Relation between External Fields and External Accelerations in QED

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Work in collaboration with J. Rafelski

Simple observation:

- QED vacuum in presence of a constant external electric field is an **accelerated state**
- must lead to Unruh-like phenomenon and vacuum structure

$\rightarrow$ specifically, we expect an effective **temperature** parameter characterizing accelerated state

Outline:

- Review of Constant External Fields: Euler-Heisenberg Effective Action
- Temperature representation of EH
- Acceleration/Unruh-Hawking Radiation
- Searching for Consistent Physics

Particle States in the External Field

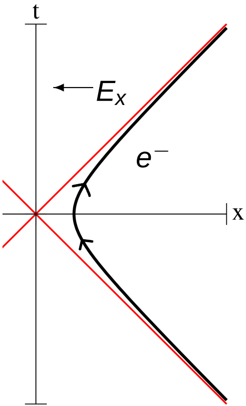
Constant homogeneous electric field:

◇ constant acceleration: $a = \frac{eE}{m}$

◇ classically hyperbolic trajectory

Klein-Gordon equation has exact solution for a linear potential $A_{\rm ext} = tE\hat{x}$:

$$[(\partial + ieA_{\rm ext})^2 + m^2]\phi = 0$$



Parabolic cylinder functions: Single particle states identified by asymptotic behavior $\phi(x) \sim e^{\pm ip\cdot x}$ as $t \to \pm\infty$

Accelerated QED Vacuum

Vacuum properties contained in generating functional

$$Z[A_{\rm ext}] = \langle {\rm vac}; A_{\rm ext} | {\rm vac}; A_{\rm ext} \rangle = \det(G[A_{\rm ext}])^{-1} = \exp\left({\rm Tr}\ \ln G[A_{\rm ext}]^{-1}\right)$$

Explicitly assume coherence retained:
no decay of field and *no detector* $\Rightarrow$ use Feynman propagator

$$G_F(x', x) = \Theta(t' - t) G^+(x', x) + \Theta(t - t') G^-(x', x)$$

Effective potential $V_{\rm eff} = i \ln Z[A_{\rm ext}]$,
for existence of electric fields in (charged-scalar) vacuum:

$$V_{\rm eff} = i {\rm Tr}\ \ln G_F^{-1} = \frac{1}{16\pi^2} \int_0^\infty \frac{ds}{s^{3+\delta}} \left(\frac{eEs}{\sin eEs} - 1 \right) e^{-m^2 s},$$

[Euler, Heisenberg, Weisskopf, Schwinger]

Temperature Representation

Using identity $\frac{x}{\sin x} = 1 + \frac{x^2}{6} + \sum_{k=1} \frac{(-1)^k}{k^2\pi^2} \frac{2x^4}{x^2 - k^2\pi^2}$

gives a “statistical” form of the effective potential

$$V_{\mathrm{eff}} = -\frac{m^3}{8\pi^2} \int_0^\infty \frac{f(\omega/m)d\omega}{e^{\omega/T}+1}, \quad T_{\mathrm{EH}} = \frac{eE}{\pi m} = \frac{a}{\pi}$$

Spectral function: $f(x) = x \ln(x^2-1+i\epsilon) - \ln\left(\frac{x+1-i\epsilon}{x-1+i\epsilon}\right) - 2x$

Alternating sign of pole source of fermi statistics:

$$\sum_{k=1} (-1)^k e^{-kx} = \frac{-1}{e^x + 1}$$

For fermions, $G_F(x', x) = \langle : \bar{\psi}(x')\psi(x) : \rangle$

$$V_{\rm eff} = \frac{-1}{8\pi^2}\int_0^\infty \frac{ds}{s^{3+\delta}}\left(\frac{eEs}{\sin eEs}\cos eEs - 1\right)e^{-m^2 s}$$

cos factor removes alternating sign:

$$\frac{x\cos x}{\sin x} = 1 - \frac{x^2}{3} + \sum_{k=1}\frac{1}{k^2\pi^2}\frac{2x^4}{x^2 - k^2\pi^2}$$

$\rightarrow$ **bosonic** statistics with Same Spectral function!

$$V_{\rm eff} = -\frac{m^3}{4\pi^2}\int_0^\infty \frac{f(\omega/m)d\omega}{e^{\omega/T}-1}, \quad T_{\rm EH} = \frac{eE}{\pi m} = \frac{a}{\pi}$$

[Mueller/Greiner/Rafelski 77]

