Gauge/Cosmology duality from AdS/CFT

Robi Peschanski [a]
(IPhT, Saclay)
Cracow School, 50th anniversary, Zakopane, 2010

- Gauge/Gravity Duality: the other facet
  Little Bang/Big Bang Relation?
- The AdS/CFT Correspondence
  Brief reminder
- The Two-Brane Geometry
  Brane-world Cosmology/Moving Isotropic Plasma
- Gauge/Cosmology Duality and Dark Energy
  Conformal Anomaly/Cosmic Acceleration
- Conclusion and Intriguing Aspects
  Bulk viscosity/e-foldings

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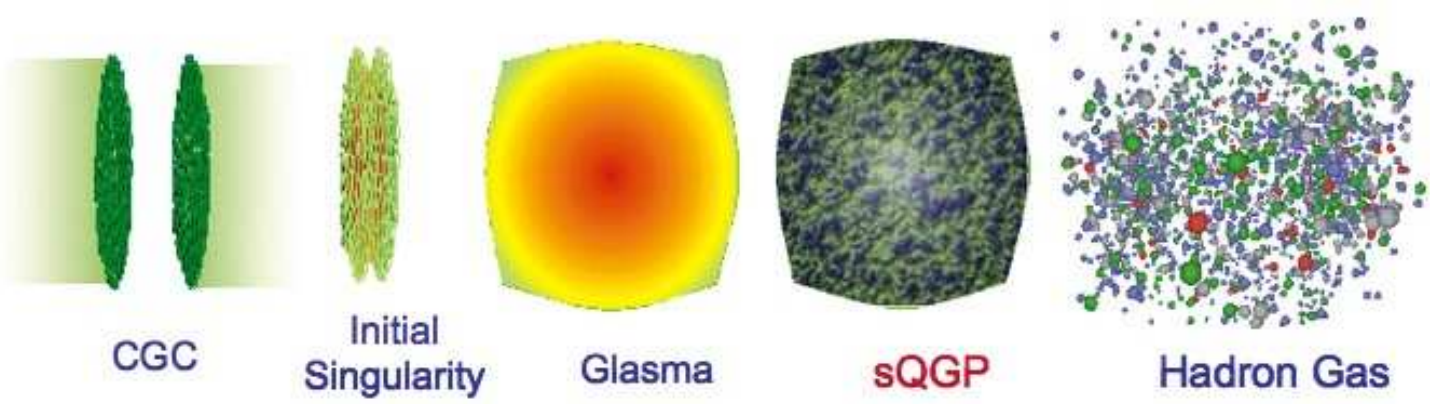
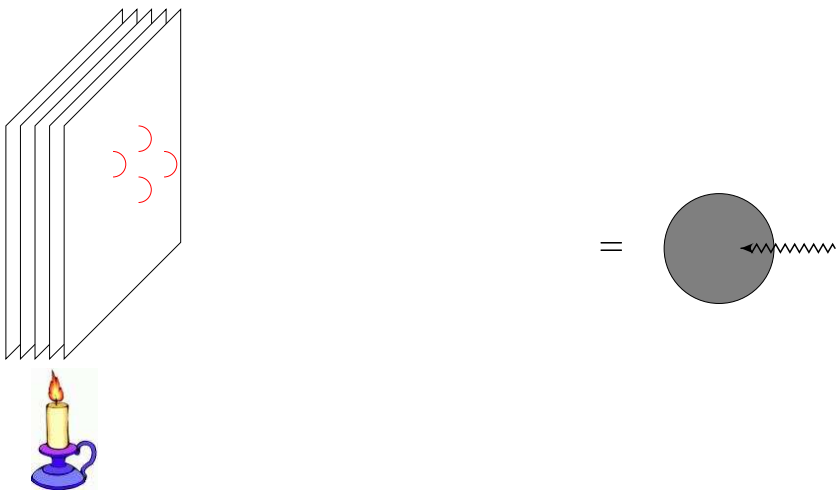
[a] with Philippe Brax, IPhT

From the Little Bang to the Big Bang ?



Viscosity on the light of duality

Consider a graviton that falls on this stack of N D3-branes
Will be absorbed by the D3 branes.
The process of absorption can be looked at from two different perspectives:



Absorption by D3 branes ($\sim$ viscosity) = absorption by black hole

Viscosity and BH/Plasma Duality: Policastro, Son, Starinets;
Perfect fluid and Expanding BH/Cooling Plasma: Janik,R.P.
Many Developments: ...

