A phenomenological study of helicity amplitudes of high energy exclusive leptoproduction of the ρ meson

Adrien Besse

Soltan Institute for Nuclear Studies - Laboratoire de Physique Théorique d'Orsay

Cracow School of Theoretical Physics
Zakopane, June 12th 2011

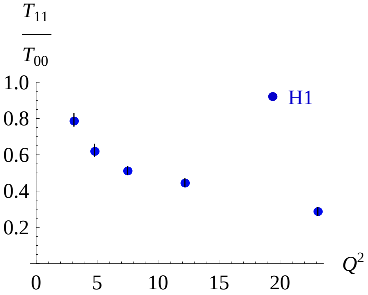
arXiv:11051761.

in collaboration with
I. V. Anikin (JINR, Dubna), D .Yu. Ivanov (SIM, Novossibirsk), B. Pire (CPhT, Palaiseau), L. Szymanowski (SINS, Warsaw) and S. Wallon (LPT, Orsay)

Introduction

Experimental data of helicity amplitudes at high energy

- Helicity amplitudes $T_{\lambda_\rho \lambda_\gamma}$: $\gamma^*_{\lambda_\gamma} + p \rightarrow \rho_{\lambda_\rho} + p$
- H1 and ZEUS data for Helicity Amplitudes at HERA:



S. Chekanov et al. (2007), F.D Aaron et al. (2010)

- Kinematics
  - High energy in the center of mass $30\,GeV < W < 180\,GeV$
  - Photon Virtuality $2.5\,GeV^2 < Q^2 < 60\,GeV^2$
  - $|t| < 1\,GeV^2$

$$\Rightarrow s_{\gamma^* p} = W^2 >> Q^2 >> \Lambda^2_{QCD}$$

Introduction

A Theoretical approach within k_T factorisation

- Perturbative Regge Limit :

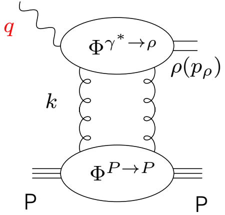
  Regge Limit : $s = W^2 >> Q^2, |t|, M^2_{\text{hadron}}$

  Hard scale : $Q >> \Lambda_{QCD}$

- k_T factorisation

  $\mathcal{M} \propto s^{\sum \sigma_i - N + 1}$

Amplitudes with gluons exchange in t-channel dominate at large s ($s = W^2$)



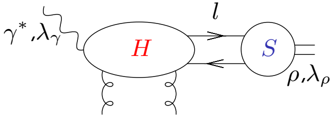
Born order: 2 t-channel gluons

Introduction
A Theoretical approach

Impact factors $\Phi^{\gamma^* \to \rho}$ and $\Phi^{P \to P}$

- $\Phi^{\gamma^* \to \rho}$ Light-Cone Collinear factorisation

$Q^2 >> \Lambda^2_{QCD}$



  - Twist t : Impact factor behaves as $1/Q^{t-1}$
  - $T_{00} \equiv \gamma^*_L \to \rho_L$ impact factor : Dominant term at twist 2
  - $T_{11} \equiv \gamma^*_T \to \rho_T$ impact factor : Dominant term at twist 3

  Recently computed at $t = t_{min} \approx 0$

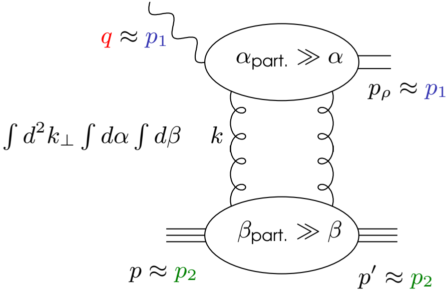
  Nucl. Phys. B **828** (2010) 1-68. by Anikin et al.

