Renaissance of Strong Field Physics

Vacuum Physics at Critical (Planck) Acceleration

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50 Cracov School of Theoretical Physics

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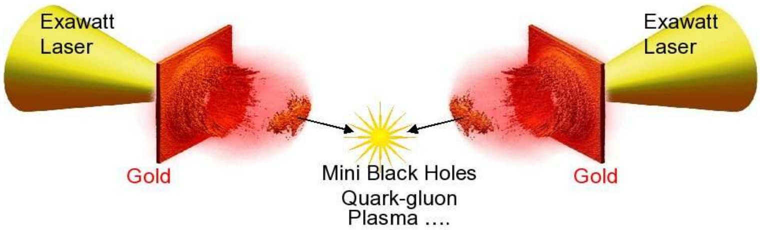
Overview

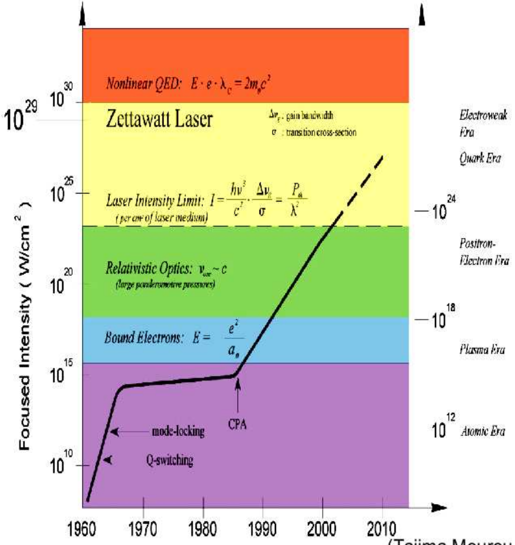
1. New beam in physics: Laser pulse
2. Acceleration: Mach, Planck, Aether
3. Lorentz-Abraham-Dirac (LAD) equations
4. Radiation Reaction Dominance
5. The Vacuum as source of laws of physics



From PRL, S.A. Bulanov et al.

Laser Pulse

– A new tool to study vacuum and high acceleration physics



(Tajima,Mourou,2002)

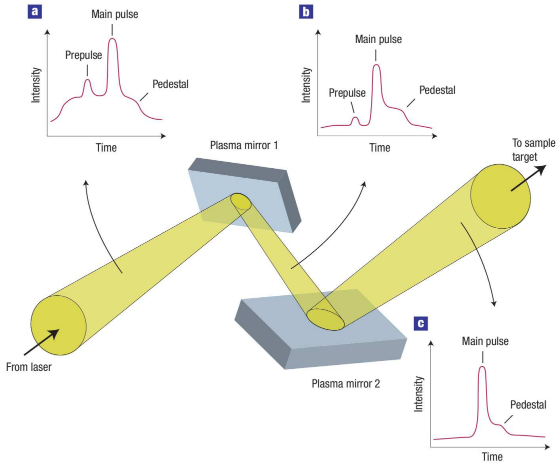
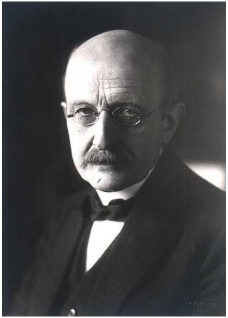


Figure 1 Laser pulse-shape conditioning with a double plasma-mirror. **a**, CPA generates a temporal output profile that consists not only of a main pulse, but often a prepulse and a wider pedestal, the presence of which can be detrimental to many of the applications for which high-intensity lasers are being developed. **b**, Reflecting the output of a CPA-laser off a plasma mirror can attenuate most of the lower-intensity components of the leading part of a pulse. **c**, Reflecting this off a second plasma mirror cleans the pulse still further, and achieves a ten-thousand-fold improvement in the contrast ratio of the pulse.

[Gibbon, Nat. Phys. 2007]

1899: Planck units



$$h/k_B = a = 0.4818\cdot 10^{-10}\,[\mathrm{sec}\times\mathrm{Celsiusgrad}]$$
$$h = b = 6.885\cdot 10^{-27}\left[\frac{\mathrm{cm}^2\mathrm{gr}}{\mathrm{sec}}\right]$$
$$c = c = 3.00\cdot 10^{10}\left[\frac{\mathrm{cm}}{\mathrm{sec}}\right]$$
$$G = f = 6.685\cdot 10^{-8}\left[\frac{\mathrm{cm}^3}{\mathrm{gr.\,sec}^2}\right]^1.$$

Wählt man nun die »natürlichen Einheiten« so, dass in dem neuen Maasssystem jede der vorstehenden vier Constanten den Werth 1 annimmt, so erhält man als Einheit der Länge die Grösse:

$$\sqrt{2\pi}\,L_{Pl} = \sqrt{\frac{bf}{c^3}} = 4.13\cdot 10^{-33}\,\mathrm{cm}, \mapsto \sqrt{2\pi}\,1.62\times 10^{-33}\,\mathrm{cm}$$

als Einheit der Masse:

$$\sqrt{2\pi}\,M_{Pl} = \sqrt{\frac{bc}{f}} = 5.56\cdot 10^{-5}\,\mathrm{gr}, \mapsto \sqrt{2\pi}\,2.18\times 10^{-5}\,\mathrm{g}$$

als Einheit der Zeit:

$$\sqrt{2\pi}\,t_{Pl} = \sqrt{\frac{bf}{c^5}} = 1.38\cdot 10^{-43}\,\mathrm{sec}, \mapsto \sqrt{2\pi}\,5.40\times 10^{-44}\,\mathrm{s}$$

als Einheit der Temperatur:

$$\sqrt{2\pi}\,T_{Pl} = a\sqrt{\frac{c^5}{bf}} = 3.50\cdot 10^{32}\,{}^{\circ}\mathrm{Cels.} \mapsto \sqrt{2\pi}\,1.42\times 10^{32}\,\mathrm{K}$$

Diese Grössen behalten ihre natürliche Bedeutung so lange bei, als die Gesetze der Gravitation, der Lichtfortpflanzung im Vacuum und die beiden Hauptsätze der Wärmetheorie in Gültigkeit bleiben, sie müssen also, von den verschiedensten Intelligenzen nach den verschiedensten Methoden gemessen, sich immer wieder als die nämlichen ergeben.

