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FROM QUANTUM DEFORMATIONS OF RELATIVISTIC SYMMETRIES TO MODIFIED KINEMATICS AND DYNAMICS

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1. INTRODUCTION

Basic symmetries of Einstein special relativity:

a) Poincaré symmetries (Poincaré group)

$$\mathbf{x}'_{\mu} = \Lambda_{\mu}^{\nu}\mathbf{x}_{\nu} + \mathbf{a}_{\mu} \qquad \mu = 0, 1, 2, 3$$

b) Poincaré algebra $(\mathbf{P}_{\mu}, \mathbf{M}_{\mu\nu})$

Mass and spin Casimirs:

$$\mathbf{P}_{\mu}\mathbf{P}^{\mu} = -\mathbf{m}^2 \qquad \mathbf{W}_{\mu}\mathbf{W}^{\mu} = \mathbf{m}^2\mathbf{s}(\mathbf{s}+1) \qquad \mathbf{s} = 0, \frac{1}{2}, 1$$

Abelian addition law of fourmomenta

$$\mathbf{P}_{\mu}{}^{(1+2)} = \mathbf{P}_{\mu}{}^{(1)} + \mathbf{P}_{\mu}{}^{(2)}$$

→ Poincaré algebra is a Hopf algebra with Abelian (primitive) coproduct $(\Delta(\vec{g}) = \widehat{g} \otimes 1 + 1 \otimes \widehat{g})$.

General relativity:

Dynamical (pseudo)-Riemannian structure of curved space-time manifold (matter can introduce torsion).

General covariance under local transformations:

$$\mathrm{x}'_{\mu} = \mathrm{x}_{\mu} + \delta\xi_{\mu}(x)$$

Einstein action:

$$S_E = \underset{\substack{\nearrow \\ \text{Planck} \\ \text{mass}}}{\frac{1}{2\kappa^2}} \int d^4x \ \sqrt{det\, g}\ \underset{\substack{\uparrow \\ \text{scalar} \\ \text{curvature}}}{R} \qquad \mathrm{R} = \mathrm{R}[\mathrm{g}_{\mu\nu}]$$

Three fundamental constants:

$$\underset{\downarrow}{c} \ , \quad \underset{\downarrow}{\hbar} \ , \qquad \kappa = \sqrt{\frac{\hbar c}{G}} \underset{\downarrow}{\simeq} 2.2 \cdot 10^{-5} g \underset{\downarrow}{\simeq} 1.2 \cdot 10^{19} \frac{GeV}{c^2}$$

geometric **dynamical** **(gravitational coupling)**

Problem: standard quantization of Einstein gravity as field theory leads to nonrenormalizable infinities - quantum gravity does not exists!

Ways out:

1) Supersymmetrization and D>4 extensions

If added additional space coordinates (Kaluza-Klein idea) one gets

D=11 supergravity (first Theory of Everything!) (**Hawking** 1982)

2) Embedding gravity in string theory ($\Rightarrow$ M-theory)

D=10 superstring (second Theory of Everything!) (**Green, Schwarz** 1984)

$\Uparrow$

SUSY + D>4 + extended object!

3) Modification of space-time structure

The renormalization problem related with short distance behavior → quantum space-time can solve the problem?

The notion of classical space-time at Planck distances is not compatible with quantization of gravity:

- measuring the position x with accuracy Δx means adding in measurement procedure to the volume $(\Delta x)^3$ the energy $E \sim \frac{1}{\Delta x}$

- due to Einstein equations adding energy E to small volume creates black holes with radius $R \sim \lambda_p \frac{E}{\kappa}$
  Fuzzy space-time structure: $\Delta x > R$

One can derive that allways in presence of gravity $\Delta x > \lambda_p$
Planck length λ_p :

The minimal value of $\Delta x \sim \lambda_p$ (in $QM \quad \Delta x$ arbitrary!)

$$\lambda_p = \frac{\hbar}{\kappa c} = \sqrt{\frac{\hbar G}{c^3}} \approx 1.6 \cdot 10^{-33}\mathrm{cm}$$