- Phenomenological modele for $\Phi^{P \to P}$

Impact factor for exclusive processes

k_T factorisation

- Light-cone vectors p_1 and p_2 : $({p_1}^2 = {p_2}^2 = 0,\ 2\,p_1 \cdot p_2 = s)$
  Sudakov decomposition : $\Rightarrow k = \alpha\, p_1 + \beta\, p_2 + k_\perp$



Non-sens polarisations:

$$\frac{g^{\mu\nu}}{k^2+i\epsilon} \to \frac{\epsilon_{NS}^{up\ \mu}\ \epsilon_{NS}^{down\ \nu}}{k_\perp^2+i\epsilon}$$

$$\epsilon_{NS}^{up} = \sqrt{\frac{2}{s}}\, p_2$$

$$\epsilon_{NS}^{down} = \sqrt{\frac{2}{s}}\, p_1$$

$$\Phi^{\gamma^* \to \rho}(\underline{k}^2) = e^{\gamma^*\mu}\, \frac{1}{2\,s} \int \frac{d\beta}{2\pi}\, \mathcal{S}_\mu^{\gamma^*\, g \to \rho\, g}(\beta, \underline{k}^2),$$

- Impact factor representation of the helicity amplitudes

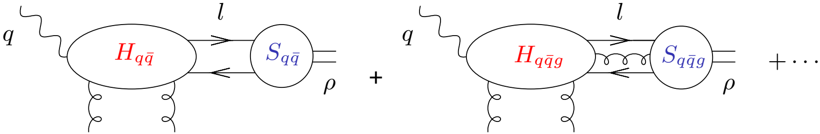
$$T_{\lambda_\rho \lambda_\gamma} = is \int \frac{d^2\,\underline{k}}{(2\pi)^2 (\underline{k}^2)^2}\, \Phi^{\gamma^*(\lambda_\gamma) \to \rho(\lambda_\rho)}(\underline{k})\ \Phi^{P \to P}(-\underline{k})$$

Collinear factorization
Light-Cone Collinear approach

- The impact factor can be written as

$$\Phi = \int d^4 l \cdots \mathrm{tr}[H(l \cdots) \quad S(l \cdots)]$$

hard part — soft part



- At the 2-body level:

$$S_{q\bar{q}}(l) = \int d^4 z \, e^{-il\cdot z} \langle \rho(p) | \psi(0) \, \bar{\psi}(z) | 0 \rangle ,$$

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

1 - Momentum factorization

- Use Sudakov decomposition in the form $(p = p_1\,,\; n = 2\,p_2/s \Rightarrow p\cdot n = 1)$

$$l_\mu \quad = \quad y\,p_\mu \quad + \quad l^\perp_\mu \quad + \quad (l\cdot p)\,n_\mu, \qquad y = l\cdot n$$

scaling: $\quad 1 \quad 1/Q \quad 1/Q^2$

- decompose $H(k)$ around the p direction:

$$H(l) = H(yp) + \left.\frac{\partial H(l)}{\partial l_\alpha}\right|_{l=yp} l^\perp_\alpha + \dots$$

- The twist 3 term $l^\perp_\alpha$ turns into a transverse derivative of the soft term $\Rightarrow$ one will deal with $\int d^4z\; e^{-il\cdot z}\langle\rho(p)|\psi(0)\; i\;\overleftrightarrow{\partial}_{\alpha\perp}\bar{\psi}(z)|0\rangle$

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

Momentum and spinorial factorization

- Integration over l of the Soft part:

$$(\tilde{S}_{q\bar{q}}, \partial_\perp \tilde{S}_{q\bar{q}})(y) = \int \frac{d\lambda}{2\pi}\, e^{-i\lambda y} \langle \rho(p)| \psi(0)\,(1,\, i \overleftrightarrow{\partial_\perp})\bar{\psi}(\lambda n)|0\rangle$$

Axial gauge condition for gluons, *i.e.* $n \cdot A = 0 \Rightarrow$ Wilson line $= 1$

- $\int dy$ performs the longitudinal momentum factorization

$$\Phi = \int dy\ \{\mathrm{tr}\,[H_{q\bar{q}}(y\,p)\, S_{q\bar{q}}(y)] + \mathrm{tr}\,[\partial_\perp H_{q\bar{q}}(y\,p)\, \partial_\perp S_{q\bar{q}}(y)]\}$$

