Semnan University

Variable flavor number parton distributions at next-to-leading order of QCD based on the Jacobi polynomials

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Outline

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- Parton distributions and hard processes
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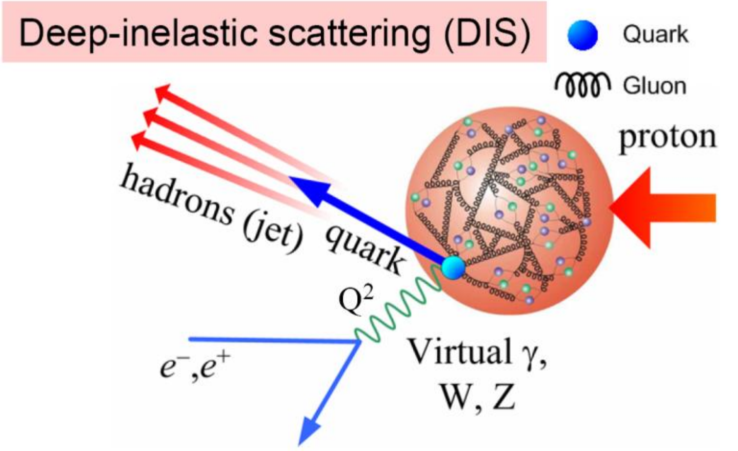
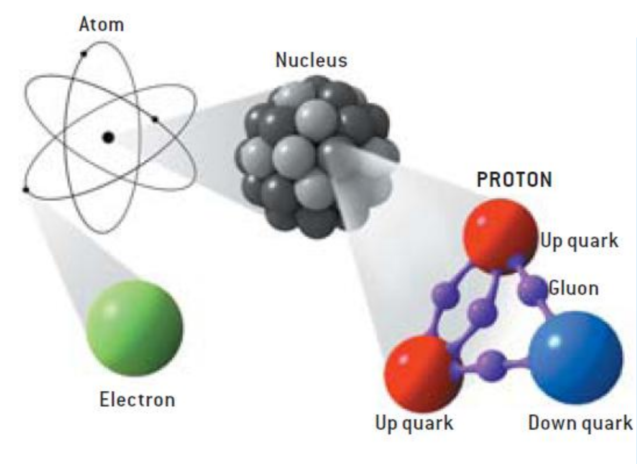
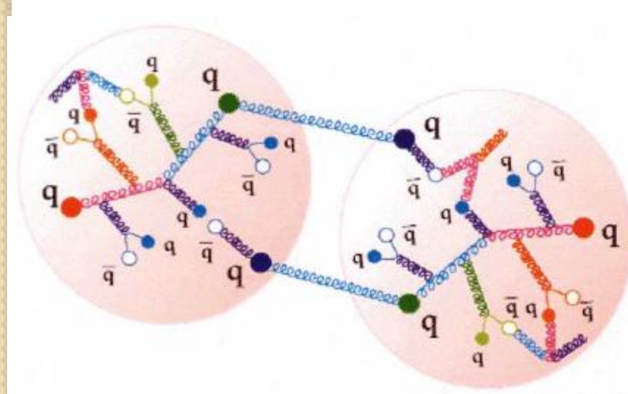
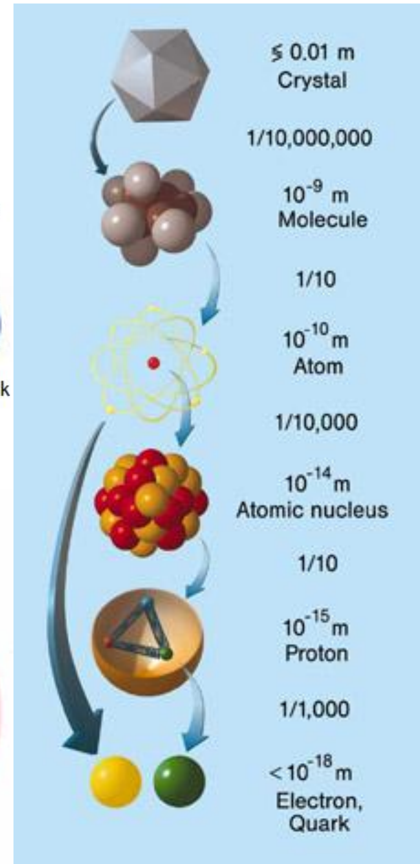
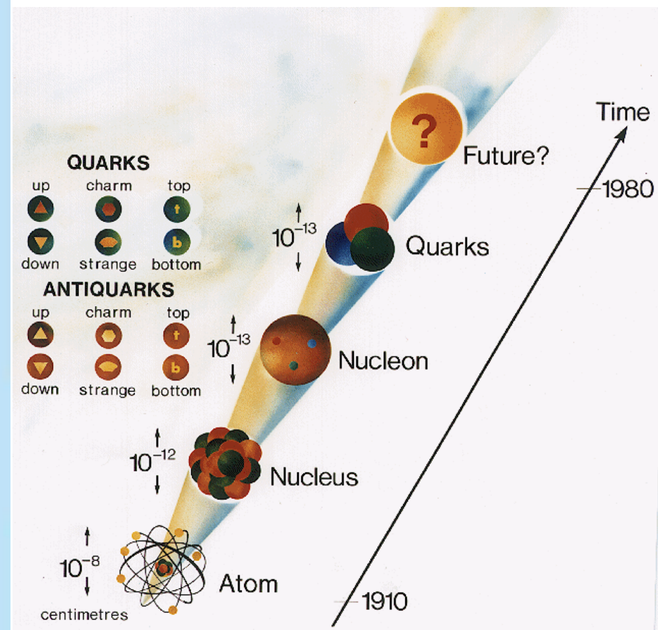


What is the matter made of?



