Hadron masses from QCD Balmer formula

In memory of

Jan Kwieciński and Leszek Łukaszuk

Józef M. Namysłowski
University of Warsaw

Singularities

- Singularities in the p-space Dyson-Schwinger equations

$$S^{-1}(p) = \not{p} - m_c + \frac{b\chi^3/(\mu a)}{\not{p} - \mu a}$$

$$D_T^{-1}(p) = p^2 + \frac{\nu^4}{p^2}$$

χ quark condensate

$$\chi_{u,d} = (0.24 \pm 0.02)\ \mathrm{GeV}$$

ν gluon condensate

$$\nu = (0.36 \pm 0.04)\ \mathrm{GeV}$$

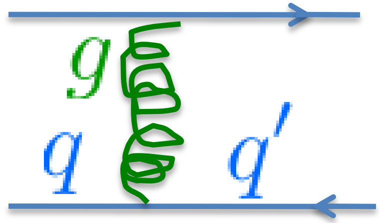
Gribov horizon

Daniel Zwanziger
Manfred Stingl

Permanent confinement

- NO asymptotic (scattering) quark or gluon states
- NO continuum $(q\bar{q})$ (qq) (qqq) $(q\bar{q}q)$
- NO colorful bound states $(q\bar{q})$ (qqq)
- PRESENCE of colorless $(q\bar{q})$ (qqq) DISCRETE, real mass states: HADRONS

Constituent gluon exchange



$$q \qquad \sim \frac{1}{r}$$

$$\bar{q}$$

r relative distance in Fourier tr. of q in {qP=0}

IRREDUCIBLE kernel

$$I(q-q')$$

for (CONSTITUENT quark, CONSTITUENT antiquark)

green nonperturbative glue

each

$$\Gamma_{qgq} \sim \frac{\nu^2}{(q-q')^2}$$

(Slavnov-Taylor)

$$D_T = \frac{(q-q')^2}{(q-q')^4 + \nu^4}$$

$$I(q-q') \sim \frac{\nu^4}{(q-q')^2[(q-q')^4 + \nu^4]}$$

$$I(q-q')|_{q_\mu P^\mu = q'_\mu P^\mu = 0} \sim \frac{1}{(q-q')^2}$$

QED & hydrogen

Balmer formula

- V. Fock, Z Phys 98 (1936) 145;
  J.Schwinger, Journ Math Phys 5 (1964) 1606
- EXACT, analytic, hydrogen an continuum (e, p)
  Green function

$$\Gamma(\Omega, \Omega') = \sum_{n,\ell,m} \frac{Y_{n\ell m}(\Omega) Y_{n\ell m}(\Omega')}{1 - \nu/n}$$

$$n = 1, 2, \ldots \qquad \nu \equiv \frac{Ze^2 m}{\sqrt{-2mE}}$$

electron mass

NEGATIVE
energy in H

BOUND state Wave Function

- The $(q\bar{q})$ CONSTITUENT-quark wave function has to be a Dirac distribution

$$\Phi = \phi\,\delta\left(\frac{q_\mu P^\mu(\mu_1+\mu_2)}{4\pi\mu_1\mu_2 M}\right)$$

- obeying constraint $q_\mu P^\mu\,\Phi(q_\mu P^\mu) = 0$
  in the hadron rest frame $q^0\Phi(q^0) = 0$
- i.e. for the RELATIVE time t: $\frac{\partial}{\partial t}\Phi(t) = 0$
  which means INDEPENDENCE of the rel. time

i) … the meson bound state amplitudes in the meson rest frame, are INDEPENDENT of the relative time of the two constituents…

in $\vec{P} = 0$ frame q^(0) = 0

H. Pagels, Phys Rev D 14 (1976) 2747

The paper which ''CHANGED my life devoted to bound states since 1964''

ii) JMN is ''ORTHOGONAL'' to almost everybody,

including the Big participants of the 50 Cracow School of Theoretical Physics

Stan Jerome Brodsky, Paul Hoyer, Staszek Glazek

because of Joseph FAULTS, not publishing results, and not even giving talks.

iii) The 50 Zakopane is ONE profound exception,

and 1 000 discussions with
Christopher Anthony MEISSNER University of Warsaw

Lagrangian Krzysztofa Meissnera

signature $(-,+,+,+)$
We start from the renormalizable lagrangian

$$\mathcal{L} = \bar{\psi}\left(\Gamma^\mu \partial_\mu - m\right)\psi - \bar{\xi}\left(\Gamma^\mu \partial_\mu - M\right)\xi + g\bar{\psi}\xi + g^\star \bar{\xi}\psi$$

Now we introduce

$$\zeta = \xi + g\left(\Gamma^\mu \partial_\mu - M\right)^{-1}\psi$$

and we get

$$\mathcal{L} = \bar{\psi}\left(\Gamma^\mu \partial_\mu - m + \frac{g^\star g}{\Gamma^\mu \partial_\mu - M}\right)\psi - \bar{\zeta}\left(\Gamma^\mu \partial_\mu - M\right)\zeta$$

They are now decoupled – we integrate over ζ (it has a wrong kinetic sign) and get the result for ψ

$$\mathcal{L} = \bar{\psi}\left(\Gamma^\mu \partial_\mu - m + \frac{g^\star g}{\Gamma^\mu \partial_\mu - M}\right)\psi$$