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Non-local interactions in renormalized Hamiltonians

Stanisław D. Głazek

Institute of Theoretical Physics, University of Warsaw

Renormalization Group Procedure for Effective Particles (RGPEP) transforms canonical Hamiltonians of local quantum field theories into non-local ones. Some generic features of renormalized non-local interaction Hamiltonian densities with a product of three fields on a light front hyperplane in space-time are illustrated and related to two-body bound-state wave functions.

S. D. Głazek, IFT/10/05. stglazek@fuw.edu.pl

Why Hamiltonians?

$$H|\psi\rangle = E|\psi\rangle$$

Relativistic ψ for proton in the Minkowski space-time?

How to define the appropriate "lattice"?

Partons at quantum amplitude level?

Formation of quantum strings of gluons?

...

$$H = \int d^3x \, \mathcal{H}(x)$$

Local Hamiltonians:

$$\mathcal{H}(x) = g \, \bar{\psi}(x) \, A(x) \, \psi(x)$$

$$\mathcal{H}(x) = g \, \bar{\psi}(x) \, \phi(x) \, \psi(x)$$

$$\mathcal{H}(x) = g \, Tr \, \partial_\mu A_\nu(x) \, [A^\mu(x), A^\nu(x)]$$

All fields at the same space-time point

What would be a non-local Hamiltonian like?

$$H = \int d^3x_1 \, d^3x_2 \, d^3x_3 \, \mathcal{H}(x_1, x_2, x_3)$$

$$\mathcal{H}(x_1, x_2, x_3) = g \, f(x_1, x_2, x_3) \, \phi_1(x_1) \, \phi_2(x_2) \, \phi_3(x_3)$$

stglazek@fuw.edu.pl

Different forms of H (Dirac 1949)

Instant: $t = 0$ Point: $x^2 = 0$ Front: $x^+ = t + z = 0$

Front: hyperplane swept by a front of a plane wave of light

Features of the Light Front (LF) form:

boost invariance (7th kinematical Poincare symmetry)

CMS and IMF ψ_{proton}

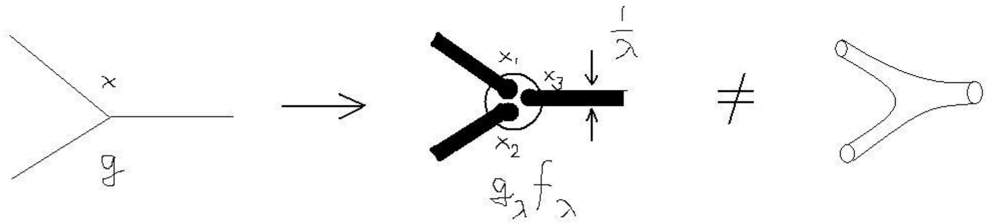
vacuum $H_{x^+=0}|0\rangle = 0$

divergences? regularization, renormalization, numerics

Renormalization of Hamiltonians (Głazek, Wilson 1993)

Renormalization Group Procedure for Effective Particles (**RGPEP**)

RGPEP (tested in various models at first, second, and third order)



$f_\lambda = f_\lambda(x_1, x_2, x_3)$ depends on RG scale λ

1st order RGPEP scale dependence $s = 1/\lambda^2$

RG equation for $H = H_0 + H_I$

$$\frac{d}{ds} H = [[H_0, H], H]$$

$$H(s=0) = H_{can} + CT$$

$$\frac{d}{ds} H = [[H_0, H_I], H_0] + O(H_I^2) \sim -H_0^2 H_I + 2H_0 H_I H_0 - H_I H_0^2$$

$$\frac{d}{ds} H_{Imn} \sim -(E_m - E_n)^2 H_{Imn} \,, \quad H_{Imn}(s) \sim e^{-s(E_m - E_n)^2} \, H_{Imn}(s=0)$$

$$\boxed{\text{IF:} \quad f_\lambda \sim e^{-(\Delta E/\lambda)^2} \qquad \longrightarrow \qquad \text{LF:} \quad f_\lambda \sim e^{-(\Delta \mathcal{M}/\lambda)^4}}$$

stglazek@fuw.edu.pl

Three fields vertex $H_I = \int d^3x \ \psi^3(x)$

$$\psi(x) = \int [p] \, a_p \, e^{-ip\,x}$$

$$\textit{bare} \qquad H_I = g \int [p_1 p_2 p_3] \ \delta_P \ (a^\dagger_{p_1} \, a^\dagger_{p_2} \, a_{p_3} + h.c.\,)$$

$$\textit{renormalized} \qquad H_{\lambda I} = g_\lambda \int [p_1 p_2 p_3] \ \delta_P \ e^{-[(p_1+p_2)^2 - p_3^2]^2/\lambda^4} \times \ (a^\dagger_{\lambda p_1} \, a^\dagger_{\lambda p_2} \, a_{\lambda p_3} + h.c.\,)$$

