Addressing giant QCD K-factors at hadron colliders

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in collaboration with Gavin Salam and Mathieu Rubin [1]

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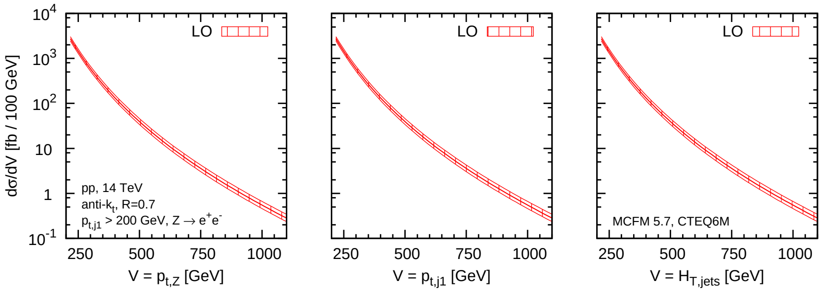
[1] M.Rubin, G.P.Salam and SS, arXiv:1006.2144 [hep-ph]

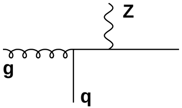
The problem of giant K factors

- Z+j at the LHC

$$H_{T,\text{jets}} = \sum_{\text{all jets}} p_{t,j}$$



LO:



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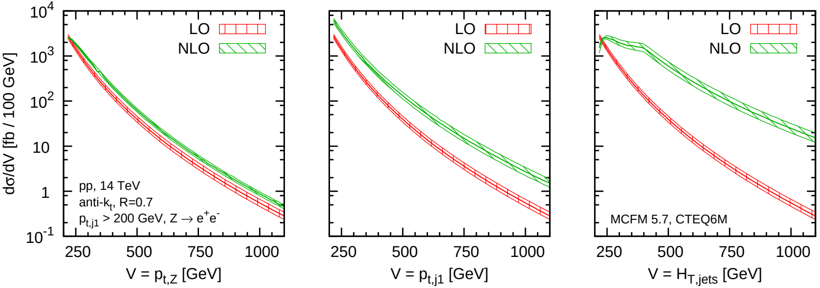

$\mathcal{O}\left(\alpha_{\text{EW}}\alpha_s^2\right)$

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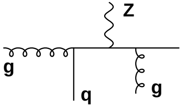
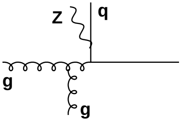
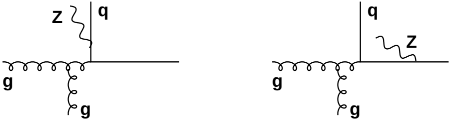
NLO:





What do we have and what is missing?

- The large K factor for the Z+jet comes from the new "dijet type" topologies that appear at NLO



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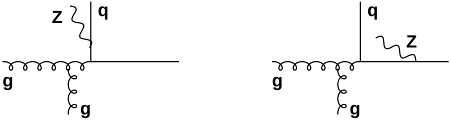

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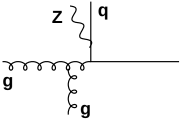

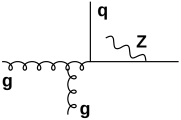

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$$\text{Z+j at NNLO} = \underbrace{\begin{array}{c}\text{Z+j at NNLO}\\ \text{Born}\end{array} + \begin{array}{c}\text{Z+j at NNLO}\\ \text{1-loop}\end{array}}_{\text{Z+2j at NLO}} + \begin{array}{c}\text{Z+j at NNLO}\\ \text{2-loop}\\ \uparrow\\ \boxed{\textbf{?}}\end{array}$$

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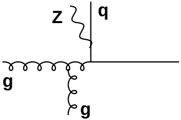

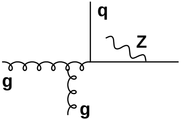

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Hence, we do not have the 2-loop part