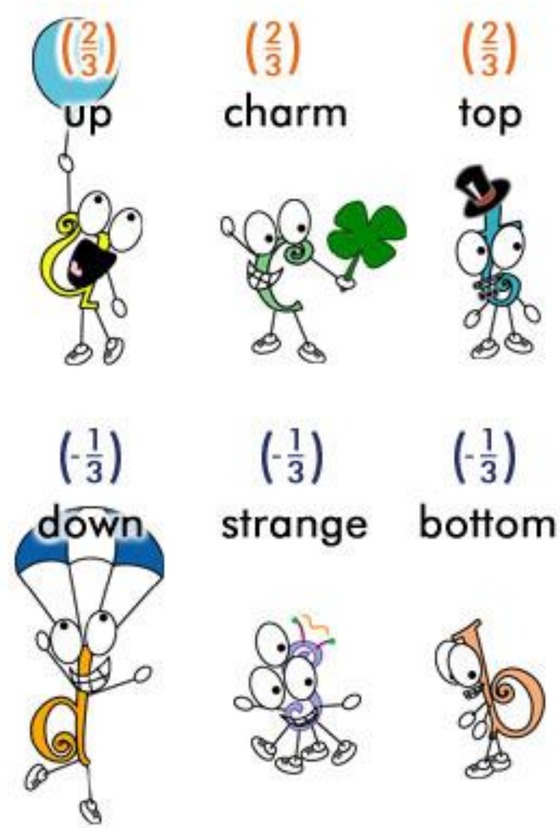


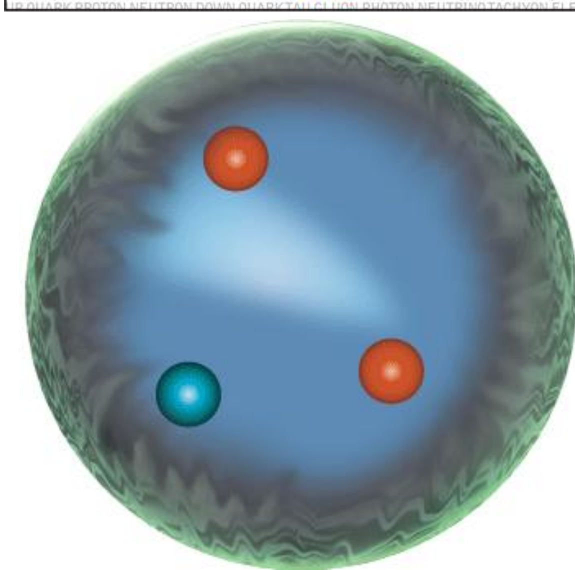




Deep inelastic scattering has revealed the most basic grains of matter, the quarks.

Proton !!?





A proton with two valence up quarks (red) and one valence down quark (green), with virtual quark-antiquark pairs (pink) being continuously created and annihilated.

Introduction

- To predict the rates of the various processes a set of universal parton distribution functions (PDFs) is required. These distributions are best determined by global fits to all the available DIS and related hard-scattering data.
- Precision phenomenology at high-energy colliders such as the LHC requires an accurate knowledge of the distribution functions of partons in hadrons.
- In this work the determination of PDFs is studied with the help of the method of the structure function reconstruction over their Mellin moments, which is based on the expansion of the structure function in terms of Jacobi polynomials as a fast and flexible method.
- The benefit of this approach is the possibility to determine parton distributions and structure function analytically and directly in x space.

Why Heavy Quarks?

- The correct treatment of heavy flavors in an analysis of parton distributions is essential for precision measurements at hadron colliders.

- Heavy quarks production is an important testing ground for quantum chromodynamics (QCD), because QCD calculations are expected to be reliable if a hard scale is present in the process. In heavy quarks production a hard scale is provided by the quark mass.

- Understanding the physics of heavy quarks gives physicists the unique opportunity to test the predictions of Quantum chromodynamics and the Standard Model. Heavy Quark Physics provides an exciting introduction to this new area of high energy physics.

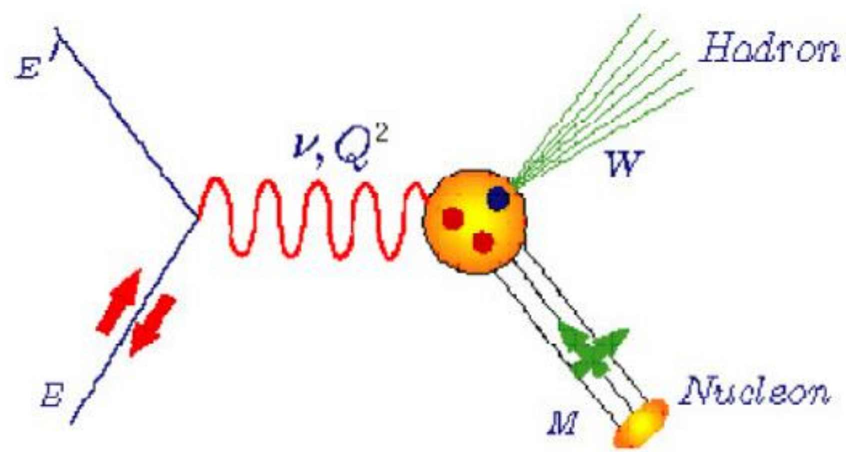
Deep Inelastic Scattering

The neutral current $e^{\pm}p \to e^{\pm}X$ unpolarised cross section is:

$$\frac{d^2\sigma_{\rm NC}(e^{\pm}p)}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[Y_+ F_2(x,Q^2) - y^2 F_L(x,Q^2) \mp Y_- xF_3(x,Q^2)\right]$$