$a_\lambda = U_\lambda \, a \, U_\lambda^\dagger$ RGPEP determines U_λ

stglazek@fuw.edu.pl

bare canonical theory

$$\psi(x) = \int [p]\, a_p\, e^{-ip\,x} \qquad\qquad a_p = \int [x]\, \psi(x)\, e^{+ip\,x}$$

renormalized effective theory

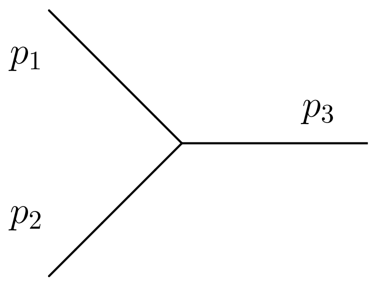
$$\psi_\lambda(x) = \int [p]\, a_{\lambda p}\, e^{-ip\,x} \qquad\qquad a_{\lambda p} = \int [x]\, \psi_\lambda(x)\, e^{+ip\,x}$$

$$\begin{aligned} H_{\lambda I} &= g_\lambda \int [p_1 p_2 p_3]\ \delta_P\ f_\lambda(p_1, p_2, p_3)\ a^\dagger_{\lambda p_1}\, a^\dagger_{\lambda p_2}\, a_{\lambda p_3} \\ &= g_\lambda \int [x_1 x_2 x_3]\ \ \bar{f}_\lambda(x_1, x_2, x_3)\ \psi^\dagger_\lambda(x_1)\, \psi^\dagger_\lambda(x_2)\, \psi_\lambda(x_3) \end{aligned}$$

non-local interaction in renormalized Hamiltonian

What to expect for $\bar{f}_\lambda(x_1, x_2, x_3)$?

Simplify to NR QM $\qquad f_\lambda(p_1, p_2, p_3) = e^{-[(p_1+p_2)^2 - m^2]/\lambda^2}$



$$(p_1 + p_2)^2 = \left(2\sqrt{m^2 + \vec{q}^{\,2}}\right)^2 \qquad \vec{p}_1 = \vec{P}/2 + \vec{q}$$

$$(p_1 + p_2)^2 - m^2 = 4\vec{q}^{\,2} + 3m^2 \qquad \vec{p}_2 = \vec{P}/2 - \vec{q}$$

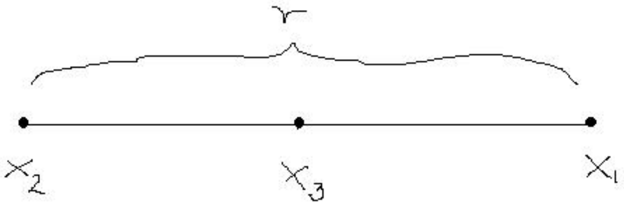
$$f_\lambda(p_1, p_2, p_3) \sim e^{-4\vec{q}^{\,2}/\lambda^2}$$

$$\int [p_1 p_2 p_3] \; \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_3) \; e^{-4\vec{q}^{\,2}/\lambda^2} \; e^{i(\vec{p}_1\vec{x}_1 + \vec{p}_2\vec{x}_2 - \vec{p}_3\vec{x}_3)}$$

$$= \int d^3P \, d^3q \; e^{-4\vec{q}^{\,2}/\lambda^2} \; e^{i\vec{q}(\vec{x}_1 - \vec{x}_2) + i\vec{P}(\vec{R} - \vec{x}_3)}$$

$$\bar{f}(x_1, x_2, x_3) \sim \delta^3\left[\vec{x}_3 - (\vec{x}_1 + \vec{x}_2)/2\right] \; e^{[\lambda(\vec{x}_1 - \vec{x}_2)/4]^2}$$