- but it will have the topology of Z+j at LO so it will not contribute much to the cross sections with giant K-factor
- we need it, however, to cancel the infrared and collinear divergences of the real part

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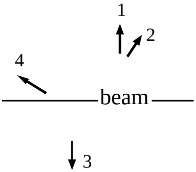
- notation: $\bar{n}$**NLO** – approx. NNLO with exact 1-loop and simulated 2-loop
- this will still not be equivalent to the full NNLO result but it should give very good approximation for the processes with large K factors

$$\sigma_{\bar{n}\mathrm{NLO}} = \sigma_{\mathrm{NNLO}} \left(1 + \mathcal{O}\left(\frac{\alpha_s^2}{K_{\mathrm{NNLO}}}\right)\right), \quad K_{\mathrm{NNLO}} \gtrsim K_{\mathrm{NLO}} \gg 1$$

The LoopSim method

Input event



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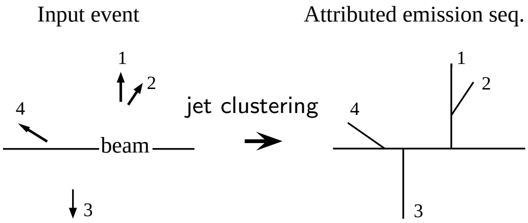

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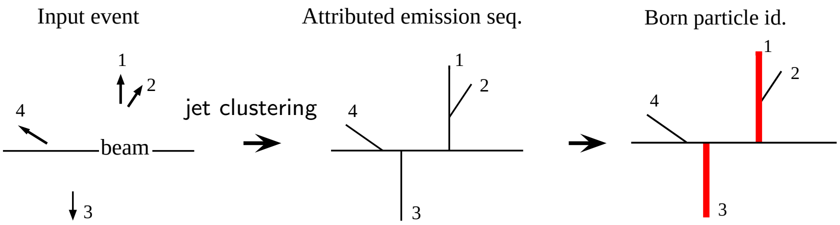

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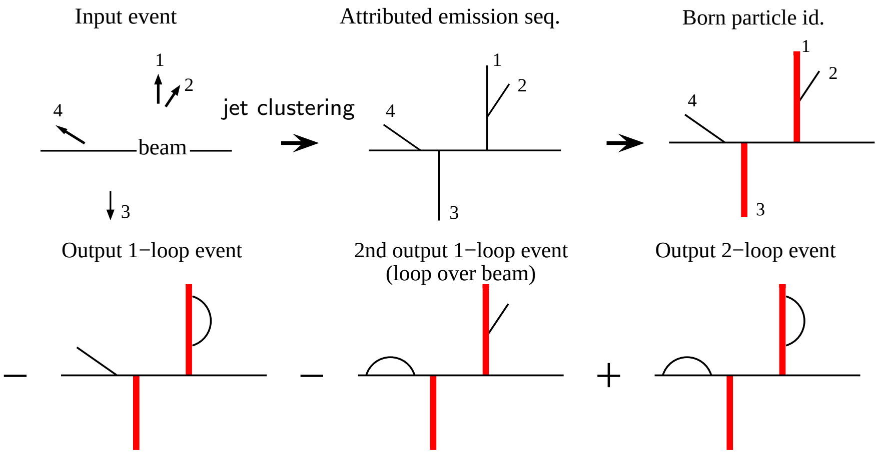