The property that $\Delta x \geq \lambda_p$ can be described algebraically by analogy with QM (κ-Planck mass)

$$[\widehat{x}_\mu, \widehat{x}_\nu] = \frac{i}{\kappa^2}\theta_{\mu\nu} \qquad \theta_{\mu\nu} = -\theta_{\nu\mu}$$

$\Longrightarrow$ **(Dopplicher, Fredenhagen, Roberts 1995):**

$\theta_{\mu\nu} = \theta^{(0)}_{\mu\nu}$ (constant) $-$ canonical deformation of space-time

2. FROM NONCOMMUTATIVE SPACE-TIME TO QUANTUM RELATIVISTIC SYMMETRIES

General deformation of space-time

$$[\widehat{x}_\mu, \widehat{x}_\nu] = \frac{i}{\kappa^2}\theta_{\mu\nu}(\kappa\widehat{x}) =$$

$$= \frac{i}{\kappa^2}\theta^{(0)}_{\mu\nu} + \frac{i}{\kappa}\theta^{(1)\rho}_{\mu\nu}\widehat{x}_\rho + i\theta^{(2)\rho\tau}_{\mu\nu}\widehat{x}_\rho\widehat{x}_\tau + \ldots$$

$$\uparrow \qquad\qquad \uparrow \qquad\qquad \uparrow$$

DFR **Lie-algebra** **quadratic**

$\theta_{\mu\nu}$ represents in algebraic form the quantum gravity effects - in future $\theta_{\mu\nu}$ will be determined dynamically: $\theta_{\mu\nu} = \theta_{\mu\nu}(g_{\mu\nu})$

1) Canonical deformation of space-time

$$[\hat{x}_\mu, \hat{x}_\nu] = \frac{i}{\kappa^2}\,\theta^{(0)}_{\mu\nu} \qquad (a)$$

Translations $\hat{x}'_\mu = \hat{x}_\mu + a_\mu$ classical

$$[\hat{x}'_\mu, \hat{x}'_\nu] = \frac{i}{\kappa^2}\,\theta^{(0)}_{\mu\nu} \;\Rightarrow\; [a_\mu, a_\nu] = [\hat{x}_\mu, a_\nu] = 0$$

Lorentz invariance - also classical but broken by constant tensor $\theta^{(0)}_{\mu\nu}$

Quantum group approach: one can modify by twisting the coalgebra structure of classical Lorentz algebra in a way that the relation (a) as describing the representation space of twisted Poincaré algebra is covariant

$\Longrightarrow$ (Wess, Chaichian et al., 2004)

2) Lie-algebraic deformation

$$[\widehat{x}_\mu, \widehat{x}_\nu] = \frac{i}{\kappa^2}\,\theta^{(1)\,\rho}_{\mu\nu}\,\widehat{x}_\rho$$

$$\widehat{x}'_\mu = \widehat{x}_\mu + \widehat{a}_\mu \qquad \Rightarrow \qquad \begin{array}{l} [\widehat{a}_\mu, \widehat{a}_\nu] = \frac{i}{\kappa^2}\,\theta^{(1)\,\rho}_{\mu\nu}\,\widehat{a}_\rho \\ [\widehat{x}_\mu, \widehat{a}_\nu] = 0 \end{array}$$

Translations $\widehat{a}_\mu$ - noncommutative!
Hopf-algebraic formula for adding $\widehat{x}_\mu$

$$\Delta(\widehat{x}_\mu) = \widehat{x}_\mu \otimes 1 + 1 \otimes \widehat{x}_\mu \qquad \leftrightarrow \qquad \widehat{x}'_\mu = \widehat{x}_\mu + \widehat{a}_\mu$$

For particular choices of $\theta^{(1)}_{\mu\nu}$ one can extend noncommutative translations to full quantum Poincaré group $(\widehat{a}_\mu, \widehat{\Lambda}_\mu{}^\nu)$

$\Longrightarrow$ Classification: Podleś, Woronowicz 1996

The most known example of Lie-algebraic deformation: κ-deformation

$\Longrightarrow$ **(J.L., Nowicki, Ruegg, Tolstoy 1991)**

One can identify noncommutative translations with κ-Minkowski coordinates: $\widehat{x}_\mu \leftrightarrow \widehat{a}_\mu$

$$[\widehat{x}_0, \widehat{x}_i] = \frac{i}{\kappa} \widehat{x}_i \qquad [\widehat{x}_i, \widehat{x}_j] = 0 \quad i, j = 1, 2, 3$$