“These scales retain their natural meaning as long as the law of gravitation, the velocity of light in vacuum and the central equations of thermodynamics remain valid, and therefore they must always arise, among different intelligences employing different means of measuring.” *M. Planck, “Über irreversible Strahlungsvorgänge.” Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin* **5**, *440-480 (1899), (last page)*

Critical (Planck) Acceleration

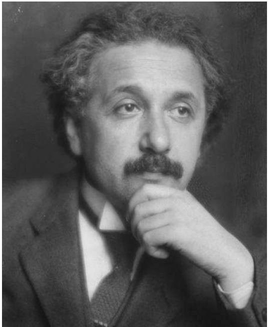
Planck mass scale corresponds to unit strength Newton force: for $m \to M_{\rm Pl}$:

$$f_{\rm grav} \simeq \frac{Gm^2}{r^2} \to \frac{\hbar c}{r^2} \quad \to 1\left[\frac{m^2c^3}{\hbar}\right] \quad \text{at} \quad r = \frac{\hbar}{mc}$$

Einstein's gravity is built upon Equivalence Principle: a relation of gravity to inertia(=the resistance to acceleration).

We study the same Planck scale physics when exposing particles to forces where acceleration is unit:

$$\boxed{m\frac{du^\alpha}{d\tau} = f^\alpha \to 1^\alpha \quad \text{in natural units} \quad \left[\frac{m^2c^3}{\hbar}\right]}$$



Laser Pulse – Energy

Coherent Light: $10^{23} \times 1$ eV photons $\equiv 16$ kJ

Comparison

7TeV proton: $\equiv 0.7 10^{-10}$ fraction of pulse

$M_{\rm Planck} = 1.22 \times 10^{28}$ eV, 100,000 more

Present Limits of technology:

Energy: 1 kJ aiming at 10kJ

Pulse length - 10 $\times$ wavelength aiming at 2-3

$$\lambda \equiv \frac{hc}{1\mathrm{eV}} \equiv 1.25\mu,\ 10\lambda \equiv 12.5\frac{\mu}{c} = 40\mathrm{fs}.$$

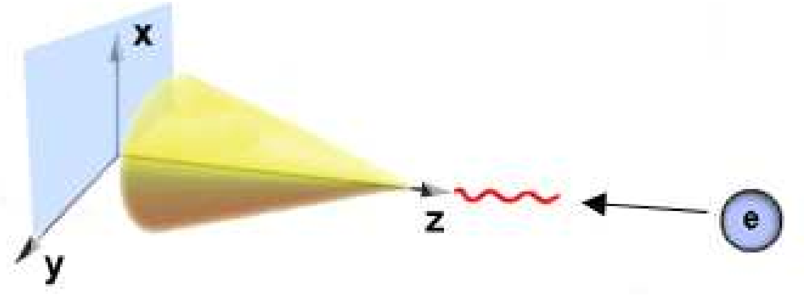
$a_0 \equiv |\vec{A}|/mc^2 = 100$ aiming at 1000–10,000

For Pulse-matter/foil interactions pre-pulse contrast very important: there must not be plasma formation before main pulse arrives.

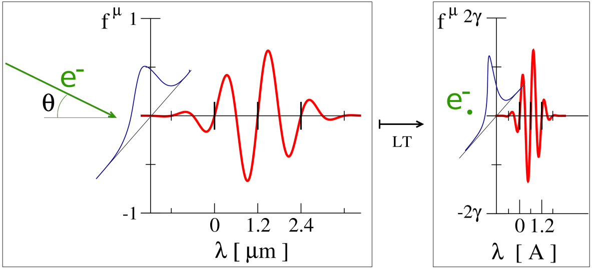
Towards critical (Planck) acceleration

To reach Plank scale:
Energy: a factor 1000 000.
Force: $a_o = mc^2/1\mathrm{eV} = 500,000$
Technology needs another factor 1000
– but difficulty grows with $a_0^2 \propto \vec{E} \times \vec{B}; \vec{E}^{\,2}; \vec{B}^{\,2}$

Simple idea: Lorentz-boost in Pulse-electron collision



Laser Pulse in Electron's Restframe



Force applied by a Gaussian photon pulse with $\gamma/\cos\theta = 2000$ – narrowed by $(\gamma\cos\theta)^{-1}$ in the longitudinal and $(\gamma\sin\theta)^{-1}$ in the transverse direction. The magnitude of the effect described by the Doppler-shift:

$$\omega \to \omega' = \gamma(\omega + \vec{v} \cdot nk)$$

Critical Pulse

In electron's rest frame: a pulse of 10^{23} photons approaching with energy 2γ eV.
For a practical electron energy of 50 GeV, $\gamma = 100,000$ a laser pulse of $a_0 = 1000$ is a Planck energy pulse in rest frame of the electron: its energy is at Planck scale and its acceleration force 20 times above the critical (unit) acceleration.

Quantum or Classical System?

The 'macroscopic' coherent laser pulse has wavelength and transverse size in micron domain, and comprises a macroscopic number of photons. Its electromagnetic field is classical. It collides with an electron which has a de Broglie wavelength 10 orders of magnitude shorter. Thus the laser field is also quasi constant. Quantum dynamics applies in this environment to electrons of micron wavelength, i.e. energy 10^{-6}eV, and thus practically never.

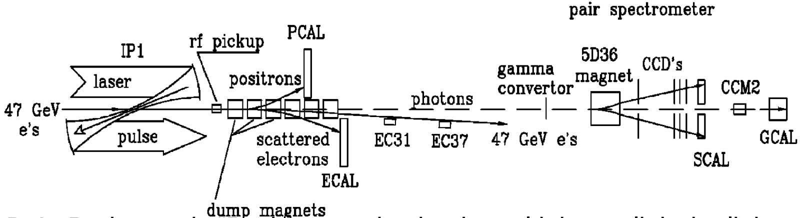
Probing EM-unit acceleration possible today

SLAC'95 experiment — *Proof of Principle*

$$p_e^0 = 46.6 \text{ GeV; in 1996/7 } a_0 = 0.4, \quad \left|\frac{du^\alpha}{d\tau}\right| = .073[m_e] \text{ (Peak)}$$

Multi-photon processes observed:

- Nonlinear Compton scattering
- Breit-Wheeler electron-positron pairs



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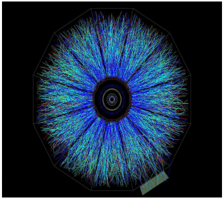
Unit Acceleration in Strong Interactions



Two nuclei smashed into each other from two sides at highest achievable energy: components can be stopped in CM frame within $\Delta\tau \simeq 1$ fm/c. Tracks show multitude of particles produced, as observed by STAR at RHIC (BNL).