Compare to Unruh Radiation

Detector under Constant acceleration a, [Hawking 75,Unruh 76]
hyperbolic motion: $x = a^{-1}\cosh(a\tau), \quad t = a^{-1}\sinh(a\tau)$

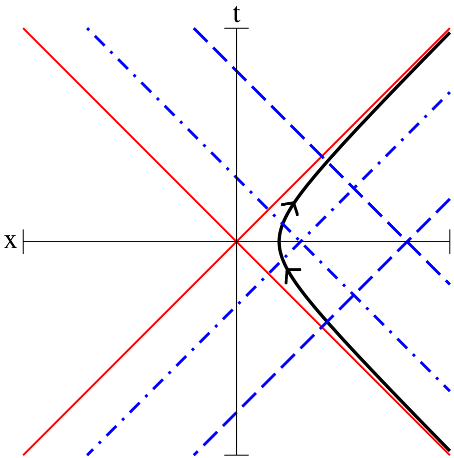
Variety of quantization models and physics pictures...

In scalar vacuum,
$\rightarrow$ thermal distribution of bosons

$$n(\omega) = \frac{1}{2\pi}\frac{1}{e^{\omega/T}-1}, \quad T_{\mathrm{HU}} = \frac{a}{2\pi}$$

Analogous result found in fermion vacuum:
$e^{\omega/T}+1$ in denominator

[see e.g. Crispini 08]



What We've Seen So Far

Acceleration Radiation	Constant Electric Field
detector accelerated against flat space vacuum	electron states under constant acceleration $a = eE/m$
detector response function $\rightarrow$ thermal excitation spectrum $T_{\rm HU} = \frac{a}{2\pi}$	sum negative energy states $\rightarrow$ effective potential $T_{\rm EH} = \frac{a}{\pi}$
statistics match (boson $\mapsto$ boson) (fermion $\mapsto$ fermion)	statistics inversion (boson $\mapsto$ fermion) (fermion $\mapsto$ boson)

- factor 2 disagreement: $T_{\rm EH} = 2T_{\rm HU}$
- statistics inconsistency

Why the apparent disagreement in physical picture?

Closer Look at Heisenberg & Euler 1936

In Fermi sea picture:

Gap width: $\Delta = 2m$

$$\frac{\text{Slope of Potential}}{\text{Scale of Wavefunction}} = \frac{eE}{m} = a$$

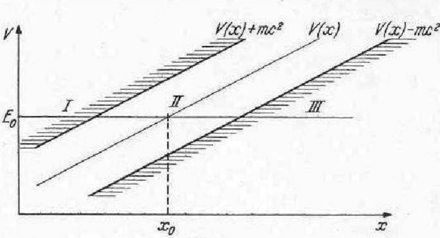


Fig. 1.

Dimensionless parameter for tunneling is $\dfrac{\Delta}{a} = \dfrac{2m^2}{eE} = \dfrac{2}{T_{EH}}\dfrac{m}{\pi}$

the state of lowest energy. In an intuitive wave theory based on equations (4) and (6), the state of lowest energy is given when all electron states of negative energy are occupied and all states of positive energy are empty. In the presence of only a magnetic field, the stationary states of an electron can be divided into those of negative and positive energy. Hence the state of the lowest energy of the matter field can be derived in the same way as for a field-free space.

$$U = \frac{1}{2}\sum_{0}^{\infty}{}^{(n)}\sum_{-1}^{+1}{}^{(\sigma)}\int_{-\infty}^{+\infty}\frac{\mathrm{d}\,p_z}{h}\int\frac{\mathrm{d}\,E}{h\,c}\,(E - e\,|\mathfrak{E}|\,x)\,u_n^2(y)\,e^{-\frac{k^2\pi}{2}} \left[\begin{matrix}|f_1^1|^2 + |g_1^1|^2 - |f_1^2|^2 - |g_1^2|^2\\ + |f_2^1|^2 + |g_2^1|^2 - |f_2^2|^2 - |g_2^2|^2\end{matrix}\right] e^{-\alpha\left(\xi^2 - \frac{1}{a}\right)}, \quad (31)$$

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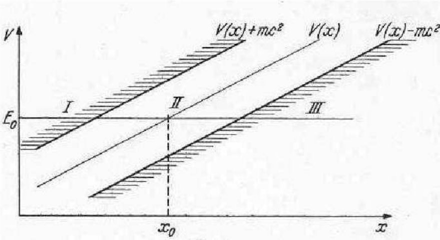

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EH note field scale m^2/e, why do they not *find* $2m^2/e$ after calculation?