The κ-deformed Poincaré group (S. Zakrzewski 1994)

$$\widehat{\Lambda}_\mu^\rho \widehat{\Lambda}_\nu^\rho = \eta_{\mu\nu} \qquad [\widehat{x}_\mu, \widehat{\Lambda}_\mu^\rho] \neq 0 \qquad [\widehat{\Lambda}_\mu^\rho, \widehat{\Lambda}_\nu^\tau] = 0$$

κ-Poincaré group $\longleftrightarrow$ **duality** **κ-Poincaré algebra**

Quantum Poincaré algebras:

a) Canonical deformation $(\theta_{\mu\nu} = \theta^{(0)}_{\mu\nu})$

- algebraic sector: classical Poincaré algebra

- coalgebraic sector: nonclassical, deformed by twist

$$\Delta_\theta(\widehat{g}) = F_\theta^{-1}\, \Delta_0(\widehat{g})\, F_\theta \qquad \widehat{g} = (P_\mu, M_{\mu\nu})$$

$$\Delta_0(\widehat{g}) = \widehat{g} \otimes 1 + 1 \otimes \widehat{g} \qquad F_\theta = \exp\{\frac{i}{2\kappa^2} P_\mu \wedge\ P_\nu\}$$

Irreducible representations of Poincaré algebra are not modified, only the composition of the representations on tensor products is not classical

Very mild deformation!

κ-deformation:

- algebraic sector $(P_\mu = (P_0, P_i),\ M_{\mu\nu} = (M_i, N_i))$

$$[M_{\mu\nu}, M_{\rho\tau}] = i(\eta_{\kappa\tau} M_{\nu\rho} - \ldots) \Leftarrow \begin{array}{c}\text{classical}\\ \text{Lorentz}\\ \text{algebra}\end{array}$$

$$[M_i, P_j] = i\epsilon_{ijk} P_k \qquad [M_i, P_0] = i\, P_i$$

$$[N_i, P_j] = i\delta_{ij}\left[\tfrac{\kappa}{2}(1 - e^{-\frac{2P_0}{\kappa}}) + \tfrac{1}{2\kappa}\,\vec{p}^{\,2}\right] - \tfrac{i}{\kappa}\, P_i\, P_j$$

$$[N_i, P_0] = iP_i$$

"Bicrossproduct basis" (Majid, Ruegg 1994)

Deformed mass Casimir → deformed KG operator:

$$P_\mu P^\kappa \ \longrightarrow\ 2\kappa\left(\sinh\frac{P_0}{2\kappa}\right)^2 - \vec{p}^{\,2} \qquad \left(\begin{array}{c}\text{in standard}\\ \text{basis}\end{array}\right)$$

κ-deformation of coalgebraic sector:

$$\begin{aligned}
\Delta(M_i) &= M_i \otimes 1 + 1 \otimes M_i \\
\Delta(N_i) &= N_i \otimes e^{-\frac{P_0}{\kappa}} + 1 \otimes N_i - \frac{1}{\kappa}\epsilon_{ijk} M_j \otimes P_k \\
\Delta(P_i) &= P_i \otimes e^{-\frac{P_0}{\kappa}} + 1 \otimes P_i \\
\Delta(P_0) &= P_0 \otimes 1 + 1 \otimes P_0
\end{aligned}$$

Modified addition of three-momenta and boosts, e.g.

$$P_i^{(1+2)} = P_i^{(1)} e^{-\frac{P_0^{(2)}}{\kappa}} + P_i^{(2)} \qquad (a)$$

Consequence of (a) → modification of bosonic and fermionic statistics

(Daszkiewicz, J.L., Woronowicz 2007)

κ-deformed field oscillators ↔ **momentum - dependent statistics** → required by nonAbelian addition law of momenta.

