R-mode oscillations and rocket effect in rotating superfluid neutron stars

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Zakopane, June 16th 2011

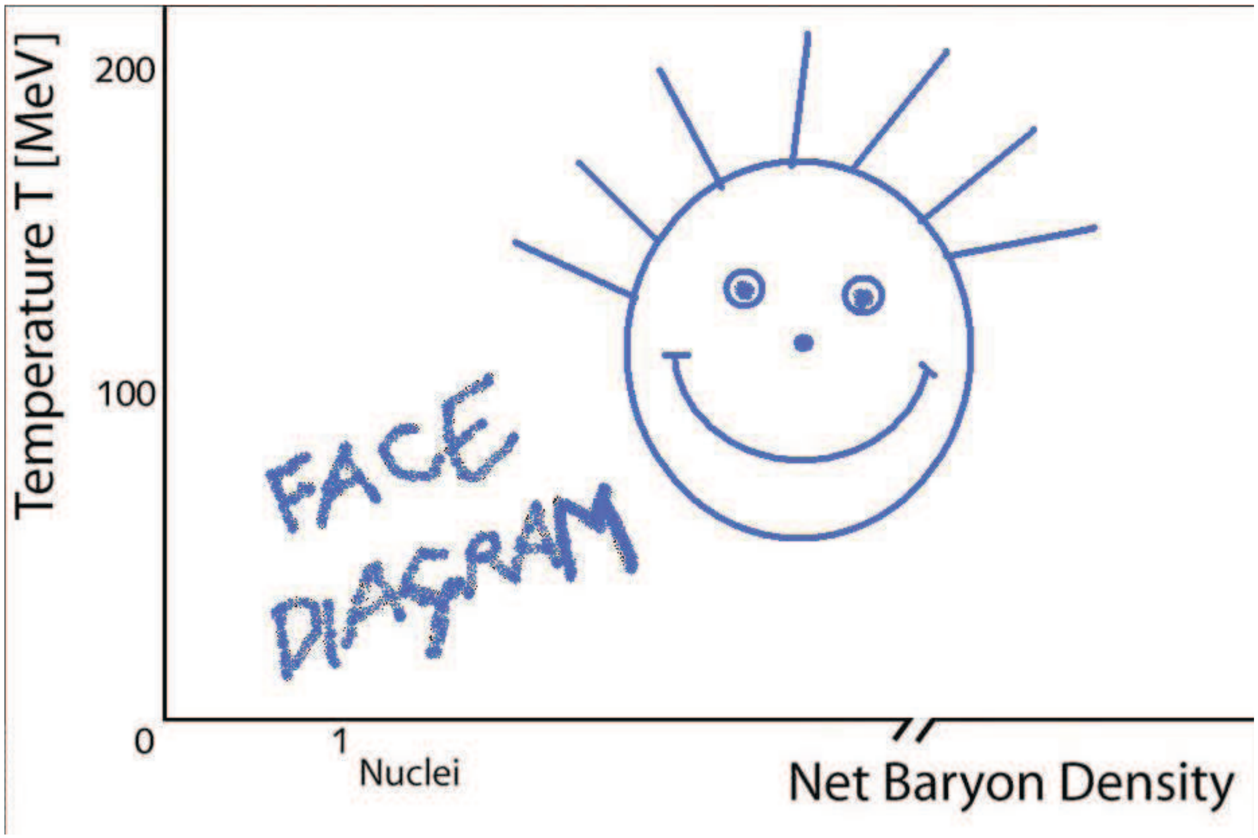
We present estimates of the damping timescale due to the presence of a novel dissipative mechanism, the rocket effect, related to processes that change the number of protons, neutrons and electrons.