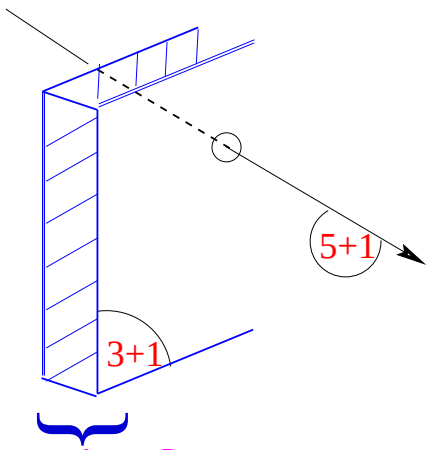
AdS/CFT Correspondence

J.Maldacena (1998)

GRAVITY

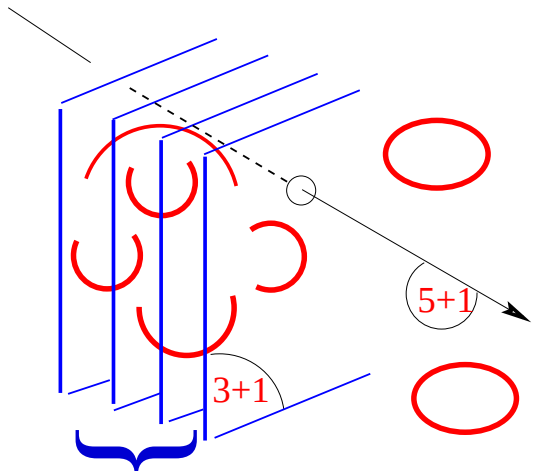
GAUGE

AdS



Gravity Source

CFT



N-Branes

Macroscopic

$(g_S N)^{-1} \to 0$

$AdS_5 \times S_5$ Superstring

Weak Gravity

Strong Gravity

Duality

Microscopic

$g_S N \to 0$

SU(N) Gauge Theory on the N-Branes

Strong Coupling

Weak Coupling

Why $AdS_5 \otimes S_5$

- D_3-brane Solution of Super Gravity:

$$\boxed{ds^2 = f^{-1/2}(-dt^2 + \sum_1^3 dx_n^2) + f^{1/2}(dr^2 + r^2 d\Omega_5)}$$

"On-Branes $\times$ Out-Branes"

