Hadron Structure

50 Cracow School of Theoretical Physics

9 – 19 June 2010

Paul Hoyer
University of Helsinki

Quark vs. Parton Model Views of Hadrons

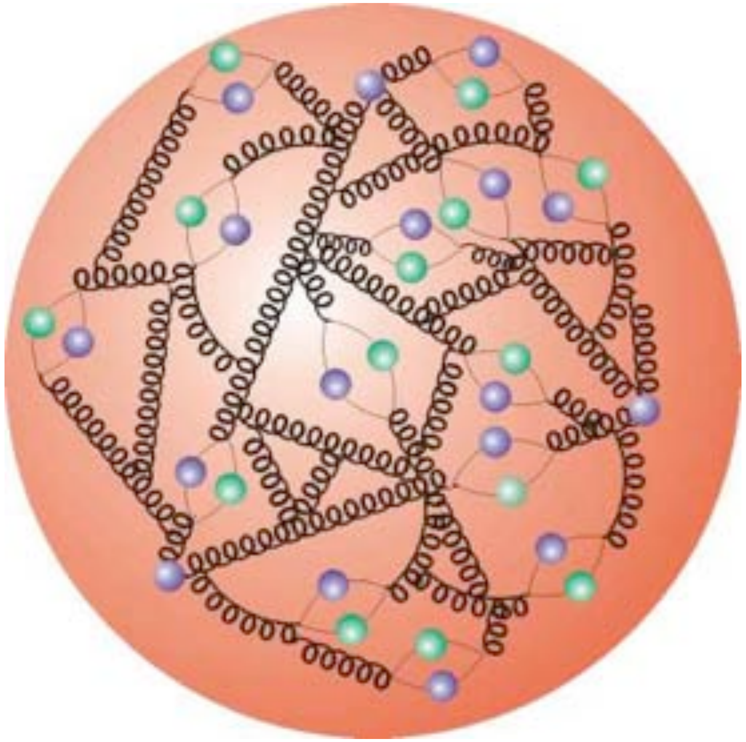
Are they incompatible?

How can we tell?

?

⇔

Quark model



Parton picture

Consider QCD bound states at lowest order in $\hbar$

PH arXiv: 0909.3045

Work in progress with Stan Brodsky

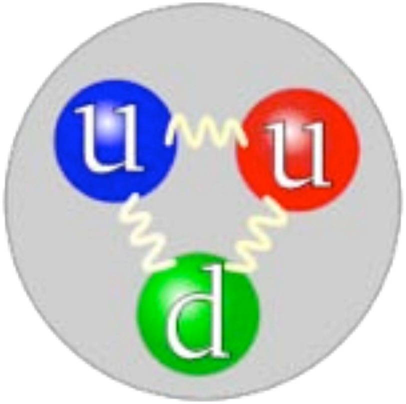
The Quark Model gets the dof's right

From the 2008 PDG Review of Particle Physics.

Mesons

$n\ {}^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $-\bar{u}s$	I = 0 f'	I = 0 f
$1\ {}^1S_0$	0^{-+}	$\boldsymbol{\pi}$	$\boldsymbol{K}$	$\boldsymbol{\eta}$	$\boldsymbol{\eta'(958)}$
$1\ {}^3S_1$	1^{--}	$\boldsymbol{\rho(770)}$	$\boldsymbol{K^*(892)}$	$\boldsymbol{\phi(1020)}$	$\boldsymbol{\omega(782)}$
$1\ {}^1P_1$	1^{+-}	$\boldsymbol{b_1(1235)}$	$\boldsymbol{K_{1B}}^\dagger$	$h_1(1380)$	$\boldsymbol{h_1(1170)}$
$1\ {}^3P_0$	0^{++}	$\boldsymbol{a_0(1450)}$	$\boldsymbol{K_0^*(1430)}$	$\boldsymbol{f_0(1710)}$	$\boldsymbol{f_0(1370)}$
$1\ {}^3P_1$	1^{++}	$\boldsymbol{a_1(1260)}$	$\boldsymbol{K_{1A}}^\dagger$	$\boldsymbol{f_1(1420)}$	$\boldsymbol{f_1(1285)}$
$1\ {}^3P_2$	2^{++}	$\boldsymbol{a_2(1320)}$	$\boldsymbol{K_2^*(1430)}$	$\boldsymbol{f_2'(1525)}$	$\boldsymbol{f_2(1270)}$
$1\ {}^1D_2$	2^{-+}	$\boldsymbol{\pi_2(1670)}$	$\boldsymbol{K_2(1770)}^\dagger$	$\eta_2(1870)$	$\boldsymbol{\eta_2(1645)}$
$1\ {}^3D_1$	1^{--}	$\boldsymbol{\rho(1700)}$	$\boldsymbol{K^*(1680)}$		$\boldsymbol{\omega(1650)}$
$1\ {}^3D_2$	2^{--}		$\boldsymbol{K_2(1820)}$		
$1\ {}^3D_3$	3^{--}	$\boldsymbol{\rho_3(1690)}$	$\boldsymbol{K_3^*(1780)}$	$\boldsymbol{\phi_3(1850)}$	$\boldsymbol{\omega_3(1670)}$
$1\ {}^3F_4$	4^{++}	$\boldsymbol{a_4(2040)}$	$\boldsymbol{K_4^*(2045)}$		$\boldsymbol{f_4(2050)}$
$1\ {}^3G_5$	5^{--}	$\rho_5(2350)$			
$1\ {}^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$
$2\ {}^1S_0$	0^{-+}	$\boldsymbol{\pi(1300)}$	$K(1460)$	$\boldsymbol{\eta(1475)}$	$\boldsymbol{\eta(1295)}$
$2\ {}^3S_1$	1^{--}	$\boldsymbol{\rho(1450)}$	$\boldsymbol{K^*(1410)}$	$\boldsymbol{\phi(1680)}$	$\boldsymbol{\omega(1420)}$

Baryons



The QM describes hadrons qualitatively as non-relativistic states of "constituent" quarks bound by a linear plus Coulomb potential:

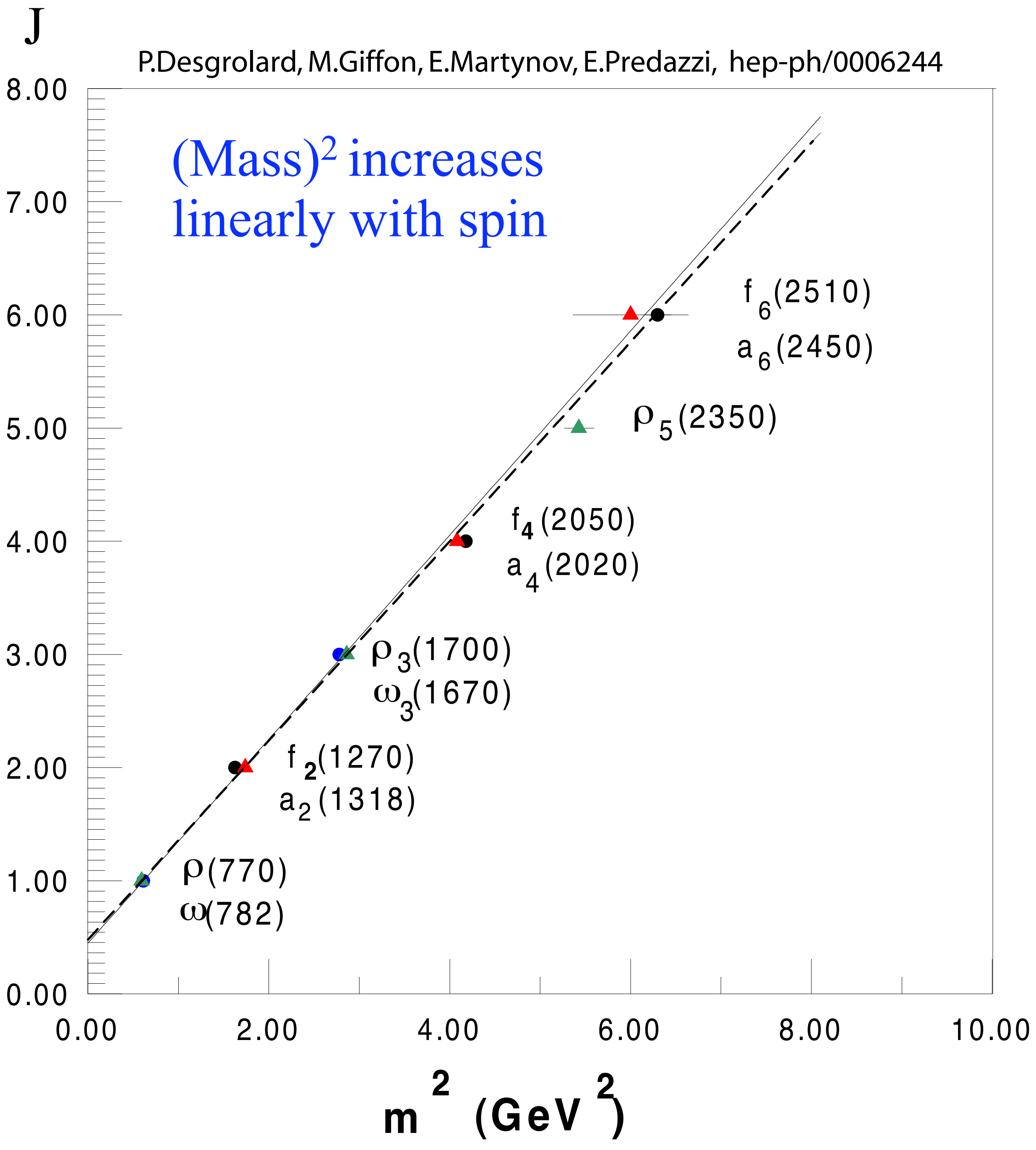
$$V(r) = c\,r - C_F \frac{\alpha_s}{r}$$

From the 2008 PDG Review of Particle Physics.