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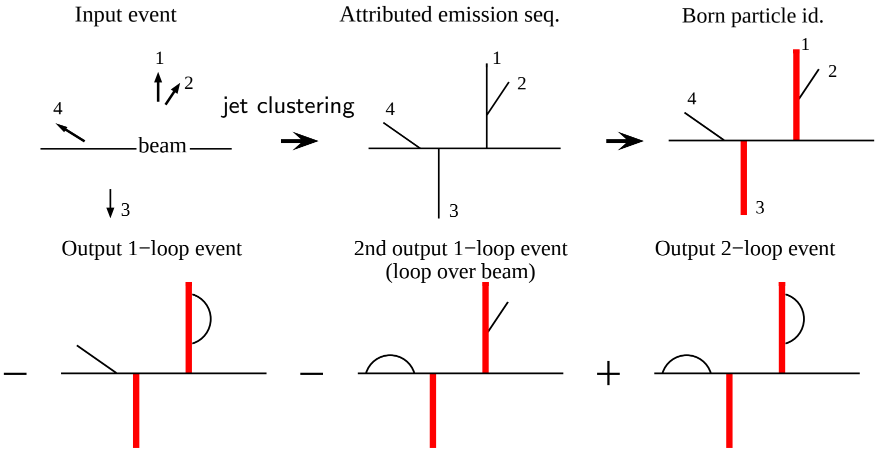

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- beware: the loops above are just a shortcut notation!

The LoopSim method: some more details

For a given input E_n event with n final state particles the weights of all diagrams generated by LoopSim sum up to zero

$$\sum_{\text{all diagrams}} w_n = \sum_{\ell=0}^{\upsilon} (-1)^{\ell} \binom{\upsilon}{\ell} = 0\,, \qquad \ell - \text{number of loops, } \upsilon - \text{maximal } \ell$$

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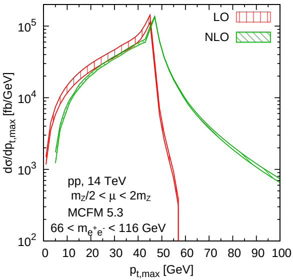
The principle of the method looks rather simple. However, there is a number of issues that need to be addressed to fully specify the procedure and make it usable:

- infrared and collinear safety
- conservation of four-momentum
- choice of jet definition (algorithm, value of R)
- treatment of flavour (e.g. for processes with vector bosons)
  - Z boson can be emitted only from quarks and never emits itself
- extension to input events with exact loops; for example:

$$Z + j@\bar{n}\text{NLO} = Z + j@\text{NLO} + \text{LoopSim} \circ (Z + 2j@\text{NLO}_{\text{only}})$$

Validation

Drell-Yan at NNLO: spectrum of harder lepton



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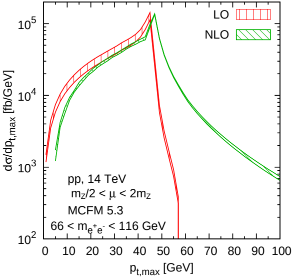

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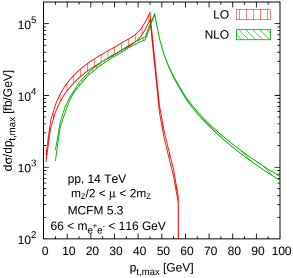

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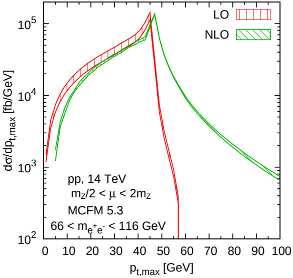

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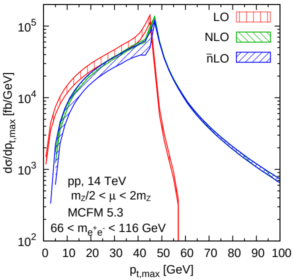

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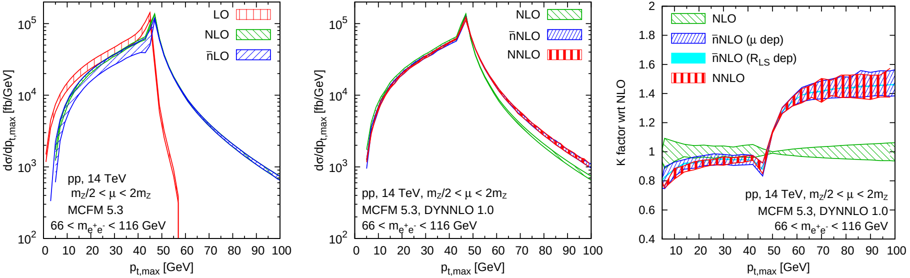