- Spinor (and color) factorisation:

$$\Phi = \int dy\ \left\{\mathrm{tr}\,[H_{q\bar{q}}(y\,p)\,\Gamma]\ S^{\Gamma}_{q\bar{q}}(y) + \mathrm{tr}\,[\partial_\perp H_{q\bar{q}}(y\,p)\,\Gamma]\ \partial_\perp S^{\Gamma}_{q\bar{q}}(y)\right\}$$

$$S^{\Gamma}_{q\bar{q}}(y) = \int \frac{d\lambda}{2\pi}\, e^{-i\lambda y} \langle \rho(p)| \bar{\psi}(\lambda n)\,\Gamma\,\psi(0)|0\rangle$$

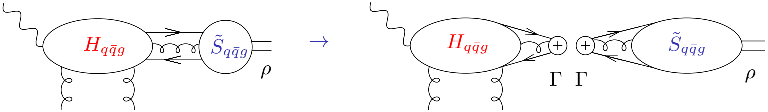
$$\partial_\perp S^{\Gamma}_{q\bar{q}}(y) = \int \frac{d\lambda}{2\pi}\, e^{-i\lambda y} \langle \rho(p)| \bar{\psi}(\lambda n)\,\Gamma\, i \overleftrightarrow{\partial_\perp}\,\psi(0)|0\rangle$$

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (3-body case)

Factorization of 3-body contributions

- 3-body contributions start at genuine twist 3
  $\Rightarrow$ no need for Taylor expansion
- Factorisation goes in the same way as for 2-body case



Collinear factorization

Parametrization of vacuum–to–rho-meson matrix elements (DAs)

2-body and 3-body non-local correlators

- Lorentz and Parity analysis $\Rightarrow$ relevant parametrisation of the non-local correlators

- vector correlator $\Gamma = \gamma_\mu$

$$\langle \rho(p)|\bar{\psi}(z)\gamma_\mu \psi(0)|0\rangle = \int dy\, e^{-i(yp)\cdot z} m_\rho\, f_\rho\, \left[\varphi_1(y)\,(e^* \cdot n)p_\mu + \varphi_3(y)\, e_\mu^{*T}\right]$$

- $\Rightarrow$ 5 2-body DAs $\{\varphi_1, \varphi_A, \varphi_3, \varphi_{1T}, \varphi_{AT}\}$

- $\Rightarrow$ 2 3-body DAs $\{B(y_1, y_2), D(y_1, y_2)\}$

- Relations between DAs : Equation of motion and n-independence
  $\Rightarrow$ 3 independent DAs : $\{\varphi_1, B(y_1, y_2), D(y_1, y_2)\}$

Collinear factorization

Wandzura-Wilczek and Genuine contributions

- Solution in the Wandzura-Wilczek Approximation (WW)

$$\varphi_1 \Rightarrow \{\varphi_3^{WW}(y), \varphi_A^{WW}(y), \varphi_{1T}^{WW}(y), \varphi_{AT}^{WW}(y)\}$$

- Genuine solutions

$$\{B(y_1, y_2), D(y_1, y_2)\} \Rightarrow \{\varphi_3^{gen}(y), \varphi_A^{gen}(y), \varphi_{1T}^{gen}(y), \varphi_{AT}^{gen}(y)\}$$

- Evolution of the DAs   P. Ball, V.M Braun, Y. Koike, K. Tanaka

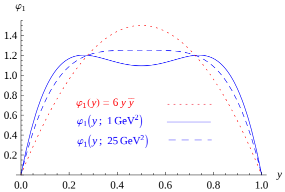
$$\begin{aligned}
\varphi_1(y,\mu^2) &= 6y\bar{y}(1 + a_2(\mu^2)\frac{3}{2}(5(y-\bar{y})^2 - 1)) \\
B(y_1, y_2; \mu^2) &= -5040 y_1 \bar{y_2}(y_1 - \bar{y_2})(y_2 - y_1) \\
D(y_1, y_2; \mu^2) &= -360 y_1 \bar{y_2}(y_2 - y_1)(1 + \frac{\omega^A_{\{1,0\}}(\mu^2)}{2}(7(y_2 - y_1) - 3))
\end{aligned}$$

