# POLARIZED DIS STRUCTURE FUNCTIONS OF NUCLEONS


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# POLARIZED DIS STRUCTURE FUNCTIONS OF NUCLEONS

- Motivation
- Theoretical framework
- The method
- QCD fit of $g_1$ data
- Results and conclusion

# Motivation

One of the most fundamental properties of elementary particles is their spin because it determines their symmetry behavior under space-time transformation. Because of that the nucleon spin still consists a developing field.

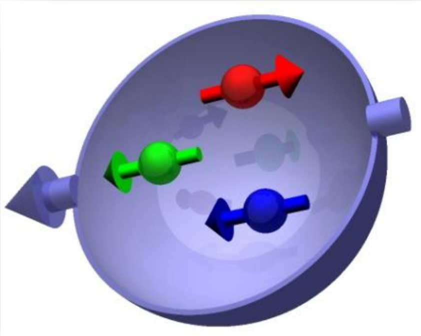
Experiments performed at CERN, SLAC, DESY and JLAB have contributed a vast amount of experimental data on inclusive polarized deeply inelastic lepton-nucleon scattering (DIS) during the last years. Having new experimental data from COMPASS, we have enough motivation to study and utilize the spin structure and quark helicity distributions.

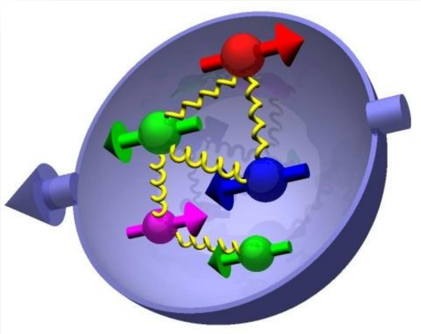
During the recent years we have done several comprehensive analysis of the polarized deep inelastic scattering data, based on the Valon model up to NLO approximation.

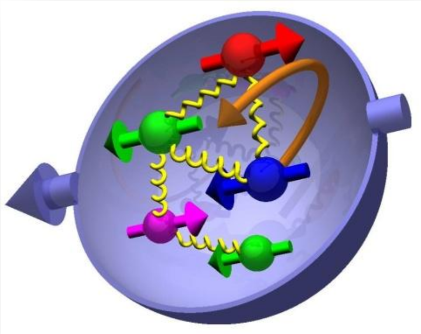
A. N. Khorramian, A. Mirjalili and S. A. Tehrani, JHEP 0410 (2004) 062
S. A. Tehrani and Ali N. Khorramian, JHEP 07 (2007) 048

# Where the ½ spin of the nucleon comes from ?

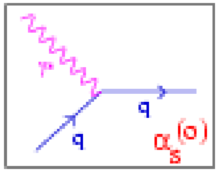
Nucleons as composite fermions obtain their spin in terms of a superposition of the spins and orbital angular momenta of their constituents, the quark and gluons. It came as a surprise when the EMC published its result more than 20 years ago,which showed that the quarks do contribute only by small fraction to the nucleon's spin. The obvious conclusion was to assume that the spin of the gluons and the orbital angular momenta of all constituents have to account for missing fraction. This result initiated activities worldwide both on the experimental and the theoretical side in order to understand this spin puzzle and, finally, the spin structure of the nucleon.





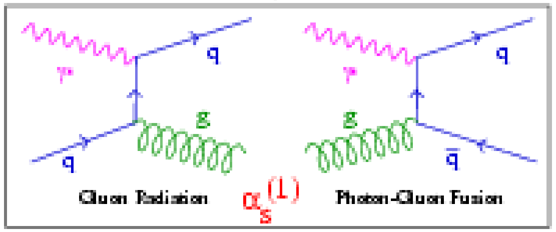


$$\frac{1}{2} = \underbrace{\frac{1}{2}(\Delta u_v + \Delta d_v + \Delta q_s)}_{\Delta\Sigma} + \Delta G + L_q + L_g$$




⟹ No gluons

$$g_1^0(x) = \frac{1}{2}\sum e_q^2\, \Delta q(x)$$

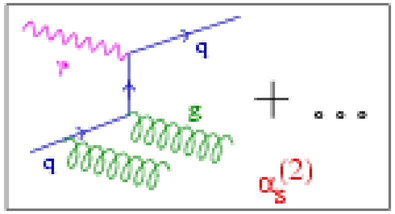



⟹ Quarks are re-defined

with the inclusion of $\Delta G$

(weak dependence)

$$g_1^{LO}(x,Q^2) = \frac{1}{2}\sum e_q^2\, \Delta q(x,Q^2)$$




⟹ $g_1$ becomes

explicitly $\Delta G$ dependent

$$g_1^{NLO}(x,Q^2) = g_1^{LO} + \frac{\alpha_s}{2\pi}\,\frac{1}{2}\sum e_q^2\left[\Delta q(x,Q^2)\otimes C_q + \Delta G(x,Q^2)\otimes C_G\right]$$