GC, M. Mannarelli, C. Manuel - arXiv:1007.2304

Outline

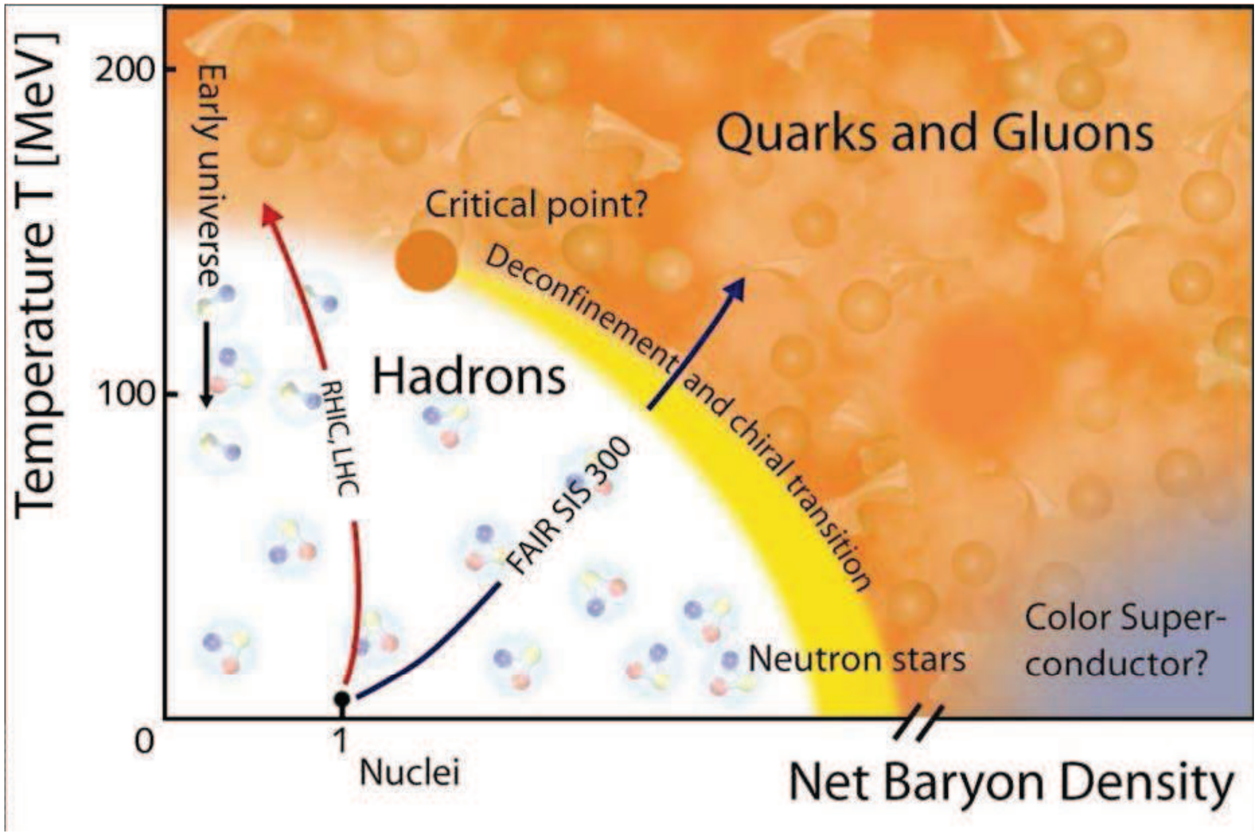
- Motivation
- Compact stars
- R-mode oscillations
- Rocket effect
- Results
  - standard r-mode
  - superfluid r-mode
- Conclusion

Motivation



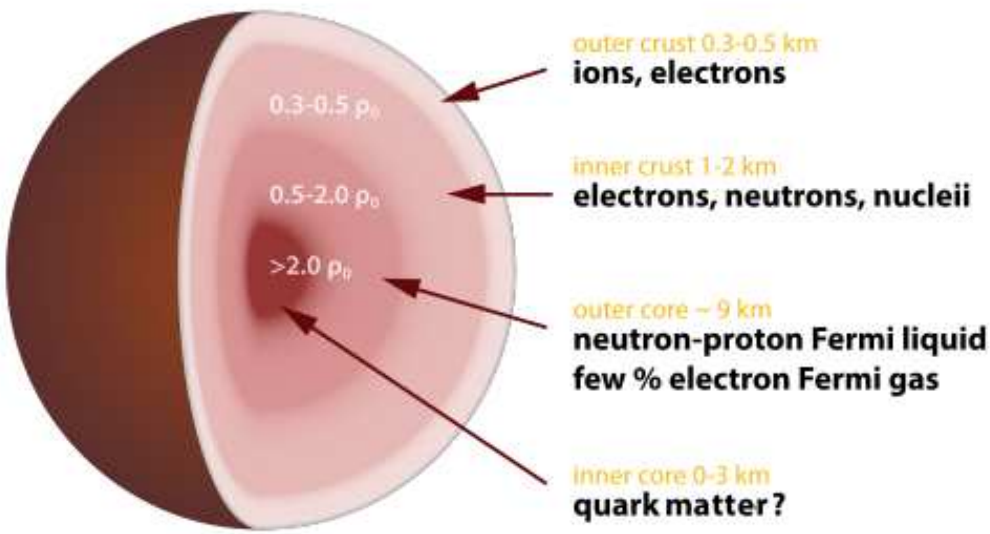
Temperature T [MeV]
200
100
0
1
Nuclei
Net Baryon Density
FACE
DIAGRAM

