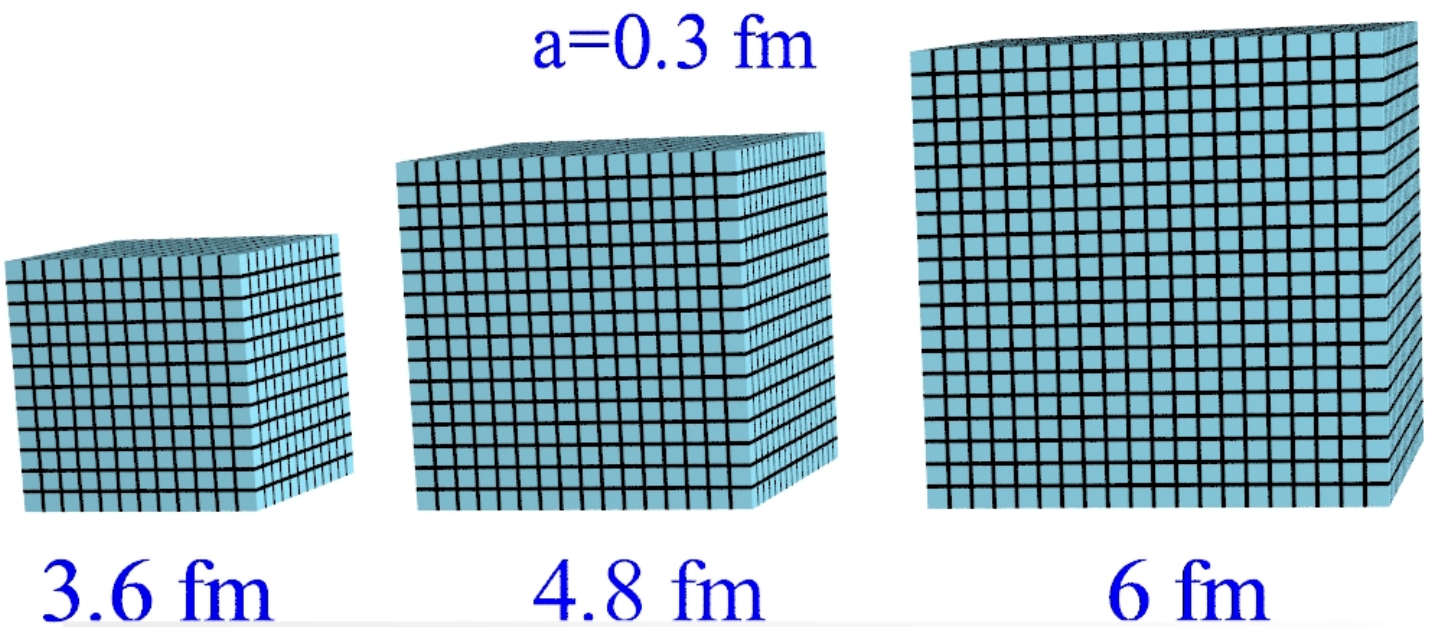
finite size scaling study of the chiral condensate (susceptibility)

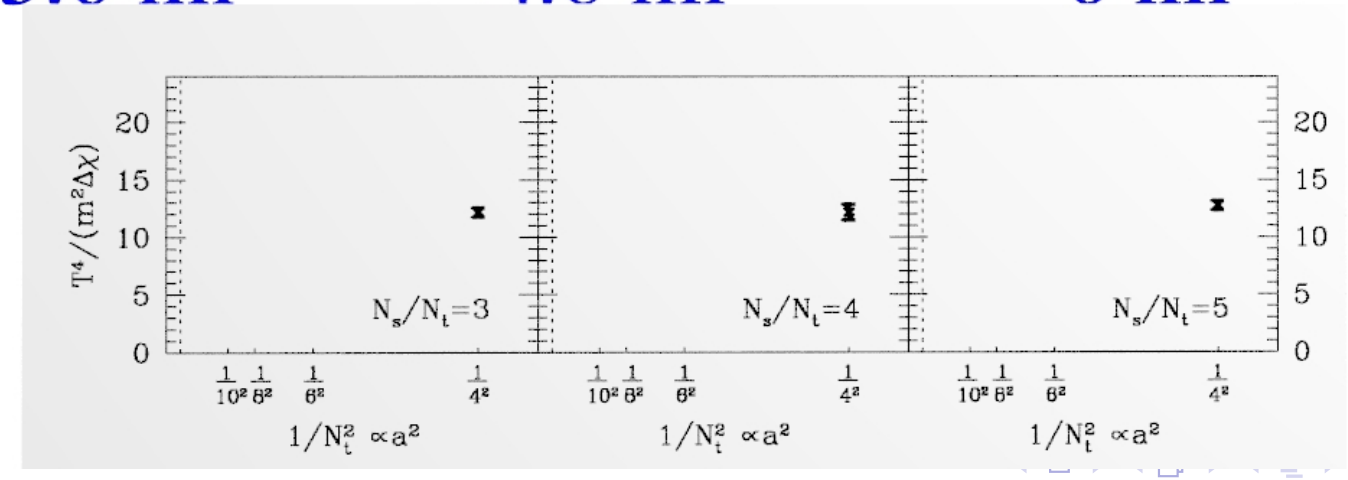
$$\chi = (T/V)\partial^2 \log Z/\partial m^2$$

phase transition: finite V analyticity $V \to \infty$ increasingly singular
(e.g. first order phase transition: height $\propto$ V, width $\propto$ 1/V)
for an analytic cross-over χ does not grow with V

two steps (three volumes, four lattice spacings):
a. fix V and determine χ in the continuum limit: a=0.3,0.2,0.15,0.1fm
b. using the continuum extrapolated χ_{max}: finite size scaling

Approaching the continuum limuit





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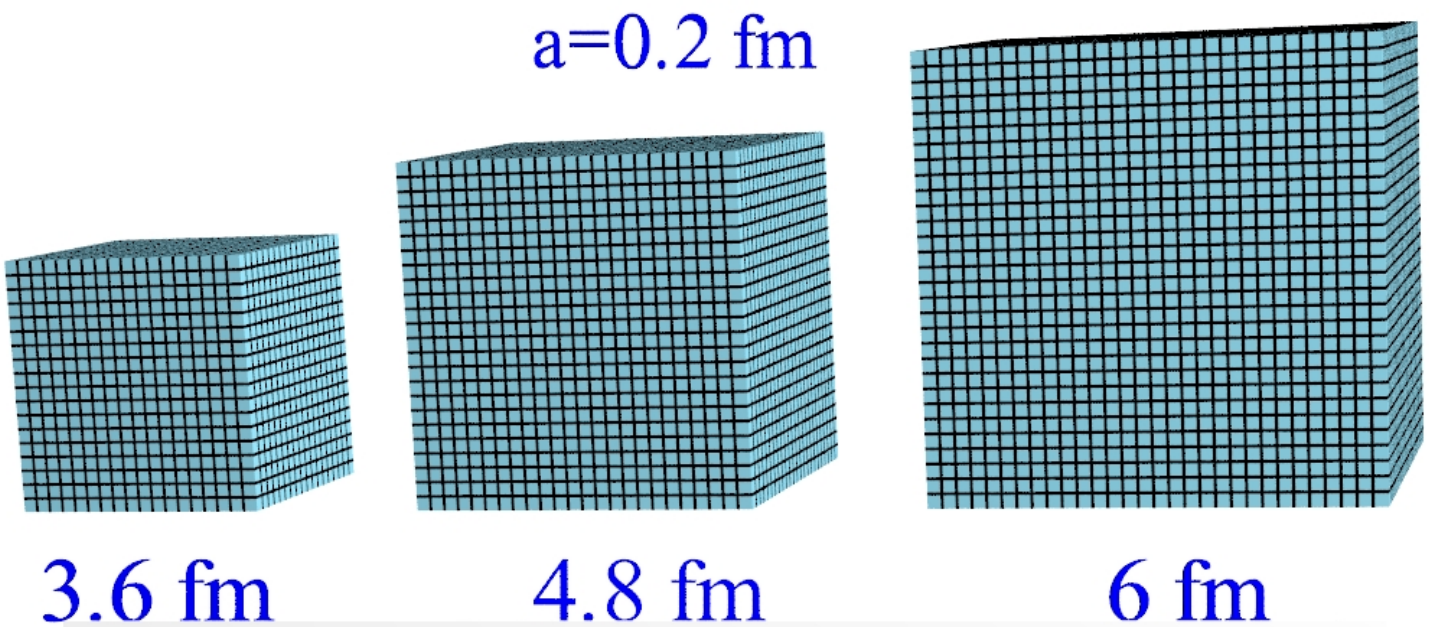