Non-relativistic intuition

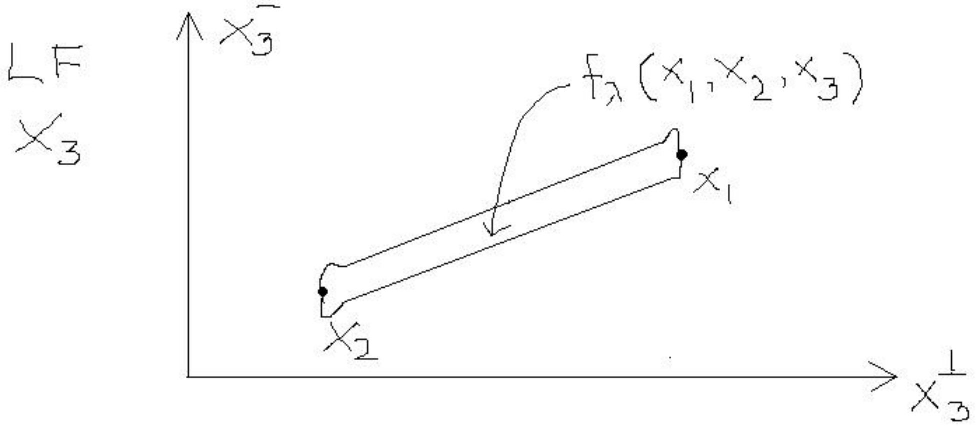


$$e^{-\frac{\lambda^2 r^2}{16}}$$

$$\delta^3\left(\vec{x}_3 - \frac{\vec{x}_1 + \vec{x}_2}{2}\right)$$

$$R = (x_1 + x_2)/2 \qquad r = x_1 - x_2$$

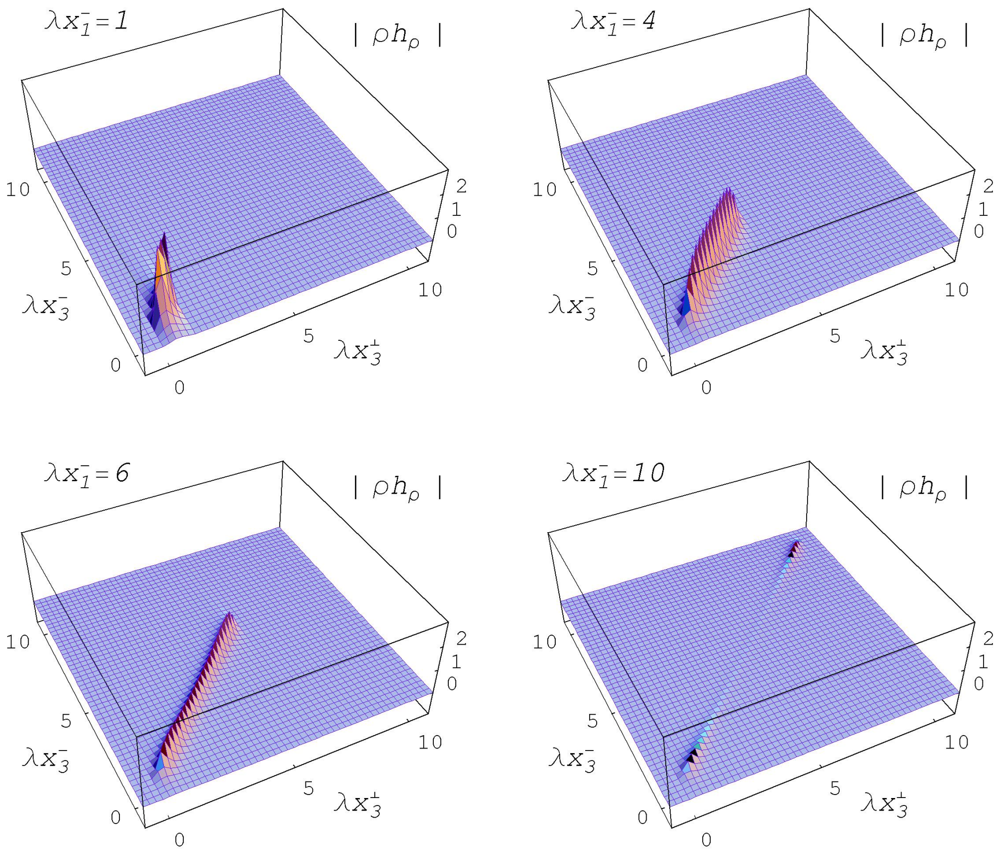
Relativistic result (qualitatively) is a density on a thin string