Motivation



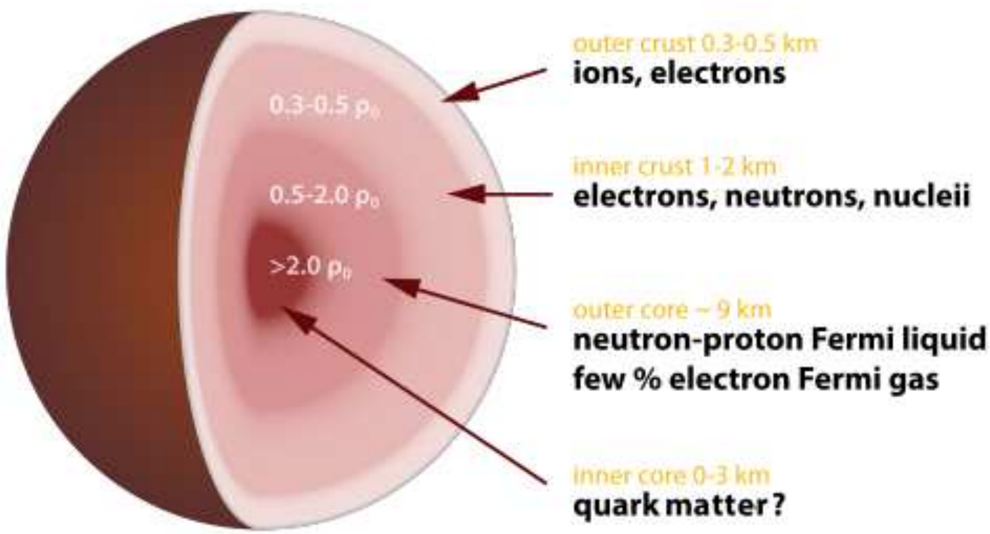
neutron stars: physics laboratories providing extremely high density

Compact stars



- mass: $M \sim 1.4 M_{\odot}$
- radius: $R \sim 10$ km
- **density**: $\rho \sim 10^{14}$ g cm^{-3}

Compact stars



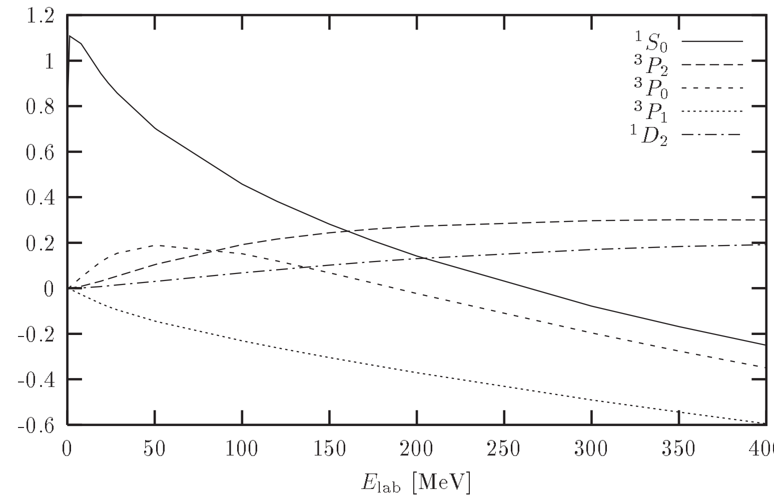
toy models used in our paper:

- newtonian framework
- npe-matter
- constant density $\rho = 2.5\rho_0$
- radius $R = 10$ km
- proton fraction: $x_p = 1/9$

Compact stars - Superfluidity

superfluidity in compact stars

- low temperature systems
- critical temperature $T_c \sim 10^{-3} T_F$
- attractive interaction
- Cooper pairing:
  - protons pair $^1S_0$
  - neutrons pair $^3P_2$