with $\mu^2 \approx Q^2$ the collinear factorisation scale

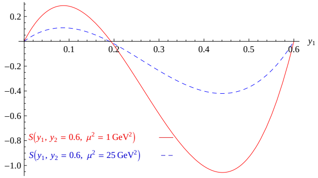
Collinear factorisation

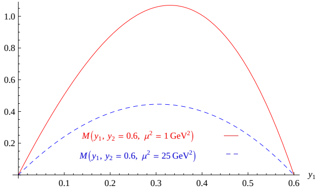
DAs dependence on μ^2

- $\varphi_1(y, \mu^2)$



- $M(y_1, y_2) = \zeta_\rho^V(\mu^2) B(y_1, y_2; \mu^2) - \zeta_\rho^A(\mu^2) D(y_1, y_2; \mu^2)$
  $S(y_1, y_2) = \zeta_\rho^V(\mu^2) B(y_1, y_2; \mu^2) + \zeta_\rho^A(\mu^2) D(y_1, y_2; \mu^2)$





Ratios of Helicity Amplitudes

A modele for the proton impact factor

- $T_{\lambda_\rho \lambda_\gamma}(Q,M) = is \int \frac{d^2\underline{k}}{(2\pi)^2} \frac{1}{(\underline{k}^2)^2} \Phi^{P\to P}(\underline{k};M^2)\Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)}(\underline{k};Q^2)$

- Phenomenological Modele for $\Phi^{P\to P}$

$$\Phi^{P\to P}(\underline{k};M^2) \propto \left[\frac{1}{M^2} - \frac{1}{M^2+\underline{k}^2}\right]$$

J.F Gunion, D.E Soper

- $\gamma^*_L \to \rho_L$ helicity amplitude:

$$T_{00} \propto \frac{is}{(2\pi)} \int_{\lambda^2}^{\infty} d\underline{k}^2 \frac{1}{(\underline{k}^2)^2} \left(\frac{1}{M^2} - \frac{1}{\underline{k}^2+M^2}\right) \frac{1}{Q}\int_0^1 dy\, \varphi_1(y,\mu^2) \frac{\underline{k}^2}{\underline{k}^2 + y\bar{y}Q^2}$$

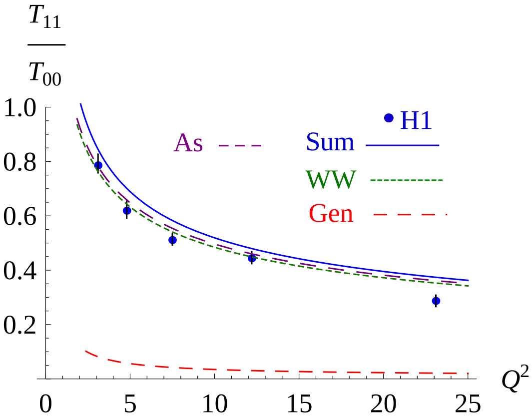
- The WW contribution:

$$T_{11}^{WW} \propto \frac{is}{2\pi} \int_{\lambda^2}^{\infty} d(\underline{k}^2) \frac{1}{(\underline{k}^2)^2} \left(\frac{1}{M^2} - \frac{1}{\underline{k}^2+M^2}\right) \left(\frac{(\epsilon_\gamma.\epsilon^*_\rho)m_\rho}{Q^2} \int_0^1 du \frac{\varphi_1(u;\mu^2)}{u} \int_0^u dy \frac{\underline{k}^2(\underline{k}^2+2y\bar{y}Q^2)}{(\underline{k}^2+y\bar{y}Q^2)^2}\right)$$

Ratios of Helicity Amplitudes

Comparison with H1 data : T_{11}/T_{00}

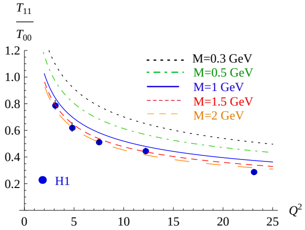
- Genuine and Wandzura-Wilczek Contributions at $M = 1GeV$