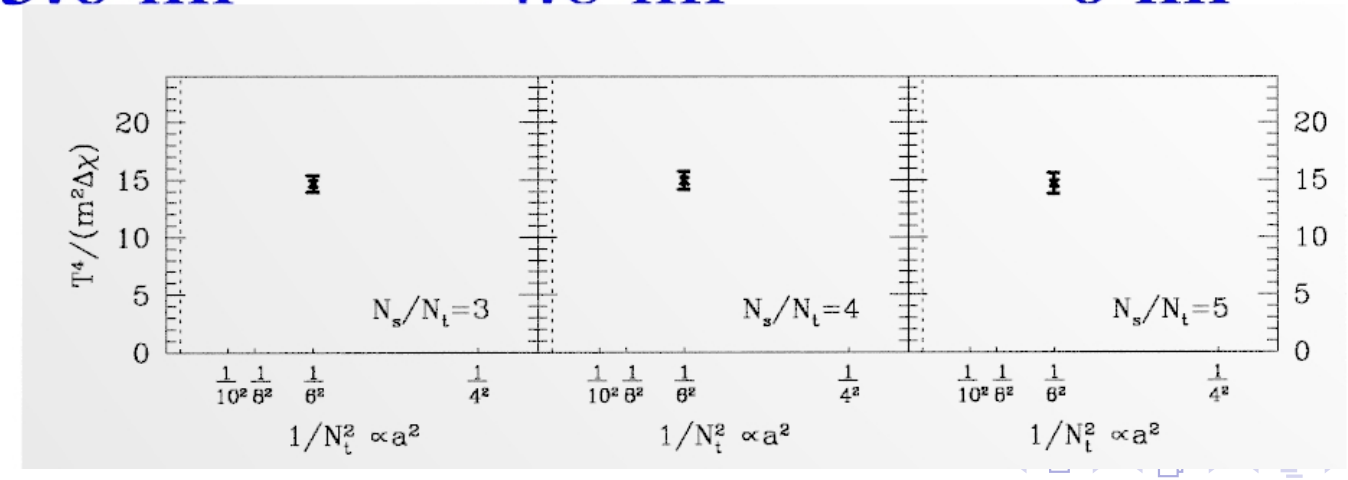

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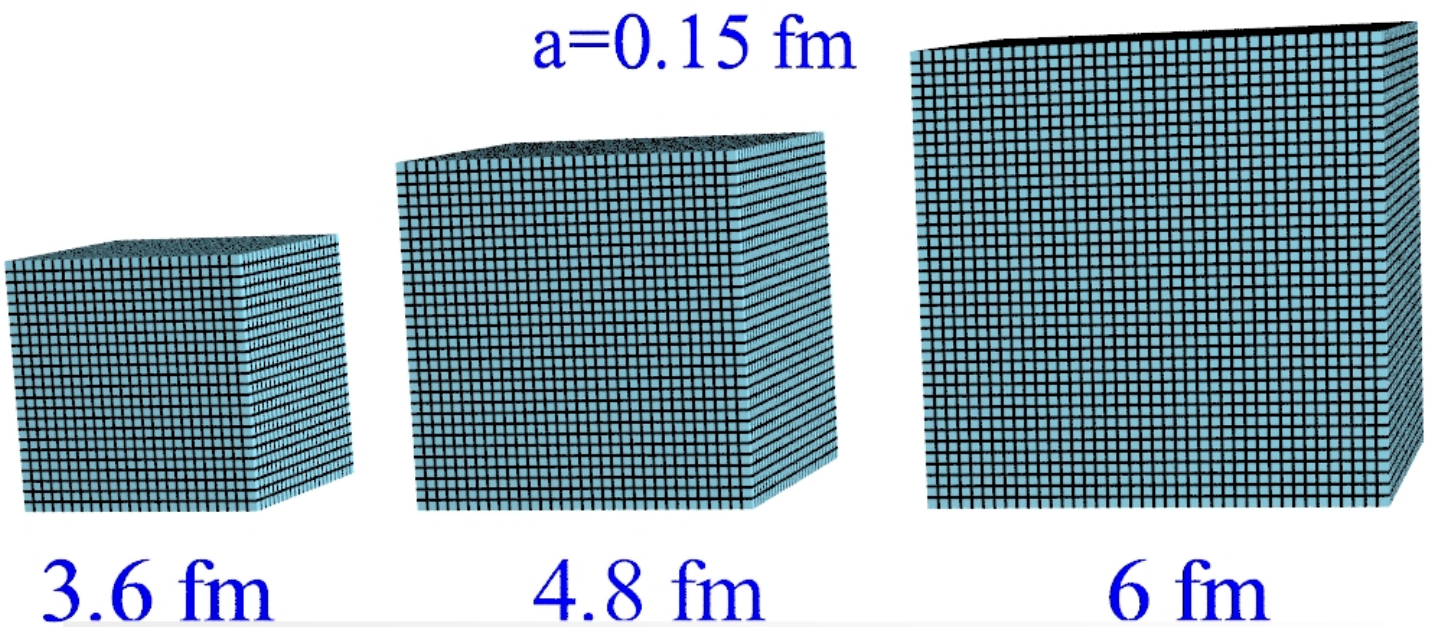
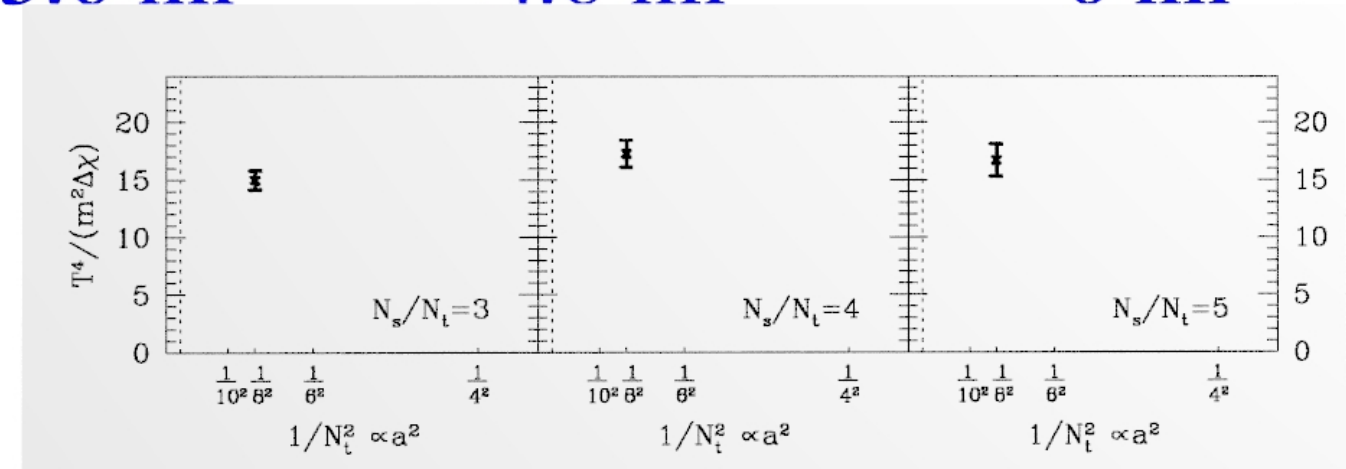
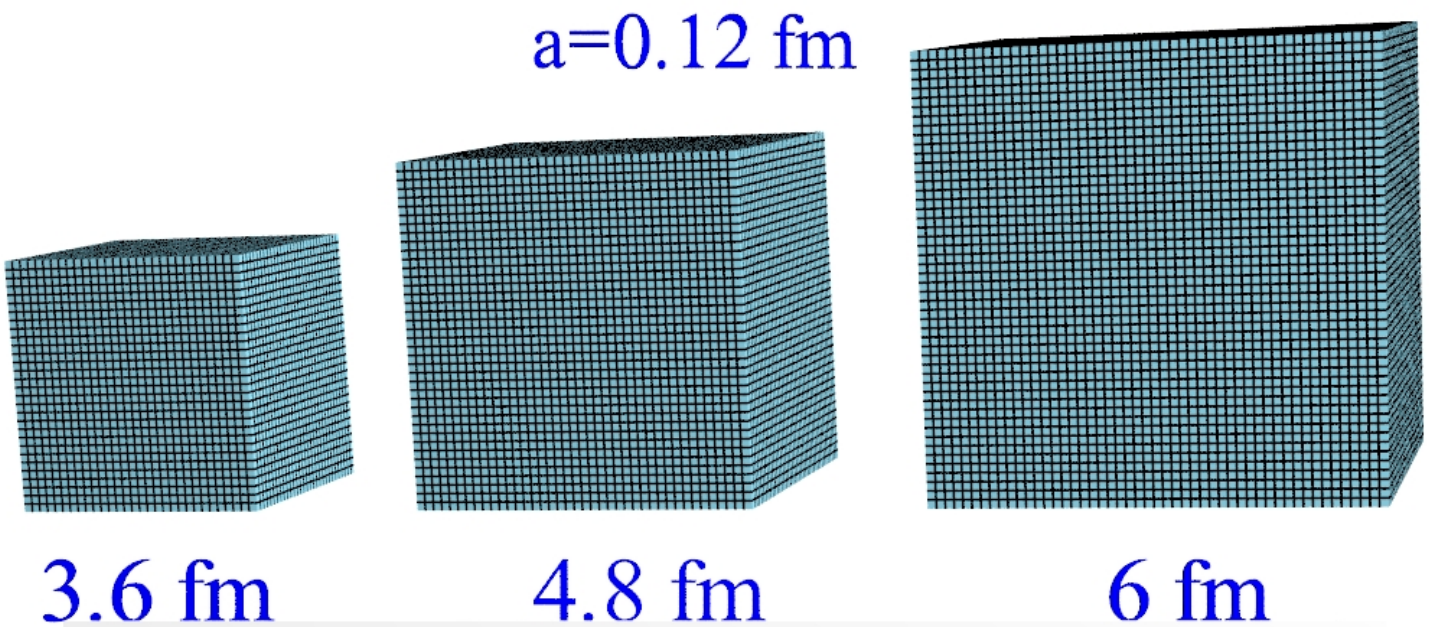
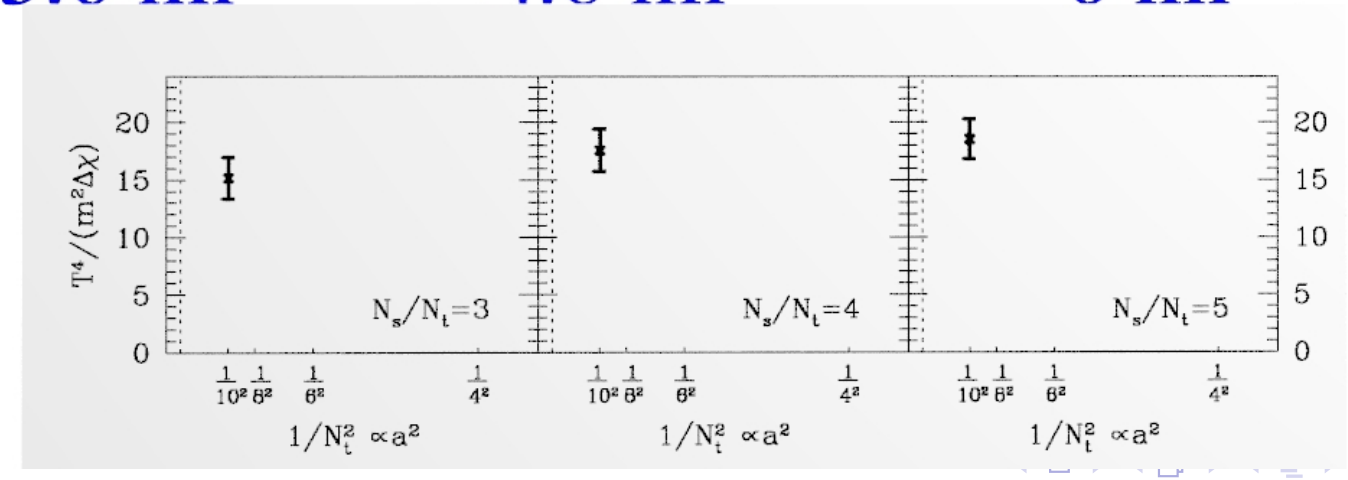




Approaching the continuum limuit





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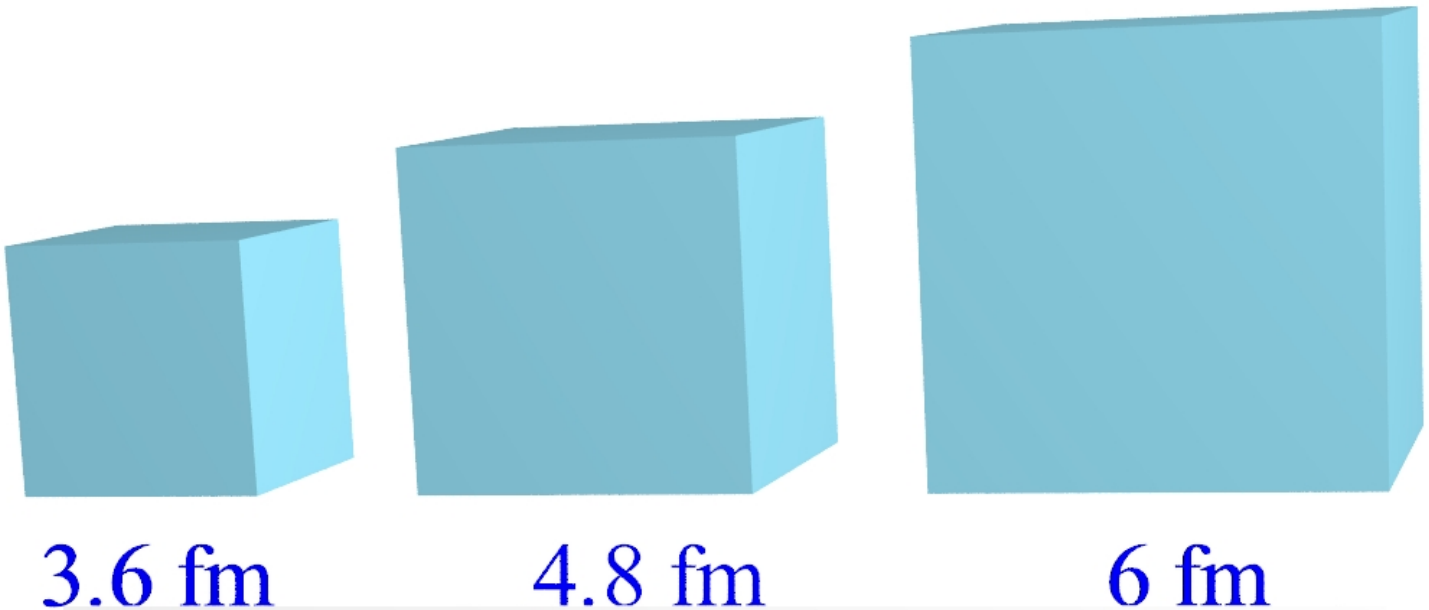
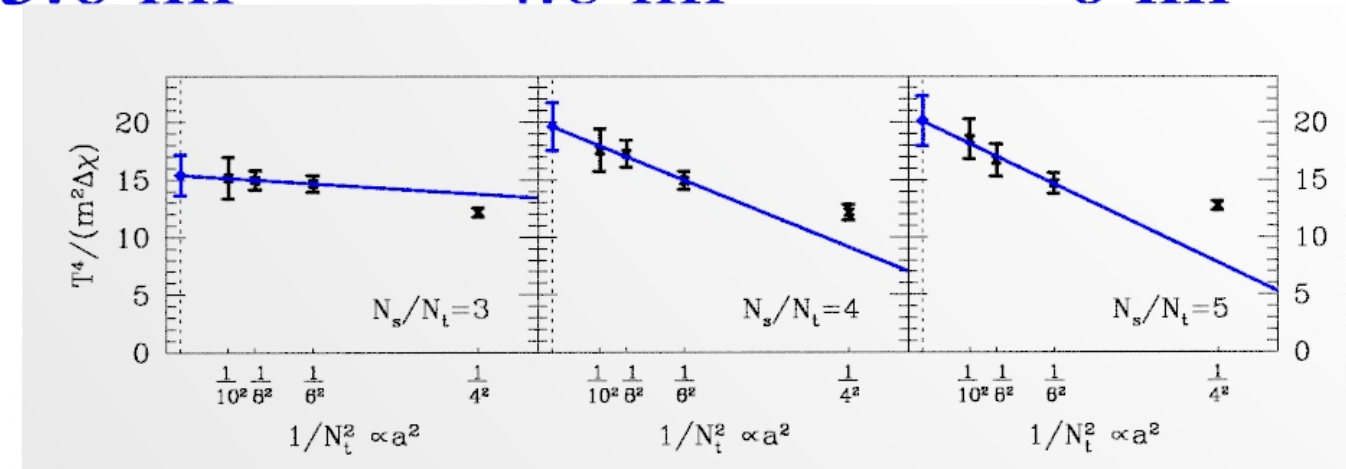
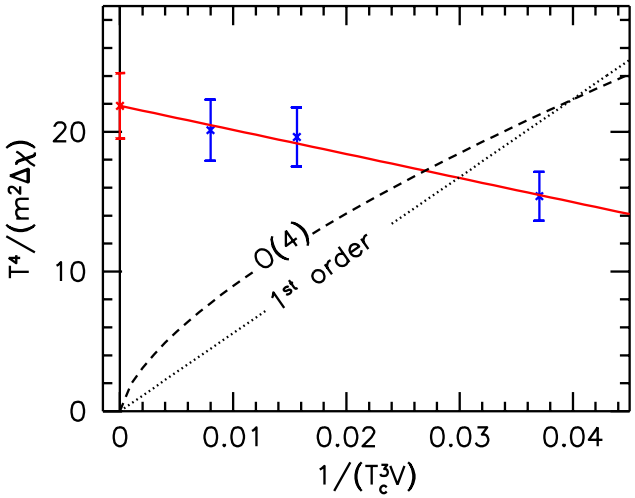