- The acceleration a required to stop some/any of the components of the colliding nuclei in CM: $a \simeq \frac{\Delta y}{M_i \Delta\tau}$. Full stopping: $\Delta y_{\rm SPS} = 2.9$, and $\Delta y_{\rm RHIC} = 5.4$. Considering constituent quark masses $M_i \simeq M_N/3 \simeq 310$ MeV we need $\Delta\tau_{\rm SPS} < 1.8$ fm/c and $\Delta\tau_{\rm RHIC} < 3.4$ fm/c to exceed critical a.
- The soft electromagnetic radiation in hadron reactions (*A. Belognni et al. [WA91 Collaboration], "Confirmation of a soft photon signal in excess of QED expectations in π–p interactions at 280-GeV/c," Phys. Lett. B* **408**, *487 (1997) [arXiv:hep-ex/9710006].*) and heavy ion reactions exceeds the perturbative QED predictions significantly

Acceleration – Physics Riddles

Framework of (electromagnetic) theory is incomplete:
Current procedure:
1) Inertial Force = Lorentz-force → get world line of particles $=$source of fields:

$$m_e \frac{du^\alpha}{d\tau} = -eF^{\alpha\beta}u_\beta \quad \rightarrow x^\alpha(\tau), u^\alpha(\tau) \quad \rightarrow j^\alpha$$

2) Source of Fields = Maxwell fields –> get fields, omitting radiated fields

$$\partial_\beta F^{\beta\alpha} = j^\alpha \quad \rightarrow F^{\beta\alpha}$$

3) Fields fix Lorentz force –> go to **1)**

As long as acceleration is small, radiation emitted can be incorporated as a perturbative additional force. For large acceleration this is a new source of resistance to acceleration.

Theoretical Framework is ad-hoc

The action $\mathcal{I}$ comprises three elements:

$$\mathcal{I} = -\frac{1}{4}\int d^4x\, F^2 + q\int_{\text{path}} d\tau \frac{dx}{d\tau}\cdot A + \frac{mc}{2}\int_{\text{path}} d\tau (u^2 - 1).$$

Path is fixed at the end points assuring gauge-invariance.

• The second and third term, when varied with respect to the form of the material particle world line, produce the Lorentz force equation.

• The two first terms upon variation with respect to the field, produce Maxwell equation including radiation emission. This is inconsistent with the Lorentz force equation, thus at least one of action terms must be modified.

• Quantum theory is proved to produce classical limit. Thus quantum theory is also inconsistent.

Both Classical and Quantum Theory require reformulation

Inertia and Mach’s Principle: ca 1895

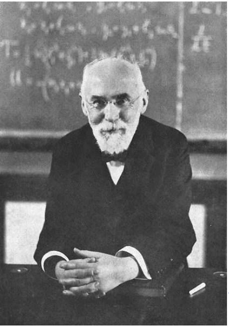
- To define acceleration another inertial reference frame, such as matter frame of the Universe, CBM-frame, the quantum vacuum, the geometric manifold - space in Einstein gravity is needed

- Einstein got rid of dynamic acceleration: point masses are in a free fall; rotating solutions and frame drag remains – "Mach’s Principle" re-introduced by Einstein in 1918

- (linear) Acceleration arises in presence of quantum matter and vnon-geometric e.g. quantum forces that create a rigid extended material body.

Newton’s absolute space is dead. Long Live Mach’s Principle.
Just that in all microscopic theory we build there is no Mach.

Radiation reaction force



energy-momentum radiated to order e^2:

$$\frac{dp^\alpha}{d\tau} = -\frac{2e^2}{3} u^\alpha \frac{du^\beta}{d\tau}\frac{du_\beta}{d\tau}$$

Recognized and further developed among others by

← Lorentz Dirac →

At unit acceleration radiation impact on charged particle dynamics require an unknown nonperturbative extension of dynamics. Quantum theory (QED) is build on this limited frame work

Lorentz-Abraham-Dirac (LAD) equations

$$m_e\,\dot{u}^\mu = -\frac{e}{c}F^{\mu\nu}u_\nu\,, \qquad u^\mu = \dot{s}^\mu(\tau)\,, \qquad \partial^\alpha\partial_\alpha A^\mu = \frac{1}{\epsilon_0 c^2}\,j^\mu$$

$$j^\mu(x) = -ec\int d\tau\, u^\mu[s(\tau)]\,\delta^4[x - s(\tau)]$$

$$A^\mu_{\rm rad} = -\frac{e}{\epsilon_0 c}\int d\tau\, u^\mu[s(\tau)]\,G_+[x - s(\tau)]$$

$$-\frac{e}{c}\,F^{\mu\nu}_{\rm rad}\,u_\nu = \frac{e^2}{\epsilon_0 c}\int d\tau\, u_\nu(x)\,\left(u^\nu[s(\tau)]\,\partial^\mu - u^\mu[s(\tau)]\,\partial^\nu\right)\,G_+[x - x(\tau)]$$

$$G_\pm = \theta[\pm X_0]\,\delta[X^2]\,, \qquad X^\mu = x^\mu - x^{\mu\,\prime}$$

$$-\frac{e}{c}\,F^{\mu\nu}_{\rm rad}\,u_\nu = \frac{2e^2}{\epsilon_0 c}\int d\tau\; u_\nu\,\left(u^{\nu\,\prime}\,X^\mu - u^{\mu\,\prime}X^\nu\right)\,\frac{\partial G_+}{\partial X^2}$$