Structure functions in deep-inelastic scattering and their scale evolution are closely related to the origins of quantum chromodynamics. DIS processes have played and still play a very important role for our understanding of QCD and nucleon structure.

Precision predictions of PDFs also are very essential for all measurements at hadron colliders.



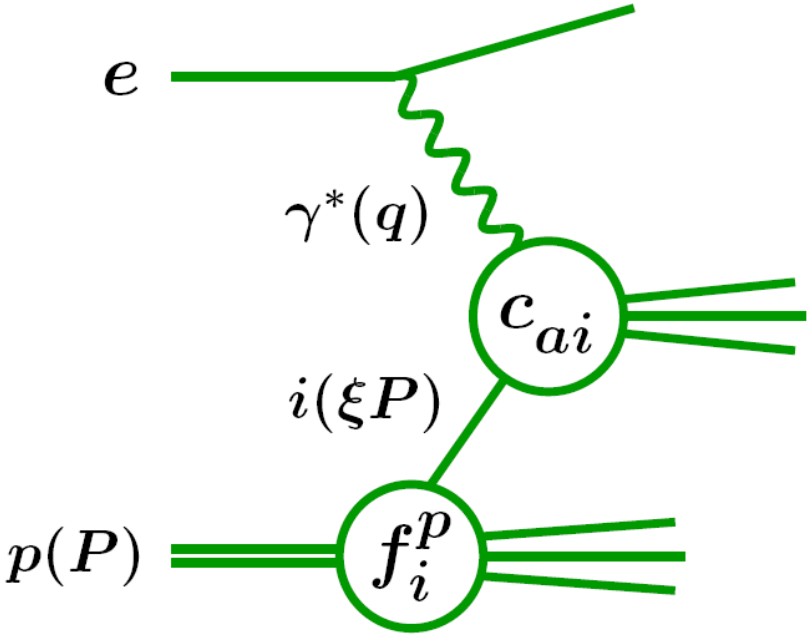
Heavy Quark Cross Section

$$\frac{d^2\sigma^{c\bar{c}}}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[(1+(1-y)^2)F_2^{c\bar{c}}(x,Q^2) - y^2 F_L^{c\bar{c}}(x,Q^2)\right]$$

For many hard processes at high energies, heavy flavor production forms a significant part of the scattering cross section. Therefore, in all precision measurements, a detailed treatment of the heavy flavor contributions is required.

The presently available DIS data allows for high precision extractions of PDFs in global fits. The treatment of the charm contribution in these fits is an important issue as it can induce potentially large effects in the PDFs of light quarks and the gluon obtained from these global fits.

Parton distributions and hard processes (I)



Parton Distribution Functions (PDFs) essential to relate theory to experiment at the LHC (and Tevatron, HERA, . . .).

inclusive photon-exchange deep-inelastic scattering (DIS)

In general, the structure functions are given by

$$x^{-1} F_a^p(x, Q^2) = \sum_i \int_x^1 \frac{d\xi}{\xi}\, c_{a,i}\left(\frac{x}{\xi}, \alpha_s(\mu^2), \frac{\mu^2}{Q^2}\right) f_i^p(\xi, \mu^2)$$

Parton distributions and hard processes (II)

$$\frac{d}{d\ln\mu^2} f_i(\xi,\mu^2) = \sum_k \left[P_{ik}(\alpha_s(\mu^2)) \otimes f_k(\mu^2)\right](\xi)$$

The initial conditions are not calculable in perturbative QCD. The task of determining these distributions can be divided in two steps. The first is the determination of the non-perturbative initial distributions at some (usually rather low) scale Q_0. The second is the perturbative calculation of their scale dependence (evolution) to obtain the results at the hard scales Q. The initial distributions have to be fitted using a suitable set of hard-scattering observables.

$\Rightarrow$ **predictions: fits of suitable reference processes**

QCD-PEGASUS [Comput. Phys. Commun. 170 (2005) 65-92]

Theoretical framework

- The NLO analysis presented here is performed in the common modified minimal subtraction (MS) factorization and renormalization scheme. Heavy quarks *(c,b,t)* are considered as massless partons within the nucleon.
- In the common MS factorization scheme the relevant structure function F^p_2 as extracted from the DIS ep process can be, up to NLO, written as:

$$F_2^p(x,Q^2) = F_{2,\mathrm{NS}}^{+}(x,Q^2) + F_{2,S}(x,Q^2) + F_2^{(c,b)}(x,Q^2,m_{c,b}^2)\,,$$

- Non–singlet contribution

$$\frac{1}{x}\,F_{2,\mathrm{NS}}^{+}(x,Q^2) = \left[C_{2,q}^{(0)} + aC_{2,\mathrm{NS}}^{(1)}\right] \otimes \left[\frac{1}{18}\,q_8^{+} + \frac{1}{6}\,q_3^{+}\right](x,Q^2)\,.$$