J^P	(D, L^P_N)	S	Octet members				Singlets
$1/2^+$	$(56,0^+_0)$	1/2	N(939)	Λ(1116)	Σ(1193)	Ξ(1318)	
$1/2^+$	$(56,0^+_2)$	1/2	N(1440)	Λ(1600)	Σ(1660)	Ξ(?)	
$1/2^-$	$(70,1^-_1)$	1/2	N(1535)	Λ(1670)	Σ(1620)	Ξ(?)	Λ(1405)
$3/2^-$	$(70,1^-_1)$	1/2	N(1520)	Λ(1690)	Σ(1670)	Ξ(1820)	Λ(1520)
$1/2^-$	$(70,1^-_1)$	3/2	N(1650)	Λ(1800)	Σ(1750)	Ξ(?)	
$3/2^-$	$(70,1^-_1)$	3/2	N(1700)	Λ(?)	Σ(?)	Ξ(?)	
$5/2^-$	$(70,1^-_1)$	3/2	N(1675)	Λ(1830)	Σ(1775)	Ξ(?)	
$1/2^+$	$(70,0^+_2)$	1/2	N(1710)	Λ(1810)	Σ(1880)	Ξ(?)	Λ(?)
$3/2^+$	$(56,2^+_2)$	1/2	N(1720)	Λ(1890)	Σ(?)	Ξ(?)	
$5/2^+$	$(56,2^+_2)$	1/2	N(1680)	Λ(1820)	Σ(1915)	Ξ(2030)	
$7/2^-$	$(70,3^-_3)$	1/2	N(2190)	Λ(?)	Σ(?)	Ξ(?)	Λ(2100)
$9/2^-$	$(70,3^-_3)$	3/2	N(2250)	Λ(?)	Σ(?)	Ξ(?)	
$9/2^+$	$(56,4^+_4)$	1/2	N(2220)	Λ(2350)	Σ(?)	Ξ(?)	

			Decuplet members			
$3/2^+$	$(56,0^+_0)$	3/2	Δ(1232)	Σ(1385)	Ξ(1530)	Ω(1672)
$3/2^+$	$(56,0^+_2)$	3/2	Δ(1600)	Σ(?)	Ξ(?)	Ω(?)
$1/2^-$	$(70,1^-_1)$	1/2	Δ(1620)	Σ(?)	Ξ(?)	Ω(?)
$3/2^-$	$(70,1^-_1)$	1/2	Δ(1700)	Σ(?)	Ξ(?)	Ω(?)
$5/2^+$	$(56,2^+_2)$	3/2	Δ(1905)	Σ(?)	Ξ(?)	Ω(?)
$7/2^+$	$(56,2^+_2)$	3/2	Δ(1950)	Σ(2030)	Ξ(?)	Ω(?)
$11/2^+$	$(56,4^+_4)$	3/2	Δ(2420)	Σ(?)	Ξ(?)	Ω(?)

Hadrons are ultra-relativistic



(Mass)^2 increases linearly with spin

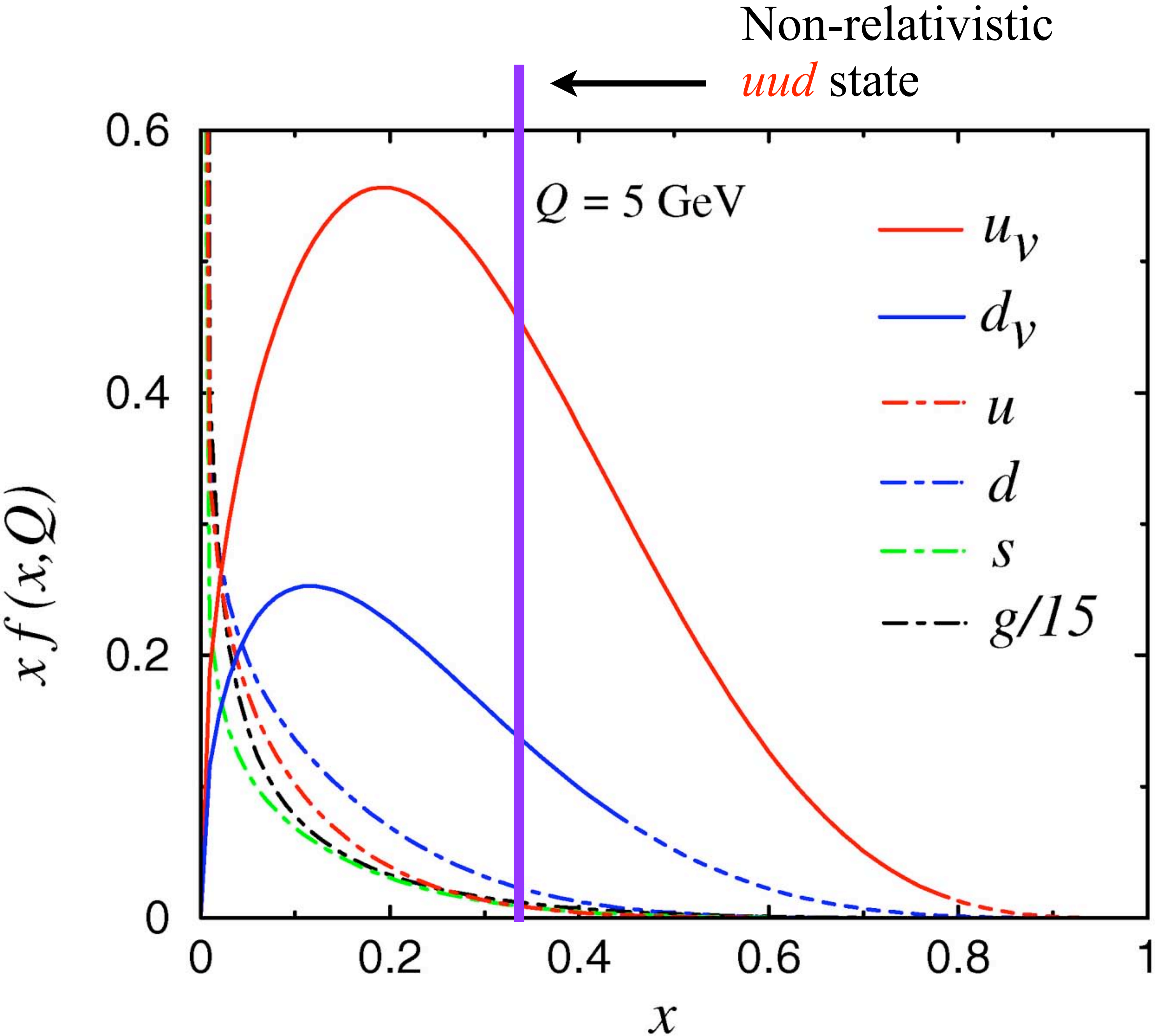
$$\frac{2m_u + m_d}{m_p} \simeq \frac{10\ \text{MeV}}{938\ \text{MeV}} \simeq 1\%$$

99% of the proton mass is dynamical

DIS reveals the relativistic internal motion

... as well as the prominence of gluons and sea quarks.

Could gluons and sea quarks be generated perturbatively, via evolution?



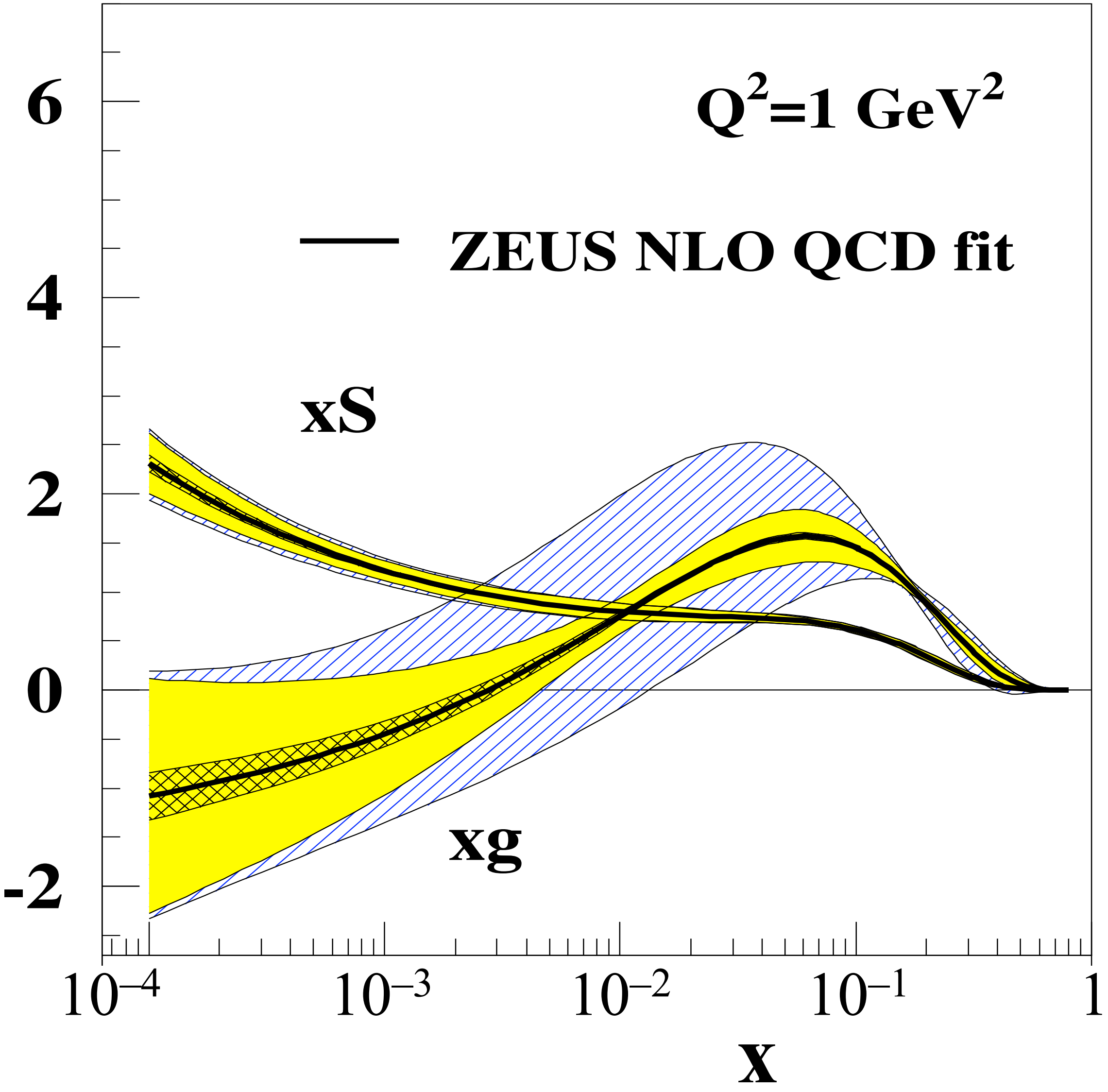
Sea Quarks come before gluons!

