Matter at Finite Temperature and Density

$$Z = Tr\ e^{-\beta H + \beta \mu N}$$

H is Hamiltonian, N is baryon number $T = 1/\beta$ μ Baryon chemical potential

$$< O > = \frac{Tr\ O\ e^{-\beta H + \beta \mu N}}{Z}$$

$$e^{-itH} \rightarrow e^{-\beta H} \qquad t \rightarrow it$$

$$Z = \int [dA][d\overline{\psi}][d\psi] e^{-\{\int_0^\beta d^4x\ \{\frac{1}{4}F^2 + \overline{\psi}(\frac{1}{i}\gamma\cdot\partial + m - i\mu\gamma^0)\psi\}}$$

$$\{\gamma^\mu, \gamma^\nu\} = -2\delta^{\mu\nu} \qquad \mu_Q = \frac{1}{N_c}\mu_B$$

Euclidean space theory, so for zero baryon number density, can be simulated using Monte Carlo methods

Periodic boundary conditions for Bosons

Anti-periodic for Fermions

Confinement:

The change in the free energy when an additional heavy test quark is added. L is operator which adds an external quark source.

$$e^{-\beta F_q}$$

0, confined

Finite, deconfined

$$e^{-\beta F_q} = < L >$$

$$L(\vec{x}) = \frac{1}{N_c} Tr\ P\ e^{i \int \ dt A^0(\vec{x},t)}$$

Polyakov Loop
or Wilson Line

Path closed by
periodicity in time

$$U(\vec{x}, \beta) = Z_N U(\vec{x}, 0)$$

$$Z_N = e^{2\pi i/N} I$$

Gauge Transformations Invariant up to the Center:

$$A^\mu(\beta, x) = A^\mu(0, \vec{x})$$

$$\begin{aligned} A^{\mu\prime}(\beta, x) &= U^\dagger(\beta, \vec{x})\{A^\mu(\beta, \vec{x}) - \frac{1}{ig}\partial^\mu\}U(\beta, \vec{x}) \\ &= U^\dagger(0, \vec{x})Z_N^\dagger\{A^\mu(0, \vec{x}) - \frac{1}{ig}\partial^\mu\}Z_N U(0, \vec{x}) \\ &= A'(0, \vec{x}) \end{aligned}$$

Wilson Line transforms as

$$L = \frac{1}{N_c} Tr\ P\ e^{i\int_0^\beta dt A^0} \rightarrow \frac{1}{N_c} Tr\ U(\beta) P\ e^{i\int_0^\beta dt A^0} U = Z_N L$$

$$< L^\dagger \cdots L^\dagger L \cdots L > \rightarrow Z_N^{N_{quarks} - N_{antquarks}} < L^\dagger \cdots L^\dagger L \cdots L >$$