$$\bar{f}_\lambda(x_1, x_2, x_3) = h_\lambda(x_3 - R, r)$$

Relativistic theory with $\lambda \gg m \to 0$

$$\rho h_\rho(x_3) \sim |r^\perp| f_\lambda(x_3, R, r)$$



stglazek@fuw.edu.pl

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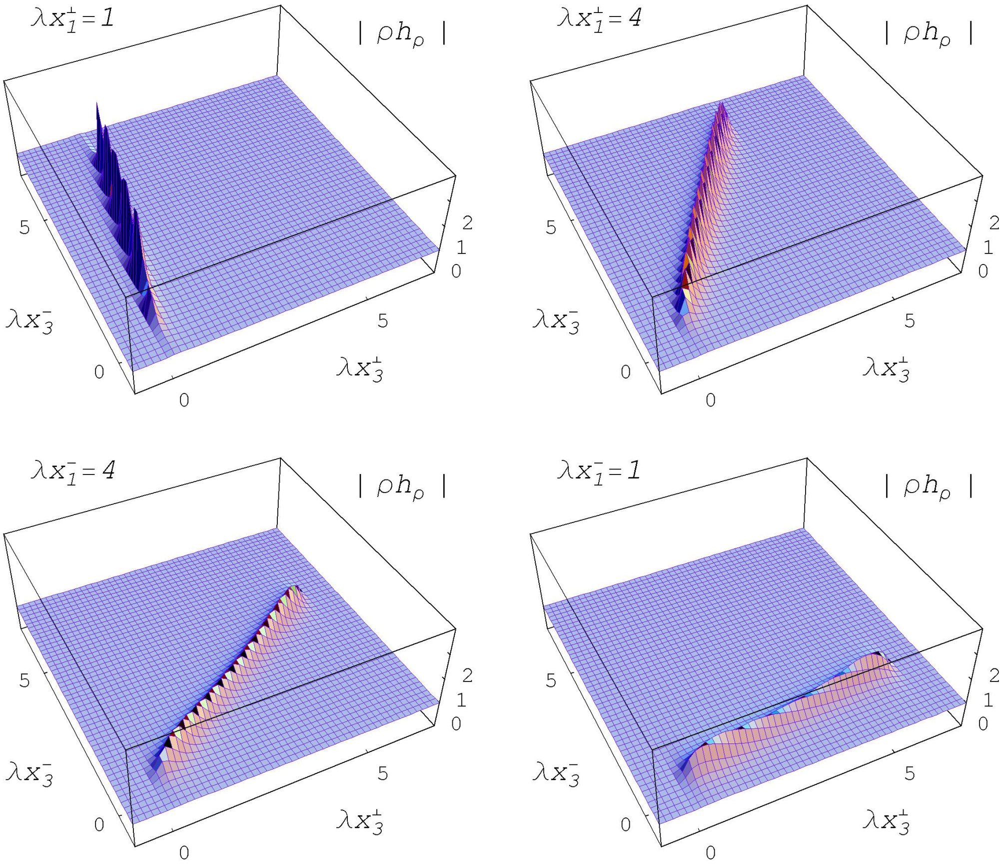
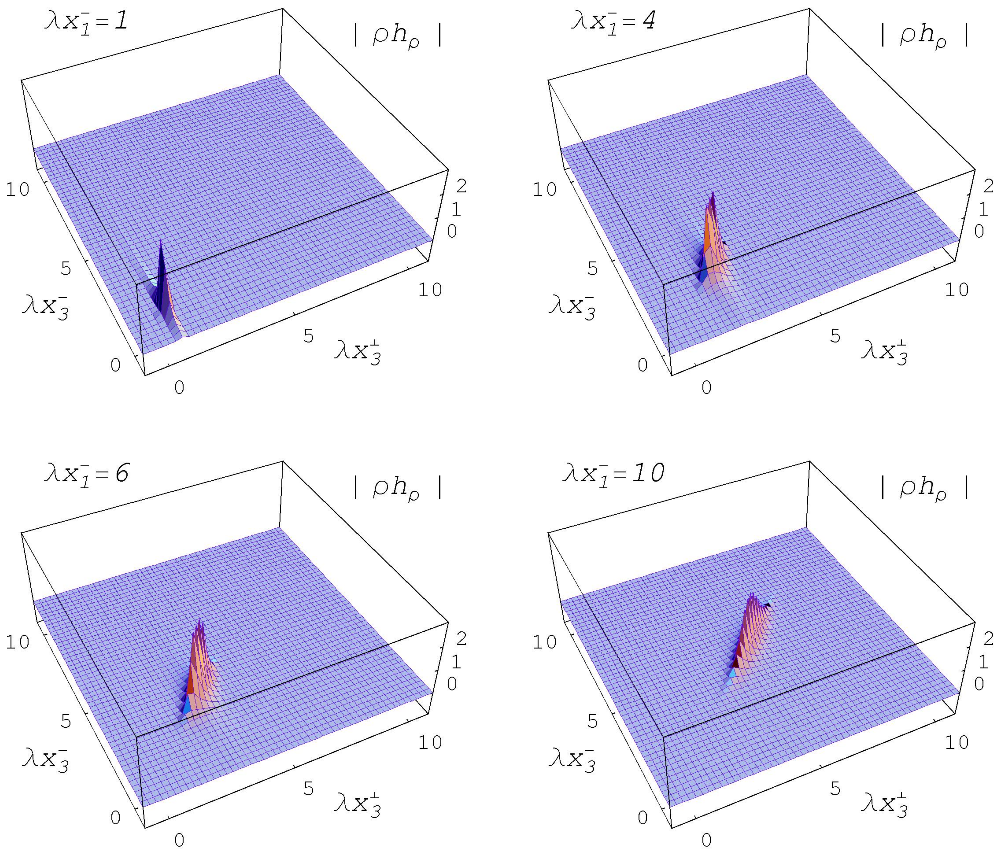