$$f = 1 + \frac{R^4}{r^4} \; ; \; R^4 = 4\pi g_{YM}^2 \alpha'^2 N$$

- "Maldacena limit": Strong coupling

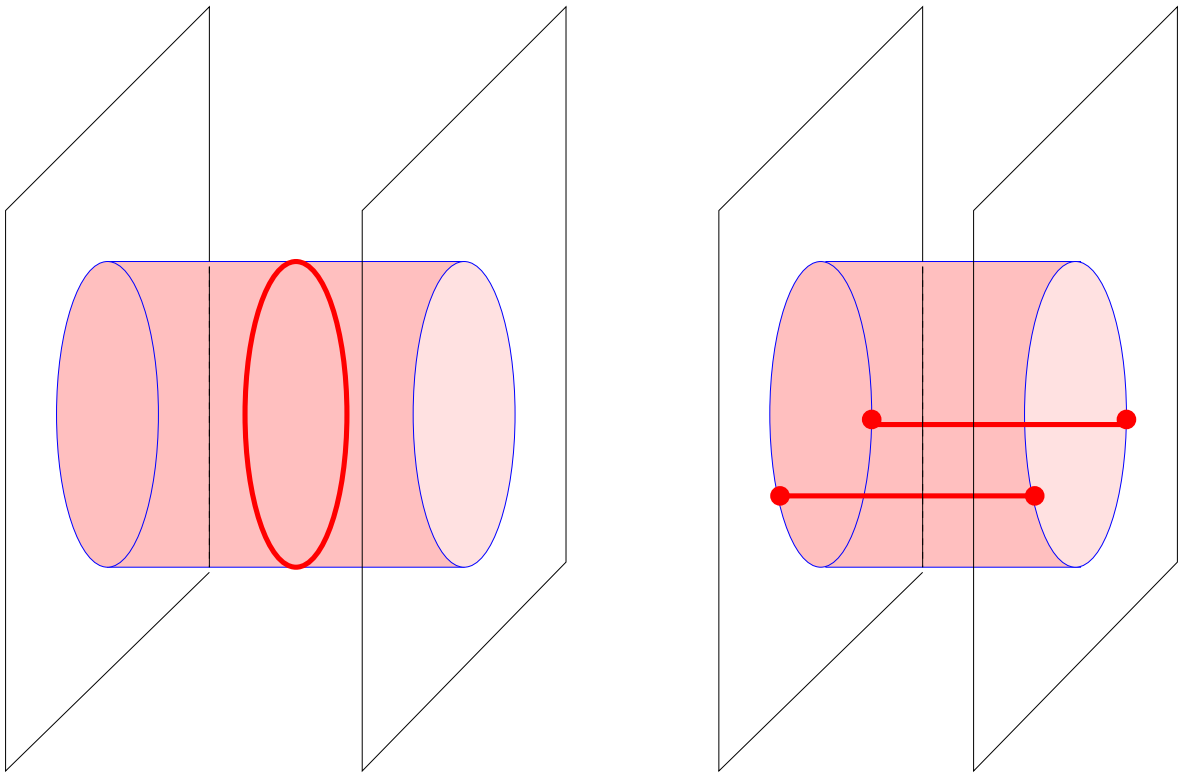
$$\frac{\alpha'(\to 0)}{r(\to 0)} \to z \; , \; R \; fixed \; \Rightarrow g_{YM}^2 N \to \infty$$

$$\boxed{ds^2 = \frac{1}{z^2}(-dt^2 + \sum_{1-3} dx_n^2 + dz^2) + R^2 d\Omega_5}$$