$$e^{2\pi i(N_q - N_{\overline{q}})/N} = 1$$

Quarks are confined to baryons and mesons in
center symmetry is not broken

Confinement and Pure Gauge Theory

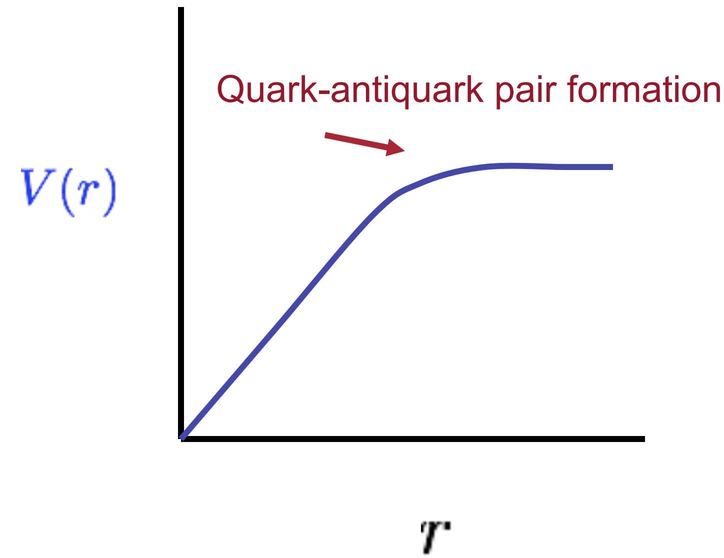
Confinement => Linear potential

Dynamical quarks => No linear potential at large distances

Can rescale fields by coupling constant so that

$$Z = e^{-\frac{1}{g^2}S[A]}$$

$$Z = \int [dA]\; e^{-\frac{1}{g^2}S[A]}$$



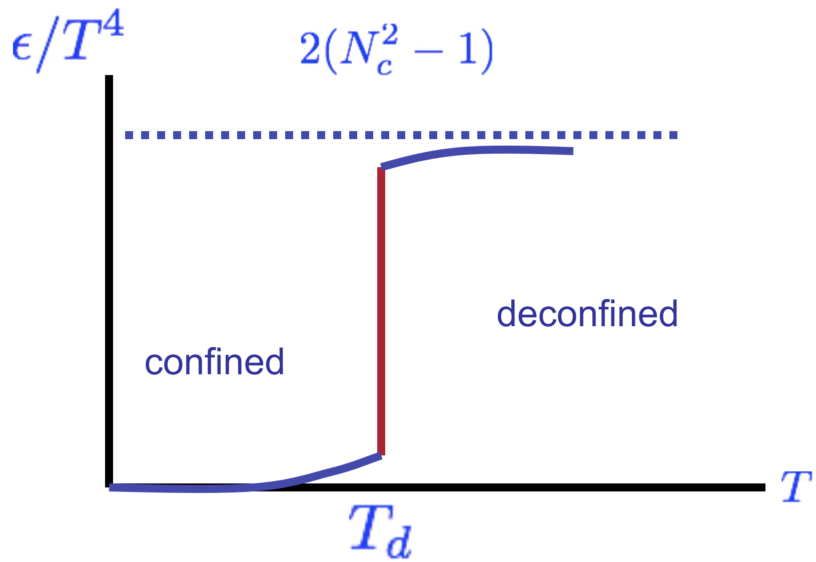
Similar to a spin system: Small g (High Temperature) => ordered => broken symmetry => deconfined

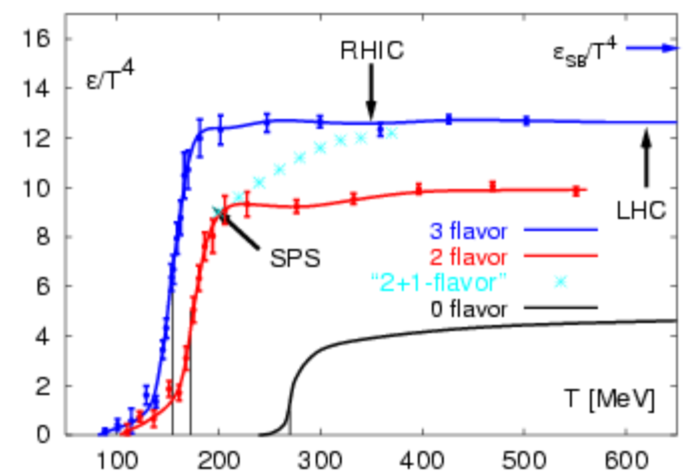
Large g (Low Temperature) => disordered => symmetric => confined

$$lim_{r \to \infty} < L(r)L(0) > \to C\; e^{-\kappa r} + < L(0) >^2$$

$$< L(0) > = 0 \quad => \quad F_{q\bar{q}}(r) = \kappa r$$

Confinement Phase Transition at Finite T





$$\epsilon_{confined}/T^4 \sim e^{-M_{glueball}}/T$$

$$\epsilon_{deconfined}/T^4 \sim 2(N_c^2 - 1)$$

$$T \sim \Lambda_{QCD}$$

$$0(1) \to N_c^2$$

Lattice Monte-Carlo:

First order phase transition for pure glue

Sharp cross over for 2-3 flavors of light quarks,

T_d ~ 170-180 MeV

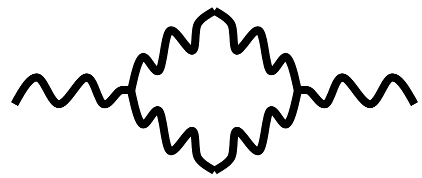
Large Numbers of Colors and De-confinement at Finite T

'‘t Hooft limit $g^2 N_c \; finite$

QCD confines at low T in large Nc limit

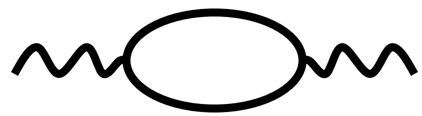
Mesons with narrow widths and small interactions

Gluon loops modify potential



$\sim g^2 N_c$

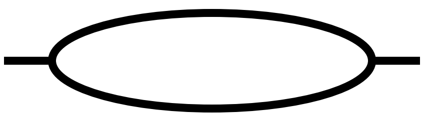
Quark loops do not



$\sim g^2$

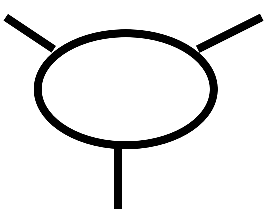
Quark Interactions:

(Nc counting not changed by interactions)



$\sim N_c \sim < 0 \mid J \mid P >< P \mid J^\dagger \mid 0 >$

Meson Interactions



$\sim N_c \sim J^3 G => G \sim 1/N_c^{1/2}$

Exercise: Show 4 meson interaction ~ 1/N_c,

3 glueball interaction ~ 1/N_c

4 gluebball interactions ~ 1/ N_c^2

The Hagedorn Spectrum:

In large Nc limit, glueballs and mesons are weakly interacting, and of vanishing width.
How might there ever be a transition to a de-confined phase?

Hagedorn: Exponential density of states:

Example:

$$Z = \int dm \; e^{(\kappa m - m/T)}$$

$$< E >= \frac{1}{1/T - \kappa}$$

Limiting temperature where the states accumulate, density becomes very high end interactions begin to become important.

This is the de-confinement temperature in large Nc

QCD: Beyond the Hagedorn temperature is a gas of quarks and gluons

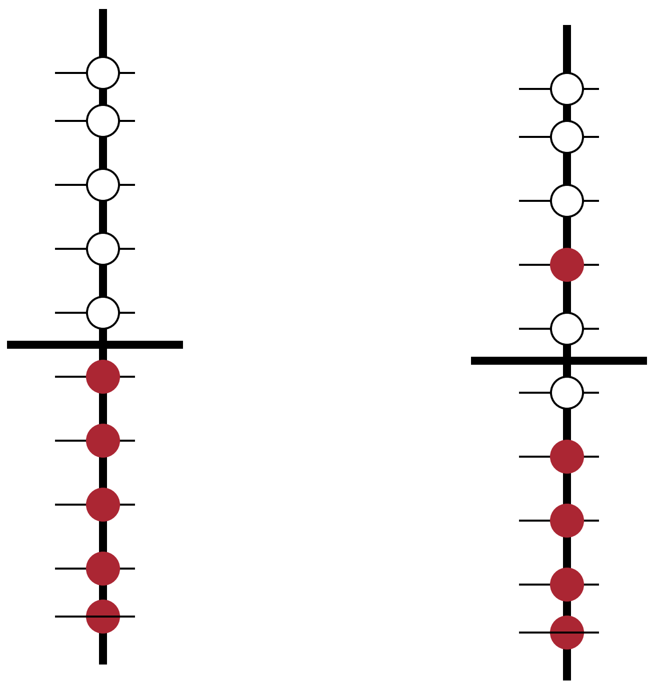
At asymptotic temperature, an almost free plasma of quarks and gluons