Inconsistency of standard bosonic commutativity:

$$a(\vec{p})a(\vec{q}) = a(\vec{q})a(\vec{p}) \longrightarrow \vec{p}e^{-\frac{q_0}{\kappa}} + \vec{q} \neq \vec{q}e^{-\frac{p_0}{\kappa}} + \vec{p}$$

κ-modification of multiplication:

$$a(\vec{p}) \cdot a(\vec{q}) \Rightarrow a(\vec{p}\,e^{-\frac{q_0}{2\kappa}}) \cdot a(\vec{q}\,e^{\frac{p_0}{2\kappa}}) \equiv a(\vec{p}) \circ a(\vec{q})$$

$$a(\vec{p}) \circ a(\vec{q}) = a(\vec{q}) \circ a(\vec{p}) \rightarrow \vec{p} + \vec{q} = \vec{q} + \vec{p} \quad ! \qquad (A)$$

We obtain **κ-deformed oscillator algebra**:

$$[a^{+}(\vec{p}), a(\vec{q})] = 2\omega\delta^3(\vec{p} - \vec{p}\,') \rightarrow [a^{+}(\vec{p}), a(\vec{q})]_{\circ} = 2\Omega_{\kappa}(\vec{p})\delta^3(\vec{p} - \vec{p}\,')$$

$\Omega_{\kappa}(\vec{p})$ - energy calculated from κ-deformed mass shell

Exchanging 1 ↔ 2 particle leads to the **modification of momentum dependence**:

$$a(\vec{p}) \cdot a(\vec{q}) = a(\vec{q}\,e^{-\frac{p_0}{\kappa}})a(\vec{p}\,e^{\frac{q_0}{\kappa}}) \qquad \sim (A)$$

3. DEFORMED KINEMATICS

a) Canonical deformations ($\theta_{\mu\nu} = \theta^{(0)}_{\mu\nu}$)

- Kinematics for single relativistic particles not modified, in particular classical energy-momentum relation $E = (\vec{p}^2 + m^2)^{\frac{1}{2}}$
- Two approaches to symmetries

  i) Classical Poincaré symmetries valid but broken by constant tensor $\theta^{(0)}_{\mu\nu}$

$$O(3,1) \longrightarrow O(2) \otimes O(1,1)$$

  ii) Quantum Poincaré symmetries with modified coproduct $\longrightarrow$ boosts for one-particle states adds in non-trivial way.

  These two approaches are further used as well in deformed gravity theory (Calmet, Aschieri, Wess).

b) κ-deformation

- Because the mass Casimir is deformed, there is modified energy-momentum relation, which depends on the chosen basis of the κ-deformed Poincaré algebra:

$$\vec{p}^{\,2} - \frac{E^2}{c^2} = -m_0^2 \longrightarrow \vec{p}^{\,2} - \left(2\kappa\, c \sinh \frac{E}{2\kappa c}\right)^2 = -m_0^2 \qquad \begin{pmatrix} \text{standard} \\ \text{basis} \end{pmatrix}$$

$$E = 2\kappa\, c \,\mathrm{arcsinh} \frac{(\vec{p}^{\,2} + m_0^2)^{\frac{1}{2}}}{2\kappa\, c} = (\vec{p}^{\,2} + m_0^2)^{\frac{1}{2}} + O(\frac{1}{\kappa^2})$$

Asymptotic behaviour:

$$E = \ln|\vec{p}| \qquad \text{in} \qquad |\vec{p}| \longrightarrow \infty \text{ limit}$$

Study of various consequences of deformed energy-momentum relation: Doubly Special Relativity (DSR) (Amelino-Camelia, Kowalski-Glikman...)

Change of ”mass-shell condition” leads to three important consequences:

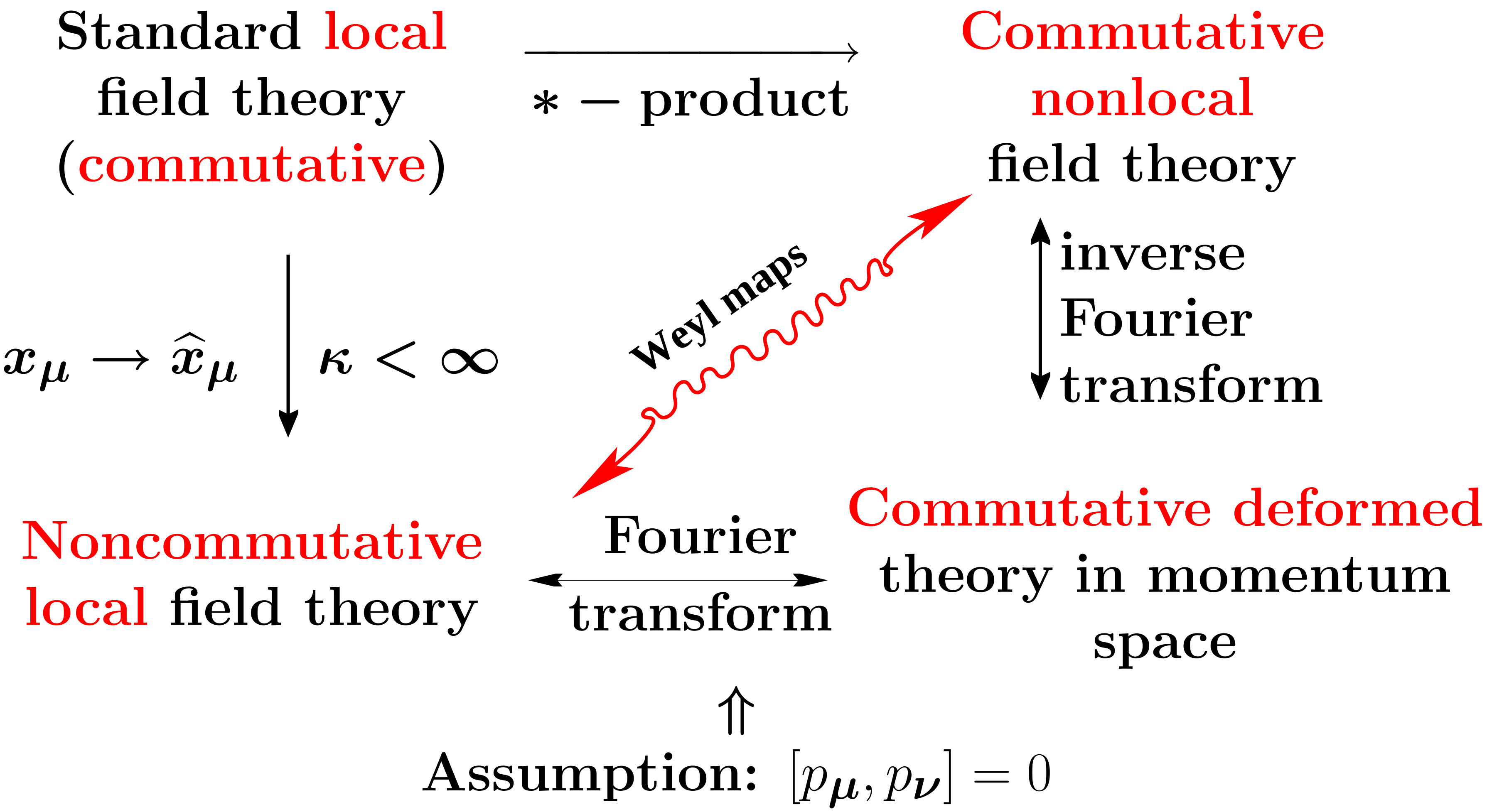
i) The notion of light-cone is modified - simpler to understand in momentum space, in space-time not clear

ii) The velocity of massless particles (photons) approaches c only if $|\vec{p}| \rightarrow \infty$ - in given basis one gets universal ”velocity curve” $v(|\vec{p}|)$

iii) Astrophysical effect: κ-deformed kinematics of absorption processes leads to modification of GKZ threshold

However experimentally there are not observed violations of Einstein kinematics. Theoretical corrections are of order $\sim \frac{1}{\kappa^2}$, at present beyond observable limits. (Domokos 1994)

4. DEFORMED (NONCOMMUTATIVE) FIELD THEORY



Standard local field theory (commutative)