The nature of the QCD transition: analytic

- finite size scaling analysis with continuum extrapolated $T^4/m^2\Delta\chi$



the result is consistent with an approximately constant behavior for a factor of 5 difference within the volume range
chance probability for 1/V is 10^{-19} for O(4) is $7 \cdot 10^{-13}$
continuum result with physical quark masses in staggered QCD:

the QCD transition is a cross-over

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675

The nature of the QCD transition

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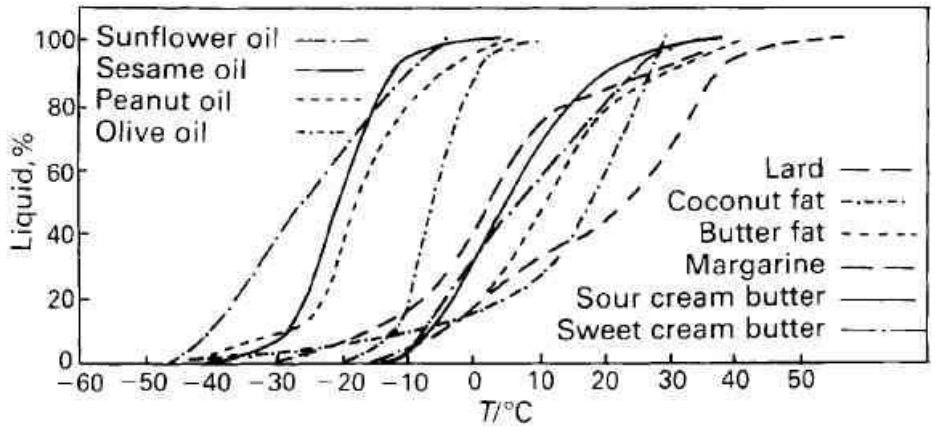
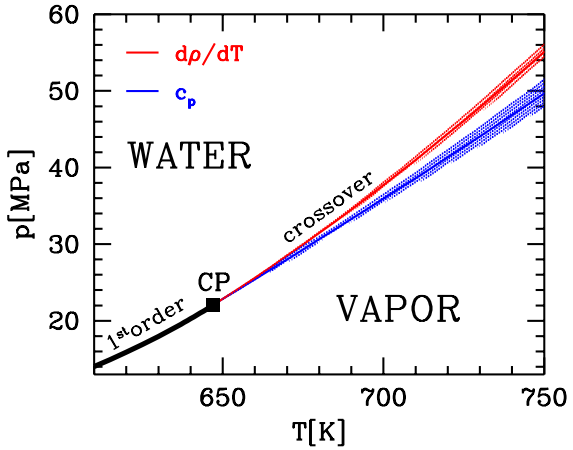
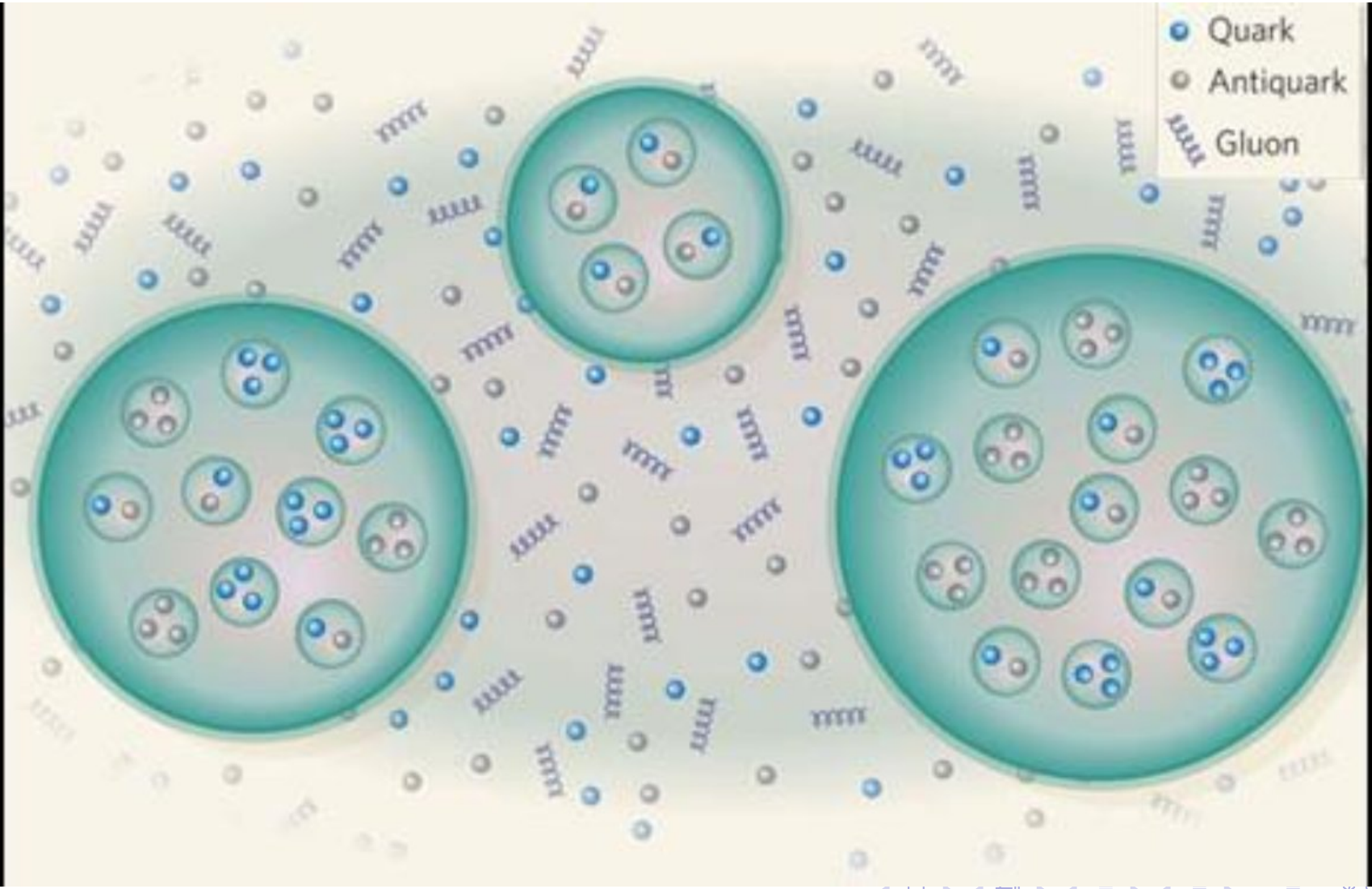
analytic transition (cross-over) $\Rightarrow$ it has no unique T_c:
examples: melting of butter (not ice) & water-steam transition



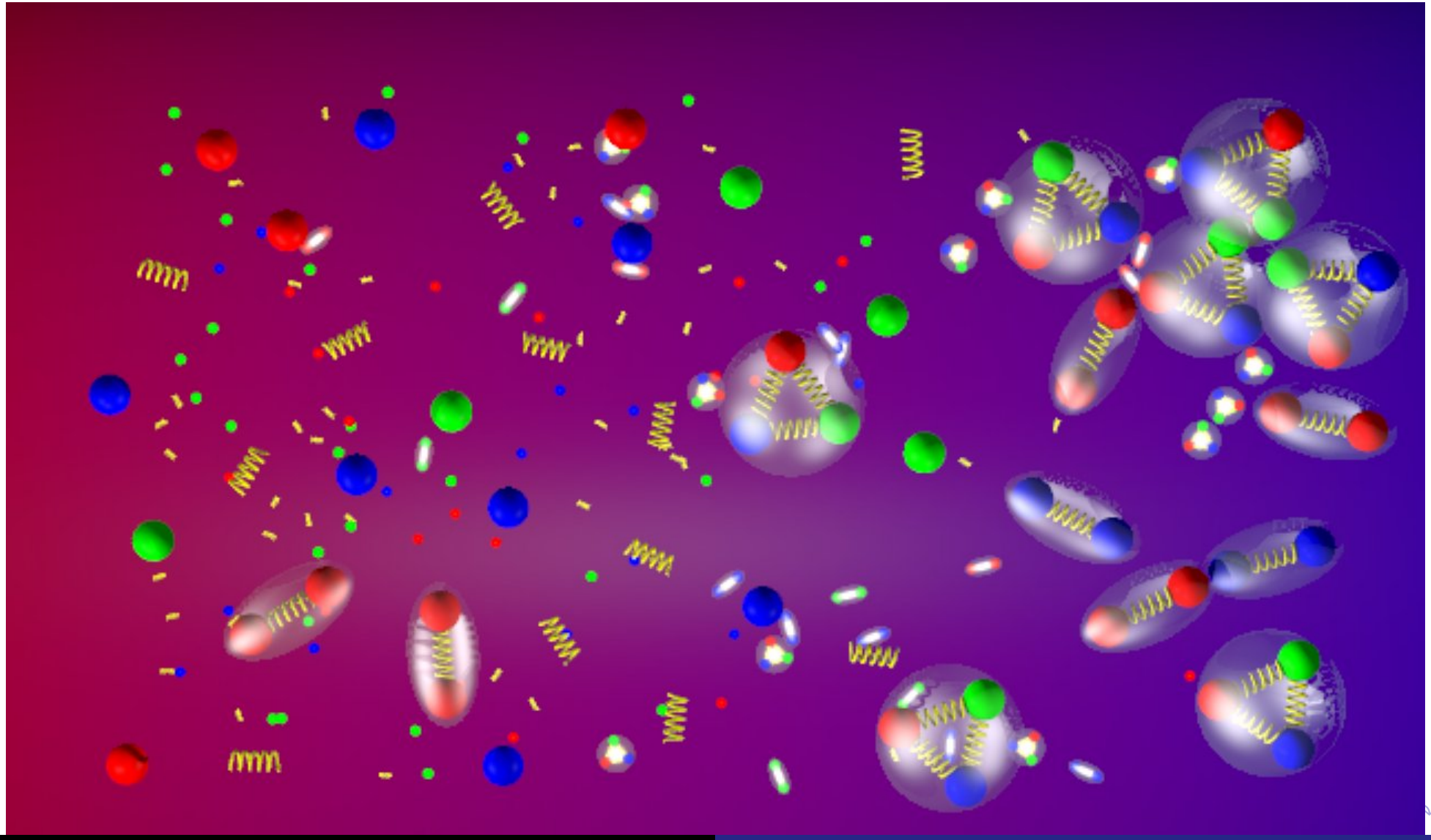


above the critical point c_p and $d\rho/dT$ give different T_cs.
QCD: chiral & quark number susceptibilities or Polyakov loop
they result in different T_c values $\Rightarrow$ physical difference

Possible first order scenario with critical bubbles



Reality: smooth analytic transition (cross-over)



Literature: discrepancies between T_c

Bielefeld-Brookhaven-Riken-Columbia Collaboration:

M. Cheng et.al, Phys. Rev. D74 (2006) 054507

T_c from $\chi_{\bar{\psi}\psi}$ and Polyakov loop, from both quantities:

T_c=192(7)(4) MeV

Bielefeld-Brookhaven-Riken-Columbia merged with MILC: 'hotQCD'

Wuppertal-Budapest group: WB

Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, Phys. Lett. B. 643 (2006) 46

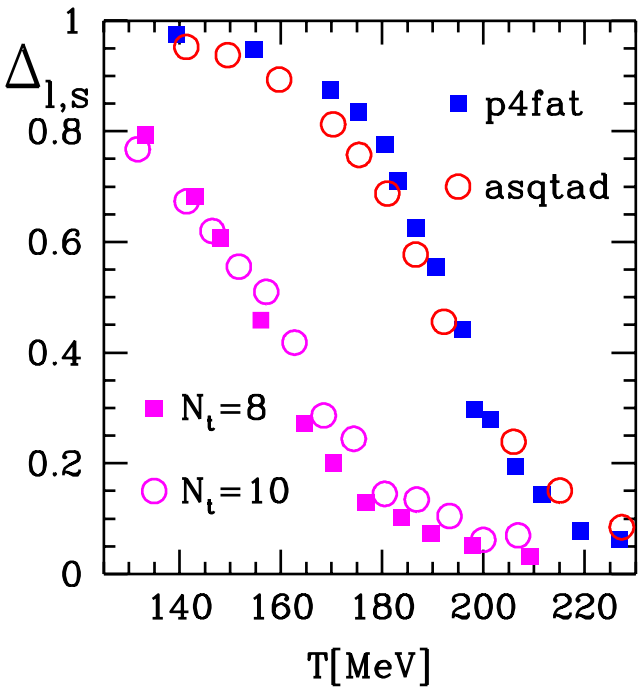
chiral susceptibility: T_c=151(3)(3) MeV
Polyakov and strange susceptibility: T_c=175(2)(4) MeV

'chiral T_c': ≈40 MeV; 'confinement T_c': ≈15 MeV difference

both groups give continuum extrapolated results with physical m_π

Literature: discrepancies between T dependencies

Reason: shoulders, inflection points are difficult to define?
Answer: no, the whole temperature dependence is shifted



for $\Delta_{l,s}$ ≈35 MeV; for the strange susceptibility ≈15 MeV
this discrepancy would appear in all quantities (eos, fluctuations)

Discretization errors in the transition region

we always have discretization errors: nothing wrong with it as long as

a. result: close enough to the continuum value (error subdominant)
b. we are in the scaling regime (a^2 in staggered)

various types of discretization errors $\Rightarrow$ we improve on them (costs)

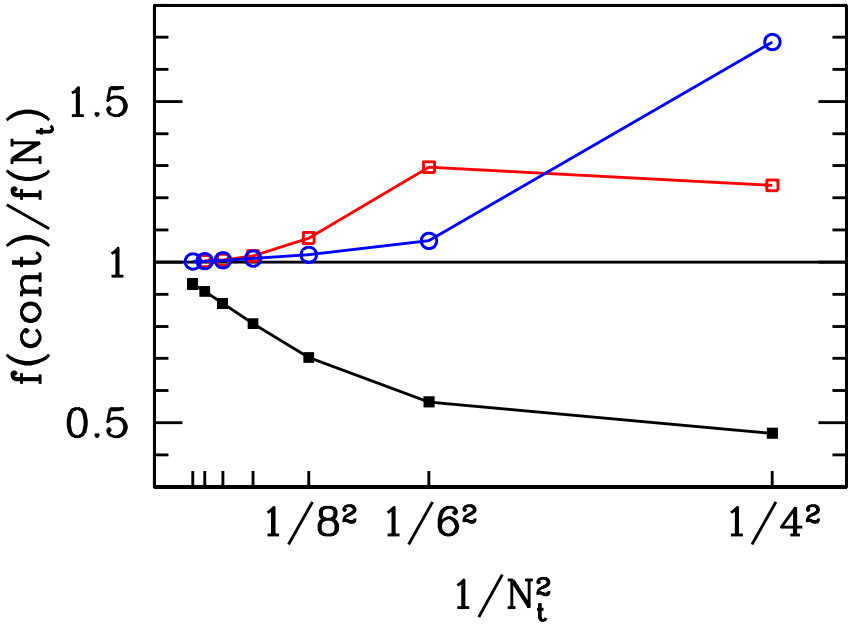
we are speaking about the transition temperature region
interplay between hadronic and quark-gluon plasma physics
smooth cross-over: one of them takes over the other around T_c

both regimes (low T and high T) are equally important
improving for one: T$\gg T_c$, doesn't mean improving for the other: T$< T_c$

example: 'expansion' around a Stefan-Boltzman gas (van der Waals)
for water: it is a fairly good description for T$\gtrsim 300^o$
claculate the boiling point: more accuracy needed for the liquid phase