Mass generation and Chiral Symmetry Breaking

QCD in limit of vanishing quark masses: $U(1) \times SU_L(2) \times SU_R(2)$

In vacuum, broken by chiral condensate of scalars

Filled Dirac sea

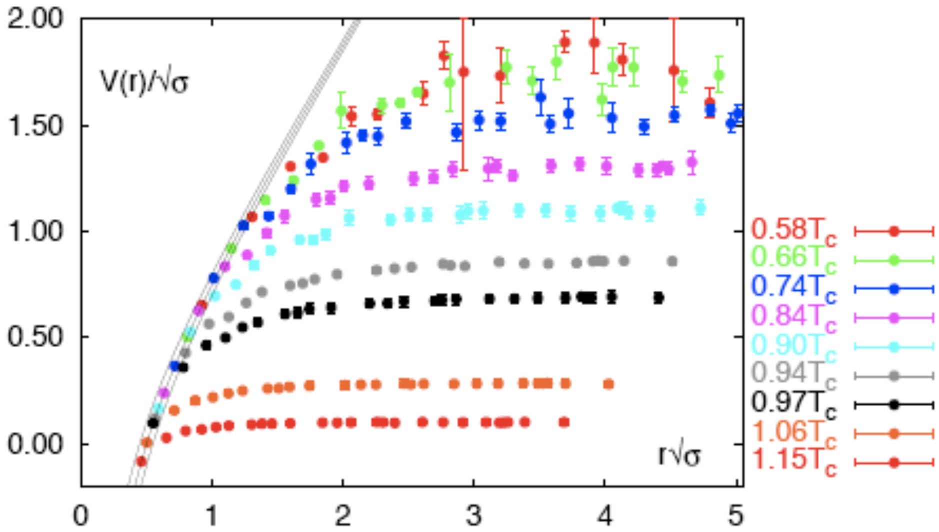


Particle-Hole excitation: Can it have lower energy than original Dirac sea?

$\bar{\psi}\tau^a\gamma_5\psi$ is made by a rotation of $\bar{\psi}\psi$

Pions are Goldstone bosons of the broken chiral symmetry

Nucleon massive and not parity doubled



Lattice-Monte Carlo:

Finite T

Nc = 3

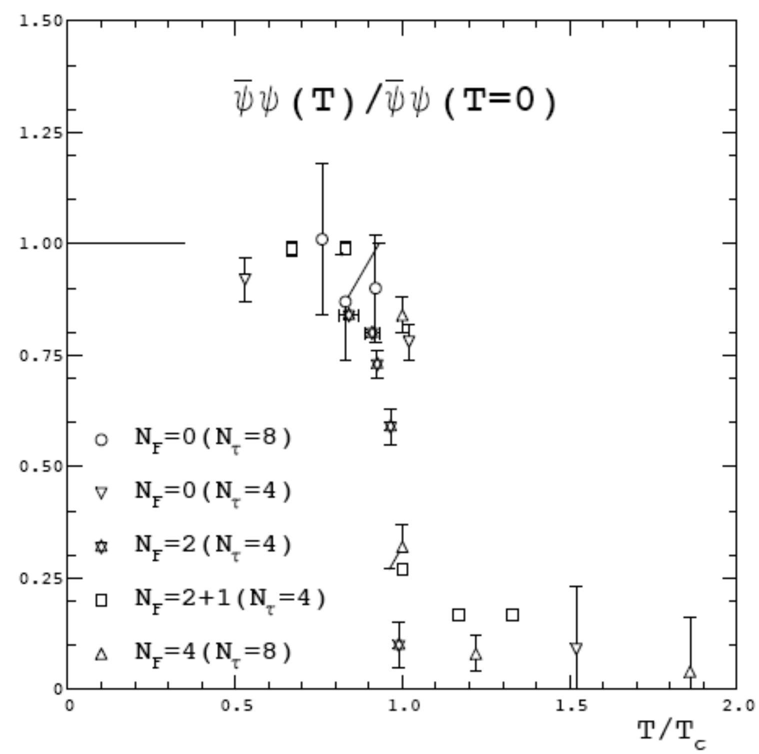
Nf = 2-3

Quark Masses?