# Neutron and Deuteron polarized structure functions

Because of isospin symmetry the polarized structure functions for proton and neutron are related by exchange of up and down quarks and anti quarks.

$$g_1^n(x,Q^2) = g_1^p(x,Q^2) - \frac{1}{6}[\delta u_v(x,Q^2) - \delta d_v(x,Q^2)]$$

The polarized deuteron spin structure function can be obtained via this relation:

$$g_1^d(x,Q^2) = \frac{1}{2}[g_1^p(x,Q^2) + g_1^n(x,Q^2)](1 - \frac{3}{2}w_D)$$

$w_D(\simeq\ 0.058)$

B. Lampe and E. Reya, Phys. Rept. 332 (2000) 1

# QCD Analysis of Structure Function

According to the following procedure determination of the PPDF's can be divided in two steps:

1. The first step is determination of the non-perturbative initial distributions at some usually rather low scale. The PPDF's are parameterized by smooth analytical functions at $Q_0^2$ as a function of $x$ with a certain number of free parameters.

2. The second step is the perturbative calculation of their scale dependence to obtain the results at the hard scales. PPDF's are evolved in $Q^2$ using the DGLAP equations. Afterwards predictions for structure functions are calculted. The free parameters are determined by performing a $\chi^2$ fit to the data.

# Parameterization of the polarized parton distributions

In the present analysis we choose the following parametrization for the polarized parton densities in the input scale of $Q_0^2 = 4$ GeV$^2$

$$
\begin{aligned}
x\delta u_v &= A_{u_v}\eta_{u_v}x^{a_{uv}}(1-x)^{b_{uv}}(1+c_{u_v}x) \\
x\delta d_v &= A_{d_v}\eta_{d_v}x^{a_{dv}}(1-x)^{b_{dv}}(1+c_{d_v}x) \\
x\delta\overline{q} &= A_s\eta_s x^{a_s}(1-x)^{b_s} \\
x\delta g &= A_g\eta_g x^{a_g}(1-x)^{b_g}
\end{aligned}
$$

$x^{a_i}$ controls the behavior of the parton density at low $x$
$(1-x^{b_i})$ controls the behavior of the parton density at large $x$
The remaining polynomial factor accounts for additional degrees of freedom at medium $x$

# Parameterization of the polarized parton distributions

The normalization constant $A_i$

$$A_i^{-1} = \left(1 + c_i \frac{a_i}{a_i + b_i + 1}\right) B\left(a_i, b_i + 1\right)$$

are chosen such that the $\eta_i$ are the first moments of $\delta q_i(x, Q_0^2)$, $\eta_i = \int_0^1 dx \delta q_i(x, Q_0^2)$.

$$B(a,b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$$

The sea–quark distribution was assumed to be described according to $SU(3)$ flavor symmetry.

$$\delta \overline{q}_s(x,Q^2) = \delta \overline{u}(x,Q^2) = \delta \overline{d}(x,Q^2) = \delta s(x,Q^2) = \delta \overline{s}(x,Q^2)$$

# Constrains on PPDF

The first moments of the polarized valence distributions, can be fixed exploiting the knowledge of the parameters F and D as measured in neutron and hyperon $\beta$-decays according to the relation:

$$a_3 = \int_0^1 dx \delta q_3 = F + D = 1.2670 \pm 0.0035$$

$$a_8 = \int_0^1 dx \delta q_8 = 3F - D = 0.585 \pm 0.025$$

$$F = 0.464 \pm 0.008 \quad \text{and} \quad D = 0.806 \pm 0.008$$

$$\eta_{u_v} = 0.928 \pm 0.014 \quad \text{and} \quad \eta_{d_v} = -0.342 \pm 0.018$$

J. Blumlein and H. Bottcher, arXiv:1005.3113 [hep-ph].

# Parameterization of the polarized parton distributions

Low - x

It is not expected that the small-x behavior of the polarized gluon and the sea quarks is much different. To achieve this it turn out that the small-x slopes of the gluon and the sea-quarks are to be related like $a_G = a_s + c$. We there fore decided to fix one of the parameter relative to the other.

High - x

In fixing the high-x slopes $b_G$ and $b_S$ we adopted a relation as derived from the unpolarized parton densities, $b_s/b_G$(pol)= $b_s/b_G$(unpol). Here, the lack of constraining power of the present data on the polarized parton densities has to be stressed.