Examples for improvements, consequences

how fast can we reach the continuum pressure at T=∞?



p4 action is essentially designed for this quantity T$\gg T_c$

asqtad designed mostly for T=0 physics (but good at high T, too)

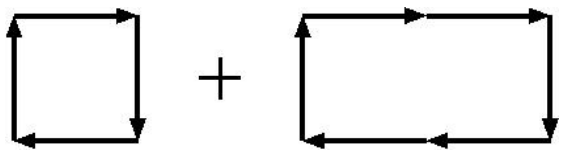
stout-smeared one-link converges slower but in the a^2 scaling regime (e.g. extrapolation from N_t=8,10 provides a result within about 1%)

Choice of the action

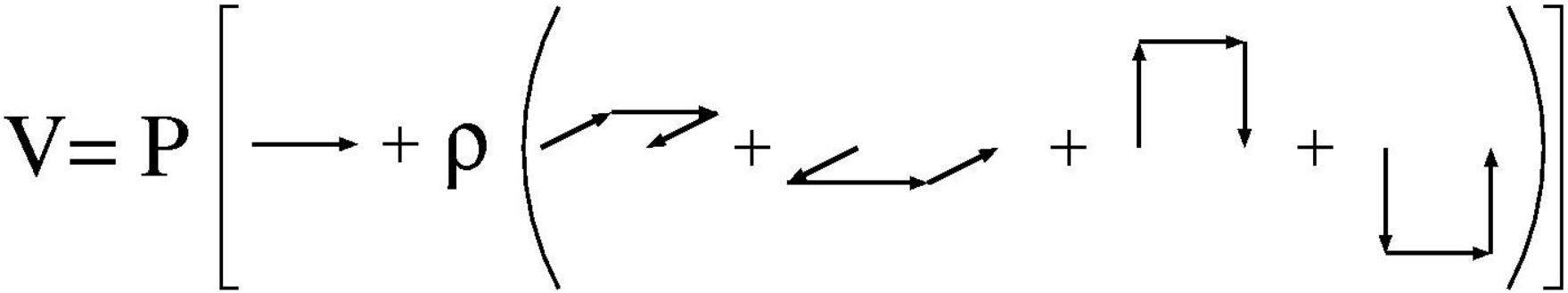
no consensus: which action offers the most cost effective approach

Aoki, Fodor, Katz, Szabo, JHEP 0601, 089 (2006) [arXiv:hep-lat/0510084]

our choice: tree-level $O(a^2)$-improved Symanzik gauge action



2-level (stout) smeared improved staggered fermions



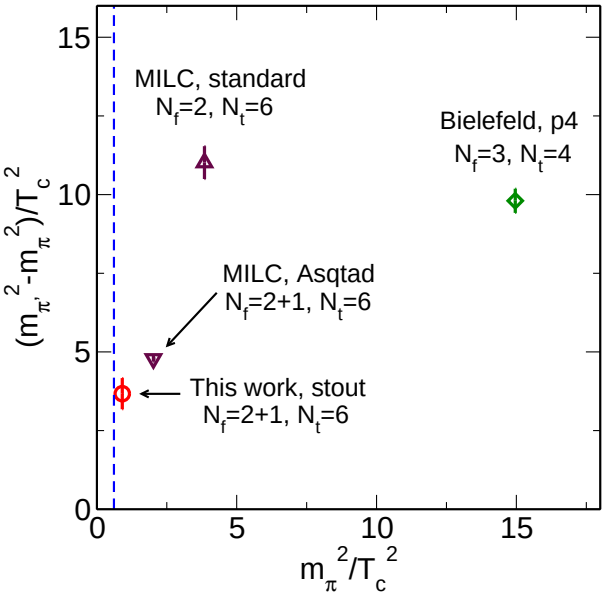
best known way to improve on taste symmetry violation

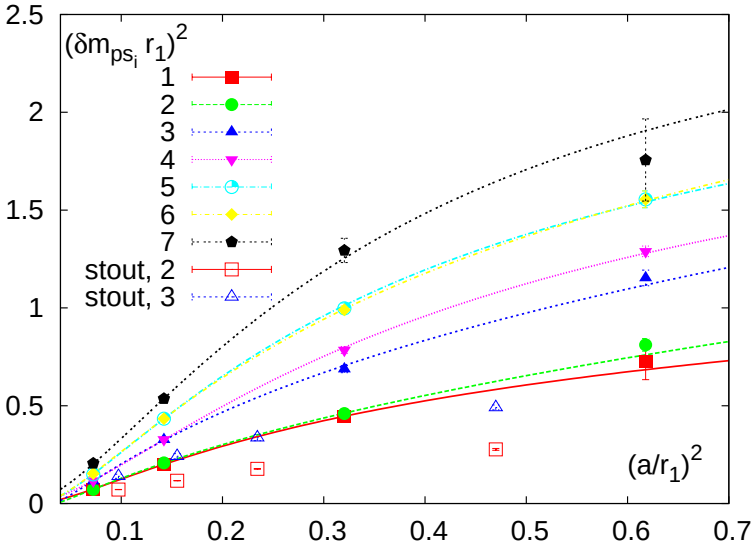
Chiral symmetry/pions

Wuppertal-Budapest: JHEP 0601 (2006) 089. [hep-lat/0510084]

transition temperature for remnant of the chiral transition:
balance between the f's of the chirally broken & symmetric sectors
chiral symmetry breaking: 3 pions are the pseudo-Goldstone bosons

staggered QCD: 1 ($\frac{3}{16}$) pseudo-Goldstone instead of 3 (taste violation)
staggered lattice artefact $\Rightarrow$ splitting disappears in the continuum limit
WB: stout-smeared improvement is designed to reduce this artefact

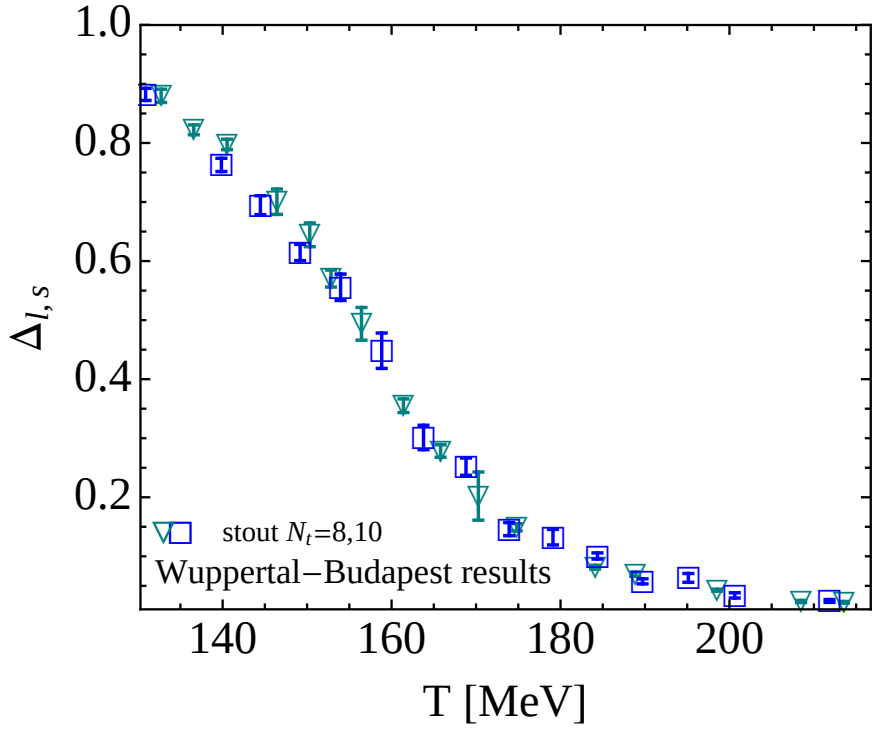




progress in the transition temperature

Wuppertal-Budapest: physical quark masses ($m_s/m_{ud} \approx$28)
gauge configs: N_t=8,10 in 2006 $\Rightarrow$ N_t=12 in 2009 $\Rightarrow$ N_t=16 in 2010

hotQCD 2009: realistic quark masses (m_s/m_{ud} =10)
hotQCD 2010: preliminary: physical quark masses (m_s/m_{ud} =20)



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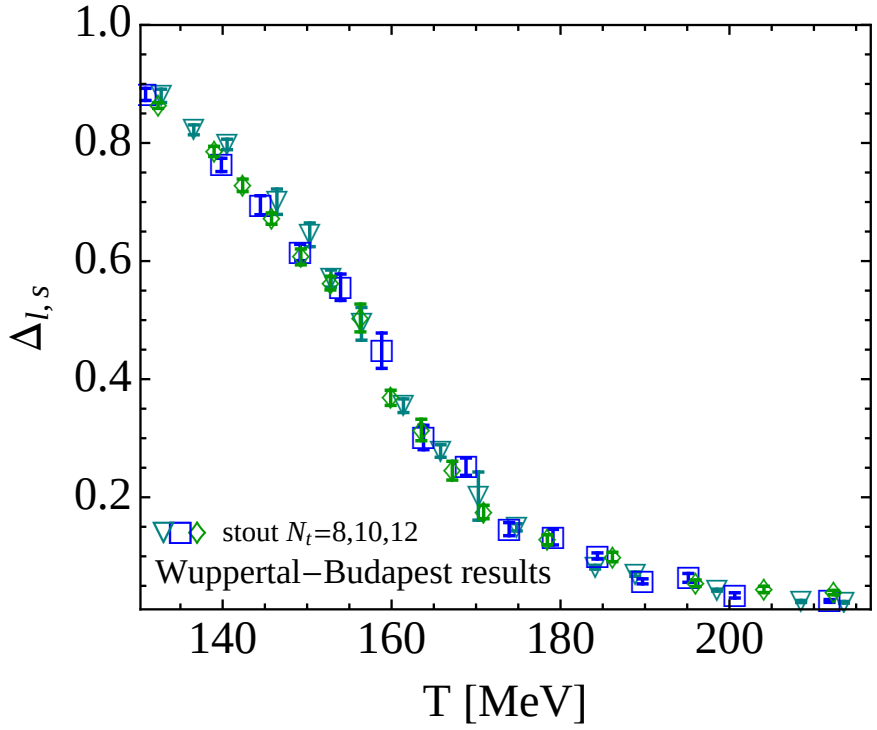

progress in the transition temperature

Wuppertal-Budapest JHEP 1009 '10 73

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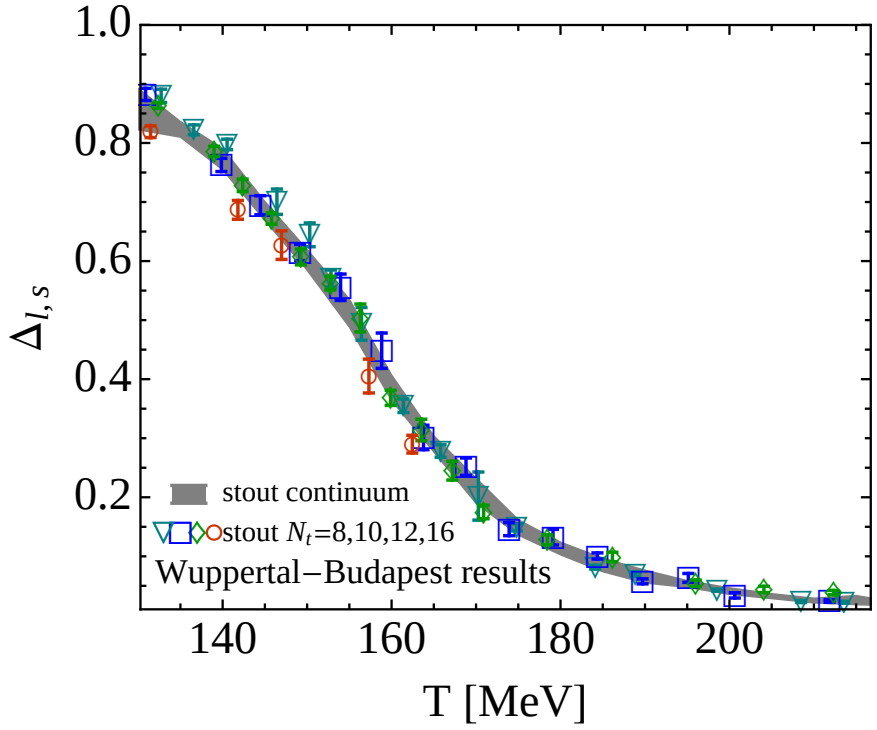