A M Cooper-Sarkar:

"...at Q ~ 1 GeV the sea input is indeed steep, but the gluon input is valence-like, with a tendency to be negative at low *x*!

(Essentially the gluon evolution must be fast in order that upward evolution can produce the extreme steepness of high Q^2 data, however this also implies that downward evolution is fast and this results in the valence-like gluon at low Q^2)."

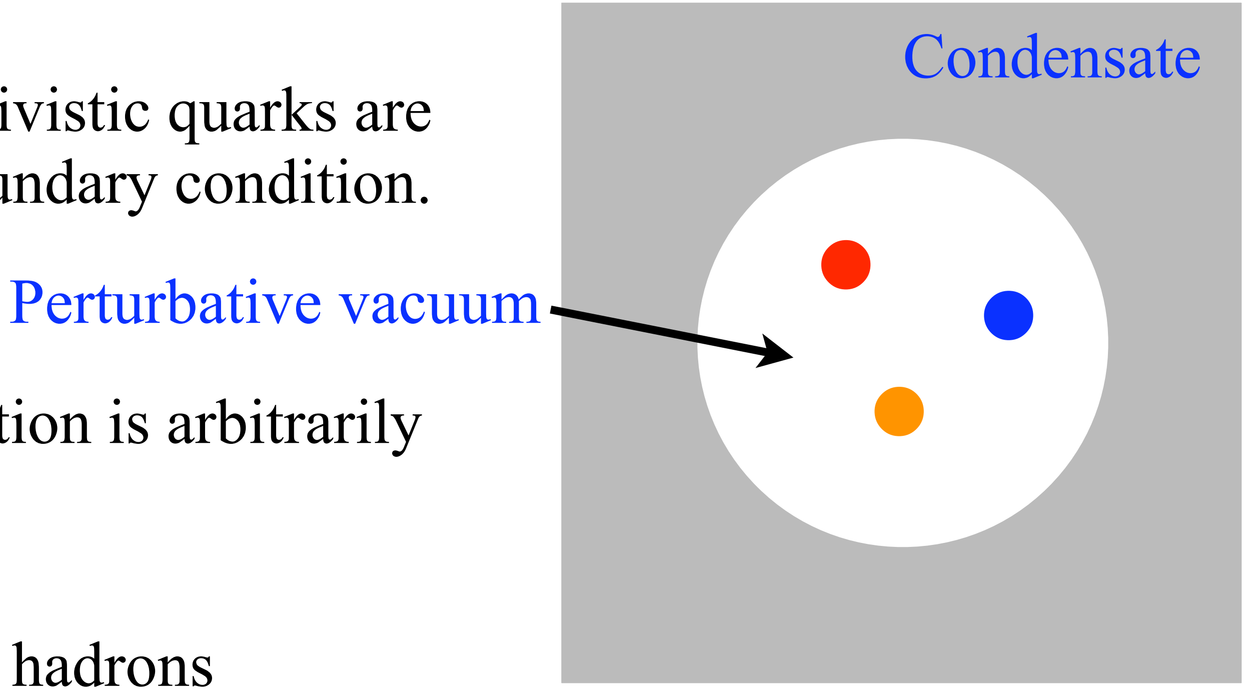
A M Cooper-Sarkar arXiv:0901.4001



Since the proton mass is dynamical, quarks are relativistic and antiquarks must be present.

But how to treat relativistic bound states? (I)

In the Bag Model, the relativistic quarks are confined by a spherical boundary condition.



– The bag boundary condition is arbitrarily imposed

– Unclear how to describe hadrons with $\mathbf{p}_{CM} \neq 0$ (Lorentz covariance)

Bag model

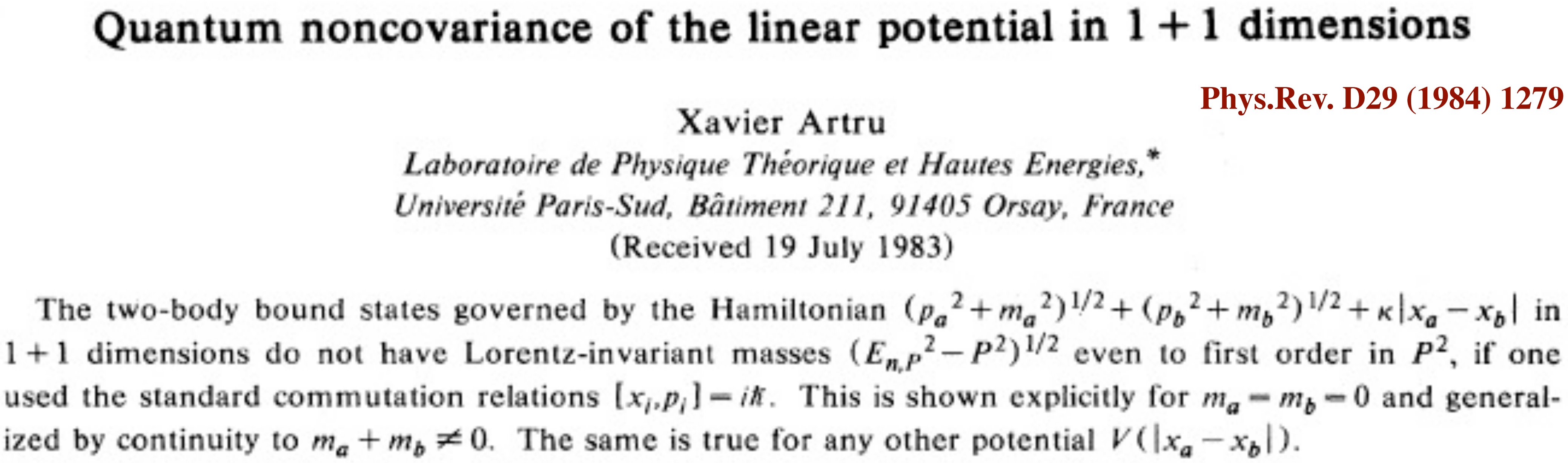
One can impose a boundary condition in a different way, which is consistent with the equations of motion and maintains boost invariance.

But how to treat relativistic bound states? (II)

Frequently, relativistic bound state equations are simply postulated, and their properties studied phenomenologically.

This may be instructive, but the lack of theoretical understanding limits further progress.

Covariant-looking equations may not actually have boost invariance:



Quantum noncovariance of the linear potential in 1 + 1 dimensions

Phys.Rev. D29 (1984) 1279

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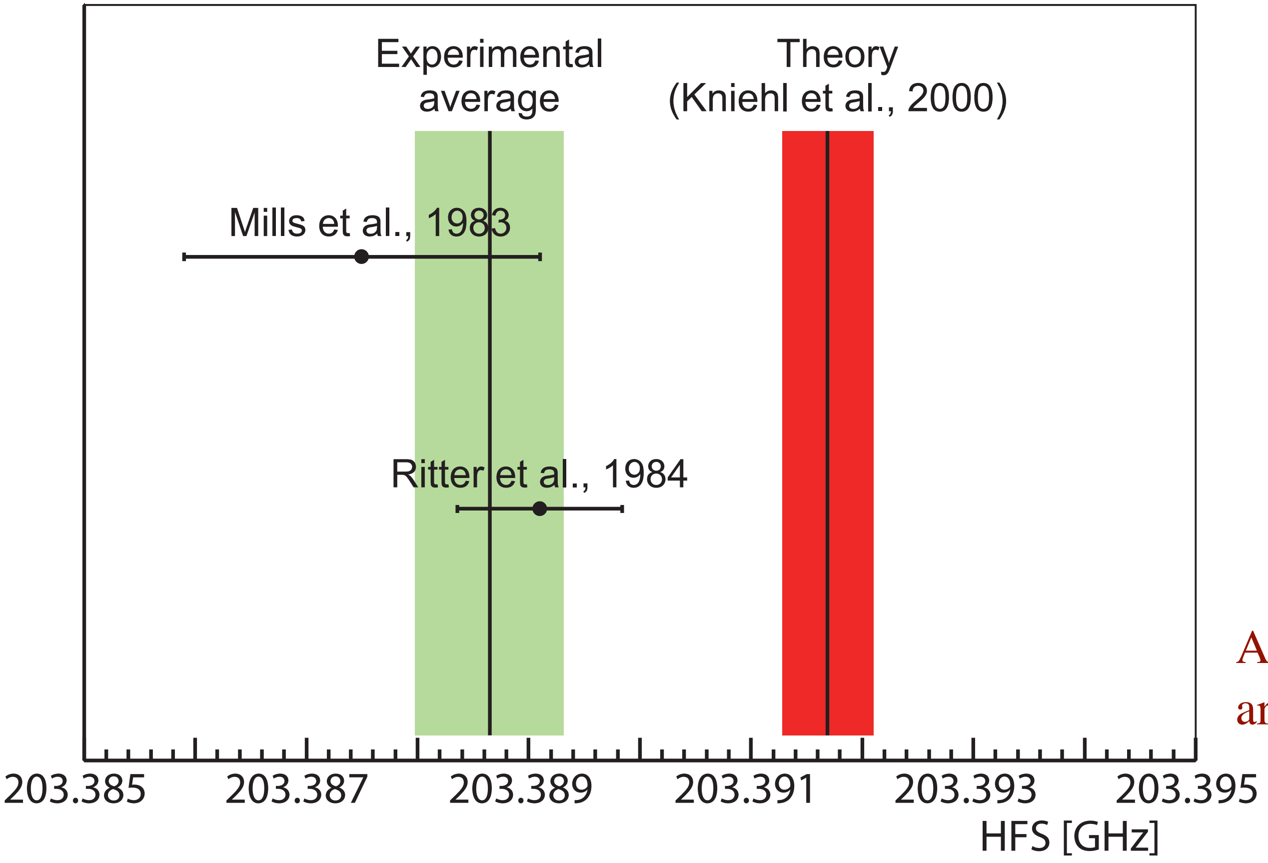
(Received 19 July 1983)

The two-body bound states governed by the Hamiltonian $(p_a{}^2+m_a{}^2)^{1/2}+(p_b{}^2+m_b{}^2)^{1/2}+\kappa|x_a-x_b|$ in $1+1$ dimensions do not have Lorentz-invariant masses $(E_{n,P}{}^2-P^2)^{1/2}$ even to first order in P^2, if one used the standard commutation relations $[x_i,p_i]=i\hbar$. This is shown explicitly for $m_a=m_b=0$ and generalized by continuity to $m_a+m_b\neq 0$. The same is true for any other potential $V(|x_a-x_b|)$.