- Flavor singlet contribution

$$\frac{1}{x}\,F_{2,S}(x,Q^2) = \frac{2}{9}\left\{\left[C_{2,q}^{(0)} + aC_{2,q}^{(1)}\right] \otimes \Sigma + aC_{2,g}^{(1)} \otimes g\right\}(x,Q^2)\,.$$

The Jacobi Polynomials QCD analysis

One of the simplest and fastest possibilities in the structure function reconstruction from the QCD predictions for its Mellin moments is Jacobi polynomials expansion.

$$xf(x,Q^2) = x^\beta(1-x)^\alpha \sum_{n=0}^{N_{max}} a_n(Q^2)\Theta_n^{\alpha,\beta}(x),$$

where N_{max} is the number of polynomials and $\Theta_n^{\alpha,\beta}(x)$ are the Jacobi polynomials of order n,

$$\Theta_n^{\alpha,\beta}(x) = \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta)x^j$$

where $c_j^{(n)}(\alpha,\beta)$ are the coefficients expressed through $\Gamma-$ functions and satisfy the orthogonality relation with the weight $x^\beta(1-x)^\alpha$ as in the following:

$$\int_0^1 dx\; x^\beta(1-x)^\alpha \Theta_k^{\alpha,\beta}(x)\Theta_l^{\alpha,\beta}(x) = \delta_{k,l}\ .$$

For the moment, we note that the Q^2 dependence is entirely contained in the Jacobi moments

$$\begin{aligned} a_n(Q^2) &= \int_0^1 dx\; xf(x,Q^2)\Theta_k^{\alpha,\beta}(x) \\ &= \sum_{j=0}^{n}\int_0^1 dx\; x^{j+1}c_j^{(n)}(\alpha,\beta)f(x,Q^2) \\ &= \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta)f(j+2,Q^2)\ , \end{aligned}$$

$$F_2^{N_{max}}(x,Q^2) = x^{\beta}(1-x)^{\alpha}\sum_{n=0}^{N_{max}}\Theta_n^{\alpha,\beta}(x) \times \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta)F_2(j+2,Q^2),$$

This method was developed and applied for non-singlet QCD analysis up to N^3LO.

[A. N. Khorramian, H. Khanpour and S. A. Tehrani, Phys. Rev. D 78 (2010) 014013.]

[A. N. Khorramian and S. A. Tehrani, Phys. Rev. D 78 (2008) 074019.]

http://particles.ipm.ir/links/QCD.htm

IPM Parton Distribution Function

Title: **Mathematica package for Non-singlet parton distribution functions at NNNLO**

KKT09-NNNLO Non-singlet PDFs

Author: **A. Khorramian, H. Khanpour** and **S. Atashbar Tehrani**

Package Version: 05 Sep. 2009

To access the latest parton distribution, for the Non-singlet QCD analysis at NNNLO please use the Mathematica package KKT09-NNNLO.tar.gz file.

Title: **Mathematica package for Non-singlet parton distribution functions**

KT08's Non-singlet PDFs

Author: **A. Khorramian** and **S. Atashbar Tehrani**

Package Version:1 May 8,2008

To access the latest parton distribution, for the Non-singlet QCD analysis please use the Mathematica package KT08.tar.gz file.

Initial distributions

$$xf_i(x,\mu_0^2) \;=\; N_i\,p_{i,1}\,x^{p_{i,2}}(1-x)^{p_{i,3}}\;\left[1+p_{i,5}\,x^{p_{i,4}}+p_{i,6}\,x\right]$$

$$xf_i(x,\mu_0^2) \;=\; N_i\,p_{i,1}\,x^{p_{i,2}}(1-x)^{p_{i,3}}\;\left[1+p_{i,4}\,x^{0.5}+p_{i,5}\,x+p_{i,6}\,x^{1.5}\right]$$

$$Q_0^2 = 2GeV^2$$

Parameters estimation & Fit procedure

- *We use the following parameterizations for input distributions:*

$$xu_v(x,Q_0^2) = N_u\, x^{\alpha_u}(1-x)^{\beta_u}(1+\gamma_u\sqrt{x}+\eta_u\, x)$$
$$xd_v(x,Q_0^2) = N_d\, x^{\alpha_d}(1-x)^{\beta_d}(1+\gamma_d\sqrt{x}+\eta_d\, x)$$
$$x\Sigma(x,Q_0^2) = N_\Sigma\, x^{\alpha_\Sigma}(1-x)^{\beta_\Sigma}(1+\gamma_\Sigma\sqrt{x}+\eta_\Sigma\, x)$$
$$xg(x,Q_0^2) = N_g\, x^{\alpha_g}(1-x)^{\beta_g}$$

$$Q_0^2 = 2GeV^2$$

Where $u_v \equiv u - \bar{u}$, $d_v \equiv d - \bar{d}$ and $\Sigma \equiv \bar{u} + \bar{d}$. The distributions are further constrained by quark number and momentum conservation sum rules:

$$\int_0^1 u_v\, dx = 2, \quad \int_0^1 d_v\, dx = 1, \quad \int_0^1 x\big(u_v + d_v + 2(\bar{u}+\bar{d}+\bar{s}) + g\big)dx = 1.$$

Determination of the Strong Coupling

- We include running coupling as a free parameter in our fits and their error estimations. Thus the running coupling is determined in our analyses together with the parton distributions of the nucleon.