Near Tc, linear potential disappears

At same temperature, chiral condensate melts

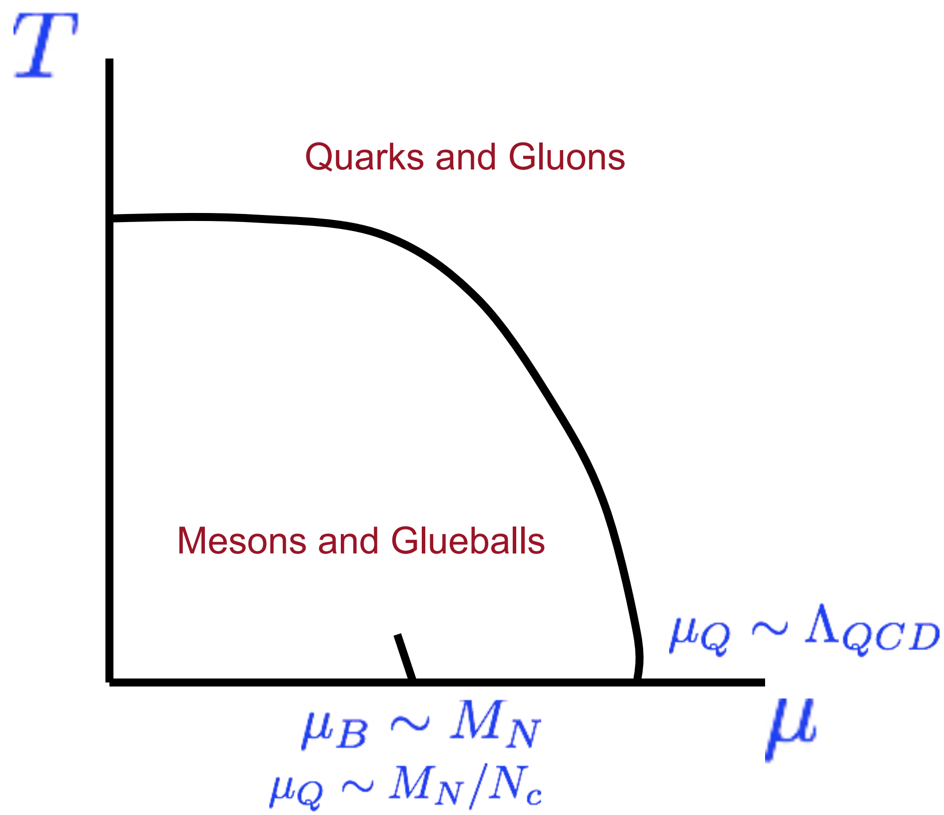
Energy density, pressure rapidly go to 70-80% of free QGP expectation



Finite Density

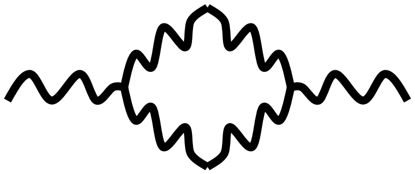
$$\delta S = i\mu \int d^4x \, \overline{\psi}\gamma^0\psi$$

Complex action makes lattice computation very complicated, except for small μ/T

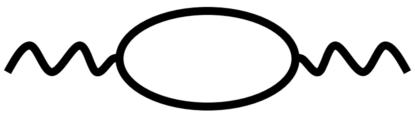


Popular Wisdom

Finite Baryon Density in the Large Nc Limit (fixed Nf)



$$\sim g^2 N_c T^2$$



$$\sim g^2 \mu_Q^2$$

For any finite baryon or quark number density, quarks cannot affect the propagation of gluons, nor the confinement potential as measured by an external source