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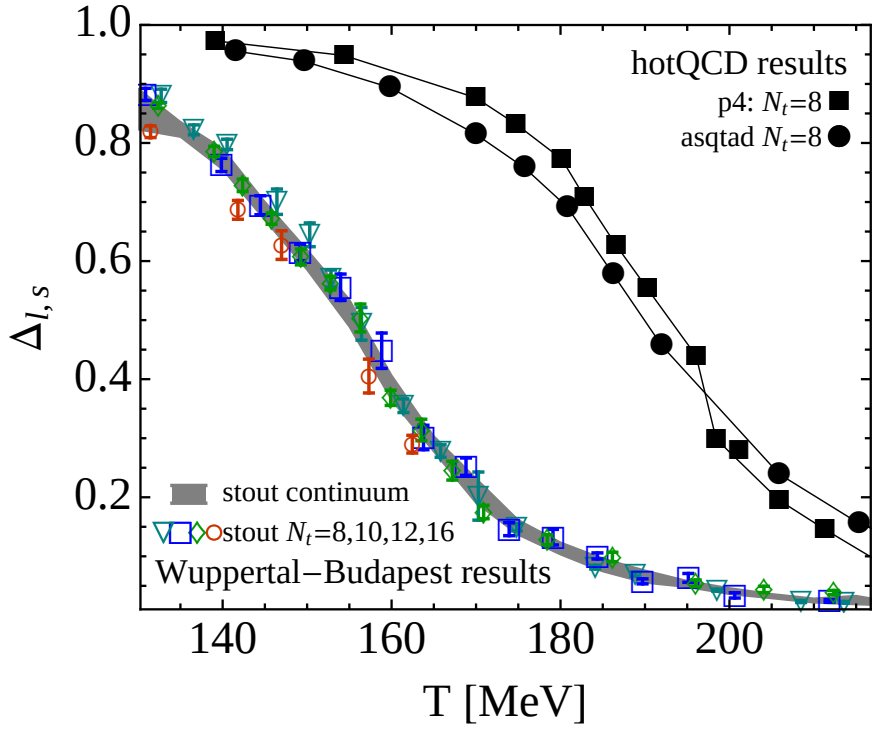

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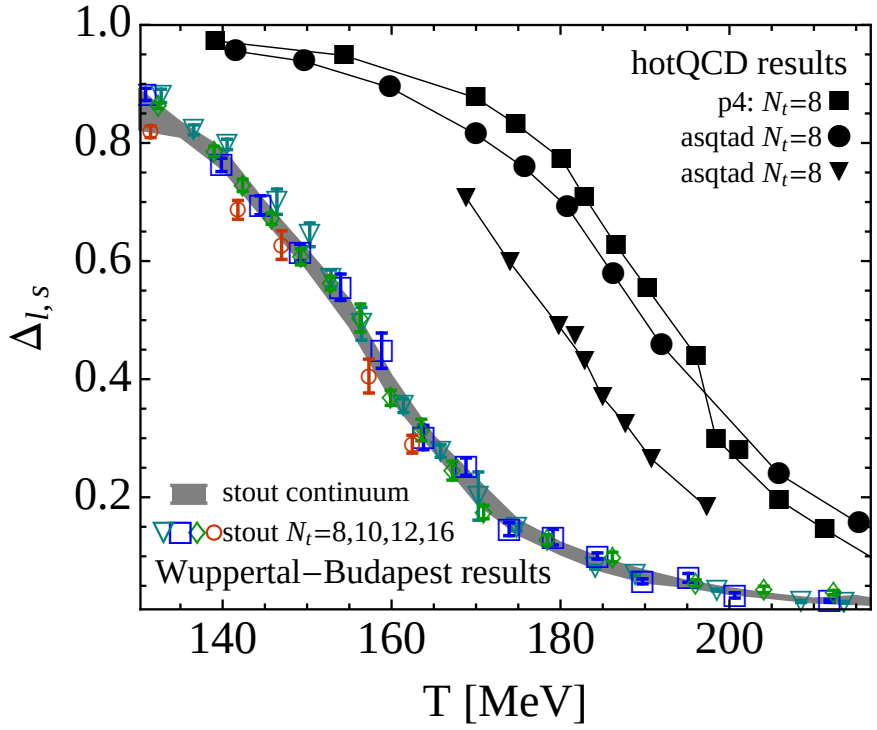

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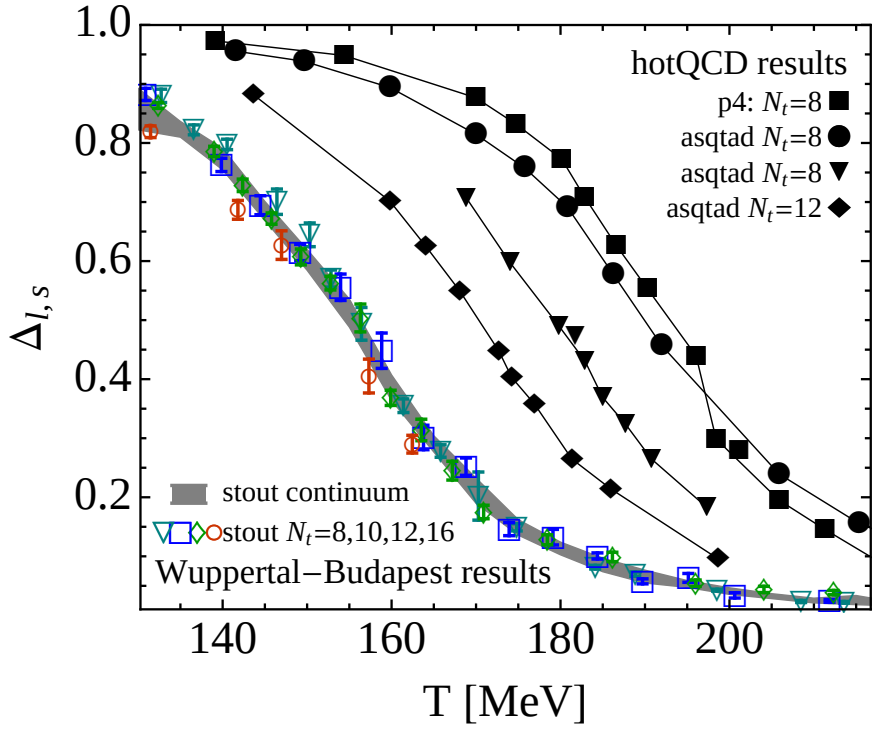

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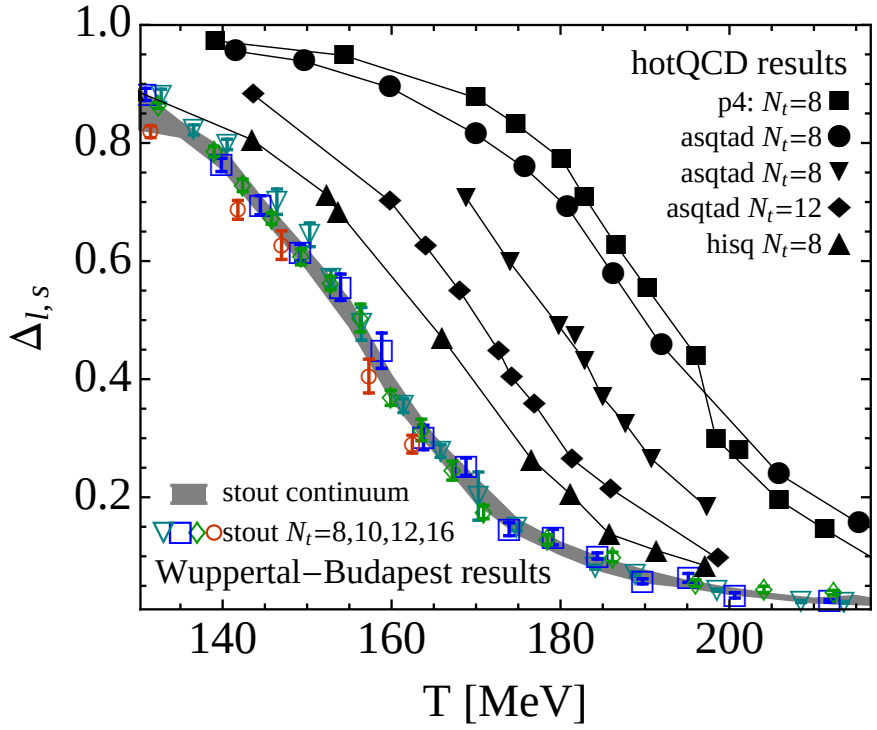


Equation of state: integral method

J. Engels et al., Phys. Lett. B252 (1990) 625

on the lattice the dimensionless pressure is given by

$$p^{\rm lat}(\beta, m_q) = (N_t N_s^3)^{-1} \log \mathcal{Z}(\beta, m_q)$$

not accessible using conventional algorithms, only its derivatives

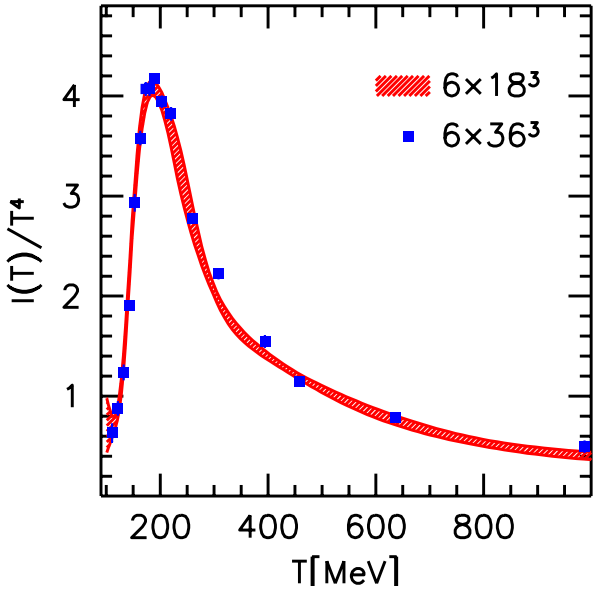
$$p^{\rm lat}(\beta, m_q) - p^{\rm lat}(\beta^0, m_q^0) = (N_t N_s^3)^{-1} \int_{(\beta^0, m_q^0)}^{(\beta, m_q)} \left(d\beta \frac{\partial \log \mathcal{Z}}{\partial \beta} + dm_q \frac{\partial \log \mathcal{Z}}{\partial m_q} \right)$$

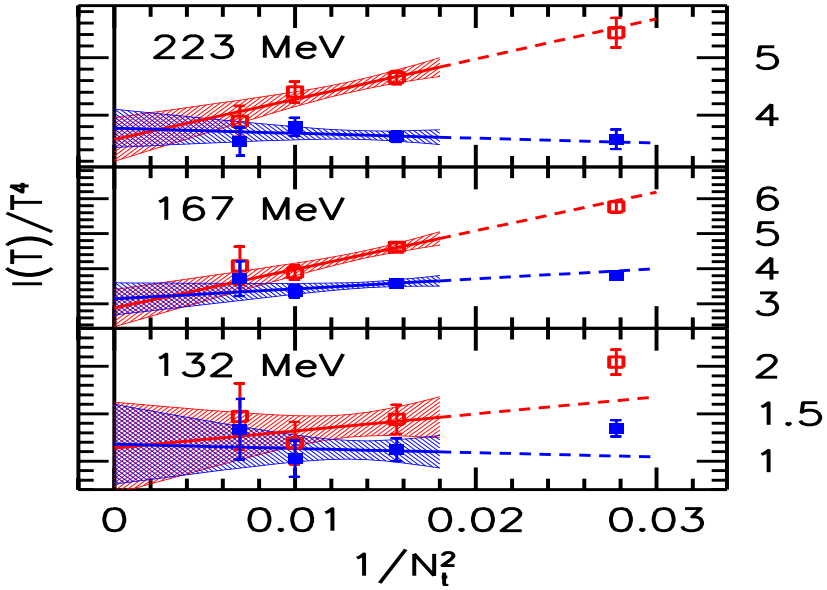
first term: gauge action & second term: chiral condensate

the pressure has to be renormalized: subtraction at T=0 (or T>0)

T$\neq$0 simulations can't go below T$\approx$100 MeV (lattice spacing is large)
physical HRG gives here 5% contribution of SB $\Rightarrow$
path of M_π=720 MeV $\Rightarrow$ distorted HRG no contribution at T=100 MeV

Finite volume and discretization effects



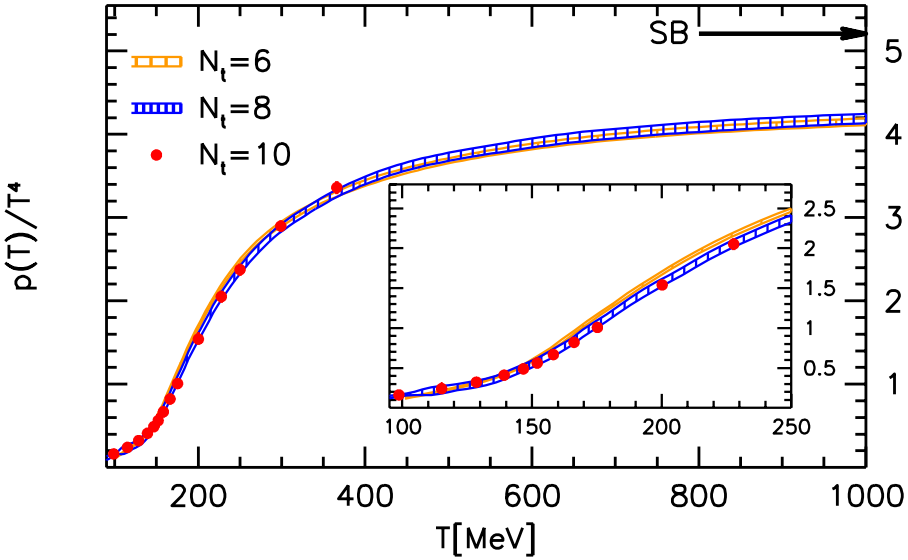


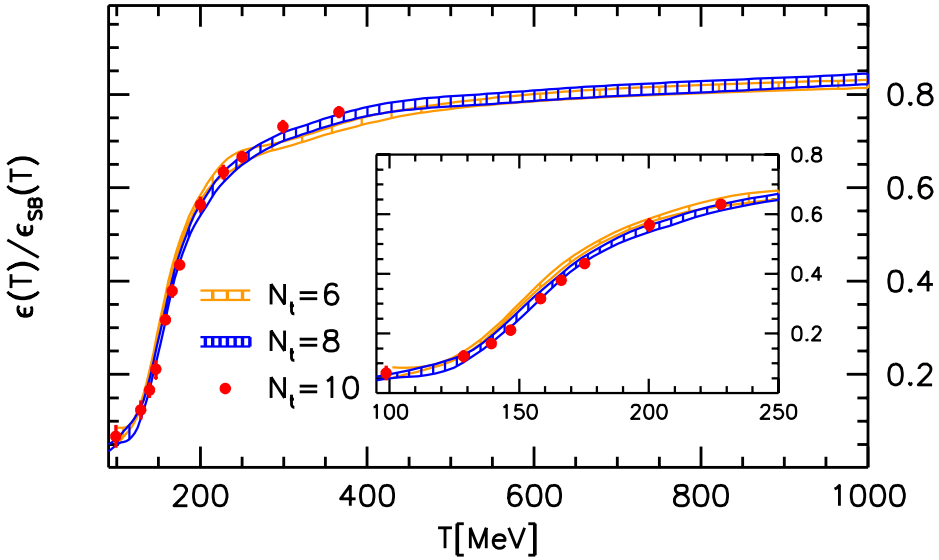
finite V: N_s/N_t=3 and 6 (8 times larger volume): no sizable difference

finite a: trace anomaly for three T-s: high T, transition T, low T
improvement program of lattice QCD (action & observables)
tree-level improvement for p (thermodynamic relations fix the others)
continuum limit N_t=6,8,10,12: same with or without improvement

improvement strongly reduces cutoff effects: slope $\approx$0 (1-2σ level)