In principle one could also expand $a_s(\mu^2)$ in a power series in $a_s(\mu_o^2)$ for some fixed μ_o^2

$$a_s(\mu^2) = a_s(\mu_o^2) - a_s^2(\mu_0^2)\,\beta_0 \ln\frac{\mu^2}{\mu_o^2} + a_s^3(\mu_0^2)\left(\beta_0^2 \ln^2\frac{\mu^2}{\mu_o^2} - \beta_1 \ln\frac{\mu^2}{\mu_o^2}\right)$$

Experimental data selection

- *The statistically most significant data that we used are the HERA (H1 and ZEUS) measurements.*
- *In addition, we have used the fixed target F_2^p data of SLAC, BCDMS, E665 and NMC.*

1. *BCDMS*
2. *E665*
3. *NMC*
4. *SLAC*
5. *H1*
6. *ZEUS*

Data sets fitted in our NLO QCD analysis

Data set	NLO
BCDMS μp F_2	164
NMC μp F_2	123
E665 μp F_2	53
SLAC ep F_2	37
H1 MB 97 e^+p NC	59
H1 low Q^2 96–97 e^+p NC	71
H1 high Q^2 94–97 e^+p NC	130
H1 high Q^2 98–99 e^-p NC	126
H1 high Q^2 99–00 e^+p NC	132
ZEUS SVX 95 e^+p NC	30
ZEUS 96–97 e^+p NC	242
All data sets	**1167**

Minimization procedure

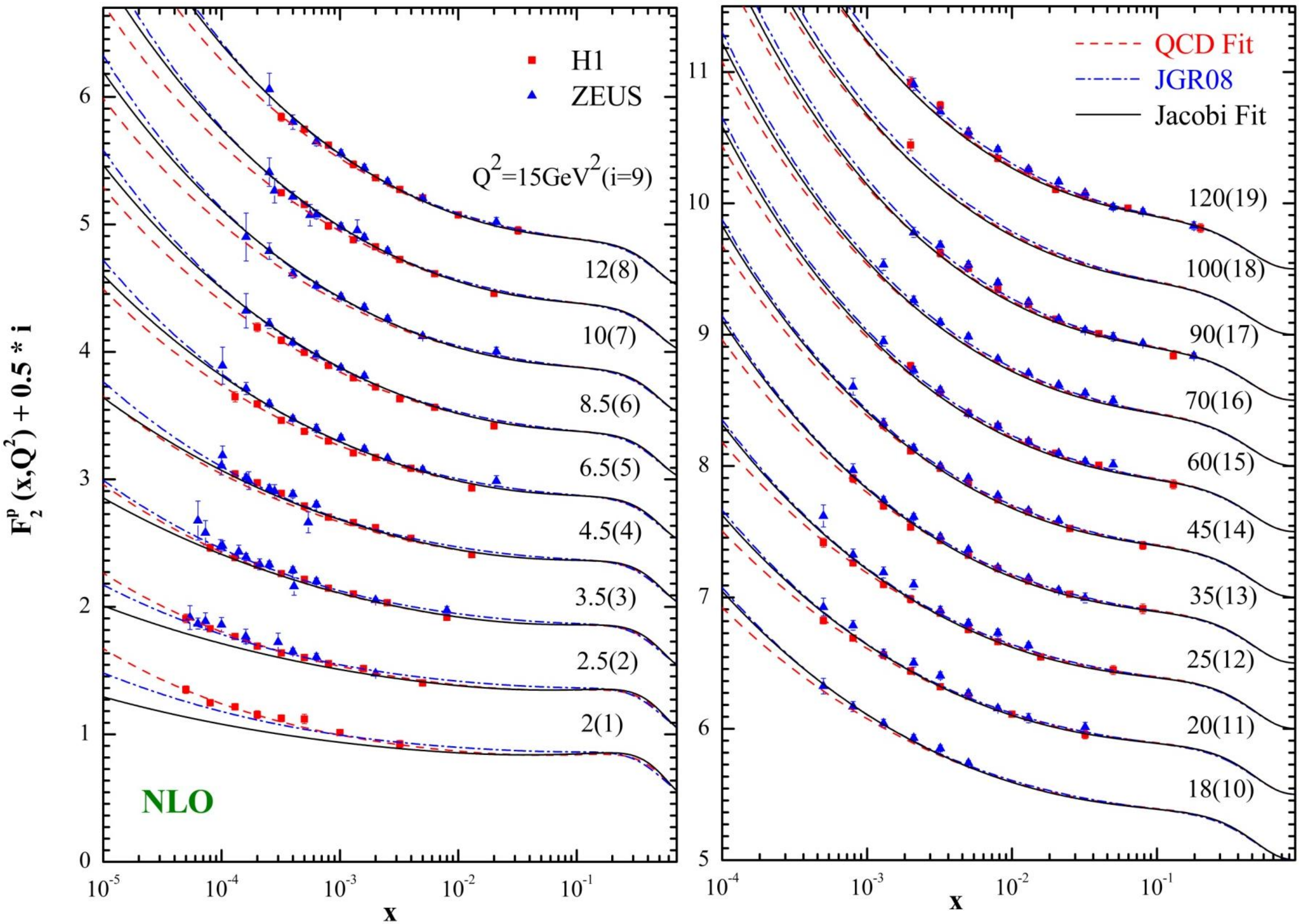
- The minimization procedure follows the usual chi–squared method with χ^2 defined as:

$$\chi^2(p) = \sum_{i=1}^{N} \left(\frac{\mathrm{data}(i) - \mathrm{theory}(i,p)}{\mathrm{error}(i)} \right)^2 ,$$