Both relations together are suited to lead to positivity for $\delta G$ and $\delta\overline{q}$

J. Blumlein and H. Bottcher, Nucl. Phys. B 636 (2002) 225

S. A. Tehrani and Ali N. Khorramian, JHEP 07 (2007) 048

# Evolution in moment space

In the present approach the QCD-evolution equations are solved in Mellin space. The Mellin transform of the parton densities is performed and Mellin $n$ moment are calculate for complex argument $n$:

$$\begin{aligned} \mathrm{M}[\delta f_i(x,Q_0^2)](n) &= \int_0^1 x^{n-2} x \delta f_i(x,Q_0^2)dx \\ &= \eta_i A_i \left(1 + c_i \frac{n-1+a_i}{n+a_i+b_i}\right) B(n-1+a_i, b_i+1) \end{aligned}$$

We can also define the Mellin moments for the polarized structure function $g_1^p(x,Q^2)$:

$$\mathrm{M}[g_1(x,Q^2)](n) = g_1^p(N,Q^2) = \int_0^1 x^{N-1} g_1^p(x,Q^2)dx$$

# Structure function in Mellin-n space

The twist–2 contributions to the structure function $g_1(n, Q^2)$ can be represented in terms of the polarized parton densities and the coefficient functions $\delta C_i^n$ in the Mellin -n space by

$$g_1^p(n,Q^2) = \frac{1}{2}\sum_q e_q^2\{(1+\frac{\alpha_s}{2\pi}\delta C_q^n)[\delta q(n,Q^2)+\delta\bar{q}(N,Q^2)] + \frac{\alpha_s}{2\pi}2\delta C_g^n \delta g(n,Q^2)\}$$

$$\alpha_s(Q^2) \cong \frac{1}{b\log\frac{Q^2}{\Lambda^2_{MS}}} - \frac{b'}{b^3}\frac{\ln\left(\ln\frac{Q^2}{\Lambda^2_{MS}}\right)}{\left(\ln\frac{Q^2}{\Lambda^2_{MS}}\right)^2}$$

$\delta q(n, Q^2)$ is the quark helicity distribution for quarks of flavor $q$. Correspondingly, $\delta\bar{q}(n, Q^2)$ is anti-quark helicity distributions. For our analysis, only the three lightest quark flavors, $q = u, d, s$, are taken into account and the number of active quark flavors $N_q$ is equal to 3. We choose $Q_0^2 = 4\ GeV^2$ as a fixed parameter and $\Lambda$ is an unknown parameter which can be obtained by fitting to experimental data. Here $\delta q(n, Q^2) = \delta q_v(n, Q^2) + \delta\bar{q}(n, Q^2)$ and $\delta C_q^n$, $\delta C_g^n$ are also the $n$-th moments of spin-dependent Wilson coefficients given by

$$\delta C_q^n = \frac{4}{3}\left[-S_2(n) + (S_1(n))^2 + \left(\frac{3}{2} - \frac{1}{n(n+1)}\right)S_1(n) + \frac{1}{n^2} + \frac{1}{2n} + \frac{1}{n+1} - \frac{9}{2}\right]$$

and

$$\delta C_g^n = \frac{1}{2}[-\frac{n-1}{n(n+1)}(S_1(n)+1) - \frac{1}{n^2} + \frac{2}{n(n+1)}]$$

# QCD-PEGASUS

To perform the evolution, we used the fortran package QCD-PEGASUS. This code provides fast, flexible and accurate solutions of the evolution equations for unpolarized and polarized parton distributions in perturbative QCD.

# Jacobi Polynomials

One of the simplest and fastest possibilities in the structure function (SF) reconstruction from the QCD predictions for its Mellin moments is Jacobi polynomials expansion. The Jacobi polynomials are especially suited since they allow one to factor out an essential part of the $x$-dependence of the SF into the weight function.


S. I. Alekhin, et al., Phys. Lett. B **452**, (1999) 402.
A. L. Kataev, et al., Nucl. Phys. B **573**, (2000) 405.
A. L. Kataev, et al., Phys. Part. Nucl. **34**, (2003) 20;

A. N. Khorramian and S. A. Tehrani, Phys. Rev. D **78**, 074019 (2008) [arXiv:0805.3063 [hep-ph]].

A. N. Khorramian, H. Khanpour and S. A. Tehrani, Phys. Rev. D **81**, 014013 (2010) [arXiv:0909.2665 [hep-ph]].


**Attention to Hamzeh Khanpour`s talk to get more information about Jacobi applications**

# Jacobi Polynomials

By inserting the Mellin-N̄ moments of $g_1(N, Q^2)$ in this Eq it is possible to extract the polarized structure function to do QCD fits using the available experimental data.