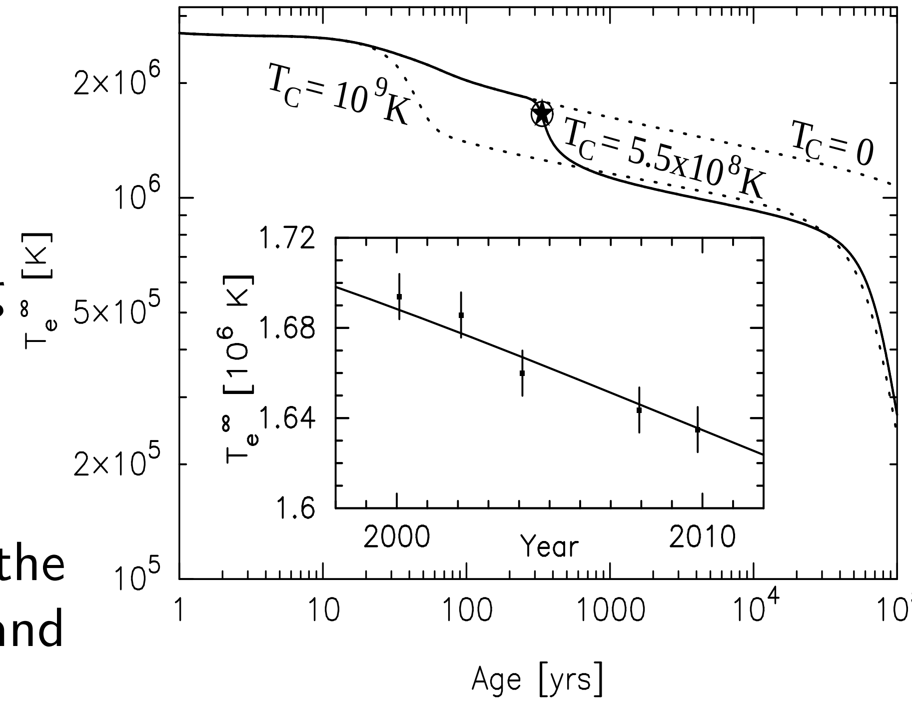


Dean & Jensen, 2003

Compact stars - Superfluidity

Cas A superfluidity

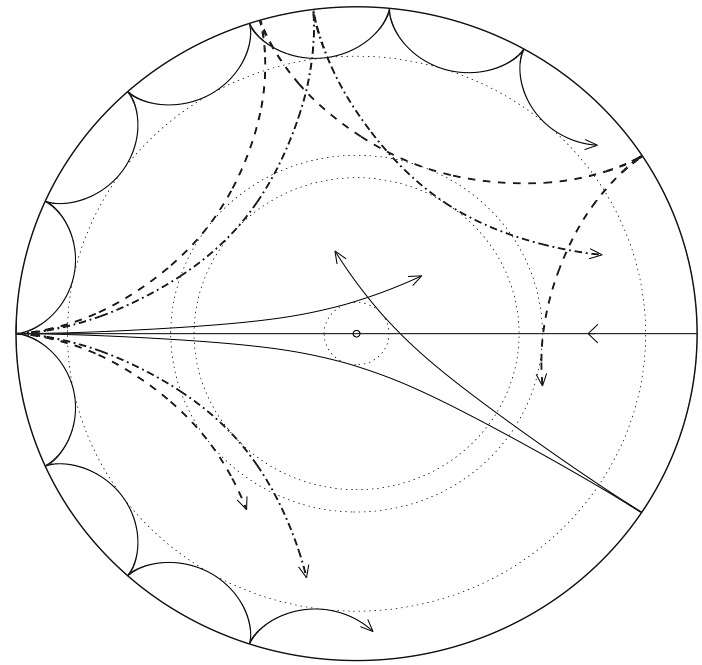
- cooling of Cas A
- 10 years of X-ray data show cooling at the rate $\frac{d \ln T_e}{d \ln t} = -1.23 \pm 0.14$ (Heinke $\&$ Ho 2010)
- enhanced neutrino emission from the recent onset of the breaking and formation of neutron Cooper pairs



Page et al., 2010

Compact stars - Oscillations

- several oscillations (*p*,*w*,*r*-modes)
- emission of GW
  (no axisymmetric modes)
- instabilities
  (growth of the amplitude)
- dissipative mechanisms
- **instability window** $\Omega = \Omega(T)$



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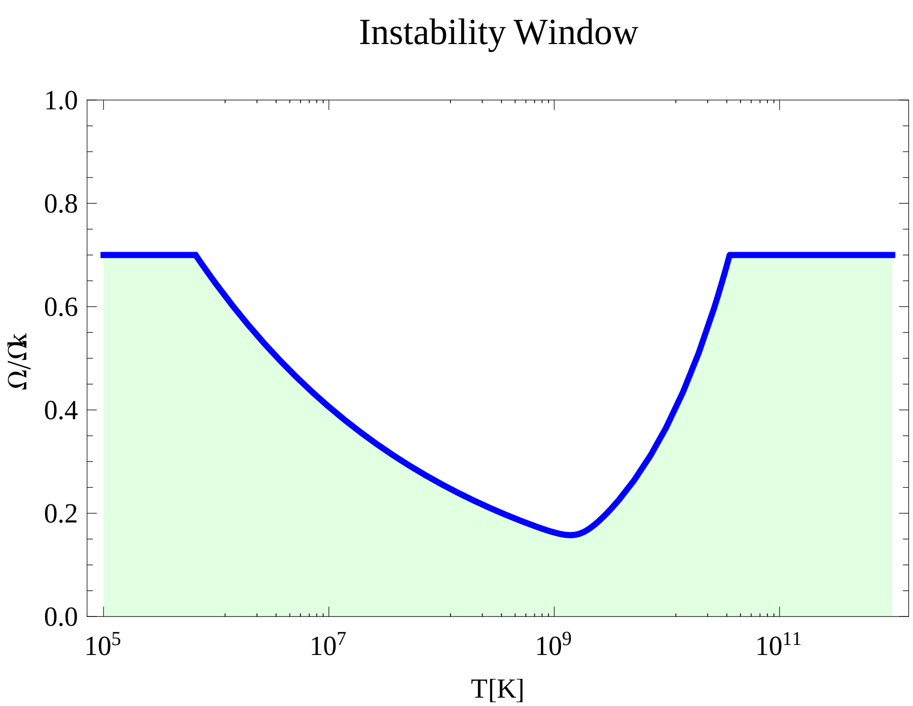