They use charge symmetry, but not symmetric states

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$$\frac{1}{2}\sum_{0}^{\infty}{}^{(n)}$$

Statistical Physics with Symmetries

[Rafelski/Danos 1980, Redlich/Turko 1980, Turko 1981]

Considering $V_{\mathrm{eff}} = i\mathrm{Tr}\ln G_F^{-1} \sim \sum_k \ln|\phi_k\rangle\langle\phi_k|$, EH-S calculate

$$V = \sum_{n<0}\langle\phi_n|H|\phi_n\rangle = -\sum_{n>0}\langle\phi_n|H|\phi_n\rangle = \frac{1}{2}\sum_{n<0}\langle\phi_n|H|\phi_n\rangle - \frac{1}{2}\sum_{n>0}\langle\phi_n|H|\phi_n\rangle$$

But these sums are independent!

For real particles, the partition function must consist only of **symmetric** states

$$Z = \sum_{\text{states } k} e^{-\beta H_k} \mapsto \sum_{Q=0 \text{ states}} e^{-\beta H_k}$$

Especially applies where particle production is possible: must have particle-anti-particle symmetry

$$\mathcal{Z} = \sum_{n \text{ particles}} \lambda^n Z^n \mapsto \sum_{n \text{ pairs}} \lambda^{2n} Z^{2n}$$

or captured by $e^{-m/T} \mapsto e^{-2m/T}$

Requiring Charge Symmetry

Project the trace onto the subspace of charge symmetric states:

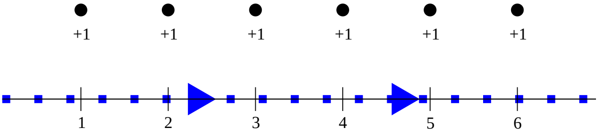
$$V_{\text{eff}} = i\mathrm{Tr}\,\ln G_F^{-1} \sim \sum_k \ln|\phi_k\rangle\langle\phi_k| \to \mathbb{P}_{(Q=0)} \sum_k \ln|\phi_k\rangle\langle\phi_k|$$

** If EHS calculation is already correct, projection is trivial **

Calculation shows that $\mathbb{P}$ amounts to modifying poles, but working loosely now, recall poles were summed as $\sum_{k=1} e^{-k/T} = \frac{1}{e^{1/T}-1}$

Without projection, pole structure is:

$$\frac{1}{x^2-k^2}, \quad k = 1, 2, \ldots$$



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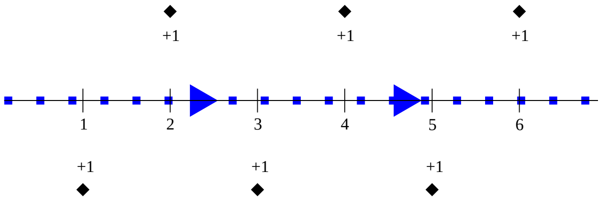
But if $k \mapsto 2k$,
then $T \mapsto T/2$
...and sum becomes

$$(e^{2/T} - 1)^{-1}$$



Consequences of Changed Potential

▶ Does **NOT** change β-function ◀

which depends only on scaling of δ-function at origin
(controlled by $s \to 0$ behavior in proper time regularization)

▶ **Does** affect non-perturbative phenomena ◀

Such as:

Pair production rate: $\Gamma = \frac{(eE)^2}{4\pi^3} e^{-\frac{2\pi m}{eE}} = \frac{(mT)^2}{\pi} e^{-m/T}, \quad T = \frac{eE}{2\pi m}$

and total vacuum persistance: $w = \frac{(mT)^2}{\pi} \sum_{k=1} \frac{1}{k^2} e^{-km/T}$

Summary

- The QED vacuum in external fields is an **accelerated** state.
- The Euler-Heisenberg effective action for the external field has a "statistical" representation in which the electron acceleration $a = eE/m$ appears as the temperature $T_{\mathrm{EH}} = a/\pi$ of an ensemble of the "wrong" particles.
- The Euler-Heisenberg and Unruh temperatures differ:

$$T_{\mathrm{EH}} = 2T_{\mathrm{HU}}$$

- T_{EH} may be made consistent with T_{HU} by modification of the pole structure of the action, specifically as may arise from enforcing charge symmetry on the quasi-particle fluctuations in the external field.