$N_{max}$ is the number of polynomials

$$xg_1(x, Q^2) = x^\beta (1-x)^\alpha \sum_{n=0}^{N_{max}} a_n(Q^2) \Theta_n^{\alpha,\beta}(x)$$

Jacobi polynomials of order $n$

$$\Theta_n^{\alpha,\beta}(x) = \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta) x^j$$

where $c_j^{(n)}(\alpha,\beta)$ are the coefficients that expressed through Γ-functions and satisfy the orthogonality relation with the weight $x^\beta(1-x)^\alpha$ as following

$$\int_0^1 dx\, x^\beta (1-x)^\alpha \Theta_k^{\alpha,\beta}(x) \Theta_l^{\alpha,\beta}(x) = \delta_{k,l}$$

# Jacobi Polynomials

For the moments, we note that the $Q^2$ dependence is entirely contained in the Jacobi moments

$$a_n(Q^2) = \int_0^1 dx\, xg_1(x,Q^2)\Theta_k^{\alpha,\beta}(x) = \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta)\mathbf{M}[xg_1, j+2]$$

moments of the structure function

$$\mathbf{M}[xg_1, N] \equiv g_1(N,Q^2) = \int_0^1 dx\, x^{N-2} xg_1(x,Q^2)$$

we can relate the PSF with its moments

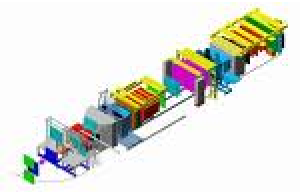
$$xg_1^{N_{max}}(x,Q^2) = x^{\beta}(1-x)^{\alpha}\sum_{n=0}^{N_{max}}\Theta_n^{\alpha,\beta}(x)\sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta)\mathbf{M}[xg_1, j+2]$$

# Experimental data

**E80, 130 (p) ; E142 (n)**
**E143 (p, d) ; E154 (n) ; E155 (p, d)**
**JLAB(n)**

**EMC(p), SMC (p, d)**

**HERMES (p, d, n)**



**COMPASS(p)**

| Experiment | x-range | $Q^2$-range[$GeV^2$] | number of data points |
|---|---|---|---|
| E143(p) | 0.031-0.749 | 1.27-9.52 | 28 |
| HERMES(p) | 0.023-0.66 | 1.01-7.36 | 19 |
| HERMES(p) | 0.23-0.66 | 2.5(Fixed) | 20 |
| SMC(p) | 0.005-0.48 | 1.30-58.0 | 12 |
| EMC(p) | 0.015-0.466 | 3.50-29.5 | 10 |
| E155 | 0.015-0.750 | 1.22-34.72 | 24 |
| HERMES06(P) | 0.0264-0.7311 | 1.12-14.29 | 66 |
| COMPAS10(P) | 0.0046-0.568 | 1.1-62.1 | 15 |
| Proton | | | 194 |
| E143(d) | 0.027-0.749 | 1.17-9.52 | 28 |
| E155(d) | 0.015-0.750 | 1.22-34.79 | 24 |
| SMC(d) | 0.005-0.479 | 1.30-54.8 | 12 |
| HERMES06(d) | 0.0328-0.7248 | 1.15-12.21 | 66 |
| Deuteron | | | 130 |
| E142(n) | 0.035-0.466 | 1.10-5.50 | 8 |
| HERMES(n) | 0.033-0.464 | 1.22-5.25 | 9 |
| E154(n) | 0.017-0.564 | 1.20-15 | 17 |
| HERMES06(n) | 0.0264-0.7311 | 1.12-14.29 | 37 |
| Neutron | | | 71 |
| total | | | 395 |

Published data points above $Q^2 = 1.0$ GeV$^2$.

# Data Analysis

A relative normalization shift, $N_i$, between the different data sets was allowed within the normalization uncertainties, $\Delta N_i$, quoted by the experiments. these normalization shifts were fitted once and then fixed. thereby the main systematic uncertainties coming from the measurements of the luminosity and the beam and target polarization were taken into account. The normalization shift for each data set enters as an additional term in the $\chi^2$ expression for the fit which then reads

$$\chi^2_{\text{global}} = \sum_n w_n \chi^2_n \,, \qquad (n \text{ labels the different experiments})$$

$$\chi^2_n = \left( \frac{1 - \mathcal{N}_n}{\Delta \mathcal{N}_n} \right)^2 + \sum_i \left( \frac{\mathcal{N}_n g_{1,i}^{data} - g_{1,i}^{theor}}{\mathcal{N}_n \Delta g_{1,i}^{data}} \right)^2 \,.$$

where the sums run over all the data sets and in each data set over all data points. The minimization of the $\chi^2$ value above to determine the best parametrization of the polarized parton distributions is performed using the program MINUIT.