Pressure and energy density

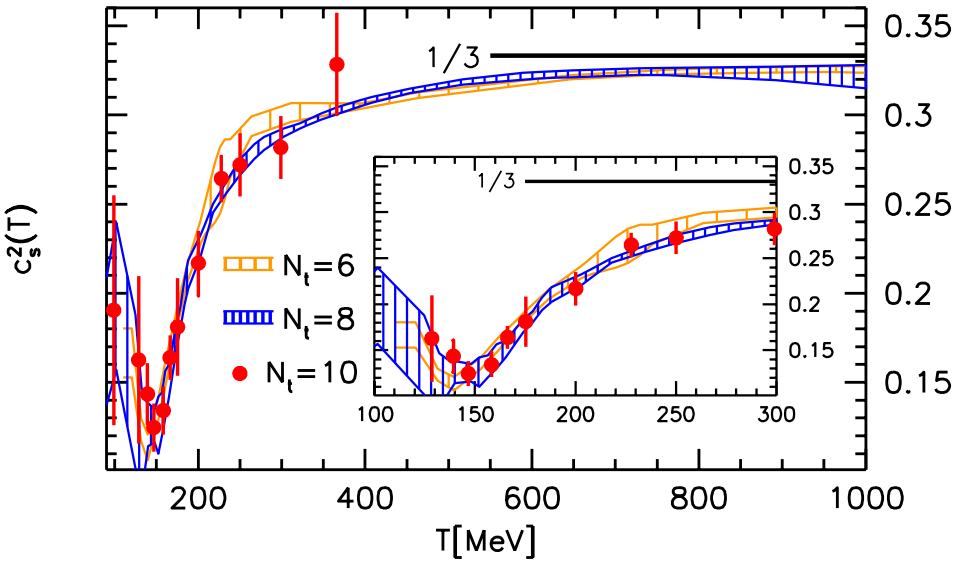


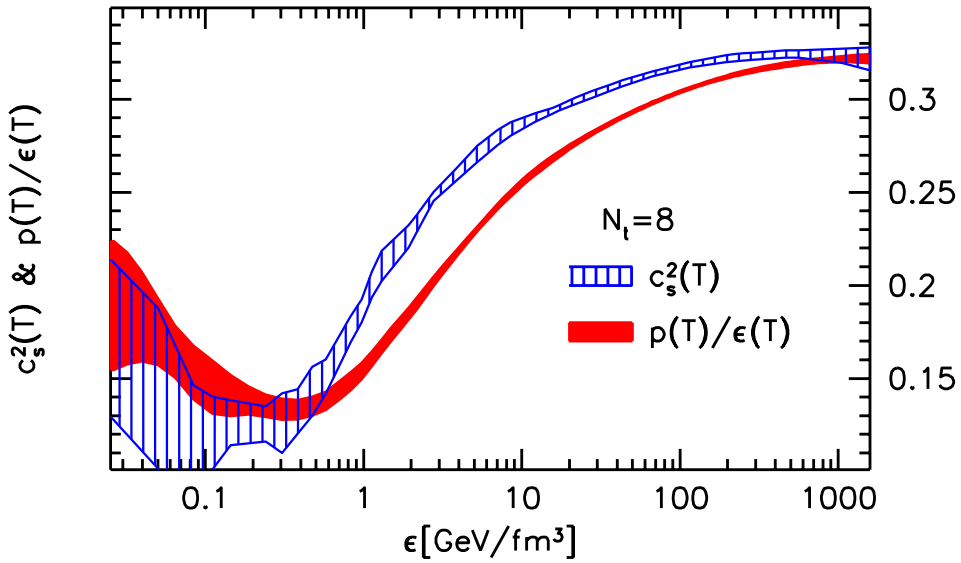


ϵ normalized to the Stefan-Boltzmann limit: $\epsilon(T \to \infty)$=15.7
at 1000 MeV still 20% difference to the Stefan-Boltzmann value

essentially perfect scaling, lines/points are lying on top of each other

Speed of sound & parametrization





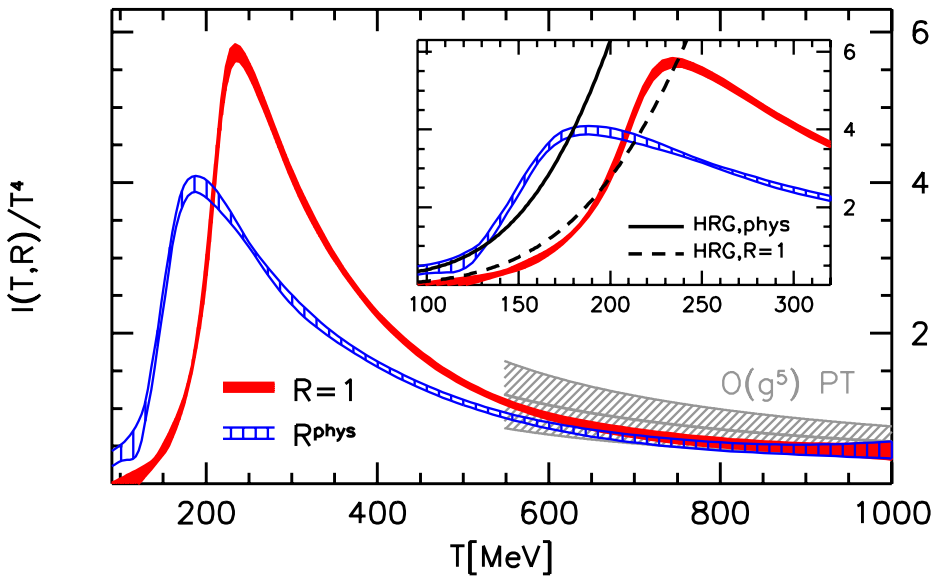
c_s minimum value is about 0.13 at T≈145 MeV

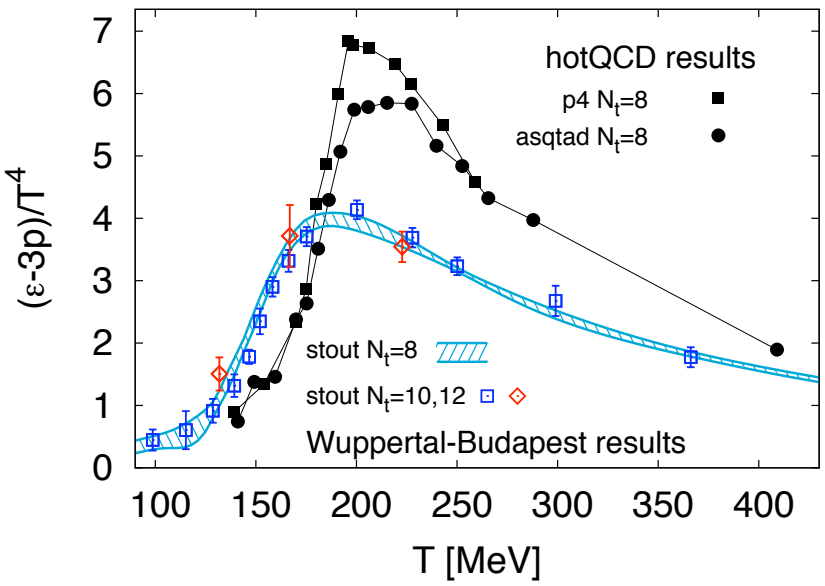
'smaller than error' parametrization T=100...1000 MeV (t=T/200 MeV)

$$\frac{I(T)}{T^4} = \exp(-h_1/t - h_2/t^2) \cdot \left(h_0 + \frac{f_0 \cdot [\tanh(f_1 \cdot t + f_2) + 1]}{1 + g_1 \cdot t + g_2 \cdot t^2} \right)$$

h_0	h_1	h_2	f_0	f_1	f_2	g_1	g_2
0.1396	-0.1800	0.0350	2.76	6.79	-5.29	-0.47	1.04

Equation of state: I(T)=ϵ-3p





two pion masses: $M_\pi \approx 720$ MeV (R=1) and $M_\pi = 135$ MeV (R^{phys})
good agreement with the HRG model up to the transition region
quark mass dependence disappears for high T
good agreement with perturbation theory

comparison with the published results of the hotQCD collaboration
discrepancy: peak at $\approx$20 MeV larger T and $\approx$70, 50, 40% higher

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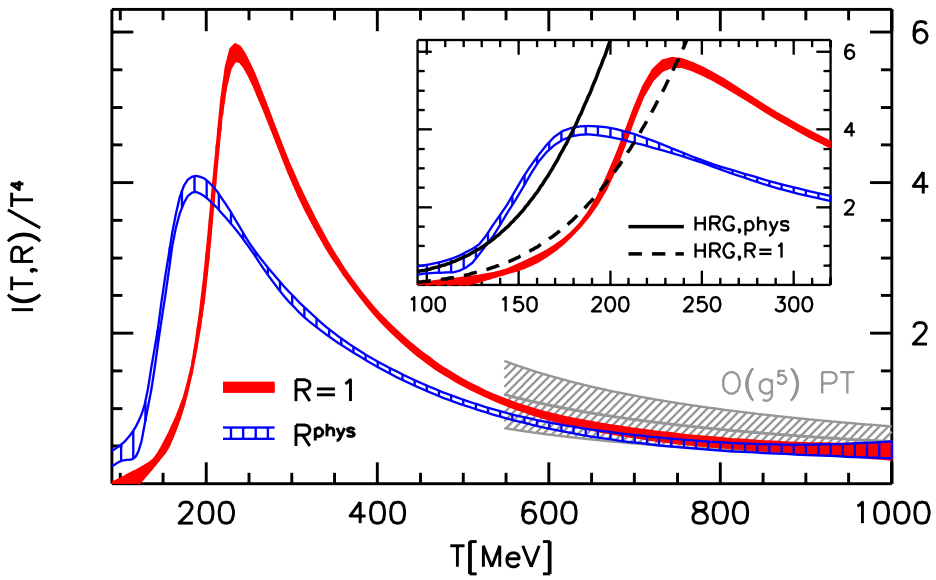

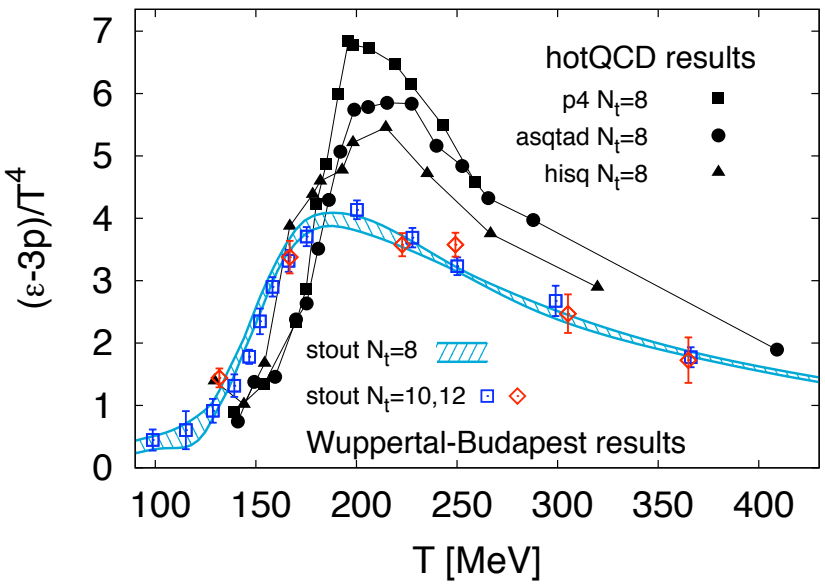

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Finite chemical potential: the sign problem

at μ=0 the fermion matrix is γ_5 hermitian: $M^\dagger = \gamma_5 M \gamma_5$
easy to check $\Longrightarrow$ eigenvalues: either real or conjugate pairs
det(M) is real, which is not true any more for non-vanishing μ

importance sampling (algorithms) for complex det(M) does not work

$$P(U \to U') = \min\left[1, \exp(-\Delta S_g)\det(M[U'])/\det(M[U])\right]$$

sign problem $\Rightarrow$ until 2001: "lattice QCD can not say anything for μ>0"

Fodor-Katz: multiparameter reweighting (hep-lat/0104001, PLB)
Bielefeld-Swansee: det(M) Taylor expanded (hep-lat/0204010, PRD)
de Forcrand-Philipsen: imaginary μ (hep-lat/0205016, Nucl.Phys.B)
D'Elia-Lombardo: imaginary μ (hep-lat/0209146, PRD)

the three methods look different, they are essentially the same

Overlap improving multi-parameter reweighting

one wants to calculate the following path integral

$$Z(\alpha) = \int [dU] \exp[-S_{bos}(\alpha, U)] \det M(U, \alpha)$$

α: parameter set (gauge coupling, mass, chemical potential)
for some parameters α_0 importance sampling can be done