$$\mu_Q \sim \sqrt{N_c}\Lambda_{QCD}$$

At zero T, deconfinement occurs at very high energy density

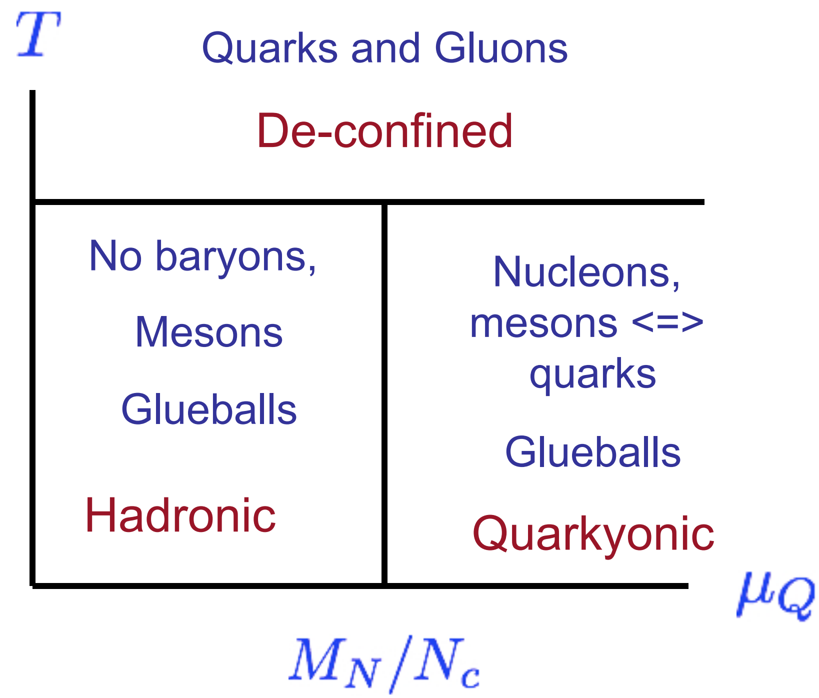
$$\epsilon \sim N_c^2 \Lambda_{QCD}^4$$

Can have a phase of very high density baryons which is confined: $\rho_B >> \Lambda_{QCD}^3$

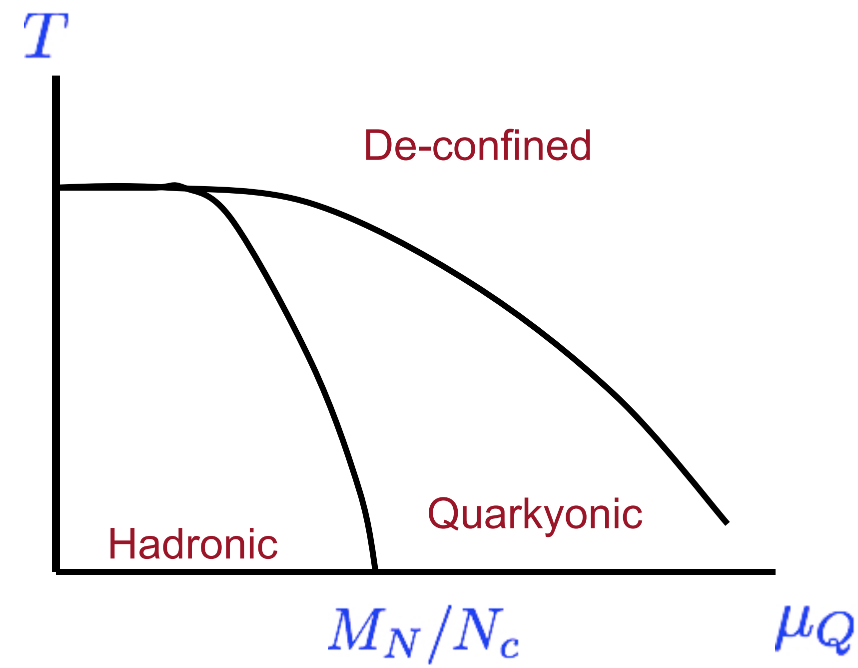
At large Nc, in the confined phase, $\rho_B \sim e^{\mu_B/T - M_B/T} \sim e^{-N_C\{M_B/N_c - \mu_Q/T\}}$

$$\mu_Q \leq M_B/T$$

Baryon density is zero. Baryon number density is an order paremeter for a phase transition



$N_c \rightarrow \infty$



$N_c \; finite$

$\epsilon \sim 0(1)$ Hadronic

$\epsilon \sim 0(N_c^2)$ De-confined

$\epsilon \sim 0(N_c)$ Quarkyonic

Quarkyonic: Confined into baryons

Continuously connected to weak coupling phase

At high baryon number density, quark bulk properties are in leading order perturbative, although quarks are confined