Background Structure: $AdS_5 \times S_5$

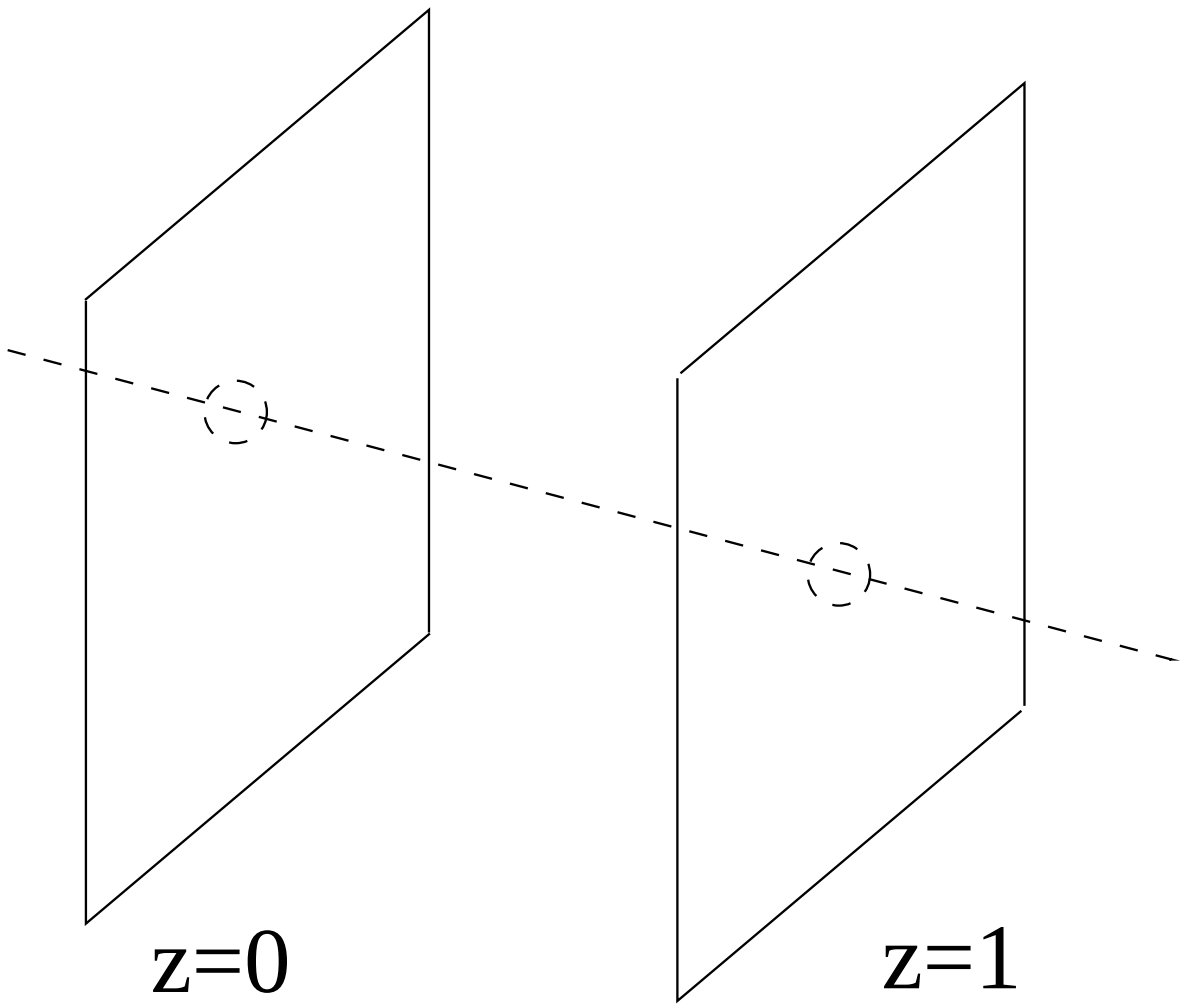
The Gauge-Gravity Correspondence

Open $\Leftrightarrow$ Closed String duality



$$
\begin{aligned}
Closed\ String &\Leftrightarrow 1-loop\ Open\ String \\
Gravity &\Leftrightarrow Gauge \\
D-Brane\ \text{“}Universe\text{”} &\Rightarrow Open\ String\ Ending \\
Small/Large\ Distance &\Rightarrow Gauge/Gravity\ Correspondence
\end{aligned}
$$

The Brane-to-Brane Geometry



$$
\begin{aligned}
z = 1 \quad &\Rightarrow \quad \textit{Cosmological Brane} \\
z = 0 \quad &\Rightarrow \quad \textit{Holographic Brane (AdS Boundary)} \\
0 < z < 1 \quad &\Rightarrow \quad \textit{Bulk } AdS_5 \textit{ Gravity} \\
z > 1 \quad &\Rightarrow \quad z \to 1/z \; AdS_5 \textit{ metric} \\
\textit{Brane} \leftrightarrows \textit{Brane} \quad &\Rightarrow \quad \textit{Gauge/Cosmology Duality}
\end{aligned}
$$

Holography at work

Brane → Bulk: Holographic Renormalization

K.Skenderis (2002)

- Bulk metric

$$\boxed{ds^2 = \frac{g_{\mu\nu}(z)\, dx^\mu dx^\nu + dz^2}{z^2}}$$

(in Fefferman-Graham Coordinates)

- $5d \Rightarrow 4d$ **metric:**

$$\boxed{g_{\mu\nu}(z) = g^{(0)}_{\mu\nu}(\neq \eta_{\mu\nu}) + z^2 g^{(2)}_{\mu\nu}(\neq 0) + z^4 g^{(4)}_{\mu\nu}(\neq \langle T_{\mu\nu}\rangle) + \mathbf{z^6}\ldots +}$$

$+\mathbf{z^6}\ldots+$: from Einstein Eqs.