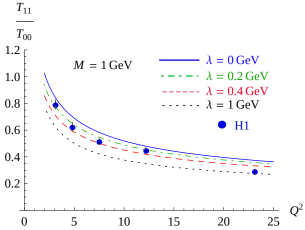
Ratios of Helicity Amplitudes

Dependence on parameters M and λ

- M dependence of the ratio T_{11}/T_{00}



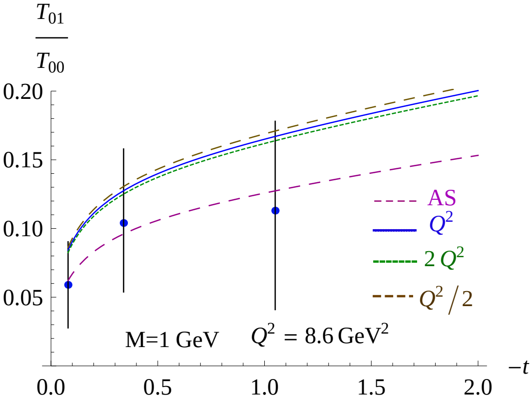
- Soft gluon effect : λ IR cut-off on k_T integrals



Ratios of Helicity Amplitudes

Comparison with H1 data : T_{01}/T_{00}

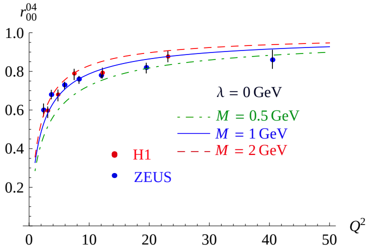
- T_{01}/T_{00} at $M = 1\,GeV$, Dependence on μ^2 $M = 1GeV$:



Conclusion : Perspectives for $\Phi^{\gamma_T^* \to \rho_T}$

- Good agreement with Experimental data
  - reasonable values of $M \approx M_p$ and $\lambda \approx 0\, GeV$
  - weak sensitivity
- Perspectives :
  - Extend the kinematic to $t \neq t_{min}$
  - Passage in coordinate space : Link with the Dipole modele
  - Implementation of Saturation effects

Comparison of r_{00}^{04} in the S-Channel Helicity Conservation approximation with ZEUS and H1 data:



Impact representation

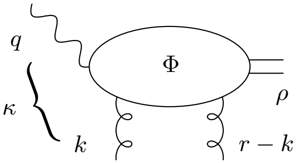
$\underline{k}$ = Eucl. $\leftrightarrow$ $k_\perp$ = Mink.

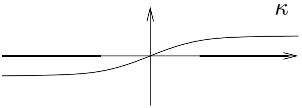
$$\mathcal{M} = is \int \frac{d^2\,\underline{k}}{(2\pi)^2 \underline{k}^2\,(\underline{r}-\underline{k})^2} \Phi^{\gamma^*(q_1)\to\rho(p_1^\rho)}(\underline{k},\underline{r}-\underline{k})\; \Phi^{\gamma^*(q_2)\to\rho(p_2^\rho)}(-\underline{k},-\underline{r}+\underline{k})$$

With $\gamma^*_{L,T}(q) g(k_1) \to \rho_{L,T}\, g(k_2)$ impact factor:

$$\Phi^{\gamma^*\to\rho}(\underline{k}^2) = e^{\gamma^*\mu}\,\frac{1}{2\,s}\int \frac{d\kappa}{2\pi}\,\text{Disc}_\kappa\, \mathcal{S}_\mu^{\gamma^*\,g\to\rho\,g}(\underline{k}^2),$$

with $\kappa = (q+k)^2 = \beta\, s - Q^2 - \underline{k}^2$





3-body non-local correlators

genuine twist 3

- vector correlator

$$\langle \rho(p)|\bar{\psi}(z_1)\gamma_\mu g A_\alpha^T(z_2)\psi(0)|0\rangle \stackrel{\mathcal{F}_2}{=} m_\rho\, f_3^V\, B(y_1, y_2)\, p_\mu\, e_\alpha^{*T},$$