But how to treat relativistic bound states? (III)

Relativistic corrections to the Schrödinger equation for atoms are calculated reliably and with high accuracy.

Hyperfine splitting in positronium atoms



A. Ishida *et al.*
arXiv:1004.5555

As we shall see, this is consistent with an expansion in $\hbar$

$\hbar$ as an expansion parameter for bound states

$\hbar$ is a fundamental constant related to quantum effects. Each order in an $\hbar$ expansion must obey all symmetries of the theory.

The $\hbar$ expansion is relevant for both relativistic and nonrelativistic, scattering and bound state dynamics.

Born terms are defined as being of lowest order in $\hbar$.

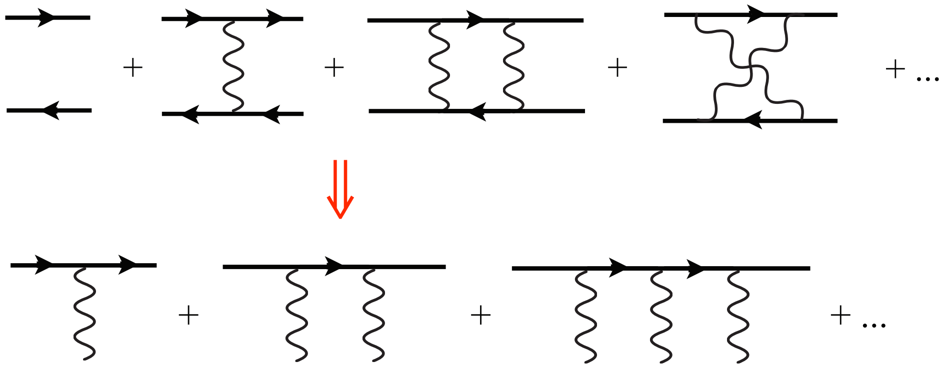
Is there a Born term for relativistic bound states?

At Born level, Quantum Field Theory reduces to Relativistic Quantum Mechanics.

Bound states from the perturbative expansion

Bound state poles appear through a divergence of the perturbative series

The Schrödinger and Dirac equations describe bound states through interactions with an external, classical potential.



The ladder (loop) sum turns into a sum of tree diagrams as one mass tends to infinity.

⇒ The $\hbar$ expansion is not trivially related to the number of loops.

Need to identify the contribution of lowest order in $\hbar$ which causes the ladder sum to diverge at the bound state energies.

$\hbar$ in the Harmonic Oscillator

$\hbar \to 0$ does not always imply classical physics. For the harmonic oscillator

$$Z = \int [dx] \exp\left[\frac{i}{\hbar}\int dt(\tfrac{1}{2}m\dot{x}^2 - \tfrac{1}{2}m\omega^2 x^2)\right]$$

$$\propto \int [d\tilde{x}] \exp\left[i\int dt(\tfrac{1}{2}m\dot{\tilde{x}}^2 - \tfrac{1}{2}m\omega^2 \tilde{x}^2)\right]$$

The $\hbar$ can be completely absorbed in $\tilde{x} \equiv x/\sqrt{\hbar}$

Bound states with $E_n = \hbar\omega(n + \frac{1}{2})$ have small $x \propto \sqrt{\hbar\, n}$ as $\hbar \to 0$ (with fixed *n*).

The classical path $x_i(t_i) \to x_f(t_f)$ is obtained when the boundary positions $x_{i,f}$ are held fixed as $\hbar \to 0$, hence $n \propto 1/\hbar$ ensuring a classical limit.

$\hbar$ expansion in QED

$$\mathcal{L}_{QED} = \bar{\psi}(i\partial\!\!\!/ - \tilde{e}A\!\!\!/ - \tilde{m})\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Dimensions: Requiring $[S] \equiv [\int d^4x\mathcal{L}] = [\hbar] = E \cdot L$ $\qquad c = \epsilon_0 = 1$

$[\psi] = E^{1/2}\ L^{-1}$ $\qquad$ also from: $\{\psi^\dagger(t,\boldsymbol{x}), \psi(t,\boldsymbol{y})\} = \hbar\,\delta^3(\boldsymbol{x}-\boldsymbol{y})$

$[A^\mu] = E^{1/2}\ L^{-1/2}$

$[\tilde{m}] = L^{-1}$ wave number! $\tilde{m} = m/\hbar$

$[\tilde{e}] = E^{-1/2}\ L^{-1/2}$

$[e] = E^{+1/2}\ L^{+1/2}$

! $\quad \alpha = \dfrac{e^2}{4\pi\hbar} = \dfrac{\tilde{e}^2\hbar}{4\pi} \simeq \dfrac{1}{137}$

(e is the classical charge) $\qquad \tilde{e} = e/\hbar$

We shall define the $\hbar \to 0$ limit by keeping the quantities $\tilde{e}, \tilde{m}$ of the "classical" action fixed

Rescaling the fields with $\hbar$

$$Z = \int [\mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A] \exp\left[\frac{i}{\hbar}\int d^4x \mathcal{L}\right] \propto \int [\mathcal{D}\tilde{\psi} \mathcal{D}\bar{\tilde{\psi}} \mathcal{D}\tilde{A}] \exp\left[i\int d^4x \tilde{\mathcal{L}}\right]$$

The rescalings $\tilde{\psi} \equiv \psi/\sqrt{\hbar}, \quad \tilde{A}^\mu \equiv A^\mu/\sqrt{\hbar}$

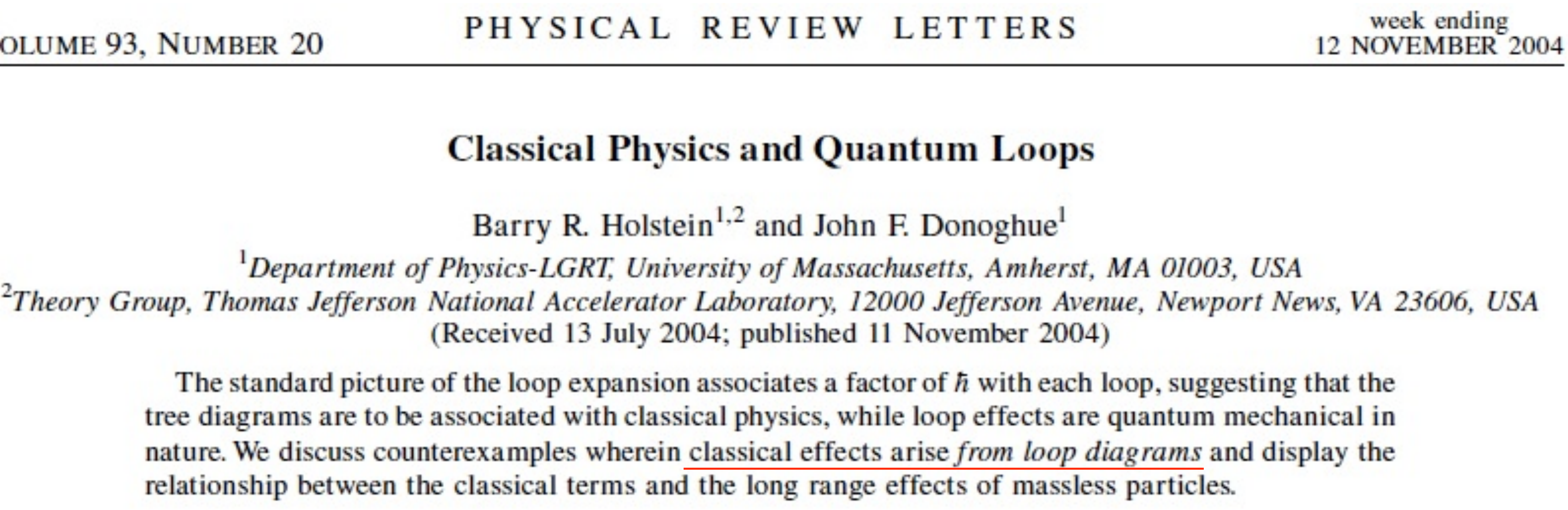
introduce an $\hbar$ dependence in the interaction term:

$$\tilde{\mathcal{L}} = \bar{\tilde{\psi}}(i\not{\partial} - \tilde{e}\sqrt{\hbar}\tilde{\not{A}} - \tilde{m})\tilde{\psi} - \tfrac{1}{4}(\partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu)^2$$

$\hbar$ now appears only in the coupling: $\alpha = \dfrac{\tilde{e}^2\hbar}{4\pi} = \mathcal{O}(\hbar)$

and the perturbative (loop) expansion is equivalent to the $\hbar$ expansion.

A comment on:



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Classical Physics and Quantum Loops

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The standard picture of the loop expansion associates a factor of $\hbar$ with each loop, suggesting that the tree diagrams are to be associated with classical physics, while loop effects are quantum mechanical in nature. We discuss counterexamples wherein classical effects arise *from loop diagrams* and display the relationship between the classical terms and the long range effects of massless particles.

This paper appears to use a different definition of the limit $\hbar \to 0$

where m and e are held fixed, hence $\tilde{m} = m/\hbar \to \infty, \quad \tilde{e} = e/\hbar \to \infty$

Then also $\alpha = e^2/4\pi\hbar \to \infty$ hence the $\hbar$ and loop expansions are not equivalent.

Much ado about nothing?

So $\hbar$ only appears via the fine structure constant α and the $\hbar$ expansion is equivalent to the standard perturbative expansion. What else is new?