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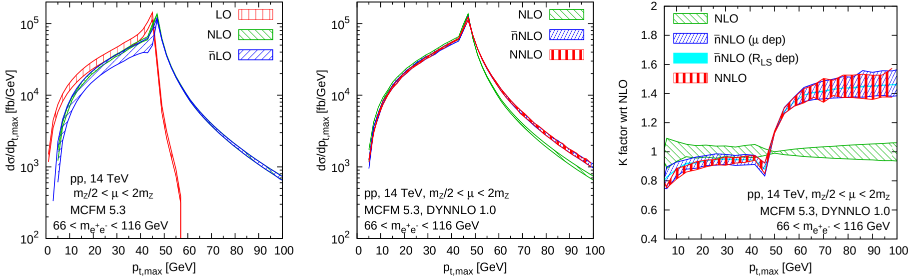

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- negligible dependence on R_{LS}

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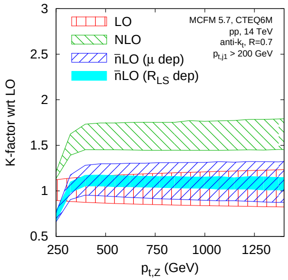

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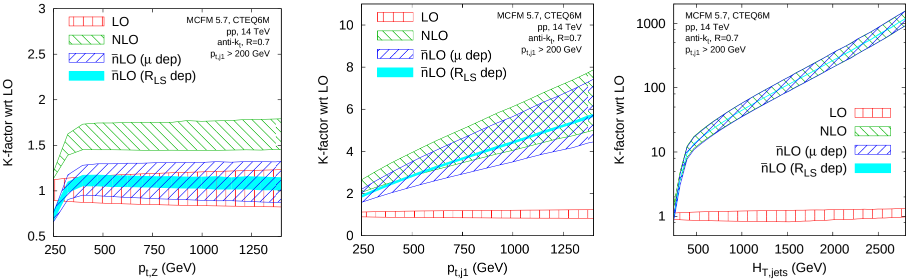

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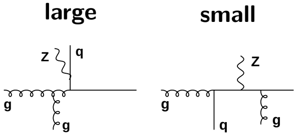

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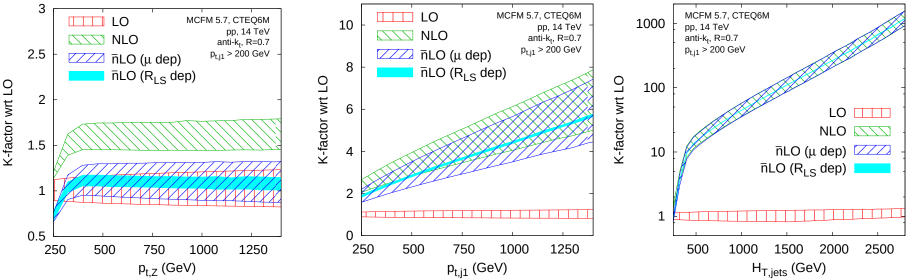

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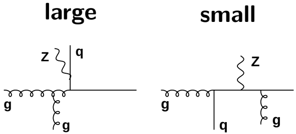

- small R uncertainties – driven only by subleading diagrams

$\bar{n}$NLO at LHC

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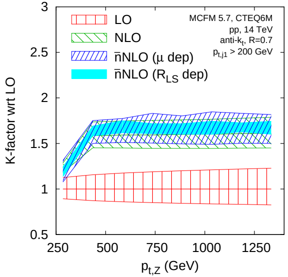

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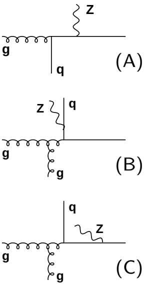