- **Energy-Momentum Tensor:**

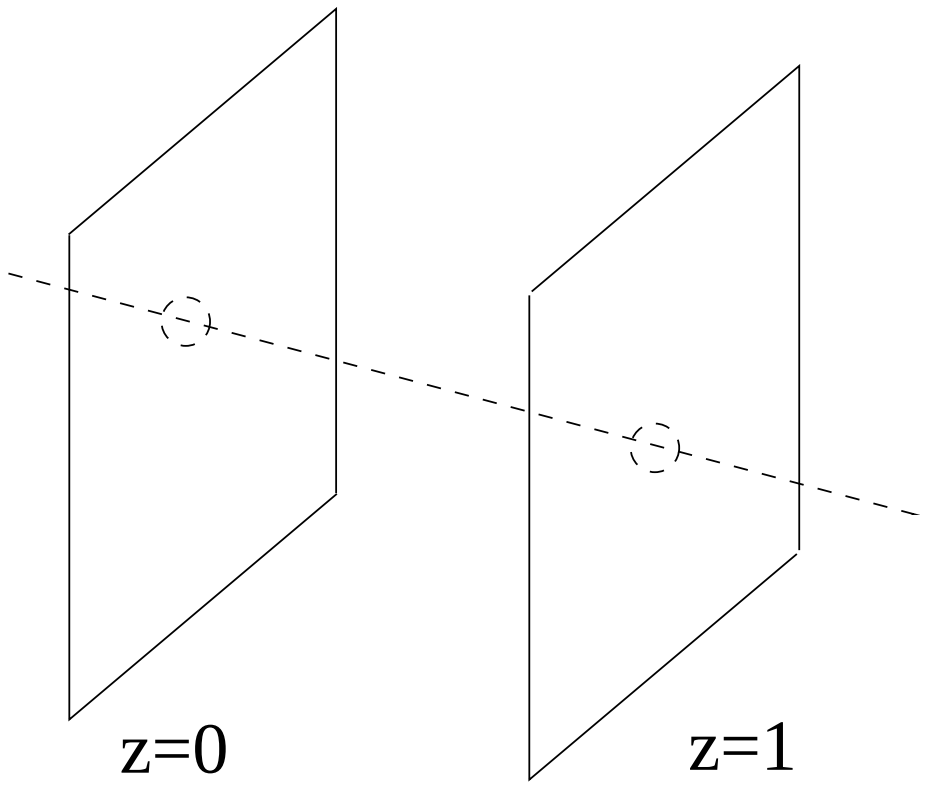
$$\boxed{\langle T_{\mu\nu}\rangle = \frac{2l^3}{\kappa_5^2}\left\{ g^{(4)}_{\mu\nu} - \frac{1}{8} g^{(0)}_{\mu\nu}\left[(\mathrm{Tr}(g^{(2)}))^2 - (\mathrm{Tr} g^{(2)})^2\right] - \frac{1}{2} g^{(2)}_{\mu\rho} g^{(0)\rho\sigma} g^{(2)}_{\sigma\nu} + \frac{1}{4}(\mathrm{Tr} g^{(2)}) g^{(2)}_{\mu\nu} \right\}}$$

Note: Curved $4d$ metric, non Minkowskian ($\rightarrow$ FRW)

The Cosmology Brane

- Brane-World Cosmology

Binetruy, Deffayet, Ellwanger, Langlois (2000)



Gauge Brane Cosmology Brane

$$\langle T \rangle_B = diag(-\rho_B, \rho_B, \rho_B, \rho_B, \rho_B) \Rightarrow \langle T \rangle_b = diag(-\rho_b, p_b, p_b, p_b)$$

(tuning Λ with Israel Conditions)