# Fit procedure

CERN Program Library Long Writeup D506

## MINUIT

Function Minimization and Error Analysis

Using the CERN subroutine MINUIT, we defined a global $\chi^2$ for all the experimental data points and found an acceptable fit with minimum $\chi^2/dof$=0.760 in NLO approximation.

# Covariance Matrix

The evolved polarized parton densities and structure functions are functions of the input densities. Let $\Delta f(x, Q^2; p_i)$ be the evolved polarized density at the scale $Q^2$ depending on the parameters $p_i$. Then the correlated statistical error as given by Gaussian error propagation is

$$\Delta\delta f(x, Q^2) = \left[\sum_{i=1}^{k}\left(\frac{\partial\delta f}{\partial a_i}\right)^2 c(a_i, a_i) + \sum_{i\neq j=1}^{k}\left(\frac{\partial\delta f}{\partial a_i}\frac{\partial\delta f}{\partial a_j}\right) c(a_i, a_j)\right]$$

| NLO | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $\alpha_s(M_z^2)$ | $a_{u_v}$ | $b_{u_v}$ | $a_{d_v}$ | $b_{d_v}$ | $\eta_{sea}$ | $a_{sea}$ | $\eta_G$ |
| $\alpha_s(M_z^2)$ | 2.342E-4 | | | | | | | |
| $a_{u_v}$ | 1.014E-4 | 8.121E-4 | | | | | | |
| $b_{u_v}$ | 4.373E-4 | 2.566E-3 | 1.000E-2 | | | | | |
| $a_{d_v}$ | 2.326E-4 | -8.513E-4 | -2.237E-3 | 5.142E-3 | | | | |
| $b_{d_v}$ | 1.980E-3 | -4.226E-3 | -4.436E-3 | 2.342E-2 | 1.777E-1 | | | |
| $\eta_{sea}$ | 7.964E-6 | 9.892E-4 | 2.908E-3 | -2.144E-3 | -9.754E-3 | 1.952E-3 | | |
| $a_{sea}$ | -3.966E-4 | 4.420E-3 | 1.325E-2 | -1.059E-2 | -4.96E-2 | 7.67E-3 | 3.568E-2 | |
| $\eta_G$ | 1.571E-4 | 3.912E-3 | 1.202E-2 | -8.70E-3 | -3.611E-2 | 5.877E-3 | 2.977E-2 | 2.767E-2 |