Expansion for far zone – Dirac 1938

$$X^\mu \approx \delta u^\mu - \frac{\delta^2}{2}\dot u^\mu + \frac{\delta^3}{6}\ddot u^\mu \pm \ldots\,, \qquad u^{\mu\,\prime} \approx u^\mu - \delta \dot u^\mu + \frac{\delta^2}{2}\ddot u^\mu \pm \ldots$$

$$\delta = t - \tau, \qquad X^2 \approx c^2\,\delta^2 \to \frac{\partial}{\partial X^2} = \frac{1}{2c^2\delta}\,\frac{\partial}{\partial\delta}$$

$$\begin{aligned} F^\mu_{\mathrm{rad}} &= \frac{e^2}{\epsilon_0 c}\int d\delta\,\frac{\partial G_+}{\partial\delta}\left(\frac{\delta}{2}\dot u^\mu - \frac{\delta^2}{3}\left[\ddot u^\mu + \frac{1}{c^2}\,\dot u^\eta \dot u_\eta u^\mu\right]\right) \\ &= -\frac{e^2}{2\epsilon_0 c^3}\dot u^\mu \int d\delta\,\frac{\Delta[\delta]}{|\delta|} + \frac{2e^2}{3\epsilon_0 c^3}\left[\ddot u^\mu + \frac{1}{c^2}\,\dot u^\eta \dot u_\eta u^\mu\right] \end{aligned}$$

$$m_r\,\dot u^\mu = -\frac{e}{c}F^{\mu\nu}u_\nu + F^\mu_{\mathrm{LAD}}, \quad m_r = m_e\left(1 + \frac{e^2}{2\epsilon_0 c^3}\int d\delta\,\frac{\Delta[\delta]}{|\delta|}\right)$$

$$F^\mu_{\mathrm{LAD}}u_\mu = \frac{2e^2}{3\epsilon_0 c^3}\left[\ddot u^\mu + \frac{1}{c^2}\,\dot u^\eta \dot u_\eta u^\mu\right]u_\mu = 0\,, \qquad u^2 = 1.$$

LAD → Landau-Lifschitz

LAD has conceptual problems (run-away solutions).

$$m_e \dot{u}^\mu = -\frac{e}{c} F^{\mu\nu} u_\nu + F^\mu_{\mathrm{LAD}} \qquad F^\mu_{\mathrm{LAD}} = \frac{2e^2}{3\epsilon_0 c^3} \left[\ddot{u}^\mu + \frac{1}{c^2} \dot{u}^\eta \dot{u}_\eta u^\mu \right]$$

Iterate using Lorentz force and its differential in LAD:

$$\begin{aligned} \ddot{u}^\mu &\to \frac{d}{d\tau}\left(-\frac{e}{m_e c} F^{\mu\nu} u_\nu\right) = -\frac{e}{m_e c}\left(\partial_\eta F^{\mu\nu} u_\nu u^\eta + F^{\mu\nu} \dot{u}_\nu\right) \\ &= -\frac{e}{m_e c}\left(\partial_\eta F^{\mu\nu} u_\nu u^\eta - \frac{e}{m_e c} F^{\mu\nu} F^{\eta}_{\nu} u_\eta\right) \end{aligned}$$

$$F^\mu_{\mathrm{LAD}} \simeq -\frac{2e^3}{3\epsilon_0 m_e c^4}\left(\partial_\eta F^{\mu\nu} u_\nu u^\eta - \frac{e}{m_e c} F^{\mu\nu} F^{\eta}_{\nu} u_\eta\right) + \frac{2e^4}{3\epsilon_0 m_e^2 c^7} F^{\eta\nu} F_{\eta\delta} u_\nu u^\delta u^\mu$$

This is equivalent to LAD only for weak acceleration. But like LAD it is a heuristic description of high acceleration domain and is often taken to be equally valid model of strong acceleration regime. No conceptual problems.

LAD → Caldirola

Caldirola notices that derivative terms comprise a nonlocality!

$$m_e\,\dot{u}^\mu = -\frac{e}{c}F^{\mu\nu}u_\nu + F^\mu_{\mathrm{LAD}}$$

$$\begin{aligned} F^\mu_{\mathrm{Caldirola}} &= \frac{m_{ed}}{2\tau_a}\left[u^\mu(\tau-2\tau_a) - \frac{1}{c^2}\,u^\mu(\tau)u^\alpha(\tau)u_\alpha(\tau-2\tau_a)\right] \\ &\approx -m_{\mathrm{ed}}\dot{u}^\mu + m_{ed}\,\tau_a\left[\ddot{u}^\mu + \frac{1}{c^2}\,\dot{u}^\alpha\dot{u}_\alpha u^\mu\right] + \ldots \end{aligned}$$

$$m_{\mathrm{ed}} = \frac{2e^2}{3d\,\epsilon_0 c^2}\,, \quad \tau_a = \frac{d}{c}$$

Caldirola used $m_{\mathrm{ed}} \to m_e$. Approach highly useful in PIC (particle in cell) radiation evaluation. d a length scale, to be chosen – e.g. so that $m_{\mathrm{ed}} = m_{\mathrm{inertial}}$,

Sample of proposed Lorentz force patches

LAD	$\mathbf{mu}^{\alpha} = \mathbf{qF}^{\alpha\beta}\mathbf{u}_{\beta} + m\tau_0\left[\ddot{u}^{\alpha} - u^{\beta}\ddot{u}_{\beta}u^{\alpha}\right]$
Landau-Lifshitz	$\mathbf{mu}^{\alpha} = \mathbf{qF}^{\alpha\beta}\mathbf{u}_{\beta} + q\tau_0\left\{F^{\alpha\beta}_{,\gamma}u_{\beta}u^{\gamma} + \frac{q}{m}\left[F^{\alpha\beta}F_{\beta\gamma}u^{\gamma} - (u_{\gamma}F^{\gamma\beta})(F_{\beta\delta}u^{\delta})u^{\alpha}\right]\right\}$
Caldirola	$\mathbf{0} = \mathbf{qF}^{\alpha\beta}(\tau)\mathbf{u}_{\beta}(\tau) + \frac{m}{2\tau_0}\left[u^{\alpha}(\tau - 2\tau_0) - u^{\alpha}(\tau)u_{\beta}(\tau)u^{\beta}(\tau - 2\tau_0)\right]$
Mo-Papas	$\mathbf{mu}^{\alpha} = \mathbf{qF}^{\alpha\beta}\mathbf{u}_{\beta} + q\tau_0\left[F^{\alpha\beta}\dot{u}_{\beta} + F^{\beta\gamma}\dot{u}_{\beta}u_{\gamma}u^{\alpha}\right]$
Eliezer	$\mathbf{mu}^{\alpha} = \mathbf{qF}^{\alpha\beta}\mathbf{u}_{\beta} + q\tau_0\left[F^{\alpha\beta}_{,\gamma}u_{\beta}u^{\gamma} + F^{\alpha\beta}\dot{u}_{\beta} - F^{\beta\gamma}u_{\beta}\dot{u}_{\gamma}u^{\alpha}\right]$
Caldirola-Yaghjian	$\mathbf{mu}^{\alpha} = \mathbf{qF}^{\alpha\beta}(\tau)\mathbf{u}_{\beta}(\tau) + \frac{m}{\tau_0}\left[u^{\alpha}(\tau - \tau_0) - u^{\alpha}(\tau)u_{\beta}(\tau)u^{\beta}(\tau - \tau_0)\right]$