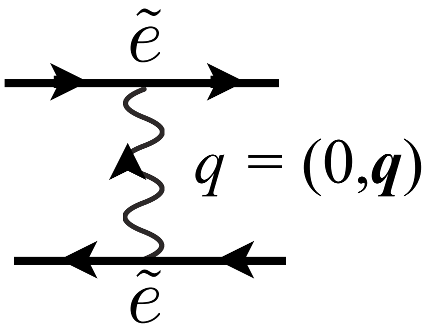
Well, consider bound states: What is the Born term for bound states?

Ladder diagrams can contribute at lowest order in $\hbar$ (and α) when the loop momentum vanishes with $\hbar$.

Extracting the $O(\hbar^0)$ contribution in ladders (I)

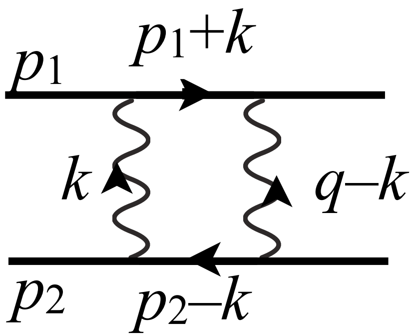
In CM elastic scattering the exchanged $q^0 = 0$ and the Born term A_1 is $\propto \tilde{e}^2\hbar$

A_1:



The $k^0 \propto \tilde{e}^2\hbar|\boldsymbol{k}|$ part of the loop integral in $A_2 \propto \hbar^0$

A_2 :

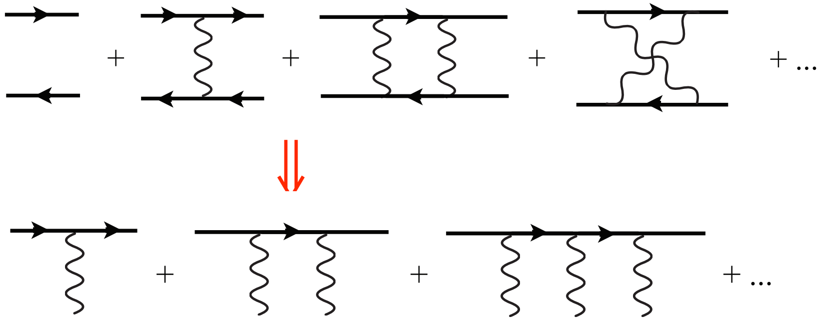


$$A_2 \sim \tilde{e}^2\hbar \int \overset{\mathcal{O}\left(\tilde{e}^2\hbar\right)}{\overset{\downarrow}{dk^0}} \frac{\overset{\mathcal{O}\left(\tilde{e}^2\hbar\right)}{\overset{\downarrow}{\tilde{e}^2\hbar}}}{\left[p_1^0+k^0-\underset{\mathcal{O}\left(\tilde{e}^2\hbar\right)}{\underset{\uparrow}{\sqrt{(\boldsymbol{p}_1+\boldsymbol{k})^2+m^2}}}+i\varepsilon\right]\left[p_2^0-k^0-\underset{\mathcal{O}\left(\tilde{e}^2\hbar\right)}{\underset{\uparrow}{\sqrt{(\boldsymbol{p}_2-\boldsymbol{k})^2+m^2}}}+i\varepsilon\right]}$$

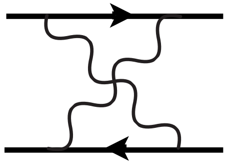
Extracting the O($\hbar^0$) contribution in ladders (II)

The loop contributions which are $\propto \hbar^0$ reduce to scattering from a classical potential

$$\sum_n A_n \; =$$

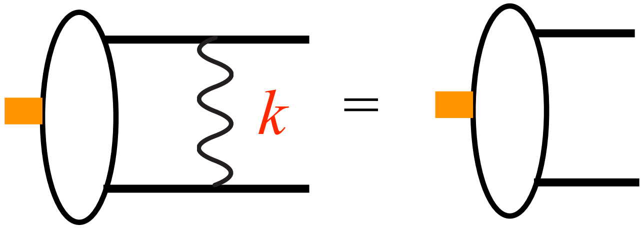


Crossed ladder diagrams do not contribute in the relevant $k^0 \propto \tilde{e}^2\hbar|\boldsymbol{k}|$ region:



$\rightarrow 0$ (no contribution $\propto \hbar^0$)

$\Rightarrow$ The wave function satisfies a BSE with a single photon kernel when $k^0 \propto \tilde{e}^2\hbar|\boldsymbol{k}|$



Furthermore: $|\boldsymbol{k}| \propto \tilde{e}^2\hbar m$ dominates (in the CM). Thus we have the standard scaling $|\boldsymbol{k}| \propto \alpha m, \quad k^0 \propto \alpha^2 m$ of the non-relativistic Schrödinger equation. No relativistic bound states at lowest order in $\hbar$!

Recap (I)

We considered the hbar expansion in QED. The coupling $\tilde{e}$ in the lagrangian has a different dimension than the classical charge e : $\tilde{e} = e/\hbar$

Keeping $\tilde{e}$ fixed and rescaling the fields we found that hbar can be made to appear only in the coupling $\alpha = \tilde{e}^2\hbar$.
Then lowest order in hbar is equivalent to lowest order in α.

Bound states derived from ladder diagrams at lowest order in α are non-relativistic and described by the Schrödinger equation.

In particular, the Dirac Coulomb equation is not of lowest order in hbar.
It has some, but not all (Lamb shift,...) higher order corrections.

Indeed, the Dirac energy levels are unphysical (complex) for Z α > 1 :

$$E_{n,j} = m\left[1 + \frac{Z^2\alpha^2}{n - j - \frac{1}{2} + \sqrt{(j+\frac{1}{2})^2 - Z^2\alpha^2}}\right]^{-1/2} \qquad (j + \tfrac{1}{2} \leq n)$$

Recap (II)

The hbar expansion has apparently led us into an impasse:

It appears not to yield the relativistic dynamics that is required to describe hadrons from first principles, yet it is in the spirit of the Quark Model.

Then again, a strong principle is needed to organize the confusing status of relativistic bound states.

The way out is to impose novel boundary conditions, different from those used in deriving the standard perturbative rules.

This will also allow us to introduce the scale Λ_{QCD} required for hadrons.

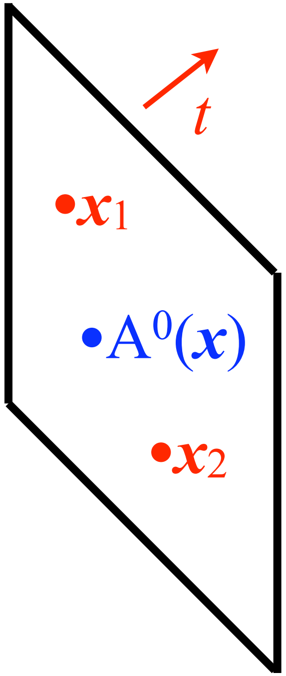
One opportunity presents itself:
Allowing a homogeneous solution of the equations of motion, which gives a linear potential that survives in the $\hbar \to 0$ limit.

It gives a relativistic and potentially relevant description of hadrons.

EOM for a non-relativistic bound state

We usually describe atoms (Hydrogen, Positronium) by solving the Schrödinger eq. for a particle with reduced mass μ in a Coulomb potential, $V(r) = -\alpha/r$. Although $V(r)$ appears as a fixed, external potential, $r = |\boldsymbol{x}_1 - \boldsymbol{x}_2|$ is actually the distance between the two constituents.

An equivalent view: For each constituent configuration $(\boldsymbol{x}_1, \boldsymbol{x}_2)$ the gauge field $A^0(\boldsymbol{x};\boldsymbol{x}_1, \boldsymbol{x}_2)$ is determined by the field equation (Gauss' law)



$$-\boldsymbol{\nabla}^2_{\boldsymbol{x}} A^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2) = e\big[\delta^3(\boldsymbol{x}-\boldsymbol{x}_1) - \delta^3(\boldsymbol{x}-\boldsymbol{x}_2)\big]$$

giving

$$A^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2) = \frac{e}{4\pi}\left(\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_1|} - \frac{1}{|\boldsymbol{x}-\boldsymbol{x}_2|}\right)$$

Note that this field depends on the configuration, i.e., on the positions $(\boldsymbol{x}_1, \boldsymbol{x}_2)$ of the constituents. The lagrangian has no time derivative $\partial_0 A^0$, hence A^0 is instantaneous (does not depend on time in Coulomb gauge).

Potential from the A^0 field

The Coulomb potential V(*r*) of a given Fock state is obtained from the potential energies of the two constituents and the energy of the field:

$$V(r) = eA^0(\boldsymbol{x} = \boldsymbol{x}_1; \boldsymbol{x}_1, \boldsymbol{x}_2) - eA^0(\boldsymbol{x} = \boldsymbol{x}_2; \boldsymbol{x}_1, \boldsymbol{x}_2) + \tfrac{1}{4}\int d^3\boldsymbol{x} F_{\mu\nu}F^{\mu\nu}$$

$$= -\frac{e^2}{4\pi}\frac{1}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|}$$

where $\frac{1}{4}\int d^3\boldsymbol{x} F_{\mu\nu}F^{\mu\nu} = \frac{e^2}{4\pi}\frac{1}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|}$ was evaluated using $A^0(\boldsymbol{x};\boldsymbol{x}_1, \boldsymbol{x}_2)$.

Irrelevant infinities of the form $1/|\boldsymbol{x}_1 - \boldsymbol{x}_1|$ were discarded in V(*r*).

This view of the interaction energy is equivalent to the standard “central potential” one, but adds some insight.

In particular, it allows us to consider a homogenous solution to Gauss’ law.