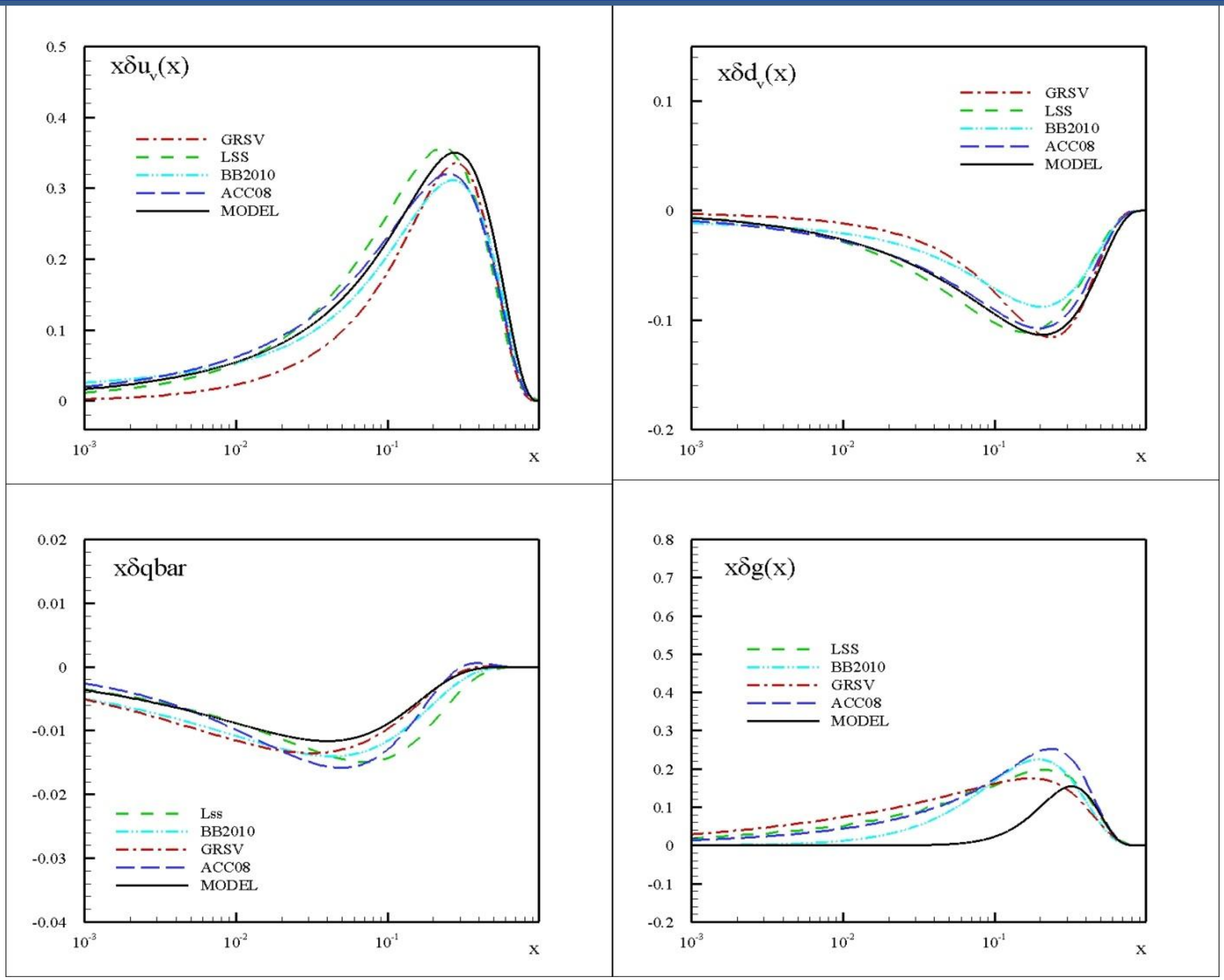
Table 1: The covariance matrix for the 7 parameter and $\alpha_s(M_Z^2)$ based on the world polarized data.

# Fit Result

| | Value | Error |
|---|---|---|
| $\eta_{u_v}$ | $0.928$ | fixed |
| $a_{u_v}$ | $0.4945$ | $0.0286$ |
| $b_{u_v}$ | $3.1764$ | $0.0099$ |
| $c_{u_v}$ | $9.666$ | fixed |
| $\eta_{d_v}$ | $-0.342$ | fixed |
| $a_{d_v}$ | $0.6006$ | $0.00717$ |
| $b_{d_v}$ | $3.8762$ | $0.4212$ |
| $c_{d_v}$ | $3.3300$ | fixed |
| $\eta_s$ | $-0.3147$ | $0.04407$ |
| $a_s$ | $0.4223$ | $0.1885$ |
| $b_s$ | $b_g * 1.44$ | fixed |
| $\eta_g$ | $0.1799$ | $0.1662$ |
| $a_g$ | $2.91 + a_s$ | fixed |
| $b_g$ | $7.010$ | fixed |
| $\alpha_s(M_z^2)$ | $0.1149$ | $0.0012$ |
| $\chi^2/ndf$ | $294/387 = 0.760$ | |