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Exact LL Solutions: di Piazza 2009, Hadad et al 2010

$$A^\alpha(x) = a_0 \mathcal{R}\left[\varepsilon^\alpha f\left(\xi\right)\right] \quad \xi = k\cdot x;\ d\xi/d\tau = k\cdot u;\ A' \equiv dA/d\xi;$$

$\varepsilon^\alpha \equiv$polarization four-vector, $f(\xi) \equiv$shape of wave, $k_\alpha F^{\alpha\beta} = 0$
$k^2 = 0;\ |\varepsilon|^2 = -1;\ k\cdot\varepsilon = 0$.Substitute $\xi = k\cdot x$ for τ:

$$k_\alpha F^{\alpha\beta}_{,\gamma} = 0; \quad \varepsilon_\alpha F^{\alpha\beta} = -(\varepsilon\cdot A')k^\beta; \quad \varepsilon_\alpha F^{\alpha\beta}_{,\gamma} = -(\varepsilon\cdot A'')k^\beta k_\gamma;$$

$$\mathcal{F}^\alpha \equiv F^{\alpha\beta}u_\beta = (u\cdot A')k^\alpha - (k\cdot u)A'^\alpha; \quad F^{\alpha\beta}F_{\beta\gamma} = -k^\alpha k_\gamma (A')^2$$

$$(k\cdot u)\,u'^\alpha = -\frac{e}{m}\mathcal{F}^\alpha - \frac{e}{m}\tau_0\left\{F^{\alpha\beta}_{,\gamma}u_\beta u^\gamma - \frac{e}{m}\left[F^{\alpha\beta}\mathcal{F}_\beta + (\mathcal{F})^2 u^\alpha\right]\right\}.$$

multiply by k: $(k\cdot u') = \tau_0\left(k\cdot u\right)^2\left(A'\right)^2$. Divide by $(k\cdot u)^2$ and integrate:

$$k\cdot u \equiv \frac{d\xi}{d\tau} = \frac{k\cdot u_0}{1-\tau_0 a_0^2 (k\cdot u_0)\psi(\xi)}; \quad \psi(\xi) = \int_0^\xi \left[\hat{A}'(y)\right]^2 dy$$

For completeness

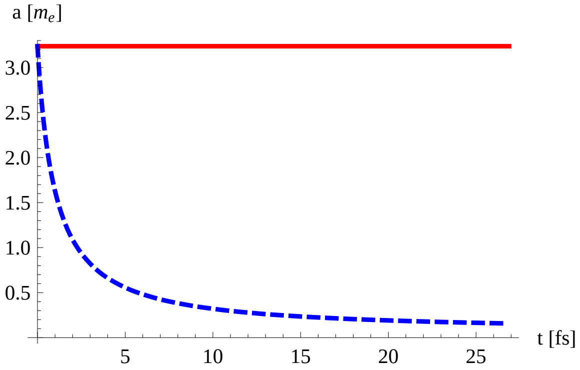
Circular plane wave

$$\varepsilon^\alpha = \frac{1}{\sqrt{2}}(0, 1, -i, 0)$$
$$k^\alpha = (\omega, 0, 0, k)$$
$$f(\xi) = \sqrt{2}e^{i(\xi-\xi_0)}$$
$$\vec{A} = a_0\left[\cos(kz - \omega t + \xi_0)\hat{x} - \sin(kz - \omega t + \xi_0)\hat{y}\right]$$
$$\vec{E} = -\omega a_0\left[\sin(kz - \omega t + \xi_0)\hat{x} + \cos(kz - \omega t + \xi_0)\hat{y}\right]$$
$$\vec{B} = -k a_0\left[-\cos(kz - \omega t + \xi_0)\hat{x} + \sin(kz - \omega t + \xi_0)\hat{y}\right]$$

Linear plane wave

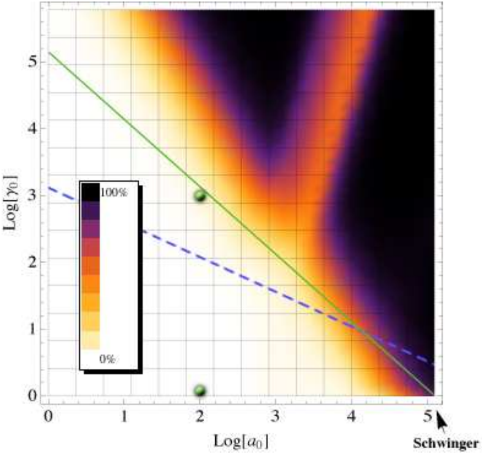
$$\varepsilon^\alpha = (0, 1, 0, 0)$$
$$k^\alpha = (\omega, 0, 0, k)$$
$$f(\xi) = \sin(\xi - \xi_0)$$
$$\vec{A} = -a_0 \sin(kz - \omega t + \xi_0)\hat{x}$$
$$\vec{E} = -\omega a_0 \cos(kz - \omega t + \xi_0)\hat{x}$$
$$\vec{B} = -k a_0 \cos(kz - \omega t + \xi_0)\hat{y}$$

Great difference in LL-LE dynamics: Radiation Reaction



The Lorentz invariant acceleration $\sqrt{-\dot{u}^\alpha \dot{u}_\alpha}$ arising from solution of dynamical equation as function of laboratory time in natural units for a collision between a circularly polarized laser wave with $a_0 = 100$ and initial $E_e = 0.5\,\mathrm{GeV}, \gamma = 1,000$ electron. The solid red line is the acceleration in the Lorentz force case, while the dashed blue line gives the acceleration according to the Landau-Lifschitz equation.