- axial correlator

$$\langle \rho(p)|\bar{\psi}(z_1)\gamma_5\gamma_\mu g A_\alpha^T(z_2)\psi(0)|0\rangle \stackrel{\mathcal{F}_2}{=} m_\rho\, f_3^A\, i\, D(y_1, y_2)\, p_\mu\, \varepsilon_{\alpha\lambda\beta\delta}\, e_\lambda^{*T}\, p_\beta\, n_\delta,$$

where $y_1, \bar{y}_2, y_2 - y_1$ = quark, antiquark, gluon momentum fraction

and $\stackrel{\mathcal{F}_2}{=} \int_0^1 dy_1 \int_0^1 dy_2 \, \exp\left[i\, y_1\, p\cdot z_1 + i(y_2 - y_1)\, p\cdot z_2\right],$ with $z_{1,2} = \lambda n$

$\Rightarrow$ 2 3-body DAs

Equations of motion

twist 2
kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

- Dirac equation leads to

$$\langle i(\overrightarrow{\not{D}}\,(0)\psi(0))_\alpha\,\bar{\psi}_\beta(z)\rangle = 0 \qquad (i\,\overrightarrow{D}_\mu = i\,\overrightarrow{\partial}_\mu + g\,A_\mu)$$

$\Rightarrow$ 2 Equations of motion:

$$\bar{y}_1\,\varphi_3(y_1) + \bar{y}_1\,\varphi_A(y_1) + \varphi_1^T(y_1) + \varphi_A^T(y_1)$$

$$+ \int dy_2 \left[\zeta_3^V\,B(y_1,\,y_2) + \zeta_3^A\,D(y_1,\,y_2)\right] = 0 \qquad \text{and} \quad (\bar{y}_1 \leftrightarrow y_1)$$

A minimal set of DAs

independence of the full amplitude with respect to the light-cone vector n

- vector correlators

$$\frac{d}{dy_1}\varphi_1^T(y_1) = -\varphi_1(y_1) + \varphi_3(y_1) - \zeta_3^V \int_0^1 \frac{dy_2}{y_2 - y_1}\left(B(y_1, y_2) + B(y_2, y_1)\right)$$

- axial correlators

$$\frac{d}{dy_1}\varphi_A^T(y_1) = \varphi_A(y_1) - \zeta_3^A \int_0^1 \frac{dy_2}{y_2 - y_1}\left(D(y_1, y_2) + D(y_2, y_1)\right)$$

$\phi_1(y)$ $\leftarrow$ 2 body twist 2 correlator
$B(y_1, y_2)$ $\leftarrow$ 3 body genuine twist 3 vector correlator
$D(y_1, y_2)$ $\leftarrow$ 3 body genuine twist 3 axial correlator

$\gamma_L^* \to \rho_L$ impact factor

$$\Phi^{\gamma_L^* \to \rho_L}(\underline{k}^2) = \frac{2\, e\, g^2\, f_\rho}{Q} \frac{\delta^{ab}}{2\, N_c} \int dy\, \varphi_1(y) \frac{\underline{k}^2}{y\, \bar{y}\, Q^2 + \underline{k}^2}$$

pure twist 2 scaling (from ρ-factorization point of view)

I.F. Ginzburg, S.L. Panfil, V.G. Serbo (1987)

Computation and results

Results: $\gamma_T^* \to \rho_T$ impact factor

$\gamma_T^* \to \rho_T$ impact factor:

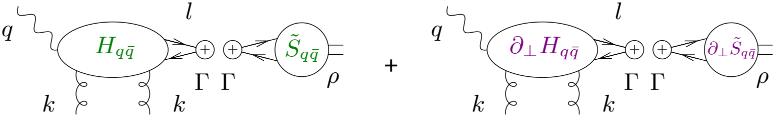
$$\Phi^{\gamma_T^* \to \rho_T}(\underline{k}^2) = \Phi_{n.f.}^{\gamma_T^* \to \rho_T}(\underline{k}^2)\, T_{n.f.} + \Phi_{f.}^{\gamma_T^* \to \rho_T}(\underline{k}^2)\, T_{f.}$$