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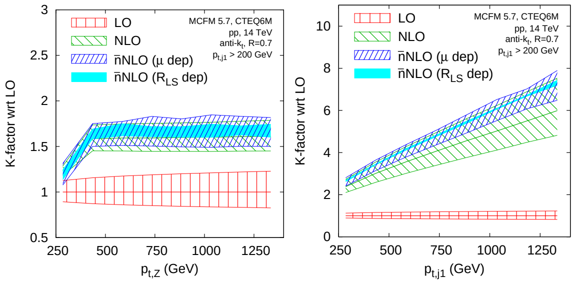

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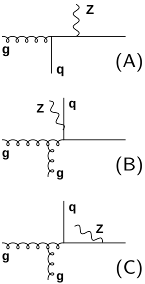

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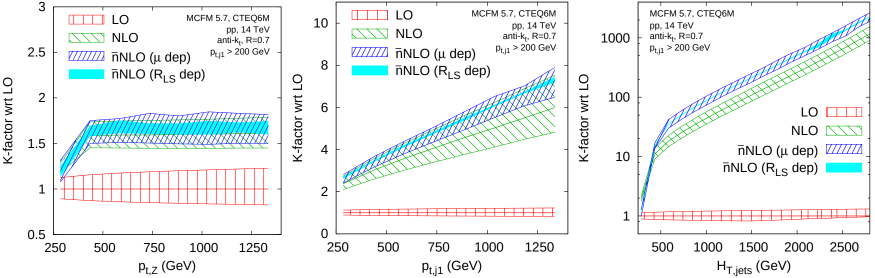

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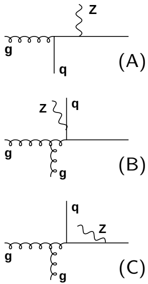
- $p_{t,j}$: small correction; $\bar{n}$NLO is like NLO for the dominant (B) and (C) configurations and it behaves like healthy NLO
- $H_{T,\text{jets}}$: significant correction; K factor ~ 2; given that its more like going from LO to NLO this may happen sometimes, especially for nontrivial observables like H_T; can be checked explicitly with jets



H_T type observables at $\bar{n}$NLO for Z+jet and for dijets

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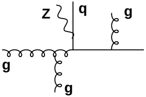
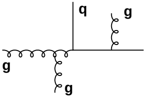
  (same topology, Z only provides the enhancement $\mathcal{O}\left(\alpha_s \ln^2 p_{t,j1}/m_{\mathrm{Z}}\right)$)





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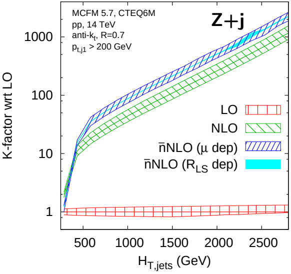

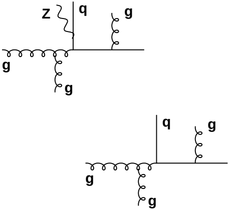

H_T type observables at $\bar{n}$NLO for Z+jet and for dijets

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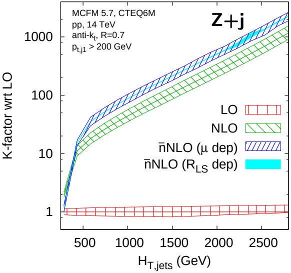
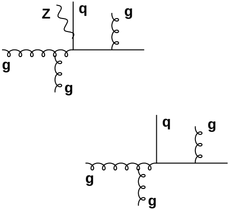
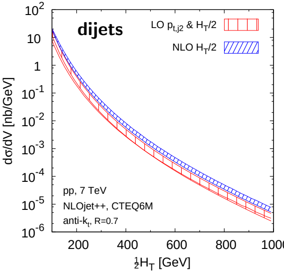
  (same topology, Z only provides the enhancement $\mathcal{O}\left(\alpha_s \ln^2 p_{t,j1}/m_Z\right)$)