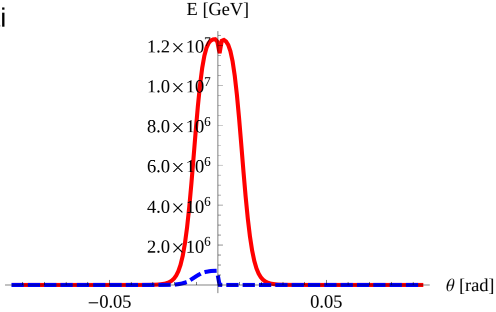
Radiation reaction (RR) regime



Deviations from Lorentz force impact significantly Lorentz dynamics in dark shaded area of the γ,a_0 plane

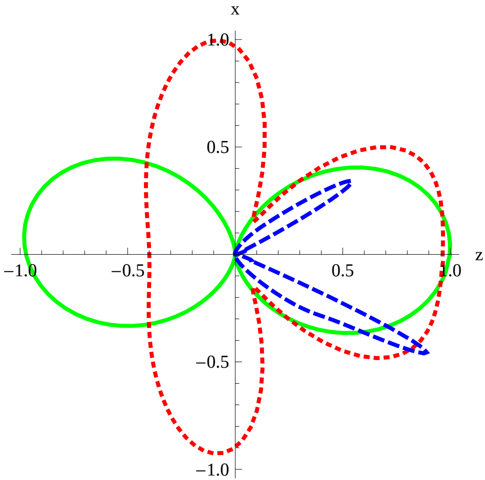
Remark on energy conservation

The RR force is not a conservative force, it acts as a friction force. Even if the force is greater for LL formulation, radiation emissions are stronger for Lorentz force dynamics, mainly because the particle keeps oscillati



Radiation emission for a circularly polarized wave with $a_0 = 100$ colliding head-on with an electron with initial $\gamma_0 = 1,000$. The angle θ is measured on the $x - z$ plane, starting from the negative z-axis. LF equation radiation (solid red line) is one order of magnitude greater than LL equation radiation (the dashed blue line).

Radiation Patterns



The angular distribution of radiation for a linearly polarized wave. This is the normalized radiation distribution for an electron initially at rest, after interacting with a laser with $a_0 = 0.1$, $a_0 = 1$ and $a_0 = 10$ plotted in solid green, dotted red and dashed blue lines respectively. This plot is identical for the LF and for the LL Equation.

Status of radiation-reaction force

- To account of emitted radiation a friction force (LAD) arises requiring knowledge of the future to damp away run-away solution; thus aside of clear indication of a missing degree of freedom, this patch involves a violation of causality.
- Modifications of LAD generate a slate of alternate models which cure the causality issue, agree with radiation reaction force at lowest order at best and usually have other 'cost' such as nonlocality, nonlinearity. None has so far been associated with an action principle and/or conservative force.
- Theoretical papers multiply in this decade with universal recognition that RR problem is foundational, and not result of errors or omissions.

- A study of the equivalence of radiation reaction between quantum and classical system at least to first order in e^2 shows expected equivalence. A. Higuchi, G.D.R. Martin, Phys.Rev. D73 (2006) 025019, A. Higuchi, P. J. Walker Phys.Rev.D79:105023,2009 *. . . we show that the radiation-reaction force derived from QED agrees with the classical counterpart. . .*
- *. . . the first rigorous derivation of the complete first order correction to Lorentz force motion...* (Gralla, Hartle, Wald, PRD80, 024031 (2009).)

CONCLUSION: (Q)ED does not describe dynamics of highly accelerated charged particles; this probably explains some of dynamic challenges arising in relativistic heavy ion collisions such as fast thermalization.

Status of acceleration

There are well known acceleration paradoxes:

- Charged electron in orbit around the Earth will not radiate if bound by gravitational field, but it will radiate had it been bend into orbit by magnets.
- A free falling electron near BH will not radiate but an electron resting on a surface of a table will.
- A micro-BH will evaporate, but a free falling observer cannot see this.

What next?

Time is ripe to seek new physics:

1) Experimental study using Laser pulse – electron collider possible.

Missing theoretical element: current gauge theory framework is NON-MACHIAN. There is no relation to an inertial (class of) frame of reference. In GR the geometry provides this. In quantum theory there is the 'vacuum' = ground state.

2) We need to bring back Einstein's æther so that there is a clear correspondence of the classical and quantum theory: Aether$\equiv$Quantum Vacuum.

1920: Einstein-Lorentz and the Vacuum (Aether)

Albert Einstein rejected æther as unobservable when formulating special relativity, but eventually changed his initial position, re-introducing what is referred to as the 'relativistically invariant' æther. In a letter to H.A. Lorentz of November 15, 1919, see page 2 in *Einstein and the Æther*, L. Kostro, Apeiron, Montreal (2000). he writes:

It would have been more correct if I had limited myself, in my earlier publications, to emphasizing only the non-existence of an æther velocity, instead of arguing the total non-existence of the æther, for I can see that with the word æther we say nothing else than that space has to be viewed as a carrier of physical qualities.

In a lecture published in May 1920 (given on 27 October 1920 at Reichs-Universität zu Leiden, addressing H. Lorentz), published in Berlin by Julius Springer, 1920, also in Einstein collected works: In conclusion:

. . . space is endowed with physical qualities; in this sense, therefore, there exists an æther. . . . But this æther may not be thought of as endowed with the quality characteristic of ponderable media, as (NOT) consisting of parts which may be tracked through time. The idea of motion may not be applied to it.

Where to look for a better theory I

Geometrize EM theory: 5-d Kaluza-Klein.

• EM potential part of 5-metric. To lowest order in charge, Lorentz force arises from 5-d geodetic. Hilbert-Einstein 5-d action reduces to 4-d Einstein-Maxwell action.

Pro: Any geometric EM theory has ‘æther’ and is Machian just like GR; charge is a property of the ‘æther; the missing degrees of freedom appear in 5th dimension.

Con: Lack of experimental input about the dynamics in 5-d. No generalized equivalence principle justifying use of 5-d geodetic.