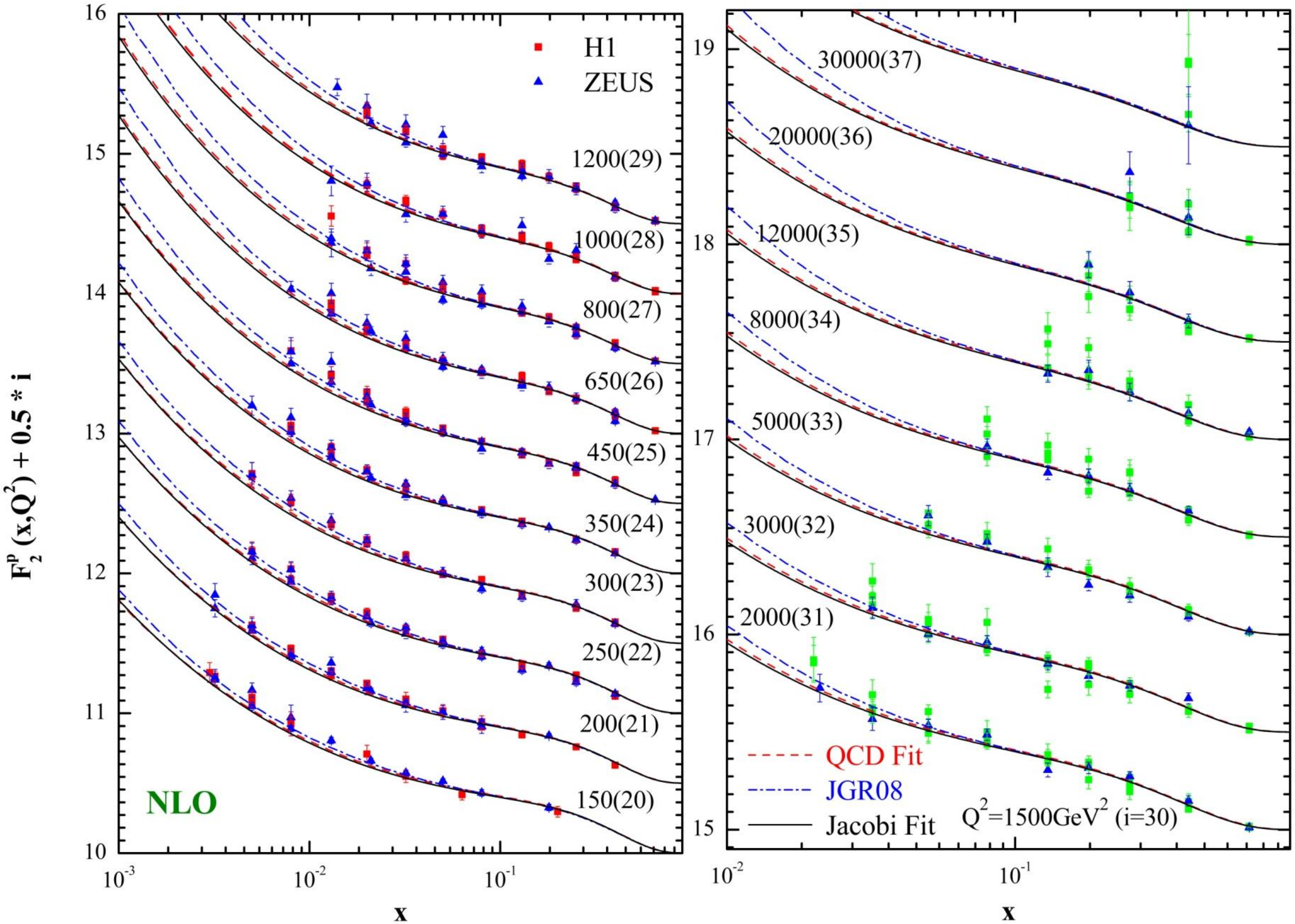
- Where **P** denotes the set of 16 independent parameters in the fit, including $\alpha_s(Q_0^2)$, and N is the number of data points included; N =1167 for the NLO fits. The errors include systematic and statistical uncertainties, being the total experimental error evaluated in quadrature.

MINUIT [F. James, CERN Program Library, Long Writeup D506 (MINUIT).]

The results of QCD analysis



Comparison of our standard NLO small-x results for $F^p_2(x,Q^2)$ with GJR08 and HERA data for $Q^2 \geq 1.5$ GeV^2.



Comparison of our standard NLO results for $F^p_2(x,Q^2)$ with GJR08 and HERA data for large values of Q^2 and larger x.