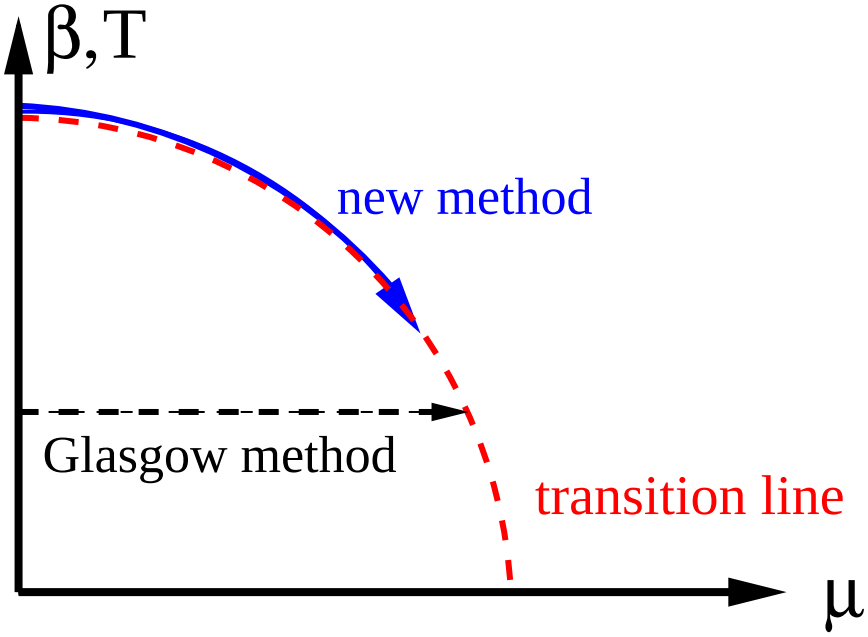
$$Z(\alpha) = \int [dU] \exp[-S_{bos}(\alpha_0, U)] \det M(U, \alpha_0)$$

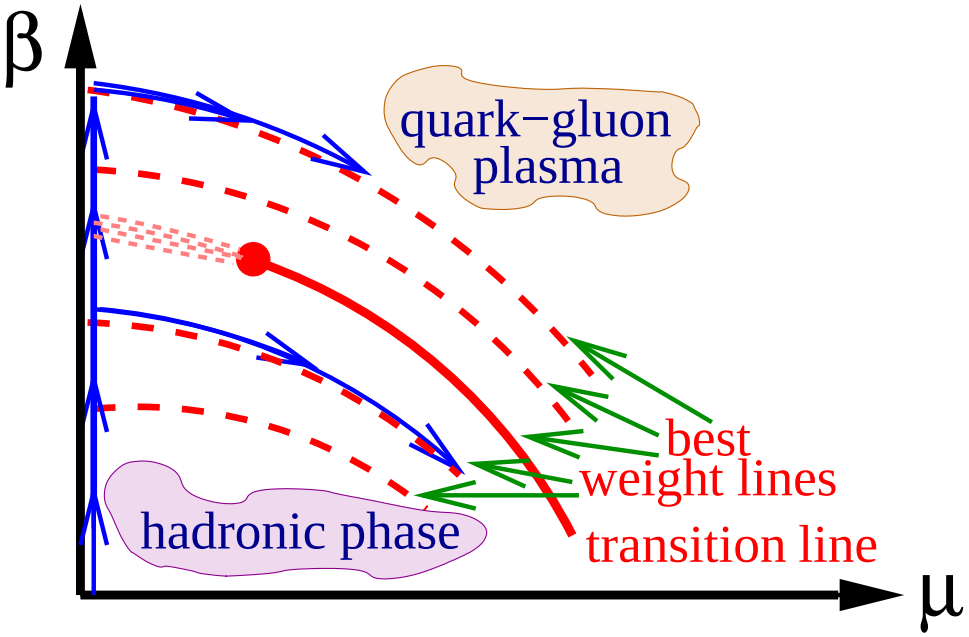
$$\{\exp[-S_{bos}(\alpha, U) + S_{bos}(\alpha_0, U)] \det M(U, \alpha) / \det M(U, \alpha_0)\}$$

first line: measure; curly bracket: observable (will be measured)
e.g. transition configurations are mapped to transition ones

reweighting factor (ratio of the determinants) can be expressed by the eigenvalues of the (reduced) fermion matrix: closed formula for any μ

Compare with Glasgow (Ferrenberg-Swendsen)





Glasgow method $\Rightarrow$ multiparameter reweighting
single parameter (μ) $\Rightarrow$ two parameters (μ and β)
purely hadronic $\Rightarrow$ transition configurations

map transition configurations to transition ones

Equivalence of the methods (formal/numerical)

(recent lattice review at μ=0 and μ>0: Fodor-Katz 0908.3341)

det(M) can be given by the eigenvalues of M' (transformed) at μ=0

$$\det M(\mu) = e^{-3V\mu} \prod_{i=1}^{6L_s^3} (e^{L_t\mu} - \lambda_i)$$

observable at μ>0 or μ_I is given by the observable and λ_i at μ=0

$$Pl(\beta,\mu) = \langle Pl \exp[\Delta\beta Pl] e^{-3V\mu} \prod_{i=1}^{6L_s^3} (e^{L_t\mu} - \lambda_i) \rangle$$

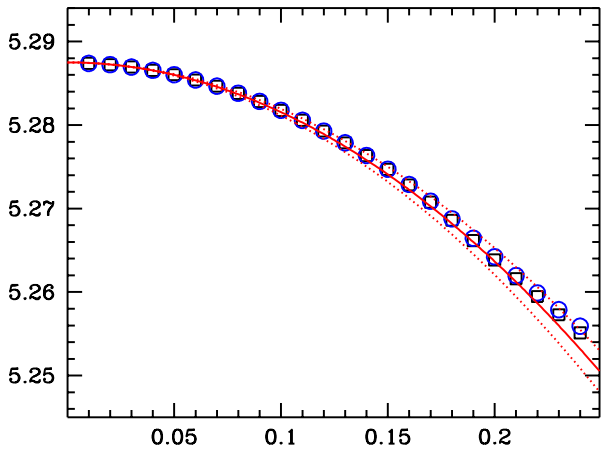
det(M) or Pl(β,μ) can be trivially Taylor expanded (Bielefeld-Swansee) termination of the series & stochastic determination of the coefficients $\Longrightarrow$ do not expect this method to work for as large μ as the full one

det(M)>0 for imaginary μ: impartance sampling still works
determine the phase line $T_c(\mu_I)$ (e.g. use a quadratic/quartic fit)
plug real μ into the same quadratic/quartic function: $c_2\mu^2 + c_4\mu^4$
formally: numerical determination of the (μ^2,μ^4) Taylor coefficients

Equivalence of the methods (formal/numerical)

$\Rightarrow$ for moderate μ Taylor and μ_I agree with reweighting

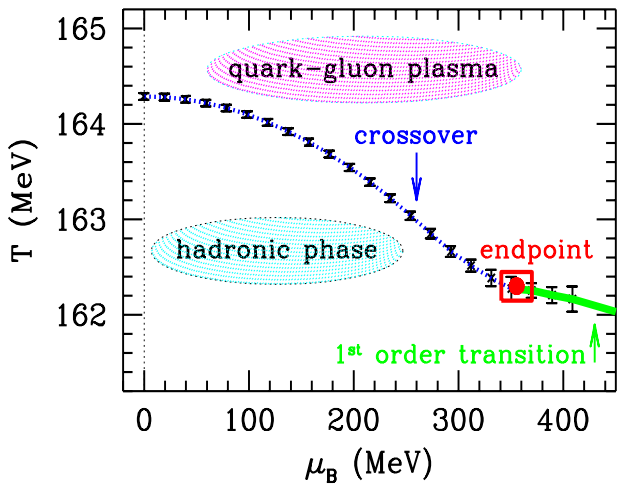
take n_f=2 setting of de Forcrand-Philipsen: $\beta_c(\mu)$ upto 4 digits

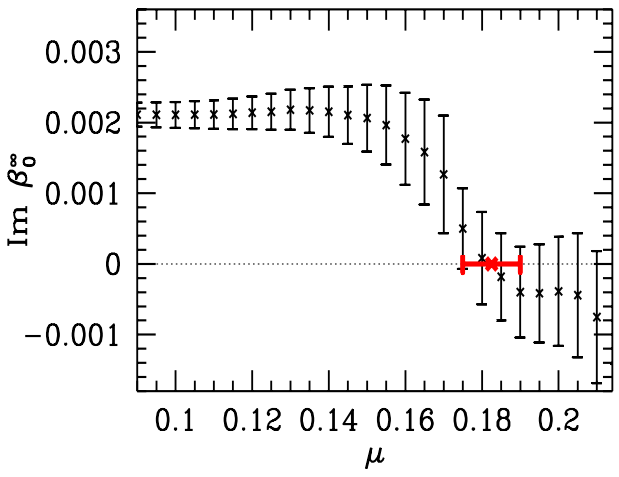


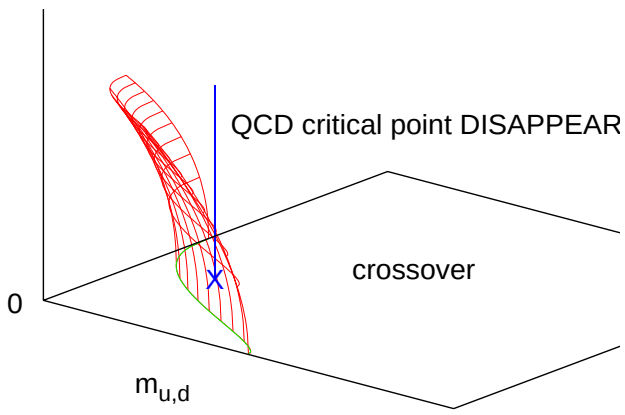
solid/dotted: imaginary μ & error; box: reweighting; circle: Taylor
for larger μ values higher order terms are getting more important

what to choose (depends on the question):
for this particular case imaginary μ has the largest CPU demand;
next one is reweighting; cheapest is Taylor (does not work for large μ)

Critical endpoint discussion (controversy?)







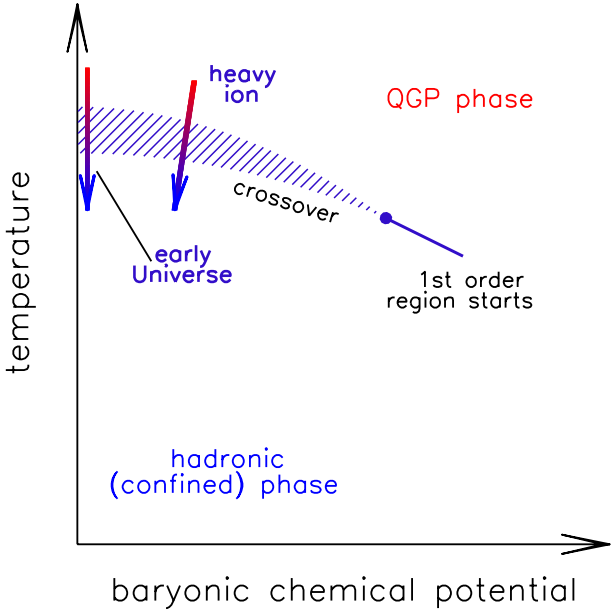
all results are from coarse lattices (a=0.3 fm, read our abstract!)

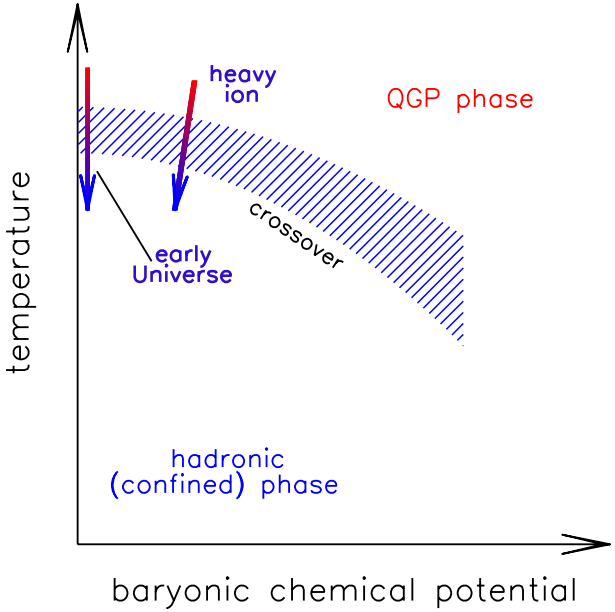
deForcrand-Philipsen: leading order $\Rightarrow$ not stronger, slightly weaker
same from reweighting: $\mu_I/T \approx$1–3 (μ_{crit}: result of the higher orders)

Taylor & radius of convergence (!) only a lower bound: Lee-Yang

full answer (all the way to the continuum) needs much more CPU

Possible scenarios





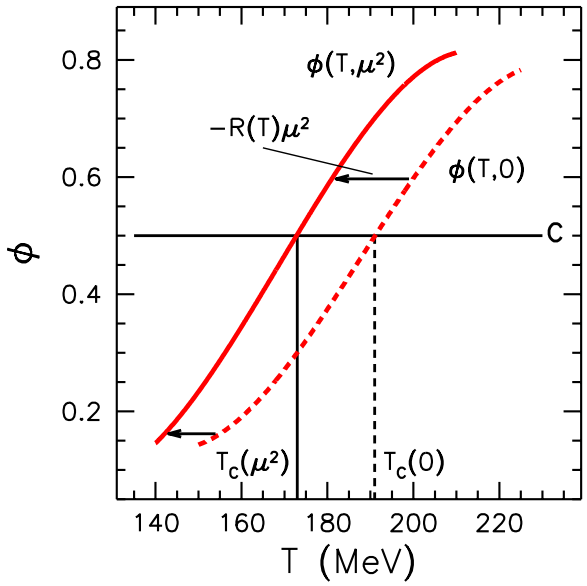
phase diagram with a transition growing stronger
even turning into a first-order phase transition at a critical endpoint

weakening transition and no critical endpoint

here we calculate the first non-trivial term: physical mass & a→0
(we do not expect any conclusion to the critical endpoint)