• Therefore, we fail (in next order in metric deformation) to find LAD type RR expression.

If theory is geometric, critical acceleration will produce quite extraordinary outcome, this is matter stability ‘Planck’ condition in 5-d theory!

Where to look for a better theory II

Without quantum theory we would not have extended bodies, without extended material bodies we cannot create critical acceleration – all this is due to interplay of quantum and EM theories.

- Rethink the classical limit of quantum field theory considering the quantum vacuum as a part of the system.

Pro: Quantum vacuum is introducing a Machian reference frame lost in ‘perturbative’ classical limit. Critical acceleration in quantum theory (critical fields) leads to new phenomena which have a good interpretation and no classical analog or obvious limit.

Con: Perturbative QFT cannot lead to any new Machian result. Nonperturbative aspects challenging, perturbative QED convincing.

Opportunity to search for missing essential ingredients such as charge quantization, flavor family hierarchy.

Even so, let me remind everyone of a few ‘old’ considerations of strong field QED which may play a role in any future search for improved related classical picture.

Laws of Physics and Quantum Vacuum

Development of quantum physics leads to the recognition that vacuum fluctuations define laws of physics (Weinberg's effective theory picture). All this is nonperturbative property of the vacuum.

- The 'quantum æther' is polarizable: Coulomb law is modified; E.A.Uehling, 1935
- New interactions (anomalies) such as light-light scattering arise considering the electron, positron vacuum zero-point energy; Euler, Kockel, Heisenberg (1930-36);
- Casimir notices that the photon vacuum zero point energy also induces a new force, referred today as Casimir force 1949
- Non-fundamental vacuum symmetry breaking particles possible: Goldstone Bosons '60-s
- 'Fundamental electro-weak theory is effective - model of EW interactions, 'current' masses as VEV Weinberg-Salam '70-s
- Color confinement and high-T deconfinement Quark-Gluon Plasma '80-s

QED Non-perturbative Vacuum

Vacuum state: $\hat{b}_e|0\rangle = \hat{d}_{\bar{e}}|0\rangle = 0$
e particle and $\bar{e}$ antiparticle 'modes'

$$\hat{\Psi} = \sum_e \psi_e \hat{b}_e + \sum_{\bar{e}} \psi_{\bar{e}} \hat{d}^\dagger_{\bar{e}},$$

Charge operator introduced such that the Dirac 'sea' charge of positrons cancels charge of electrons (net zero charge in groundstate):

$$\hat{Q} = \int d^3x \frac{e}{2}\left[\Psi(x)^\dagger, \Psi(x)\right], \qquad \langle 0|\hat{Q}|0\rangle = \frac{e}{2}\left[\sum_e - \sum_{\bar{e}}\right] \to 0$$

Up-to-date the nonperturbative computation of zero point energy vacuum fluctuations is carried out for independent Dirac particle seas: electron, positron, spin $\pm$.This may need improvement – seminar lecture by Lance Labun.

$$\mathcal{E} = -\frac{1}{2V}\sum_{e,s}\varepsilon_e - \frac{1}{2V}\sum_{\bar{e},s}(-\varepsilon_{\bar{e}}) \to (g_{\rm Bos} - g_{\rm Ferm})M^4_{\rm Planck} + d_1\bar{m}^2 M^2_{\rm Pl} + \mathcal{L}(E,B)$$

Last nonperturbative term proportional to particle degeneracy, may be constrained by the need to assure that charge and spin of the vacuum vanish.

Euler-Heisenberg Z. Physik 98, 714 (1936) evaluate Dirac zero-point energy with (constant on scale of $\hbar/mc$) EM- fields, transform to action $\mathcal{E}(D,H) \to \mathcal{L}(E,B) = \mathcal{E} - ED$, $E \equiv \frac{\partial\mathcal{E}}{\partial D}$

$$\mathcal{L}(E,B) = -\frac{1}{8\pi^2}\int_{i\epsilon}^{\infty}\frac{ds}{s^3}\mathrm{e}^{-m^2 s}\left[\frac{sE}{\tan sE}\,\frac{sB}{\tanh sB} - 1 + \frac{1}{3}(E^2 - B^2)s^2\right].$$

E, B relativistic definition in terms of invariants

$$\mathcal{L}(E,B) \to \frac{2\alpha^2}{45m^4}\left[\left(\vec{E}^{\,2} - \vec{B}^{\,2}\right)^2 + 7(\vec{E}\cdot\vec{B})^2\right] = \text{light} - \text{light scattering}$$

Schwinger 1951 studies in depth the imaginary part – see zeros in $\tan sE$:

$$|\langle 0_+|0_-\rangle|^2 = e^{-2\mathcal{I}m\,\mathcal{L}}, \qquad 2\mathcal{I}m\,\mathcal{L} = \frac{\alpha^2}{\pi^2}\sum_{n=1}^{\infty} n^{-2} e^{-n\pi m^2/eE}$$

is the vacuum persistence probability in adiabatic switching on/off the E-field. For $E \to 0$ essential singularity.

Note: vacuum unstable for any finite value of E and there is no analytic $E \to 0$ limit!. The light–light perturbative term is first term of semi convergent expansion which fails in a subtle way.

Temperature for vacuum fluctuations at $a =$Const.

Using identity [Müller/Greiner/Rafelski 77]:

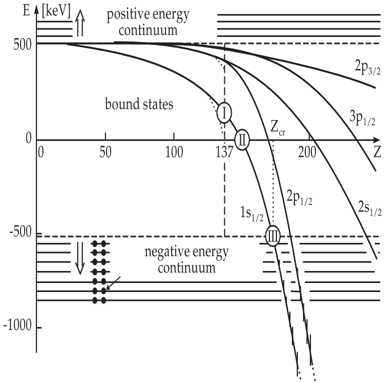
$$\frac{x\cos x}{\sin x} = 1 - \frac{x^2}{3} + \sum_{k=1} \frac{1}{k^2\pi^2}\frac{2x^4}{x^2-k^2\pi^2}$$

give after resummation a “statistical” form with $T = eE/\pi m = a/\pi$:

$$\mathcal{L}(E) = -2_s \frac{m^2 T}{8\pi^2}\int_0^\infty \rho(\omega)\ln(1-e^{-\omega/T})d\omega, \quad \rho = \ln(\omega^2 - m^2 + i\epsilon);$$

Same sign of pole residues source of Bose statistics. For scalar loop particles we have $\frac{x}{\sin x} = 1 + \frac{x^2}{6} + \sum_{k=1} \frac{(-1)^k}{k^2\pi^2}\frac{2x^4}{x^2-k^2\pi^2}$ which yields same expression up to: 1) statistics turns from Bose into Fermi: $-\ln(1-e^{-\omega/T}) \to \ln(1+e^{-\omega/T})$ and 2) factor $2_s \to 1$. Note that the statistics is reversed compared to expectations and temperature is twice larger compared to Hawking-Unruh observer. Potential improvements needed in the nonperturbative approach (Lance Labun seminar) to constrain symmetry of the vacuum state.