NLO

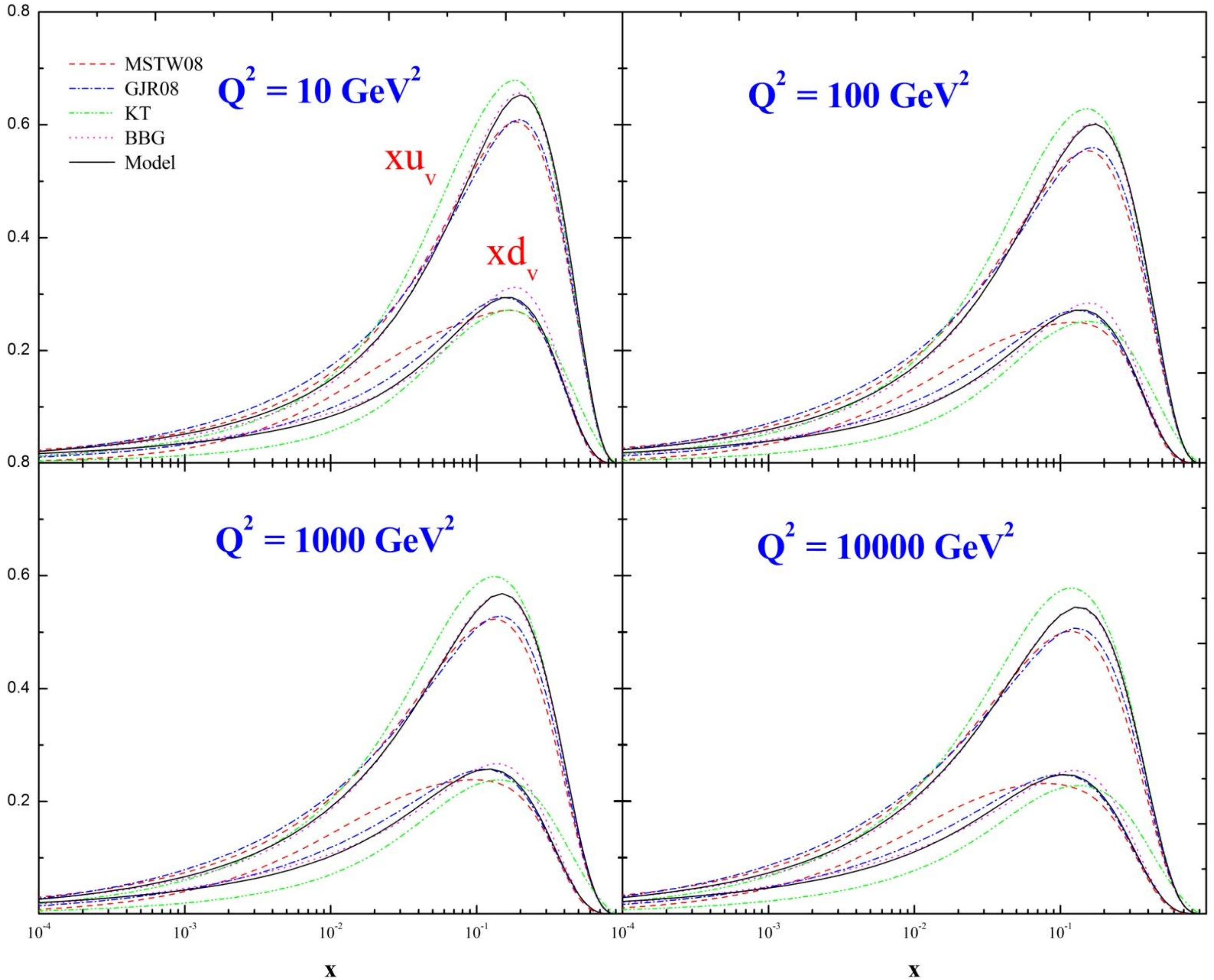
	u_v	d_v	$\bar{q}+\bar{d}$	g
N	0.1747	0.2603	0.1997±0.0033	6.1983
α	0.4190±0.0029	0.2841±0.0039	-0.2667±0.0041	0.2446±0.0061
β	3.5839±0.0085	6.8998±0.066	10.104±0.3469	6.9498±0.0420
γ	29.9418	1.9978	4.8059	-
η	25.4905	25.5864	13.0965	-
$\chi^2/\mathrm{dof} = 1.15$				
$\alpha_s(M_Z^2) = 0.1203 \pm 0.00014$				

Parameters of our standard input distributions at an input scale $Q_0^2 = 2$ GeV^2.

Since the input gluon distribution turned out to be insensitive to the polynomial terms , we have set them to zero.

NLO	$\alpha_s(M_Z^2)$
CTEQ6	0.1165 ± 0.0065
MRST03	0.1165 ± 0.0020
A02	0.1171 ± 0.0015
ZEUS	0.1166 ± 0.0049
H1	0.1150 ± 0.0017
GRS	0.112
BBG	0.1148 ± 0.0019
KA	0.1149 ± 0.0021
GJR08	0.1178 ± 0.0021
MSTW08	$0.1202 \begin{array}{l} +0.0012 \\ -0.0015 \end{array}$
QCD Fit	0.1198 ± 0.0002
Jacobi Fit	0.1203 ± 0.00014

Our results for strong coupling in comparison with the results obtain by the other groups.