$$T_{n.f.} = -(e_\gamma \cdot e^*) \Rightarrow \pm \to \pm \quad \text{and} \quad T_{f.} = \frac{(e_\gamma \cdot k)(e^* \cdot k)}{\underline{k}^2} + \frac{(e_\gamma \cdot e^*)}{2} \Rightarrow \pm \to \mp$$

$$\begin{aligned}
&\Phi_{n.f.}^{\gamma_T^* \to \rho_T}(\underline{k}^2) \\
&= -\frac{e\, g^2 m_\rho f_\rho}{2\sqrt{2}\, Q^2} \frac{\delta^{ab}}{2\, N_c} \left\{ -2 \int dy_1 \frac{\left(\underline{k}^2 + 2\, Q^2\, y_1\, (1-y_1)\right) \underline{k}^2}{y_1\, (1-y_1)\, \left(\underline{k}^2 + Q^2\, y_1\, (1-y_1)\right)^2} \left[(2y_1 - 1)\, \varphi_1^T(y_1) + \varphi_A^T(y_1)\right] \right. \\
&+2 \int dy_1\, dy_2 \left[\zeta_3^V\, B(y_1, y_2) - \zeta_3^A\, D(y_1, y_2)\right] \frac{y_1\, (1-y_1)\, \underline{k}^2}{\underline{k}^2 + Q^2\, y_1\, (1-y_1)} \left[\frac{(2 - N_c/C_F) Q^2}{\underline{k}^2\, (y_1 - y_2 + 1) + Q^2\, y_1\, (1-y_2)} \right. \\
&\left. - \frac{N_c}{C_F} \frac{Q^2}{y_2 \underline{k}^2 + Q^2\, y_1\, (y_2 - y_1)} \right] - 2 \int dy_1\, dy_2 \left[\zeta_3^V\, B(y_1, y_2) + \zeta_3^A\, D(y_1, y_2)\right] \left[\frac{2 + N_c/C_F}{1 - y_1} \right. \\
&+ \frac{y_1\, Q^2}{\underline{k}^2 + Q^2 y_1\, (1-y_1)} \left(\frac{(2 - N_c/C_F)\, y_1\, \underline{k}^2}{\underline{k}^2\, (y_1 - y_2 + 1) + Q^2 y_1\, (1 - y_2)} - 2 \right) \\
&\left.\left. + \frac{N_c}{C_F} \frac{(y_1 - y_2)\, (1 - y_2)}{1 - y_1} \frac{Q^2}{\underline{k}^2\, (1 - y_1) + Q^2\, (y_2 - y_1)\, (1 - y_2)} \right] \right\}
\end{aligned}$$

2 - Spinorial (and color) factorization

- Use Fierz decomposition of the Dirac (and color) matrices $\psi(0)\,\bar{\psi}(z)$ and $\psi(0)\,i\,\overleftrightarrow{\partial_\perp}\,\bar{\psi}(z)$:



$$\Phi = \int dy \left\{ \mathrm{tr}\left[H_{q\bar{q}}(y\,p)\,\Gamma\right]\; S^{\Gamma}_{q\bar{q}}(y) + \mathrm{tr}\left[\partial_\perp H_{q\bar{q}}(y\,p)\,\Gamma\right]\; \partial_\perp S^{\Gamma}_{q\bar{q}}(y) \right\}$$

$$S^{\Gamma}_{q\bar{q}}(y) = \int \frac{d\lambda}{2\pi}\, e^{-i\lambda y} \langle \rho(p)|\bar{\psi}(\lambda n)\,\Gamma\,\psi(0)|0\rangle$$

$$\partial_\perp S^{\Gamma}_{q\bar{q}}(y) = \int \frac{d\lambda}{2\pi}\, e^{-i\lambda y} \langle \rho(p)|\bar{\psi}(\lambda n)\,\Gamma\, i\,\overleftrightarrow{\partial_\perp}\,\psi(0)|0\rangle$$

- Axial gauge condition for gluons, *i.e.* $n \cdot A = 0 \Rightarrow$ Wilson line $= 1$