Local critical fields: High Z Atoms and QED Vacuum



Single Particle Dirac Equation

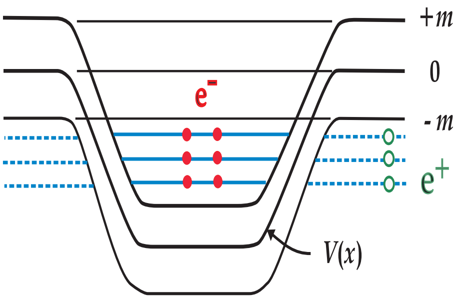
$$(\vec{\alpha}\cdot i\vec{\nabla}+\beta m+V(r))\Psi_n(\vec{r}) = E_n\Psi_n(\vec{r})$$

$$V(r) = \begin{cases} -\frac{Z\alpha}{r} & r > R_N \\ -\frac{3}{2}\frac{Z\alpha}{R_N} + \frac{r^2}{2}\frac{Z\alpha}{R_N^3} & r < R_N \end{cases}$$

The bound states drawn from one continuum move as function of $Z\alpha$ across into the other continuum.

References: The large volume of work from 1968-85 is reviewed in W. Greiner, B. Müller and JR "Quantum Electrodynamics of Strong Fields," (Springer Texts and Monographs in Physics, 1985), ISBN 3-540-13404-2.

Formation of Charged Vacuum

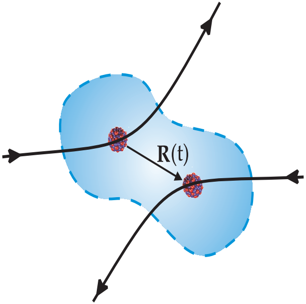


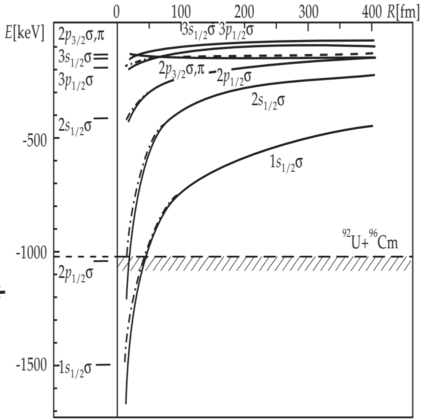
Pair production across a nearly constant field fills the additional 'dived' states available in the localized domain. There is real localized charge density in the vacuum. Formation of the charged vacuum ground state observable in positron emission.

Back-reaction of real vacuum charge and screening of the external field (Müller and JR, PRL34, 349 (1975)):

$$-\vec{\nabla}^2 V = \rho_N + \langle \rho_e \rangle, \qquad \langle \rho_e \rangle \simeq -\frac{e^2}{3\pi^2}(2mV + V^2)^{3/2}\Theta(-V - 2m)$$

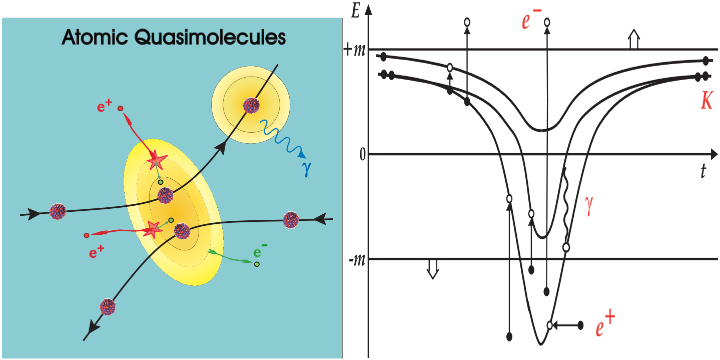
Experimental Effort (presently resurrected)





The velocity of electrons is near that of light, that of ions about c/10: Born-Oppenheimer quasi-molecular two-center orbitals. For $R < 30$ fm Supercritical σ-states.

Time Dependent Processes



The natural vacuum life span (vacuum resonance width) longer than typical duration of collision. The time dependence induces particle production process with energy extracted from collision: ‘assisted pair production’ (invented 1975, reinvented in 2008). Phys.Rev.D78:061701,2008 *Pair Prod. Beyond Schwinger Formula in t-dep. E-fields*. Too many pairs, not possible to identify which are relevant for study of vacuum structure

Epilog: Strong Fields and Heavy Ion Collisions

1. Supercritical fields in heavy ion collisions exist, but for too short a time, positron dynamically spread in energy and the total yield is small
2. Positron production from other time dependent processes offers a dominant background with a broad spectrum
3. Efforts to 'glue' heavy nuclei in grazing collisions resulted in nuclear contributions to particle production but no sharp lines

Today: Renewed effort under consideration. However, alternate technology can produce a new source of supercritical fields: femto second laser pulsed lasers today with $N_\gamma = 0.610^{22}$ photons and pulse length is 10-70 fs, focus diffraction limited at $(2\mu\mathrm{m})^3$. The energy density: $\epsilon \simeq 100\mathrm{J}/\mu\mathrm{m}^3 =$ $= 600\mathrm{MeV}/(10^{-10}\mathrm{m})^3$ is very high on atomic scale: pulse contains nearly $E = mc^2$ hydrogen gas energy.

Summary

Critical acceleration possible in electron-laser pulse collisions.

Opportunity to study foundational new physics involving acceleration and radiation reaction as dominant force.

New Search for generalization of laws of physics – motivated by need for better understanding of inertia, Mach's principle, Einstein's Aether.