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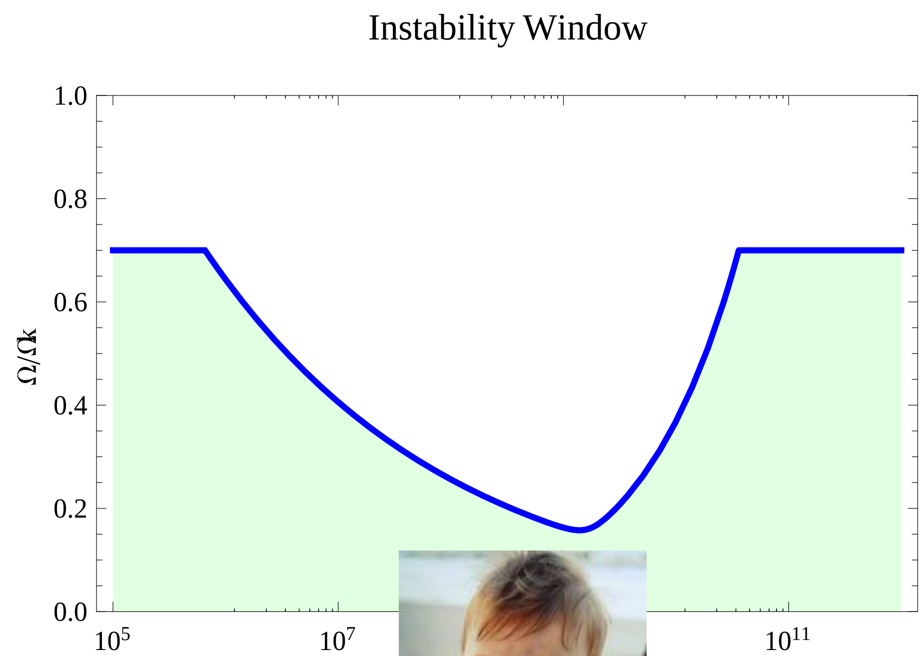

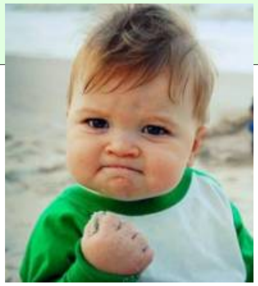

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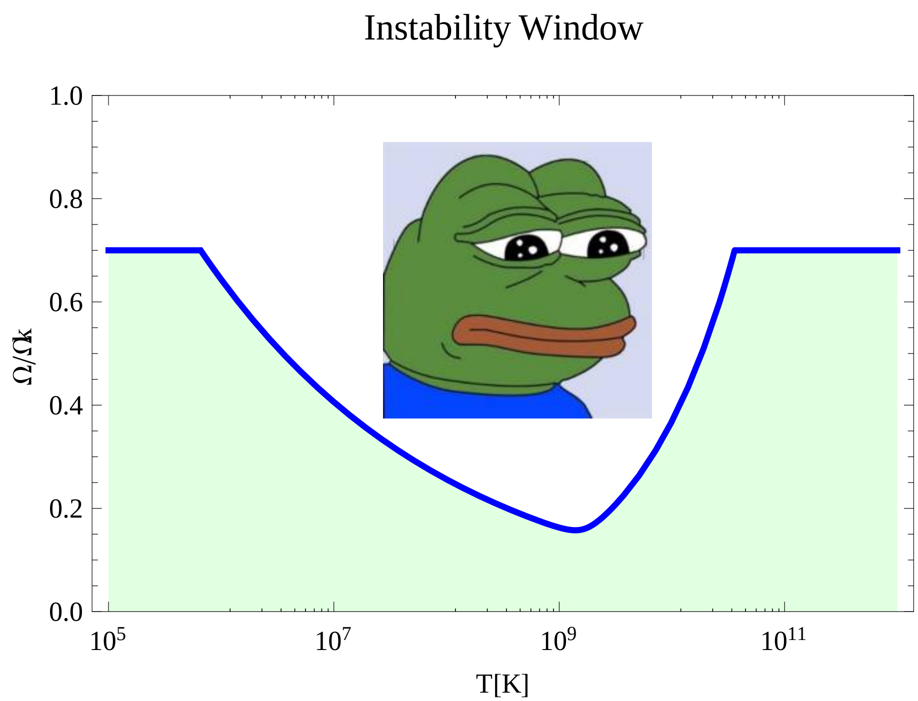
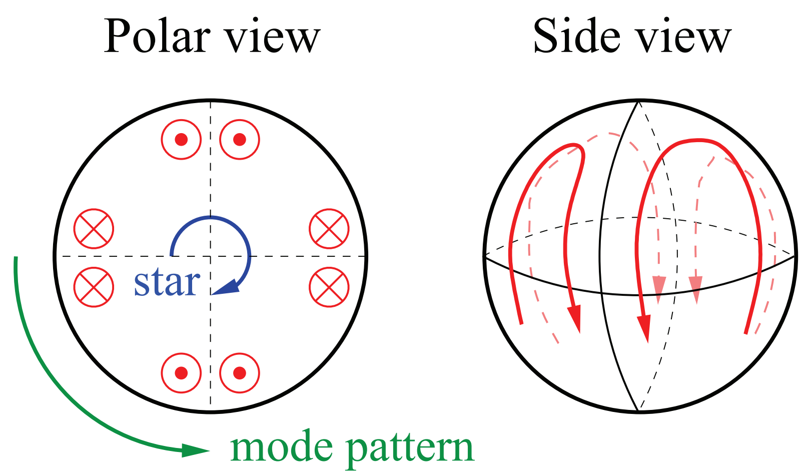


R-mode oscillations - Single fluid

r-mode:

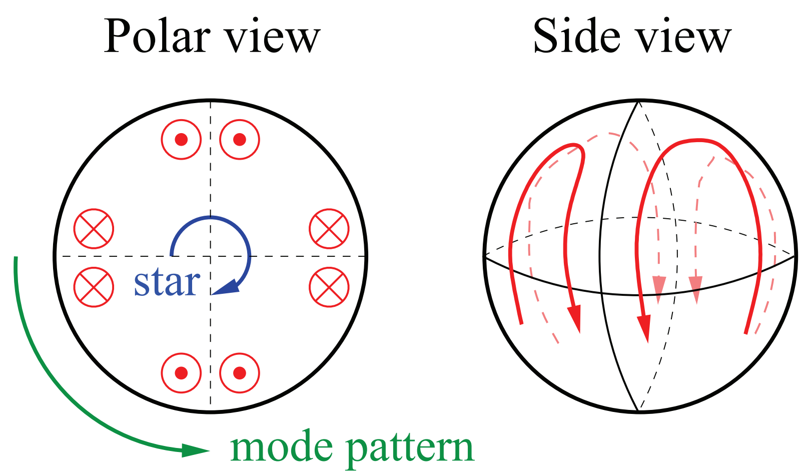
- nonradial oscillation which emits gravitational radiation

- CFS instability



for a star spinning at high frequency, r-mode oscillations are **unstable** if dissipative mechanisms are **slow**

R-mode oscillations - Single fluid



slow-rotation approximation:

- expansion in terms of $\Omega/\Omega_K \ll 1$

vector displacement:

$$\frac{\vec{\xi}}{r} = \sum_{l,m} \left(S_{lm}, Z_{lm}\partial_\theta, \frac{Z_{lm}}{\sin\theta}\partial_\phi \right) Y_l^m + \sum_{l,m} \left(0, \frac{K_{lm}}{\sin\theta}\partial_\phi, -K_{lm}\partial_\theta \right) Y_l^m.$$

spheroidal part $\sim \Omega^2$ toroidal part $\sim \Omega^0$

R-mode oscillations - Superfluid matter

r-modes in superfluid matter

two degrees of freedom:

- charged component (*p*,*e*)
- neutral component (*n*)

hydrodynamics described by:

$$\mathbf{v} = \frac{\rho_n \mathbf{v}_n + \rho_c \mathbf{v}_c}{\rho}$$

$$\mathbf{w} = \mathbf{v}_c - \mathbf{v}_n$$

- comoving **v**
- countermoving **w**

$$\delta\mathbf{v} = \partial_t \xi_+ \sim \Omega \xi_+ \qquad \delta\mathbf{w} = \partial_t \xi_- \sim \Omega \xi_-$$

R-mode oscillations - Superfluid matter

$$\frac{\vec{\xi}_+}{r} = (0, \frac{K_{lm}}{\sin\theta}\partial_\phi, -K_{lm}\partial_\theta)Y_l^m + \sum_{\nu,\mu}(S_{\nu\mu}, Z_{\nu\mu}\partial_\theta, \frac{Z_{\nu\mu}}{\sin\theta}\partial_\phi)Y_\nu^\mu$$

$$\frac{\vec{\xi}_-}{r} = (0, \frac{k_{lm}}{\sin\theta}\partial_\phi, -k_{lm}\partial_\theta)Y_l^m + \sum_{\nu,\mu}(s_{\nu\mu}, z_{\nu\mu}\partial_\theta, \frac{z_{\nu\mu}}{\sin\theta}\partial_\phi)Y_\nu^\mu$$

type of r-mode	K_{lm}	k_{lm}	ζ_{lm}	τ_{lm}	$S_{lm}, s_{lm}, Z_{lm}, z_{lm}$
standard r-mode	$\mathcal{O}(\Omega^0)$	$\mathcal{O}(\Omega^2)$	$\mathcal{O}(\Omega^2)$	$\mathcal{O}(\Omega^4)$	$\mathcal{O}(\Omega^2)$
superfluid r-mode	$\mathcal{O}(\Omega^2)$	$\mathcal{O}(\Omega^0)$	$\mathcal{O}(\Omega^4)$	$\mathcal{O}(\Omega^2)$	$\mathcal{O}(\Omega^2)$

$$\delta p = \rho g r \sum_{l,m} \zeta_{lm} Y_l^m \quad \delta\beta = g r \sum_{l,m} \tau_{lm} Y_l^m$$

Rocket effect

the mass and the velocity of the rocket change as it consumes its fuel

$$F = \dot{p} = m\dot{v} + \dot{m}v$$

mass conservation: $\partial_t \rho_x + \nabla_i(\rho_x v_x^i) = \Gamma_x \quad x = n, p, e$

mass creation rate: $\Gamma_p = \Gamma_e = -\Gamma_p$

for standard *npe*-matter: $n \to p + e^- + \bar{\nu}_e \quad p + e^- \to n + \nu_e$

Urca processes

Shoenberg to Gamow in the casino Urca in Rio de Janeiro:
the energy disappears in the nucleus of the supernova as quickly as the money disappeared at that roulette table:

$$
\begin{aligned}
n &\rightarrow p + e^- + \bar{\nu}_e \\
p + e^- &\rightarrow n + \nu_e
\end{aligned}
$$

- critical density (Haensel&Gnedin, 1994):

$$\rho_{\rm crit}(n \rightarrow p + e^- + \bar{\nu}_e) = 4.2 \times 10^{14} \mathrm{gcm}^{-3}$$

- critical proton fraction of the order 11% in npe-matter.