$\xrightarrow[\ast - \text{product}]{}$

Commutative nonlocal field theory

$x_\mu \to \widehat{x}_\mu$ $\kappa < \infty$

Weyl maps

inverse Fourier transform

Noncommutative local field theory

Fourier transform

Commutative deformed theory in momentum space

⇑

Assumption: $[p_{\boldsymbol{\mu}}, p_{\boldsymbol{\nu}}] = 0$

Star product: homomorphic mapping of noncommutative fields into classical fields

$$\varphi(\widehat{x}) \cdot \chi(\widehat{x}) \quad \xrightarrow[\substack{\text{realization} \\ \text{of NC fields algebra}}]{\qquad\qquad} \quad \varphi(x) \star \chi(x)$$

a) Canonical deformation ⇒ Moyal star product

Product of noncommutative plane waves

$$e^{ip_\mu \widehat{x}^\mu} \cdot e^{iq_\mu \widehat{x}^\mu} \longrightarrow e^{ip_\mu x^\mu}\, e^{iq_\mu x^\mu}\, e^{\frac{i}{2} p^\mu \theta^{(0)}_{\mu\nu} q^\nu}$$

induces the definition of Moyal star product

$$\varphi(x) \star_\theta \chi(x) \doteq \varphi(x) \exp\left\{\frac{i}{2} \overleftarrow{\partial}^{\,\mu} \theta^{(0)}_{\mu\nu} \overrightarrow{\partial}^{\,\nu}\right\} \chi(x)$$

Canonical deformation - very mild:

- **no modification of mass-shell condition - the same free field equations**

- **no modification of Abelian addition law for three-momenta**

- **the phase factor $\exp \frac{i}{2} p\, \theta\, q$ enters into momentum space vertices in Feynmann diagrams (Filk 1995)**

- **definition of θ-deformed bosonic and fermionic statistics (Abe; Balachandran 2006)**

$$a(\vec{p}) \circ a(\vec{q}) = e^{\frac{i}{2}\theta^{(0)}_{\mu\nu} p^{\mu} q^{\nu}} \cdot a(\vec{p}) \cdot a(\vec{q})$$

Every known field theory (e.g. gauge theories - QED, QCD; gravity) has been canonically deformed

b) κ-deformation ⇒ BCH star product (Birkhoff, Campbell, Hausdorf)

Product of noncommutative plane waves:

$$e^{i\,p_\mu\,\widehat{x}^\mu}\,e^{i\,q_\mu\,\widehat{x}^\mu}\longrightarrow e^{i\,p_\mu\,x^\mu}\,e^{i\,q_\mu\,x^\mu}\,e^{\frac{i}{2}x_\rho\,p^\mu\,\theta^{(1)\,\rho}_{\mu\nu}q^\nu}$$

More complicated deformation:

- Free field equations modified (modified Casimirs of Poincaré algebra)
- The numerical phase factor in momentum space vertices (Feynmann diagrams) becomes differential operator in momentum space

$$e^{\frac{i}{2}\,p^\mu\,\theta^{(1)\,\rho}_{\mu\nu}\,q^\nu\,\frac{\partial}{\partial p^\rho}}$$

- The conservation of fourmomenta modified
- The bosonic and fermionic statistics modified

(Daszkiewicz, J.L., Woronowicz 2007)

Question: can one formulate perturbative κ-deformed field theory?

For that purpose needed κ-deformed c-number free propagator. Indeed if we introduce κ-deformed $\star$-product for quantized fields $\widehat{\phi}$

$$\widehat{\phi}_\kappa(\widehat{x})\widehat{\phi}_\kappa(\widehat{y}) \Rightarrow \widehat{\phi}_\kappa(x) \star \widehat{\phi}_\kappa(y) \quad \Leftarrow \quad \begin{array}{c}\kappa\text{-deformed}\\ \text{oscillators } \widehat{\phi}_\kappa(\widehat{x})\end{array}$$

one can show that (Daszkiewicz, J.L., Woronowicz 2008)

$$\begin{aligned}[\widehat{\phi}_\kappa(\widehat{x}), \widehat{\phi}_\kappa(\widehat{y})]_\star &= i\Delta_\kappa(x - x') = \\ &= \frac{i}{(2\pi)^3}\int \frac{d^3p}{2\Omega_\kappa} \sin\frac{\omega_\kappa t}{2\kappa} e^{i\vec{p}\vec{x}}\end{aligned}$$

Interesting equivalence of noncommutative and braided fields (for canonical deformation - Oeckl (2001))

$$\underset{\text{noncommutative}}{\phi_\kappa(x) \star \phi_\kappa(y)} = \underset{\text{braided}}{\phi_\kappa(x) \circ \phi_\kappa(y)}$$

Valid also for quantized field $\widehat{\phi}_\kappa(\widehat{x})$.