stglazek@fuw.edu.pl

Relativistic theory with $\lambda \sim m$

$$\rho h_\rho(x_3) \sim |r^\perp| f_\lambda(x_3, R, r)$$



stglazek@fuw.edu.pl

$f_\lambda(x_1, x_2, x_3)$ is related to a 2-body wave function

$\psi_{\lambda P} = G_{0P^-}\,\phi_{\lambda P}$

ϕ is the vertex fuction (think Lee model)

$$g_\lambda \bar{f}_\lambda(x_1, x_2, x_3) = \int_0^\infty \frac{dP^+}{2(2\pi)} \int \frac{d^2P^\perp}{(2\pi)^2}\, \phi_{\lambda P}(x_1, x_2)\, e^{iPx_3}$$

$$\phi_{\lambda P}(x_1, x_2) = g_\lambda \int d^3x_3\, \bar{f}_\lambda(x_1, x_2, x_3)\, e^{-iPx_3}$$

$\boxed{\lambda \gg m \to 0}$ partons in the IMF

$$\phi_{\lambda P}(x_1, x_2) = 3g_\lambda \left(\frac{\lambda}{4\pi}\right)^2 P^+ e^{-iPR} \times \int_0^1 dz\, z(1-z)\, e^{-i(z-1/2)Pr - \frac{1}{4}z(1-z)\,\lambda^2 r^{\perp 2}}$$

$\boxed{\lambda \lesssim m}$ constituents in the CMF

$$\phi_{\lambda P}(x_1, x_2) = 3g_\lambda \left(\frac{\lambda}{4\pi}\right)^2 P^+ e^{-iPR} \times C(\lambda/m) \quad e^{-\frac{\lambda^4}{96m^2}\left[\left(\frac{Pr}{2m}\right)^2 + r^{\perp 2}\right]}$$

$$\phi_{\lambda P} \sim P^+ \lambda^2 \phi_\lambda(Pr, r^2) \qquad r^+ = 0$$

No such construction in other forms of Hamiltonian dynamics

Pr **is dimensionless,** r^2 **has dimension,** λ **is necessary**

Conclusion

- RGPEP opens a frontier to explore
- $H_\lambda|\Psi_\lambda\rangle = M^2|\Psi_\lambda\rangle$
- Dynamics in 5 dimensions: space $\vec{x}$, time t, scale λ
- Insight into high-energy non-locality