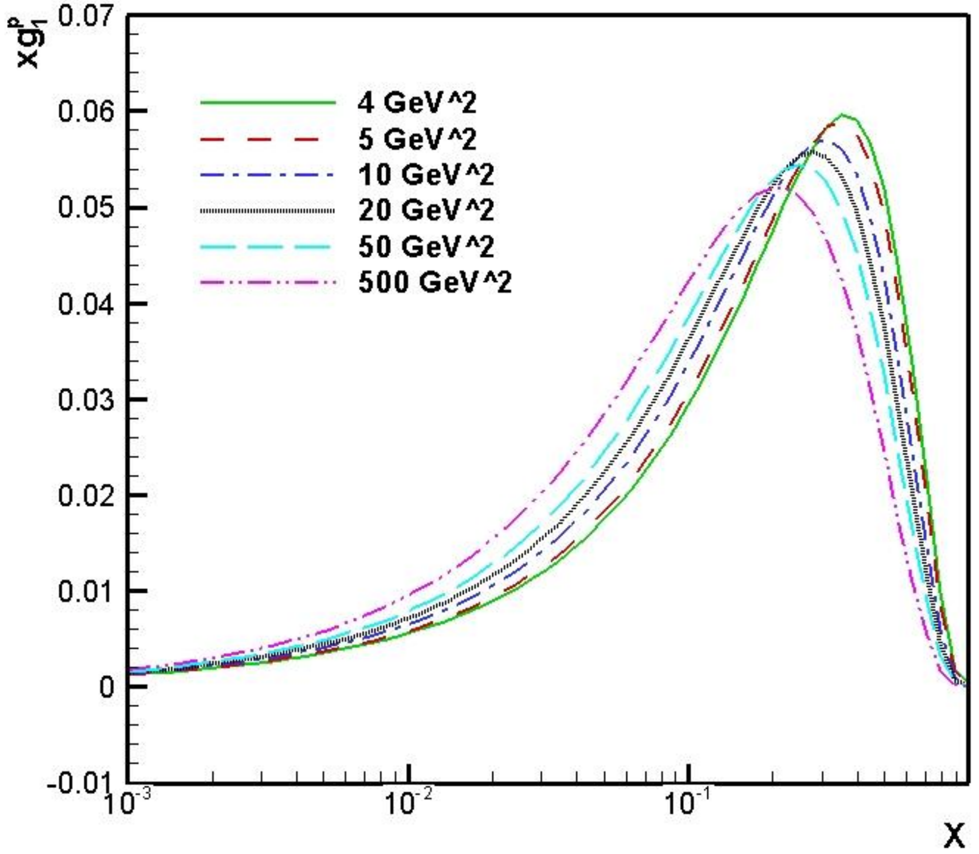
**Table 2:** Final parameter values and their errors at the input scale.

# Polarized parton distribution comparison



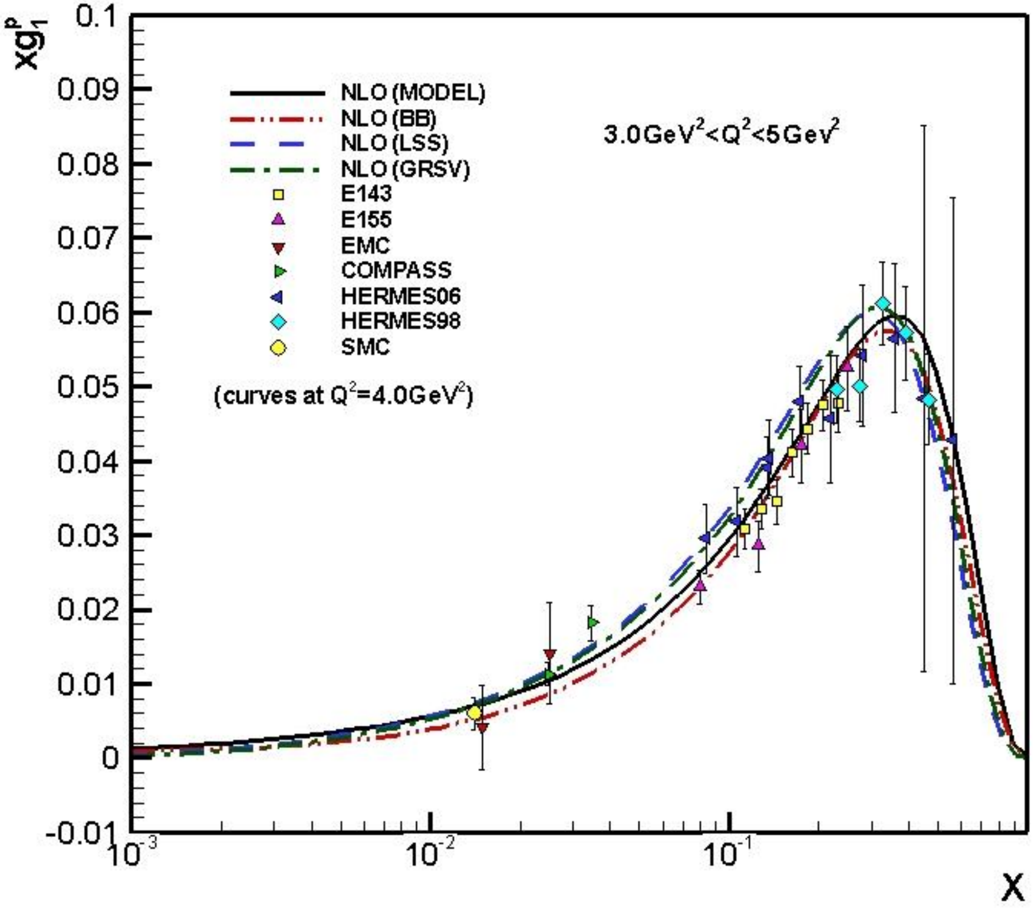

Polarized parton distributions at $Q_0^2 = 4\ \mathrm{GeV}^2$ as a function of x in NLO approximation. The solid curve is our model and dashed, dotted, dashed dot and dashed dot dot are LSS, ACC, GRSV and BB2010 respectively.