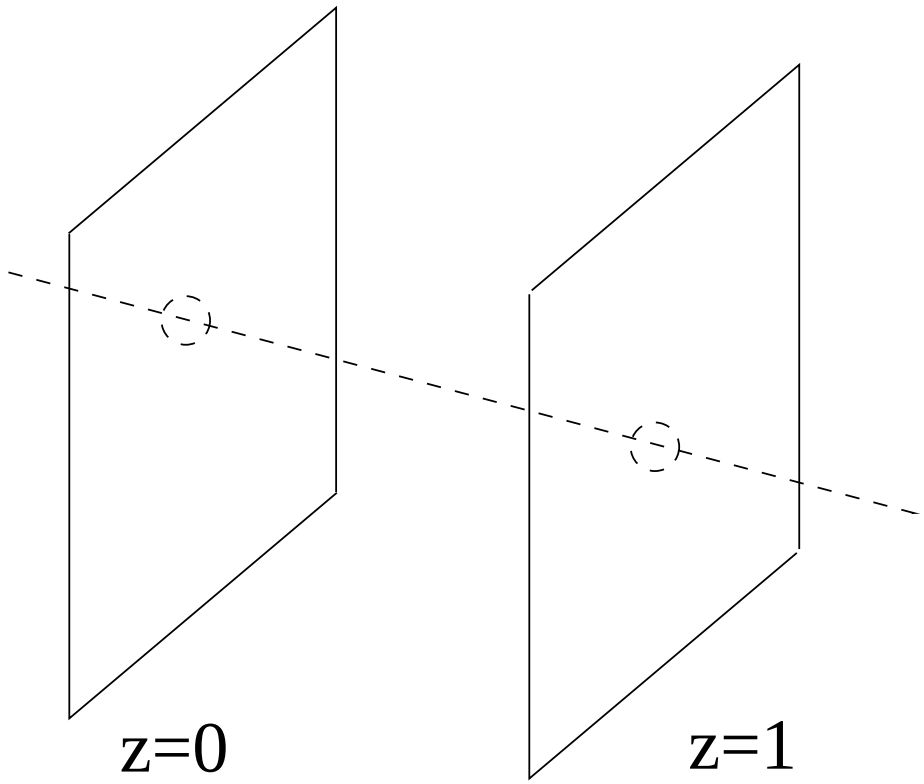
- Induced Friedmann equation

$$H^2 l^2 = -1 + \frac{\rho_b^2}{\rho_\Lambda^2} + \frac{\mathcal{C}}{a_0^4}$$

The Gauge Brane

- Isotropically Expanding Plasma in AdS/CFT

Kajantie, Tahkokallio (2007)



Gauge Brane Cosmology Brane

$$ds_5^2 = \frac{l^2}{z^2}\left[dz^2 - \frac{dt_K^2}{l^2}\frac{h^2 r^2}{b(t_K,z)}\left(1 + A_2 z^2 + A_4 z^4\right)^2 + \frac{dx^2}{l^2} b(t_K,z)\right]$$

$b(t_K,z), A_2, A_4$, functions of $r, h,$ and t_K derivatives

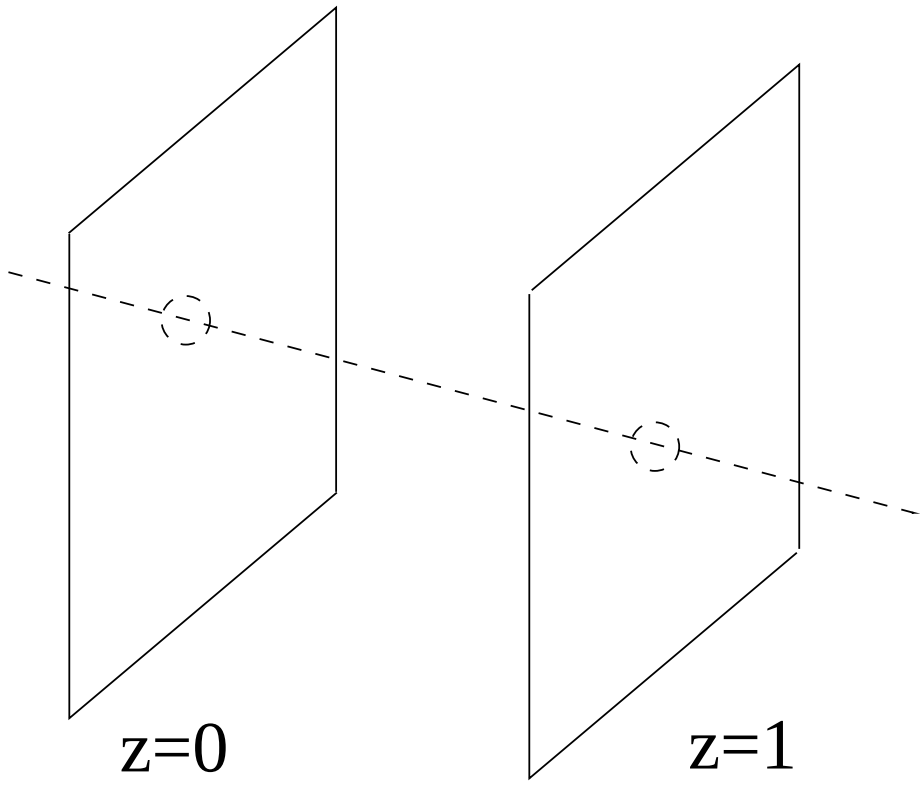
- Gauge/Gravity duality

$$\frac{l^3}{\kappa_5^2} = \frac{N_c^2}{4\pi^2}$$

Brane-to-Brane Duality

- Isotropically Expanding Plasma in AdS/CFT

Brax, R.P. (2010), to appear



Gauge Brane Cosmology Brane

$$ds_5^2 = \frac{l^2}{z^2}\left[dz^2 - \frac{dt_B^2}{l^2}\left(\frac{H^2l^2/2 + \dot{H}l^2}{2} - z^2\right)^2 + \frac{dx^2}{l^2}a_0^2(t_B)\left(\frac{H^2l^2}{4} - z^2\right)^2\right]$$