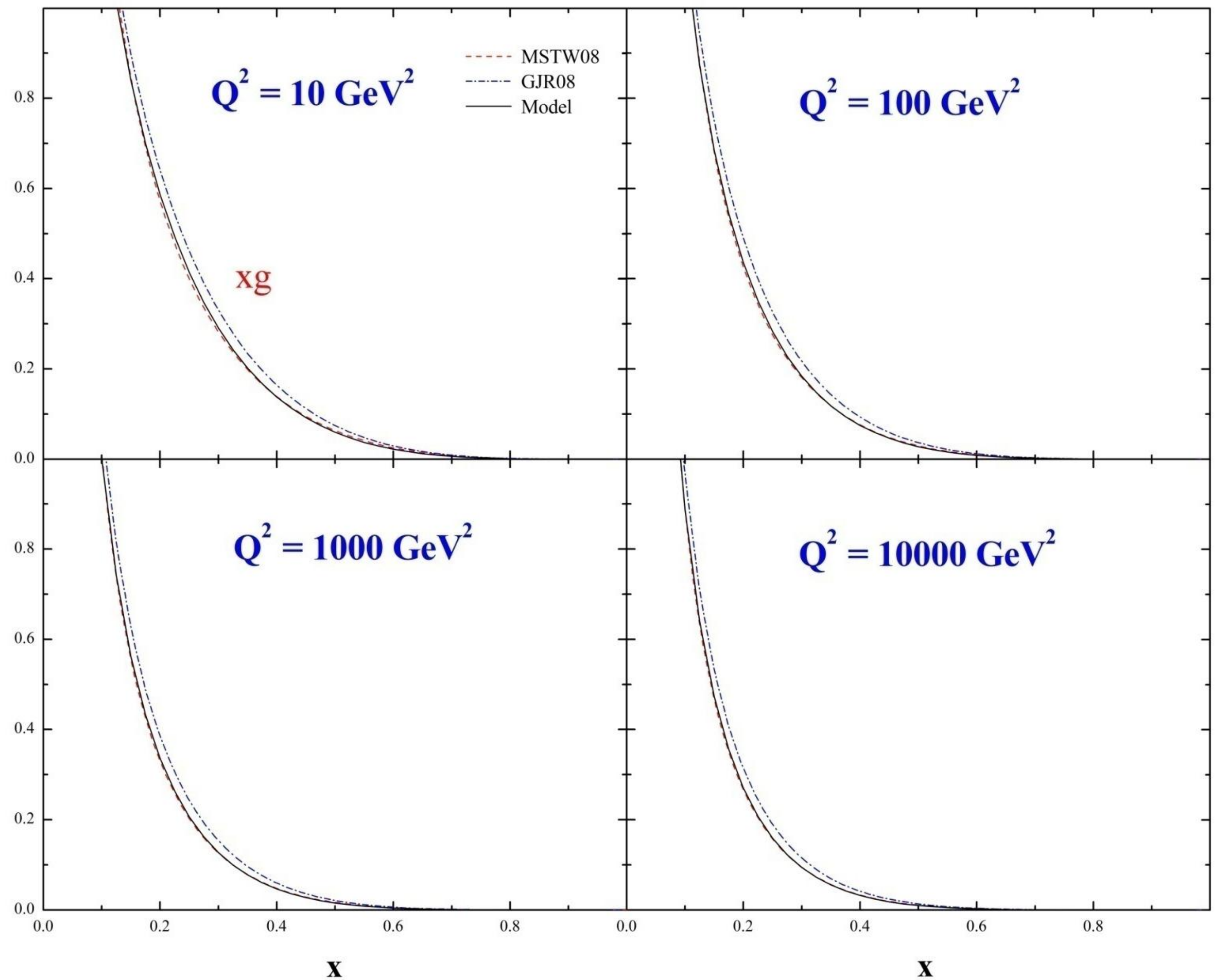
Our standard NLO(MS) valence distributions evolved up to $Q^2 = 10^4$ GeV^2 together with the results obtained by MSTW08, GJR08, BBG and KT.

[Eur. Phys. J. C (2009) 63: 189–285]

[Eur. Phys. J. C (2008) 53: 355–366]

[Nucl. Phys. B 774 (2007) 182]

[Phys. Rev. D 78 (2008) 074019]



Our standard NLO(MS) gluon distribution evolved up to $Q^2 = 10^4$ GeV^2 together with the results obtained by MSTW08 and GJR08.

[Eur. Phys. J. C (2009) 63: 189–285]

[Eur. Phys. J. C (2008) 53: 355–366]

NNLO calculation

During the last years, our understanding of PDFs has steadily improved at the NNLO level, and upcoming high-precision data from hadron colliders will continue in this direction. So in order to perform a consistent QCD analysis of the DIS world data and other hard scattering data, a next-to-next-to-leading order (NNLO) analysis is required and this is our motivation to do the NNLO QCD analysis.

We hope our results of QCD analysis of structure functions in terms of Jacobi polynomials could be able to describe the hadron structure functions. We also hope to be able to consider massive quark contributions by using the structure function expansion in terms of the Jacobi polynomials at next-to-next-to-leading order (NNLO).

Conclusion

- We have performed the standard next-to-leading order (NLO) parton distribution functions QCD analysis of the inclusive neutral-current deep-inelastic-scattering (DIS) world data.
- We restrict the analysis to the NLO heavy flavor corrections and extract heavy quark flavors distributions.
- We generate VFNS parton distributions where the heavy quark flavors $h = c, b, t$ are considered as massless partons within the nucleon.
- The analysis was performed using the Jacobi polynomials method to determine the parameters of the problem in a fit to the data.
- By studying the role of these distributions in the production of heavy particles at high energy ep and pp colliders, we show that our results for parton distribution functions and proton structure function are in agreement with the DIS data and other theoretical models.


Dziekuje za uwage


Thank
you!