Urca processes

rates of the processes:

$$\Gamma^d_{\mathrm{Urca}} = \int \prod_{i=n,p,e,\nu} \left[\frac{d^3p_i}{(2\pi)^3 2E_i}\right] f_n\,(1-f_p)\,(1-f_e) \sum_{\mathrm{spin}} |M|^2 (2\pi)^4 \delta^{(4)}(P_i - P_f)$$

$$\Gamma^c_{\mathrm{Urca}} = \int \prod_{i=n,p,e,\nu} \left[\frac{d^3p_i}{(2\pi)^3 2E_i}\right] f_p f_e\,(1-f_n) \sum_{\mathrm{spin}} |M|^2 (2\pi)^4 \delta^{(4)}(P_i - P_f)$$

$$\Gamma_n = \Gamma^c_{\mathrm{Urca}} - \Gamma^d_{\mathrm{Urca}}$$

at equilibrium $\Gamma^c_{\mathrm{Urca}} = \Gamma^d_{\mathrm{Urca}} \equiv \bar{\Gamma}_{\mathrm{Urca}}$

linear analysis $f_i = \bar{f}_i + \delta f$

$$\Gamma_n = -\frac{1}{T}\left(\delta\mu_n - \delta\mu_c + \frac{m}{2}(1-\epsilon_n-\epsilon_p)(\delta\mathbf{w})^2\right)\bar{\Gamma}_{\mathrm{Urca}}$$

Urca processes

R_χ: reduced phase space for the interaction

$$\bar{\Gamma}_{\rm Urca} = R_\chi \times 1.9 \times 10^{33} (1 - \epsilon_c) \left(1 - \epsilon_c \frac{x_c}{1 - x_c}\right) \left(\frac{\rho_c}{\rho_0}\right)^{1/3} T_9^5 {\rm cm}^{-3} {\rm s}^{-1}$$

reduction factors for some superfluid phases (Haensel et al., 2000)

$$\begin{aligned}
R_A &= \left(0.2787 + \sqrt{(0.7213)^2 + (0.1564 v_A)^2}\right)^{3.5} \exp\left(2.9965 - \sqrt{(2.9965)^2 + v_A^2}\right) \\
R_B &= \left(0.2854 + \sqrt{(0.7146)^2 + (0.1418 v_B)^2}\right)^{3} \exp\left(2.0350 - \sqrt{(2.0350)^2 + v_B^2}\right) \\
R_C &= \frac{0.5 + (0.1086 v_C)^2}{1 + (0.2347 v_C)^2 + (0.2023 v_C)^4} + 0.5 \exp\left(1 - \sqrt{1 + (0.5 v_C)^2}\right)
\end{aligned}$$

where $v_X = \frac{\Delta_X(T)}{k_B T}$

Euler equations

two-fluids hydrodynamics:

$$\begin{aligned}(\partial_t + v_n^j \nabla_j)(v_i^n + \epsilon_n w_i) + \nabla_i(\tilde{\mu}_n + \Phi) + \epsilon_n w^j \nabla_i v_j^n &= +\frac{f_i^{\mathrm{MF}}}{\rho_n} \\ (\partial_t + v_c^j \nabla_j)(v_i^c - \epsilon_c w_i) + \nabla_i(\tilde{\mu}_c + \Phi) - \epsilon_c w^j \nabla_i v_j^c &= -\frac{f_i^{\mathrm{MF}}}{\rho_c} + (1 - \epsilon_n - \epsilon_c)\frac{\Gamma_n}{\rho_c} w_i\end{aligned}$$

mutual friction **rocket effect**

one can use as degrees of freedom the c.m. and the relative motion in any case one has two fluids that can oscillate

Euler equations

perturbed continuity equation

$$\partial_t \delta\rho_X + \nabla_i(\rho_X \delta v_X^i) = \Gamma_X, \quad X = n, c$$

hydrodinamical system in term of the comoving and countermoving velocities:

$$\begin{aligned}
\partial_t \delta\rho + \nabla_i(\rho\delta v^i) &= 0 \\
\partial_t \delta x_c + \frac{1}{\rho}\nabla \cdot [x_c(1-x_c)\rho\delta\vec{w}] + \delta\vec{v}\cdot\nabla x_c + \frac{\Gamma_n}{\rho} &= 0 \\
\partial_t \delta v_i + 2\epsilon_{ijk}\Omega^j\delta v^k + \frac{1}{\rho}\nabla_i\delta p - \frac{\delta\rho}{\rho^2}\nabla_i p + \nabla_i\delta\Phi &= 0 \\
\partial_t(1-\bar{\epsilon})\delta w_i + \nabla_i(\delta\beta) + 2\bar{B}'\epsilon_{ijk}\Omega_j\delta w^k - 2\bar{B}\epsilon_{ijk}\hat{\Omega}^j\epsilon^{klm}\Omega_l\delta w_m &= 0
\end{aligned}$$