Gauss' law with a non-vanishing boundary condition

Consider adding a homogeneous solution to Gauss' law:

$$-\boldsymbol{\nabla}_{\boldsymbol{x}}^2 A^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2) = e\big[\delta^3(\boldsymbol{x}-\boldsymbol{x}_1) - \delta^3(\boldsymbol{x}-\boldsymbol{x}_2)\big]$$

$$A^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2) = \Lambda^2\,\hat{\boldsymbol{\ell}}\cdot\boldsymbol{x} + \frac{e}{4\pi}\left(\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_1|} - \frac{1}{|\boldsymbol{x}-\boldsymbol{x}_2|}\right)$$

where Λ and ℓ are $\boldsymbol{x}$-independent, but may depend on $\boldsymbol{x}_1, \boldsymbol{x}_2$

The action for the $(\boldsymbol{x}_1, \boldsymbol{x}_2)$ configuration becomes

$$-\frac{1}{4}\int d^3\boldsymbol{x}\,F_{\mu\nu}F^{\mu\nu} = \tfrac{1}{2}\Lambda^4\int d^3\boldsymbol{x} + \frac{1}{3}e\Lambda^2\hat{\boldsymbol{\ell}}\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2) - \frac{e^2}{4\pi}\frac{1}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}$$

The first (divergent) term must not depend on $(\boldsymbol{x}_1, \boldsymbol{x}_2)$, hence $\Lambda \neq \Lambda(\boldsymbol{x}_1, \boldsymbol{x}_2)$

Stationarity of the second term requires $\hat{\boldsymbol{\ell}} = \hat{\boldsymbol{\ell}}(\boldsymbol{x}_1,\boldsymbol{x}_2) \parallel \boldsymbol{x}_1 - \boldsymbol{x}_2$

The linear potential

With Λ and ℓ thus determined the A^0 field gives the potential

$$V(\boldsymbol{x}_1, \boldsymbol{x}_2) = \frac{2}{3} e\Lambda^2 |\boldsymbol{x}_1 - \boldsymbol{x}_2| - \frac{e^2}{4\pi|\boldsymbol{x}_1 - \boldsymbol{x}_2|}$$

The linear term involves the non-perturbative scale $e\Lambda^2$ set by the non-vanishing boundary condition on $F_{\mu\nu} F^{\mu\nu}$ for $|\boldsymbol{x}| \to \infty$ for each configuration.

Stationarity of the action ensured rotational invariance. Also boost covariance is fulfilled, in a non-trivial way as we shall see later.

Each Fock state $(\boldsymbol{x}_1, \boldsymbol{x}_2)$ has a constant electric field extending to $|\boldsymbol{x}| = \infty$. A distant observer sees the coherent field summed over Fock states which vanishes for neutral states.

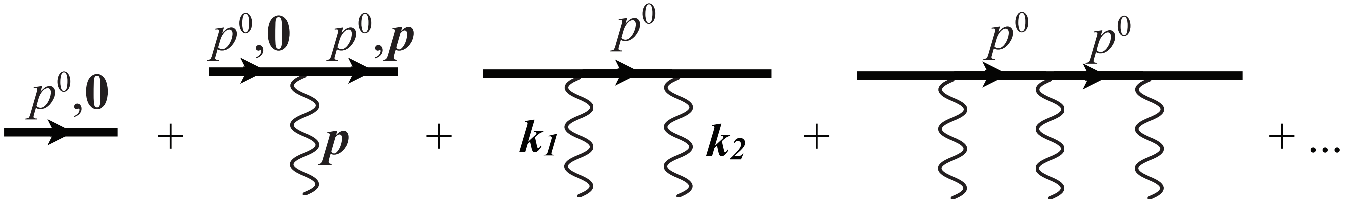
The coefficient of the linear potential may be taken to be independent of $\hbar$, allowing relativistic bound states at lowest order in $\hbar$.

Using retarded propagators at $\hbar^0$

Tree diagrams are independent of the $i\varepsilon$ prescription.

The bound state energies E_R of a fermion in an instantaneous potential

$$G(p^0,\boldsymbol{p}) =$$



$$= \frac{R(E_R,\boldsymbol{p})}{p^0-E_R}+\ldots$$

may be evaluated using retarded propagators (since $p^0 \neq -E_p$)

$$S_R(p^0,\boldsymbol{p}) = i\,\frac{\not{p}+m_e}{(p^0-E_p+i\varepsilon)(p^0+E_p+i\varepsilon)}$$

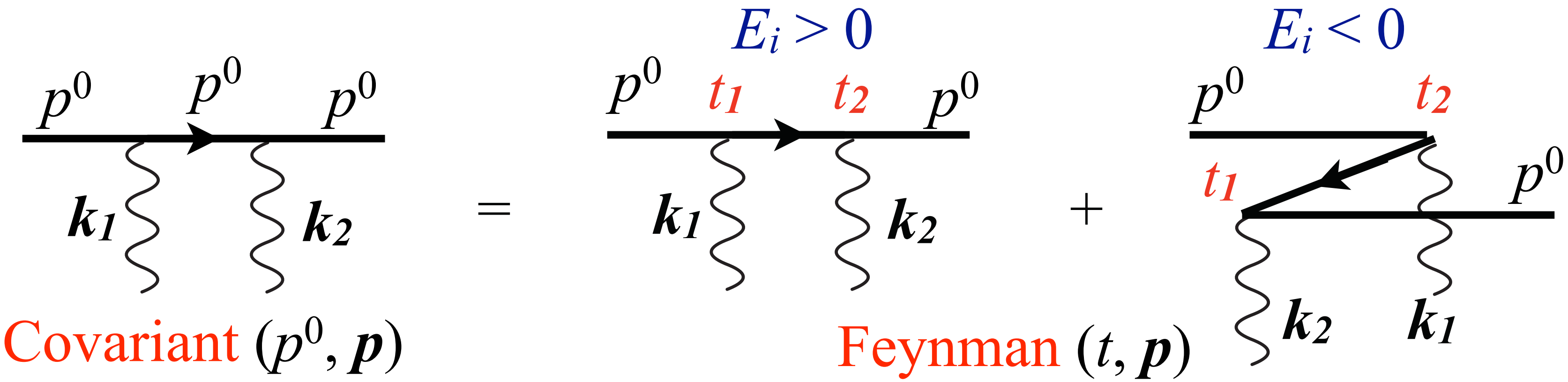
which only propagate forward in time,

$$S_R(t,\boldsymbol{p}) = \frac{\theta(t)}{2E_p}\left[(E_p\gamma^0-\boldsymbol{p}\cdot\boldsymbol{\gamma}+m_e)e^{-iE_pt}+(E_p\gamma^0+\boldsymbol{p}\cdot\boldsymbol{\gamma}-m_e)e^{iE_pt}\right]$$

thus allowing a hamiltonian description.

Wave function dependence on iε

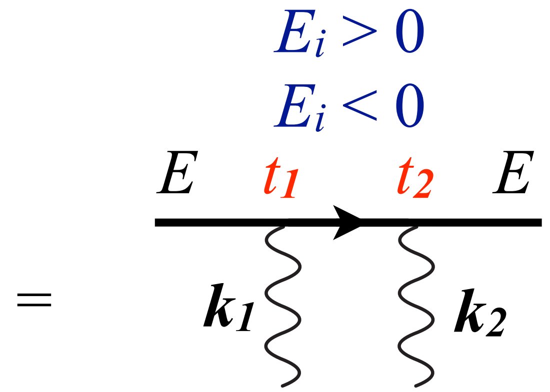
The time-ordered diagrams, and hence also the equal-time wave functions of bound states, depend on the $i\varepsilon$ prescription,



Covariant $(p^0, \boldsymbol{p})$

Feynman $(t, \boldsymbol{p})$

Text



Retarded $(t, \boldsymbol{p})$

The Dirac "single particle" states with E > 0 and E < 0 are obtained with retarded propagators.

The E < 0 states correspond to intermediate states with extra particle pairs using the causal (Feynman) prescription.

Bound state energies are independent of $i\varepsilon$ only at lowest order in $\hbar$!

Operator description of retarded propagation

The retarded propagator: $S_R(p) = i\,\dfrac{\not{p}+m}{(p^0-\sqrt{\boldsymbol{p}^2+m^2}+i\varepsilon)(p^0+\sqrt{\boldsymbol{p}^2+m^2}+i\varepsilon)}$

can be expressed as: $S_R(x-y) = {}_R\langle 0|\,T[\psi(x)\bar{\psi}(y)]\,|0\rangle_R$

where $|0\rangle_R = N^{-1}\prod_{\boldsymbol{p},\lambda} d^\dagger_{\boldsymbol{p},\lambda}|0\rangle$ is the "retarded vacuum", for which

$$\psi(x)|0\rangle_R = \int\sum_\lambda\left[u(\boldsymbol{p},\lambda)e^{-ip\cdot x}b_{\boldsymbol{p},\lambda} + v(\boldsymbol{p},\lambda)e^{ip\cdot x}d^\dagger_{\boldsymbol{p},\lambda}\right]|0\rangle_R = 0$$

Hence in the Interaction Picture:

$$H_I(t)|0\rangle_R = e\int d^3\boldsymbol{x}\,A^0(\boldsymbol{x})\,\psi^\dagger(t,\boldsymbol{x})\psi(t,\boldsymbol{x})|0\rangle_R = 0$$

No particle production in the retarded vacuum.

Describes physics at lowest order in hbar.

Dirac equation from Hamiltonian formulation

The bound state: $$|E,t=0\rangle = \int d^3\boldsymbol{x}\psi^\dagger(t=0,\boldsymbol{x})\varphi(\boldsymbol{x})|0\rangle_R$$

where φ(***x***) is the Dirac wave function.