If we introduce braided vertices

$$\lambda\,\phi^4(\widehat{x}) \longrightarrow \lambda\widehat{\phi}(x)\circ\widehat{\phi}(x)\circ\widehat{\phi}(x)\circ\widehat{\phi}(x)$$

one obtains at every vertex classical conservation law of fourmomenta. One can define consistently the braided products "$\circ$" of n oscillators for any n and by using κ-deformed Wick theorem one can calculate the Feynmann diagrams.

Conjecture (J.L., Woronowicz; in preparation)
κ-deformed quantum $\lambda\phi^4$ theory formulated as braided field theory has the same perturbative expansion as "standard" $\lambda\phi^4$ theory, only with κ-deformed propagators $((p^2 - m^2) \rightarrow (C_2^\kappa(\vec{p}) - m^2))$.

(For canonical deformation analogous result:
Fiore, Wess (2007))

5. MODIFICATION OF EINSTEIN GRAVITY

(Chamseddine 2001, Wess et al. 2004, Aschieri et al. 2006)
Mostly canonical deformation, but also obtained for Lie-algebraic deformation (Banerjee 2008)

After introducing $\star$-multiplication one gets

- **Differential calculus → $\star$-deformed differential calculus**
- **Diffeomorphisms → $\star$-deformed diffeomorphisms**

$$\delta_\xi = \xi^\mu \partial_\mu \qquad [\delta_\xi, \delta_\eta] = \delta_{\xi \times \eta} \qquad \text{undeformed}$$

- **Deformed composition of diffeomorphisms**

$$\underset{\substack{\Uparrow \\ \text{standard} \\ \text{Leibnitz rule}}}{\Delta \delta_\xi} = \delta_\xi \otimes 1 + 1 + \delta_\xi \xrightarrow[\text{deformation}]{} \underset{\substack{\Uparrow \\ \text{modified} \\ \text{Leibnitz rule}}}{\Delta_\theta \delta_\xi} = \delta_\xi^{(1)} \otimes \delta_\xi^{(2)}$$

Gravity framework:

metric: $$g_{\mu\nu} = e^a_\mu\, e_{\nu a} \to g^\theta_{\mu\nu} = \frac{1}{2}(e^a_\mu \star e_{\nu a} + \mu \Leftrightarrow \nu)$$

Deformed Einstein action:

$$S^\theta_E = \frac{1}{2\kappa^2} \int d^4x (\det \star\, e^\mu{}_a) \star R^\theta$$

where one can expand

$$R^{(\theta)} = R + \theta^{\alpha\beta}\, R^{(1)}_{\alpha\beta} + \frac{1}{2}\theta^{\alpha\beta}\, \theta^{\gamma\delta} R^{(2)}_{\alpha\beta\gamma\delta} + \ldots$$

Important result:

$$R^{(1)}_{\alpha\beta} = 0 \quad \Longrightarrow$$ only second order corrections nonvanishing!

Conjecture: it is also valid for κ-deformation!

The modified Einstein action in commutative space-time:

- nonlocal

- invariant under deformed diffeomorphisms parametrized by four functions ξ_μ as in standard gravity

- one can stay with standard Leibnitz rule, but in such a way the invariance is only with respect to subclass of general coordinate transformations (Calmet 2006)

$$\xi_\mu = \theta_{\mu\nu}\,\partial^\nu\,\phi \quad \Leftarrow$$

deformed
unimodular gravity

- there were described $\star$-deformations of many Einstein solutions (Schwarzschild solutions, Rindler spaces etc.)

6. FINAL REMARKS

Noncommutative geometry in quantum gravity appears to be necessary for short distances ($\sim \lambda_p$)

continuous space-time $\xrightarrow[\text{quantization}]{}$ discrete cell structure (lattice, foam) of size λ_p

In astrophysics there are Planck windows in which Planck length effects can be observable (e.g. GZK threshold). In cosmology:

- **Big Bang singularity is modified**
- **description of inflation period in the evolution of Universe modified**

Interesting - compare "noncommutative corrections" with "string corrections" (Alvarez-Gaume et al. 2006)