Stability analysis

timescale of dissipative processes

$$\frac{1}{\tau_{\rm diss}} = \frac{\left(\frac{dE}{dt}\right)_{\rm diss}}{2E}$$

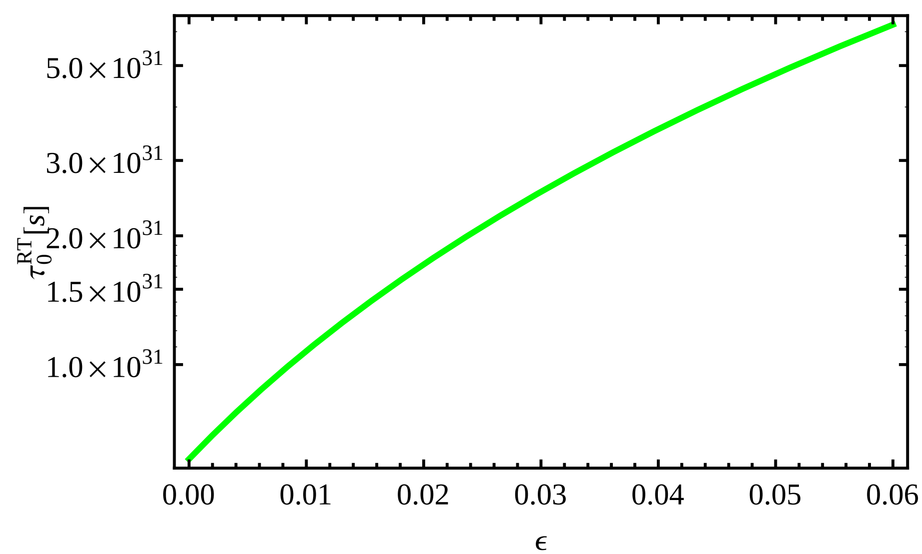
- solve the free equations of the oscillations
- E: approximated by the kinetic energy of the free oscillations
- $\left(\frac{dE}{dt}\right)_{\rm diss}$: integrate the Euler eqs and insert the free solution

Stability analysis

critical condition of stability:

$$-\frac{1}{\tau_{\rm gw}} + \frac{1}{\tau_{\rm sv}} + \frac{1}{\tau_{\rm bv}} + \frac{1}{\tau_{\rm MF}} + \frac{1}{\tau_{\rm RT}} = 0$$

1) standard r-mode oscillations are not efficiently damped by the rocket term. in that case mutual friction, bulk and shear viscosity dominate



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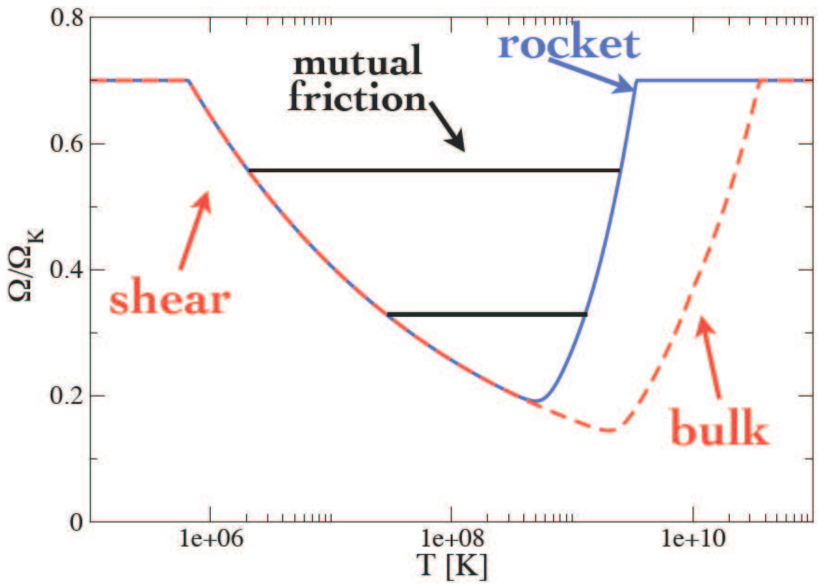
$$-\frac{1}{\tau_{\rm gw}} + \frac{1}{\tau_{\rm sv}} + \frac{1}{\tau_{\rm bv}} + \frac{1}{\tau_{\rm MF}} + \frac{1}{\tau_{\rm RT}} = 0$$

2) the instability window for the superfluid r-mode is reduced. the rocket effect acts as an effective bulk viscosity in the range of high temperature



Conclusions

- asterosiesmology and star's structure
- r-modes and CFS instability
- reduction of superfluid r-mode instability with the rocket effect acting as an effective bulk viscosity
- what's going on: different processes of matter transformation

thank you all for the attention