Fock amplitude: $$\begin{aligned}\phi(t,\boldsymbol{x}) &= {}_R\langle 0|\psi(t,\boldsymbol{x})|E,t\rangle \\ &= \varphi(\boldsymbol{x})\ \exp(-iEt)\end{aligned}$$ for a stationary state

From

$$i\frac{d\phi(0,\boldsymbol{x})}{dt} = {}_R\langle 0|i\frac{d\psi(0,\boldsymbol{x})}{dt}|E,0\rangle + {}_R\langle 0|\psi(t,\boldsymbol{x})H_I|E,t\rangle = E\phi(0,\boldsymbol{x})$$

follows the Dirac equation: $$(-i\boldsymbol{\nabla}\cdot\boldsymbol{\gamma} + e\gamma^0 A^0(\boldsymbol{x}) + m)\varphi(\boldsymbol{x}) = E\gamma^0\varphi(\boldsymbol{x})$$

Remember: There are Fock states with virtual pairs in the standard vacuum | 0 ⟩

Determination of A^0 for an $e^-\mu^+$ bound state

Define an | e⁻ μ⁺ ⟩ state in terms of a 4 × 4 wave function χ

$$|E, t=0\rangle = \int d\boldsymbol{y}_1 d\boldsymbol{y}_2 \psi_e^\dagger(t=0,\boldsymbol{y}_1)\chi(\boldsymbol{y}_1,\boldsymbol{y}_2)\psi_\mu(t=0,\boldsymbol{y}_2)|0\rangle_R$$

All matrix elements of the QED operator equation of motion must vanish:

$$\partial_\mu F^{\mu\nu}(x) - e\sum_{i=e,\mu}\bar\psi_i(x)\gamma^\nu\psi_i(x) = 0 \qquad \text{(operator EOM)}$$

$$_R\langle 0|\psi^\dagger_{\mu\beta}(0,\boldsymbol{x}_2)\psi_{e\alpha}(0,\boldsymbol{x}_1)\,(\text{EOM})\,|E,0\rangle = 0 \qquad \big(\text{matrix element} = 0\big) \Rightarrow$$

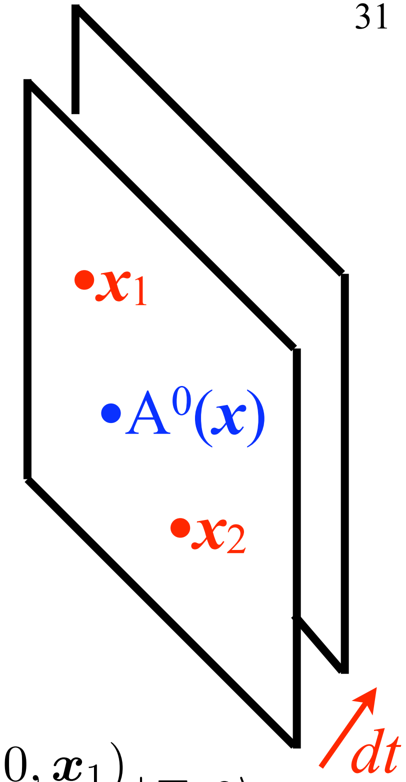
$$-\boldsymbol{\nabla}^2_{\boldsymbol{x}} A^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2) = e\big[\delta^3(\boldsymbol{x}-\boldsymbol{x}_1) - \delta^3(\boldsymbol{x}-\boldsymbol{x}_2)\big]$$

Relativistically moving charges generate $A^i \neq 0$ at $\mathcal{O}(e)$.

Neglecting $\mathcal{O}(e^2)$ only A^0 survives and the potential is purely linear:

$$V(\boldsymbol{x}_1,\boldsymbol{x}_2) = \frac{2}{3}e\Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2|$$

Bound state equation for a linear potential



Keeping only the linear, $\mathcal{O}(\hbar^0)$ A^0 field in H_I ,
and requiring stationarity:

$$\begin{aligned}\phi_{\alpha\beta}(t;\boldsymbol{x}_1,\boldsymbol{x}_2) \equiv_R \langle 0|\psi_{\mu\beta}(t,\boldsymbol{x}_2)\psi^\dagger_{e\alpha}(t,\boldsymbol{x}_1)|E,t\rangle \\ = e^{-iEt}\phi_{\alpha\beta}(t=0;\boldsymbol{x}_1,\boldsymbol{x}_2)\end{aligned}$$

imposes:

$$\begin{aligned}i\frac{d\phi_{\alpha\beta}(0;\boldsymbol{x}_1,\boldsymbol{x}_2)}{dt} = \langle 0|i\frac{d\psi^\dagger_{\mu\beta}(0,\boldsymbol{x}_2)}{dt}\psi_{e\alpha}(0,\boldsymbol{x}_1)|E,0\rangle \quad &+ \quad i\psi^\dagger_{\mu\beta}(0,\boldsymbol{x}_2)\frac{d\psi_{e\alpha}(0,\boldsymbol{x}_1)}{dt}|E,0\rangle \\ +\langle 0|\psi^\dagger_{\mu\beta}(0,\boldsymbol{x}_2)\psi_{e\alpha}(0,\boldsymbol{x}_1)[H_I(0)+E_A]|E,0\rangle \quad &= \quad E\,\phi_{\alpha\beta}(0;\boldsymbol{x}_1,\boldsymbol{x}_2)\end{aligned}$$

where $E_A = -\frac{1}{3}e\Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2|$ is the energy stored in the field, which gives

$$\boxed{\begin{aligned}\gamma^0(-i\boldsymbol{\nabla}_1\cdot\boldsymbol{\gamma}+m_e)\chi(\boldsymbol{x}_1,\boldsymbol{x}_2) \quad &- \quad \chi(\boldsymbol{x}_1,\boldsymbol{x}_2)\gamma^0(i\boldsymbol{\nabla}_2\cdot\boldsymbol{\gamma}+m_\mu) \\ &= \quad [E-V(\boldsymbol{x}_1,\boldsymbol{x}_2)]\chi(\boldsymbol{x}_1,\boldsymbol{x}_2)\end{aligned}}$$

Remarks

We have derived an $\mathcal{O}\left(\hbar^0\right)$ “Born level” bound state equation for QED (an analogous equation applies to QCD, see next slides).

The linear potential results from a non-trivial boundary condition on A^0.

The bound state is described by a natural extension of the Dirac equation to two particles, which was already studied phenomenologically.

Breit (1929)
Suura *et al* (1977)
Krolikowski *et al*

The explicit derivation allows to explore in detail the properties of the bound states and the inclusion of higher order corrections.

The bound states have some unusual and welcome features:

- The bound state energies are covariant under boosts (this holds only for a linear potential)
- Abundant virtual pair production resembles features of the parton model

u$\bar{\text{d}}$ meson states in in QCD

$$\mathcal{L}_{QCD} = -\tfrac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu} + \textstyle\sum_f \bar{\psi}_f^A(i\not{\partial} - g\not{A}_a T^a_{AB} - m_f)\psi_f^B$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - g f_{abc} A_b^\mu A_c^\nu$$

$$|E, t=0\rangle = \int d^3\boldsymbol{y}_1 d^3\boldsymbol{y}_2\, \psi_u^{A\dagger}(t=0,\boldsymbol{y}_1)\chi^{AB}(\boldsymbol{y}_1,\boldsymbol{y}_2)\psi_d^B(t=0,\boldsymbol{y}_2)|0\rangle_R$$

Under time-independent gauge transformations $\psi(t,\boldsymbol{x}) \to U(\boldsymbol{x})\psi(t,\boldsymbol{x})$ the wave function transforms as

$$\chi(\boldsymbol{y}_1,\boldsymbol{y}_2) \to U(\boldsymbol{y}_1)\chi(\boldsymbol{y}_1,\boldsymbol{y}_2)U^\dagger(\boldsymbol{y}_2)$$

In a gauge where $\chi^{AB}(\boldsymbol{y}_1,\boldsymbol{y}_2) = \delta^{AB}\chi(\boldsymbol{y}_1,\boldsymbol{y}_2)$

only the diagonal color fields A_a^0 with $a = 3,8$ can be nonzero.

Since $f_{a38} = 0$ the commutator terms do not contribute at $O(g)$.

Fock states with quarks of color *C* give the EOM for A_a^0

$$-\boldsymbol{\nabla}^2 A_a^0(\boldsymbol{x}) = g\, T_a^{CC}\left[\delta^3(\boldsymbol{x}-\boldsymbol{x}_1)-\delta^3(\boldsymbol{x}-\boldsymbol{x}_2)\right]$$

$$A_a^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2,C) = \Lambda_a^2\,\hat{\ell}_a\cdot\boldsymbol{x} + \frac{gT_a^{CC}}{4\pi}\left(\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_1|}-\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_2|}\right) \qquad (a=3,8)$$

$$-\frac{1}{4}\sum_a\int d^3\boldsymbol{x}F_{\mu\nu}^a F_a^{\mu\nu} = \sum_{a=3,8}\left[\frac{1}{2}\Lambda_a^4\int d^3\boldsymbol{x}+\frac{1}{3}g\Lambda_a^2\,T_a^{CC}\hat{\ell}_a\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2)+\mathcal{O}\left(g^2\right)\right]$$

$\Lambda^4 \equiv \sum_{a=3,8}\Lambda_a^4$ must be independent of $\boldsymbol{x}_1$, $\boldsymbol{x}_2$, and $\hat{\boldsymbol{\ell}}_a \parallel \boldsymbol{x}_1-\boldsymbol{x}_2$

Determining Λ_3/Λ_8 from stationarity it turns out that
the potential is independent of the quark color *C*,

$$V(\boldsymbol{x}_1,\boldsymbol{x}_2) = \frac{2g\Lambda^2}{3\sqrt{3}}|\boldsymbol{x}_1-\boldsymbol{x}_2|$$

and the bound state equation for the color singlet wave function χ has the same form as in QED.

uds baryon states in in QCD

$$|E,t=0\rangle = \int \prod_{j=1}^{3} d^3\boldsymbol{y}_j\, \psi_{u\alpha_1}^{A\dagger}(t=0,\boldsymbol{y}_1)\psi_{d\alpha_2}^{B\dagger}(t=0,\boldsymbol{y}_2)\psi_{s\alpha_3}^{C\dagger}(t=0,\boldsymbol{y}_3)\chi_{ABC}^{\alpha_1\alpha_2\alpha_3}(\boldsymbol{y}_1,\boldsymbol{y}_2,\boldsymbol{y}_3)|0\rangle_R$$