- Brane-to-Brane Space-Time Relations

$$r(t_K) = \frac{H^2l^2}{4}\, a_0(t_B) \qquad \frac{1}{h}\,\frac{dr}{dt_K} = \frac{da_0}{dt_B}$$

Gauge/Cosmology Duality

- $\mathcal{N} = 4$ SYM Energy-Momentum Tensor

$$\boxed{T_b^{\mu\nu} = (\epsilon_b + p_b)u^\mu u^\nu - p_b \eta^{\mu\nu}}$$

- Duality Relations

$$\boxed{\frac{r^2}{a_0^2} = \frac{1}{4}\left\{\frac{\rho^2}{\rho_\Lambda^2} + \frac{\mathcal{C}}{a_0^4}\right\} :} \quad \rho_b \equiv \rho_\Lambda + \rho$$

$$\boxed{\rho_K = \frac{3N_c^2}{8\pi^2}\frac{a_0^4}{r^4}\left(\frac{\mathcal{C}}{a_0^4} + \frac{H^4 l^4}{4}\right) :} \quad \text{Energy density}$$

$$\boxed{\rho_K - 3p_K = \frac{\ddot{a}_0}{a_0}\ \frac{3N_c^2}{8\pi^2}\ \left(\frac{a_0}{r}\right)^3\ \epsilon_K \frac{da_0}{dr} :} \quad \text{Trace Anomaly}$$

- Covariant Acceleration/Anomaly Relation

$$\boxed{\sqrt{-g_B}\, dt_B\ H^2\ \frac{\ddot{a}_0}{a_0} = \sqrt{-g_K}\, dt_K\ \frac{8\pi^2}{3N_c^2}\ (\rho_K - 3p_K)}$$

Plasma/Cosmology Duality

- Equations of State: Holographic brane

$$\boxed{w_{eff} \equiv \frac{p_H}{\rho_H} = -\frac{w}{2+3w}}$$

- Duality Relations

$$\begin{aligned} Matter\ Cosmology\ \omega = 0 &\Leftrightarrow \omega_{eff} = 0 \\ Dark\ energy\ \ \omega = -1 &\Leftrightarrow \omega_{eff} = -1 \\ No\ acceleration\ \ \omega = -1/3 &\Leftrightarrow \omega_{eff} = 1/3 = Perfect\ Fluid \end{aligned}$$

- Holographic expansion/contraction

$$\boxed{a_0 \sim t_B^{\frac{2}{3(1+w)}} \quad \Leftrightarrow \quad a_H(t) \sim t_K^{\frac{2}{3(1+w_{eff})}} = t_K^{-\frac{2(2+3w)}{3(1+w)}}}$$

$$\begin{aligned} Matter\ Cosmology\ \omega > -2/3 &\Leftrightarrow Plasma\ Contraction \\ Dark\ energy\ \ \omega < -2/3 &\Leftrightarrow Plasma\ Expansion \end{aligned}$$

Conclusions

- Brane-to-Brane Duality
  Through a Bulk 5d metric
- Conformal Anomaly $\Leftrightarrow$ Cosmic acc- (*vs.* de-) celeration
  Conformal Anomaly $\sim$ Bulk Viscosity
- Gauge/Cosmology Duality
  Plasma expansion (*vs.* contraction) $\Leftrightarrow$ Cosmological Cst. (*vs.* Matter/Radiation) eras

An intriguing Relation:

$$\boxed{\mathcal{A} \equiv \lim_{N_c \to \infty} \frac{8\pi^2}{3} \frac{\rho_H - 3p_H}{N_c^2} = \mathcal{E}^4}$$ Anomaly by d.o.f.

$$\boxed{dN_B = H\ dt_B = d \log a_0}$$ Number of e-foldings

$$\boxed{\Delta\tau \Delta\mathcal{E} \sim \Delta N_B = O(1)}$$

Quantum/Classical Identity