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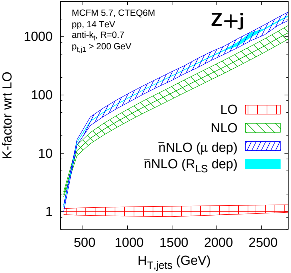

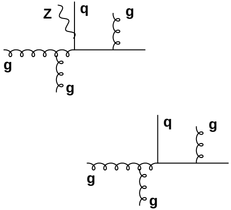

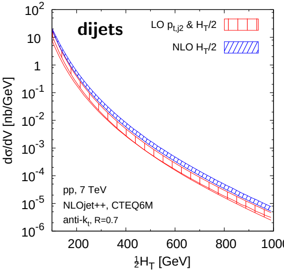

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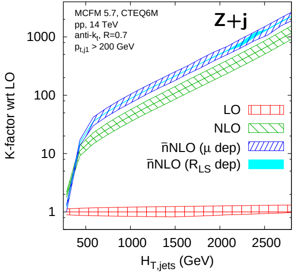

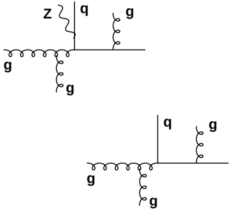

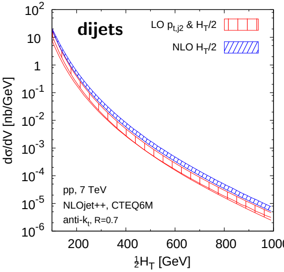

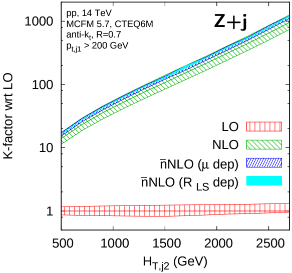

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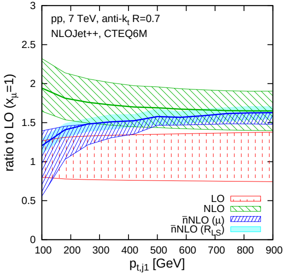
  - and indeed it is small!

Dijets at $\bar{n}$NLO



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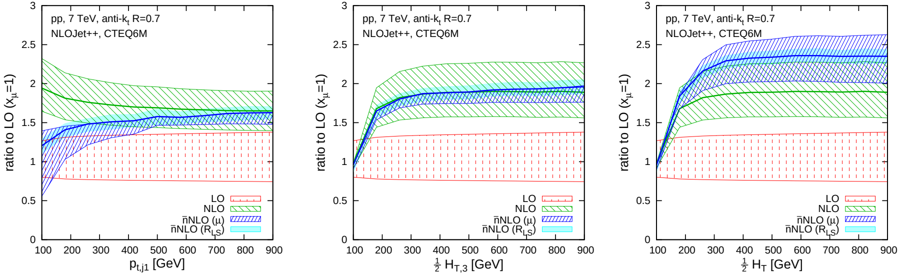

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- **$H_{T,3}$ converges, H_T does not:** again caused by the initial state radiation, this time a second emission which shifts the distribution of H_T to higher values and causes no effect for the $H_{T,3}$ distribution

Summary

- several cases of observables with giant NLO K factor exist
- those large corrections arise due to the appearance of new topologies at NLO
- we developed a method, called *LoopSim*, which allows one to obtain approximate NNLO corrections for such processes
- the method is based on unitarity and makes use of combining NLO results for different multiplicities
- we gave arguments why the method should produce meaningful results and we validated it against NNLO Drell-Yan and also NLO Z+j and NLO dijets
- we computed approximated NNLO corrections to Z+j and dijets at the LHC finding, depending on observable, either indication of convergence of the perturbative series or further corrections
- the latter has been understood and attributed to the initial state radiation

Outlook

- processes with W, multibosons, heavy quarks, ...