In a gauge where

$$\chi_{ABC}^{\alpha_1\alpha_2\alpha_3}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3) = \epsilon_{ABC}\chi^{\alpha_1\alpha_2\alpha_3}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3)$$

the relevant gauge fields are, for quark colors $ABC = 123$

$$\begin{aligned} A_3^0(\boldsymbol{x};\{\boldsymbol{x}_i\},ABC=123) &= \Lambda_3^2\,\hat{\ell}_3\cdot\boldsymbol{x} + \frac{g}{4\pi}\frac{1}{2}\left(\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_1|} - \frac{1}{|\boldsymbol{x}-\boldsymbol{x}_2|}\right) \\ A_8^0(\boldsymbol{x};\{\boldsymbol{x}_i\},ABC=123) &= \Lambda_8^2\,\hat{\ell}_8\cdot\boldsymbol{x} + \frac{g}{4\pi}\frac{1}{2\sqrt{3}}\left(\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_1|} + \frac{1}{|\boldsymbol{x}-\boldsymbol{x}_2|} - 2\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_3|}\right) \end{aligned}$$

and the interference term of $O(g)$ in the action is

$$S_{int}^{123} = \frac{g\Lambda_3^2}{6}\,\hat{\boldsymbol{\ell}}_3\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2) + \frac{g\Lambda_8^2}{6\sqrt{3}}\,\hat{\boldsymbol{\ell}}_8\cdot(\boldsymbol{x}_1+\boldsymbol{x}_2-2\boldsymbol{x}_3)$$

and is stationary for

$$\hat{\ell}_3 \parallel \boldsymbol{x}_1 - \boldsymbol{x}_2, \qquad\qquad \hat{\ell}_8 \parallel \boldsymbol{x}_1 + \boldsymbol{x}_2 - 2\boldsymbol{x}_3$$

$$\frac{\Lambda_3^2}{\Lambda_8^2} = \sqrt{3}\,\frac{|\boldsymbol{x}_1 - \boldsymbol{x}_2|}{|\boldsymbol{x}_1 + \boldsymbol{x}_2 - 2\boldsymbol{x}_3|}$$

For different colors *ABC* = 213, *etc*., the result is given by $\boldsymbol{x}_1 \leftrightarrow \boldsymbol{x}_2$, *etc*.

When expressed in terms of the universal strength
the potential obtained for stationary action is
the same for all color choices *ABC*,

$$\Lambda^4 \equiv \sum_{a=3,8} \Lambda_a^4$$

$$V(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) = \frac{\sqrt{2}g\Lambda^2}{3\sqrt{3}}\sqrt{(\boldsymbol{x}_1 - \boldsymbol{x}_2)^2 + (\boldsymbol{x}_2 - \boldsymbol{x}_3)^2 + (\boldsymbol{x}_3 - \boldsymbol{x}_1)^2}$$

and the bound state equation for the color singlet wave function is

$$\sum_{j=1}^{3} \left[\gamma^0(-i\boldsymbol{\nabla}_j \cdot \boldsymbol{\gamma}_j + m_j)\right]\chi = (E - V)\chi$$

Interesting properties of the meson solutions

- Lorentz covariance: $E = \sqrt{\boldsymbol{k}_{CM}^2 + M^2}$ χ transforms in a novel way

  This only holds for a purely linear potential.

- Rotational invariance: $\boldsymbol{J} = \boldsymbol{S} + \boldsymbol{L}$ commutes with the Hamiltonian. Allows separation of angular dependence in CM.

- Linear Regge trajectories: $\alpha' = 1/8g\Lambda^2$

- High relative momentum components with oscillating phase ("Klein paradox"). Related to Z-diagrams, i.e., to multi-particle Fock states: ⇒ Sea quarks?

Frame dependence ($t_1 = t_2$ in all frames!)

The wave function of a bound state with CM momentum $\boldsymbol{k}$ has

$$\chi(\boldsymbol{x}_1, \boldsymbol{x}_2) = \exp\left[i\boldsymbol{k}\cdot(\boldsymbol{x}_1 + \boldsymbol{x}_2)/2\right] \phi(\boldsymbol{x}_1 - \boldsymbol{x}_2)$$

The equation for $\phi(\boldsymbol{x})$ becomes (for $m_1 = m_2 = m;\quad \boldsymbol{x} = \boldsymbol{x}_1 - \boldsymbol{x}_2$):

$$-i\boldsymbol{\nabla}\cdot[\boldsymbol{\alpha}, \phi] + \tfrac{1}{2}\boldsymbol{k}\cdot\{\boldsymbol{\alpha}, \phi\} + m\left[\gamma^0, \phi\right] = (E - V)\phi$$

where the solutions $\phi(\boldsymbol{x})$ and E depend on the CM momentum $\boldsymbol{k}$.

$E = \sqrt{\boldsymbol{k}^2 + M^2}$ holds only for a purely linear potential $V(|\boldsymbol{x}|)$!

PH (1986)

Boost covariance of the wave function

How should relativistic, equal-time wave functions transform under Lorentz boosts? The above bound state equation gives, for $\boldsymbol{k} = (0,0,k)$:

$$\gamma^0 \phi_k(s) = e^{\zeta\alpha_3/2}\gamma^0\phi_{k=0}(s)e^{-\zeta\alpha_3/2}$$

for $\phi_k(\boldsymbol{s}) \equiv \phi_k(x_1=0, x_2=0, x_3(\boldsymbol{s}))$ (and its transverse derivative) on the z-axis and with the "invariant distance" s defined by

$$\frac{ds}{dx_3} = \tfrac{1}{2}[E - V(x_3)]$$

and

$$\tanh\zeta(s) = -\frac{k}{E-V}$$

$$s(x_3) = \tfrac{1}{2}x_3\left[E - \tfrac{1}{2}V(x_3)\right]$$

Note: For $V << E$ this reduces to standard Lorentz contraction,
but in general the boost depends on the canonical energy $p^0 - eA^0$.

Rotational covariance in the CM

Geffen and Suura (1977)

For $\boldsymbol{k} = 0$ and $m_1 = m_2$ the bound state equation becomes

$$-i\boldsymbol{\nabla}\cdot[\boldsymbol{\alpha},\phi(\boldsymbol{x})] + m\left[\gamma^0,\phi(\boldsymbol{x})\right] = (M - V)\phi(\boldsymbol{x})$$

Use a direct product of Pauli matrices $\{\rho_i\}$, $\{\sigma_i\}$: $\alpha_i = \rho_1 \times \sigma_i$, $\gamma^0 = \rho_3 \times 1$. to express the 4 × 4 wave function $\phi(\boldsymbol{x})$ in terms of four 2 × 2 χ_μ :

$$\phi(\boldsymbol{x}) = \Sigma_i\, \rho_i \times \chi_i = \begin{pmatrix} \chi_4 + \chi_3 & \chi_1 - i\chi_2 \\ \chi_1 + i\chi_2 & \chi_4 - \chi_3 \end{pmatrix}$$

The angular momentum operator $\boldsymbol{J}$:

$$\boldsymbol{J}\phi(\boldsymbol{x}) = \tfrac{1}{2}\left[\boldsymbol{1} \times \boldsymbol{\sigma},\phi(\boldsymbol{x})\right] + \boldsymbol{L}\phi(\boldsymbol{x})$$

where $\boldsymbol{L} \equiv -i\boldsymbol{x} \times \boldsymbol{\nabla}$

satisfies $[J_i, J_j] = i\epsilon_{ijk}J_k$ and commutes with the Hamiltonian.

Separation of variables in the CM

There are four independent eigenfunctions of J^2 and J^z:

$$
\begin{aligned}
\chi_{jm}^{(1)} &= Y_{jm}(\theta,\varphi)\,F_1(r)\\
\chi_{jm}^{(2)} &= \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\,Y_{jm}(\theta,\varphi)\,F_2(r)\\
\chi_{jm}^{(3)} &= \boldsymbol{\sigma}\cdot\boldsymbol{x}\,Y_{jm}(\theta,\varphi)\,F_3(r)\\
\chi_{jm}^{(4)} &= \boldsymbol{\sigma}\cdot\boldsymbol{L}\,Y_{jm}(\theta,\varphi)\,F_4(r)
\end{aligned}
$$

Considering parity and charge conjugation, the 2 × 2 wave functions χ_μ may be expressed as linear combinations of (some of) the above structures. This way one can identify states on the π, A_1 , and ρ trajectories, and obtain the corresponding radial equations for the $F_i(r)$.

The radial functions are potentially singular at $x = 0$ and at $M = V$.

The regular solutions have a discrete mass (M) spectrum.

Wave function properties (in CM, k = 0)

Separating the angular dependence, the wave function may be described by a set of radial functions $F(r)$. For the pion trajectory, with $P = (-1)^{J+1}$, $C = (-1)^{J}$:

$$F_1(r) = -\frac{2im}{E-V}\,F_2(r)$$

Geffen and Suura, PR D16 (1977) 3305

$$F_2''(r) + \left(\frac{2}{r} + \frac{V'}{E-V}\right)F_2'(r) + \left[\tfrac{1}{4}(E-V)^2 - \frac{J(J+1)}{r^2} - m^2\right]F_2(r) = 0$$

- $E = V(r)$ is a singular "turning point"
- Requirement that $F_1(r)$ is locally normalizable at $E = V$ quantizes E
- $F_2(r \to \infty) \propto \exp[\, iV'\, r^2]$: "Klein Paradox" corresponds to virtual pair production in a strong field. The wave function in the retarded vacuum implicitly describes virtual pairs of constant density per unit separation.
- High relative momenta between quarks can contribute to end-points of distribution amplitudes and high energy Regge exchange.

Comparison with the Quark Model

The quark model uses a potential $V(r) = g\Lambda^2 r - C_F \frac{\alpha_s}{r}$

where the Coulomb term (one gluon exchange) is perturbative.

In the present approach the linear (non-perturbative) term emerges as a homogenous solution of the equations of motion.

Perturbative gluon exchange is of order g^2 , hence is subdominant to the order linear term. Terms of order g^2 were dropped in the bound state equation.

This is the why boost covariance at equal time is expected to, and in fact does, hold only for a purely linear potential.

Summary of Talk

- Is there a systematic approximation of QCD which gives the quark model?
- Consider an hbar expansion for bound states: Born term at $\mathcal{O}\left(\hbar^0\right)$
- Determine A^0 from equation of motion (for each constituent configuration)
- Allow homogeneous solution: linear potential $A^0 = \boldsymbol{l} \cdot \boldsymbol{x}$
- Fix direction of $\boldsymbol{l}$ by stationarity of action (for each Fock state)
- Ignore $\mathcal{O}(g^2)$ (Coulomb exchange) – hence use purely linear potential
- Find meson and baryon states with interesting phenomenology
- Observe non-trivial Lorentz covariance for a linear potential
- Sea quarks generated implicitly, through use of retarded vacuum (no new degrees of freedom).