# $xg_1^p$ structure function at different $Q^2$



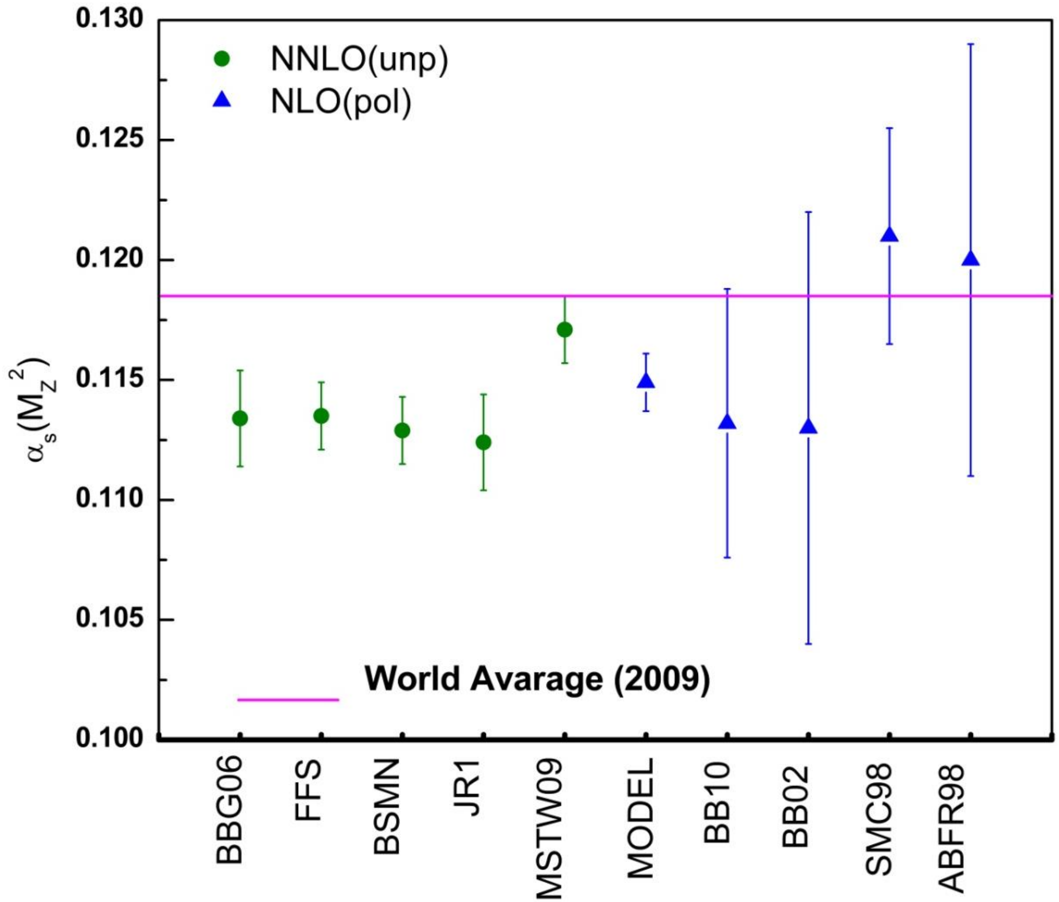

Our result on polarized structure function at different value of $Q^2$ as a function of x.

# $xg_1$ comparison with experimental data




Polarized proton structure function measured in the interval $3.0GeV^2 < Q^2 < 5GeV^2$ as a function of x. Also shown are the QCD NLO curves at the same value of $Q^2$ obtained by BB, LSS and GRSV groups for comparison.

# Coupling constant comparison




The strong coupling constant from different DIS measurements. The pink line describes the weighted average of a wide range of coupling constant measurements.

# Conclusion

We performed QCD analysis of the polarized structure functions world data up to NLO by using Jacobi polynomials expansion.

In deriving polarized distributions some unknown parameters are introduced which should be determined by fitting to experimental data.

After calculating polarized distributions based on Jacobi polynomials, all polarized parton density in a proton are calculable. The results are used to evaluate the spin component of the proton.

Our results for polarized structure functions are in good agreement with available experimental data on $g_1^p$, $g_1^n$ and $g_1^d$.

Now, by having the polarized proton and also neutron structure function, we have enough motivation to extract the polarized nuclei structure function such as ${}^3H$ and ${}^3He$.

# Thanks for your paying attention