The curvature

we change μ and look at the transition curve
it shifts to the left, we look at its value of a fixed C



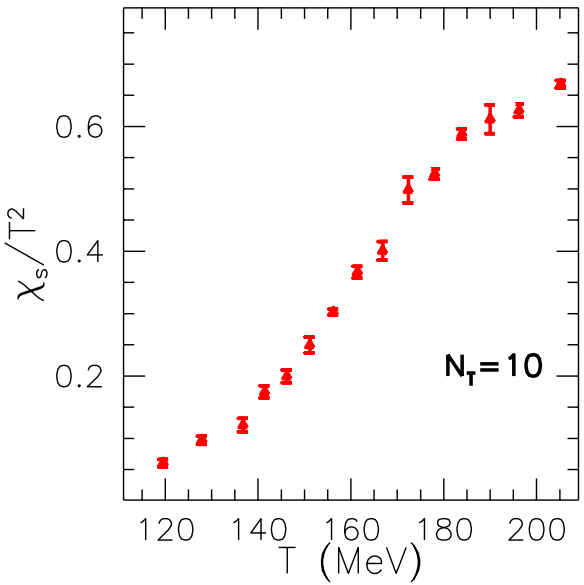
the dimensionless curvature is defined as $\kappa(T) = -T_c(\mu = 0) \cdot R(T)$

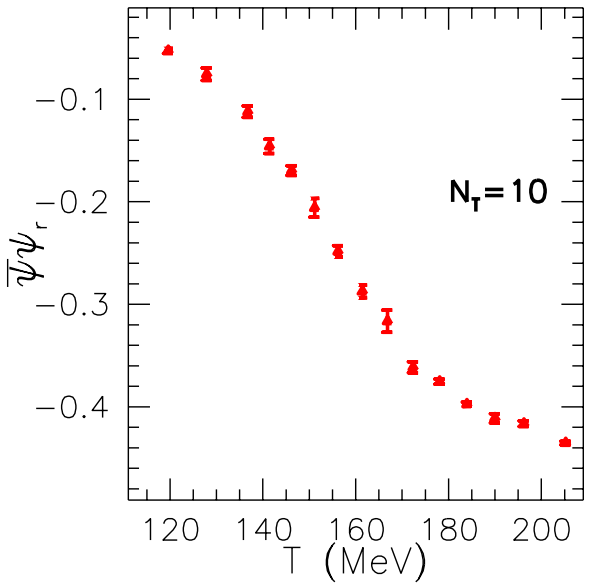
$d\kappa/dT$ at T_c tells if the transition is broadening or narrowing
(a point below T_c has a larger or smaller curvature)

Observables

strange susceptibility: $\chi_s = (T/V)\partial^2 \log Z/\partial\mu_s^2$

chiral condensate (bare): $\bar\psi\psi = (T/V)\partial \log Z/\partial m$
renormalize it: $\bar\psi\psi_R = [\bar\psi\psi - \bar\psi\psi(T=0)] \cdot m/M_\pi^4$

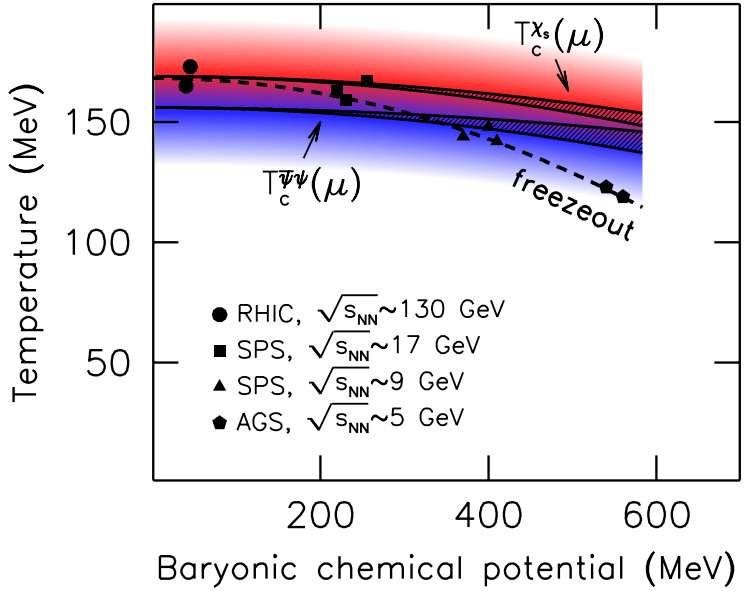




we study how fast it moves to the left if we increased μ

Continuum prediction for the curvature: full result

G. Endrodi, Z. Fodor, S.D. Katz, K.K. Szabo, JHEP 1104 2011 001 [1102.1356]



lower solid line: T_c from the chiral condensate
upper solid line: T_c from the strange susceptibility

bands (red and blue) indicate the widths of the transition lines
the widths remain in this order approximately the same
in leading order: no critical point (can be anything)

Historical background

1972 Lagrangian of QCD (H. Fritzsch, M. Gell-Mann, H. Leutwyler)

1973 asymptotic freedom (D. Gross, F. Wilczek, D. Politzer)
at small distances (large energies) the theory is “free”

1974 lattice formulation (Kenneth Wilson)
at large distances the coupling is large: non-perturbative

Nobel Prize 2008: Y. Nambu, & M. Kobayashi T. Masakawa

spontaneous symmetry breaking in quantum field theory
strong interaction picture: mass gap is the mass of the nucleon

mass eigenstates and weak eigenstates are different

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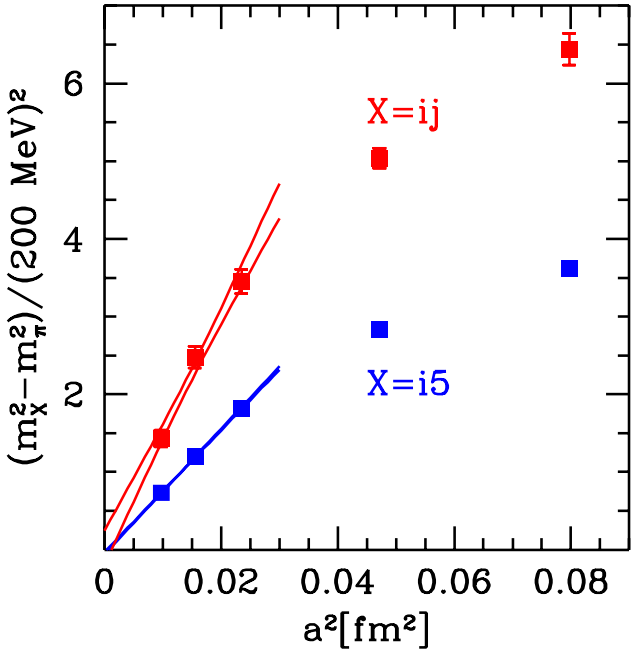
Scientific Background on the Nobel Prize in Physics 2008

"Even though QCD is the correct theory for the strong interactions, it can not be used to compute at all energy and momentum scales ... (there is) ... a region where perturbative methods do not work for QCD."

true, but the situation is somewhat better: new era
fully controlled non-perturbative approach works (took 35 years)

Scaling for the pion splitting



scaling regime is reached if a^2 scaling is observed

asymptotic scaling starts only for $N_t \gtrsim 8$ ($a \lesssim 0.15$ fm): two messages

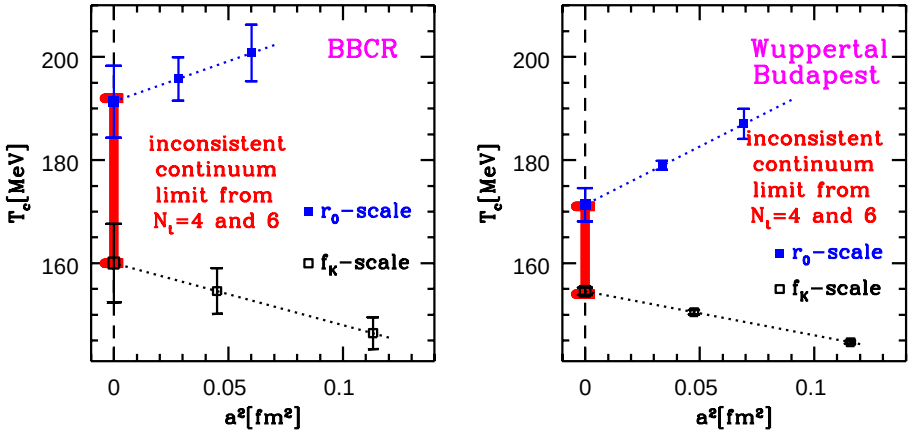
a. N_t=8,10 extrapolation gives 'p' on the $\approx$1% level: good balance

b. stout-smeared improvement is designed to reduce this artefact

most other actions need even smaller 'a' to reach scaling

Consequences of the non-scaling behaviour

for large 'a' no proper a^2 scaling (e.g. due to large m_π splitting)
how do we monitor it, how to be sure being in the scaling regime?
dimensionless combinations in the a→0 limit:
$T_c r_0$ or T_c/f_K for the remnant of the chiral transition

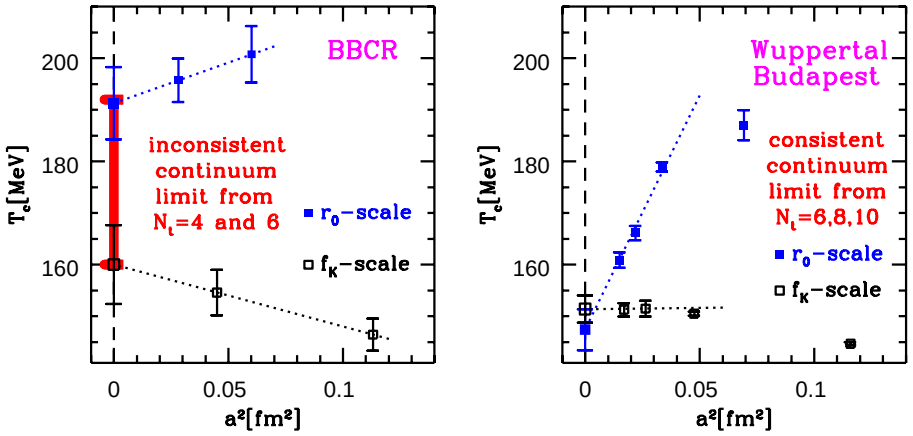


N_t=4,6: inconsistent continuum limit
N_t=6,8,10: consistent continuum limit (stout-link improvement)

independently which quantity is taken one obtains the same T_c signal: extrapolation is safe, we are in the a^2 scaling regime

Consequences of the non-scaling behaviour

no proper a^2 scaling for large 'a' (e.g. due to large m_π splitting)
how do we monitor it, how to be sure being in the scaling regime?
dimensionless combinations in the a→0 limit:
$T_c r_0$ or T_c/f_K for the remnant of the chiral transition



N_t=4,6: inconsistent continuum limit
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independently which quantity is taken one obtains the same T_c
signal: extrapolation is safe, we are in the a^2 scaling regime

Setting the scale in lattice QCD

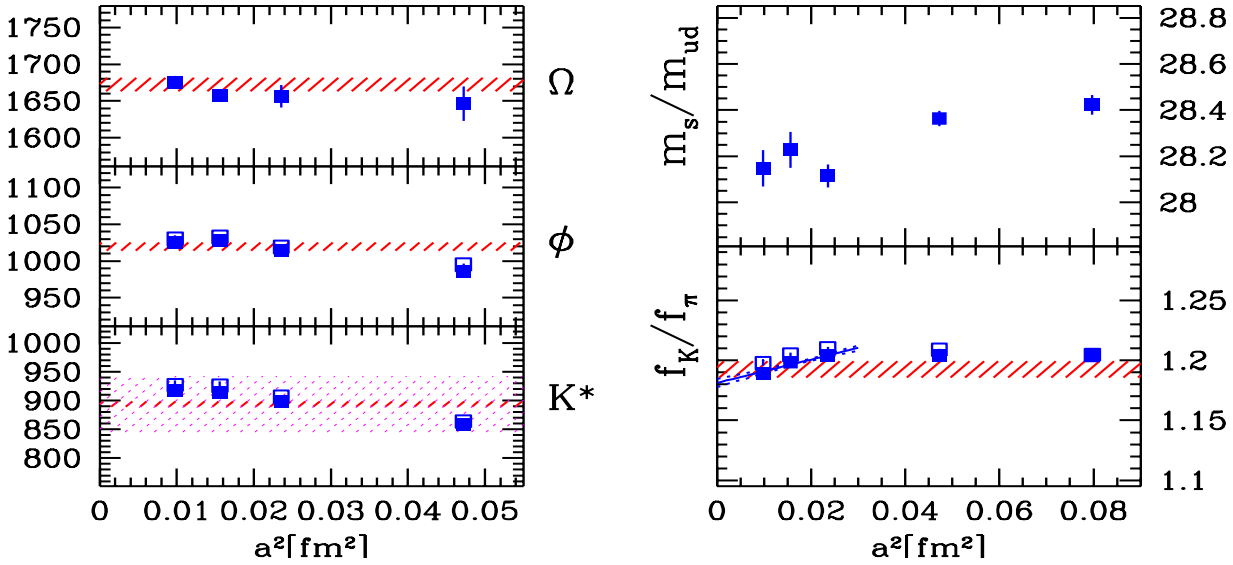
in meteorology, aircraft industry etc. grid spacing is set by hand

in lattice QCD we use g,m_{ud} and m_s in the Lagrangian ('a' not)

measure e.g. the vacuum mass of a hadron in lattice units: M_Ωa

since we know that M_Ω=1672 MeV we obtain 'a' and T=1/N_ta

Y.Aoki et al. [Wuppertal-Budapest Collaboration] arXiv:0903.4155



independently which quantity is taken (we used physical masses)

$\Rightarrow$ one obtains the same 'a' and T, result is safe