QCD

AT THE DAWN OF THE LHC

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LECTURE III

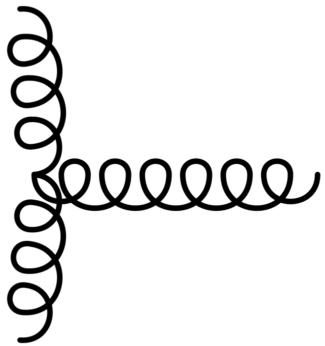
LARGE LOGS FROM GLUON RADIATION

- PRODUCE FINAL STATE OF MASS M^2 WITH C.M. ENERGY $s = \frac{M^2}{\tau}$

- UPON GLUON RADIATION THE CROSS SECTION $\hat{\sigma}(y, Q^2)$ GETS A CORRECTION
$\sigma(\tau, M^2) = \int_y^1 \frac{dy}{y} P\left(\frac{x}{y}\right) \int_{\mu^2}^{(s-M^2)^2/s} \frac{dk_t^2}{k_t^2} \hat{\sigma}(y, M^2)$

THE GLUON SPLITTING FUNCTION

$$P_{gg}(x) = 2C_A\left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right] + \beta_0 \delta(1-x)$$



LOGARITHMICALLY ENHANCED TERMS

- INFRARED LOGS: $\int_\tau^1 dy \frac{1}{1-y}_+ \sim \ln(1-\tau)$
- UV LOGS: $\int_\tau^1 dy \frac{1}{y} \sim \ln(\tau)$
- COLLINEAR LOGS: $\int_{\mu^2}^{(s-M^2)^2/s} \frac{dk_t^2}{k_t^2} \sim \ln\left[\frac{Q^2}{\mu^2}\frac{(1-\tau)^2}{\tau}\right] = \ln\frac{Q^2}{\mu^2} + \ln(1-\tau)^2 + \ln\tau$

SOFT-COLLINEAR $\Rightarrow$ DOUBLE LOGS AT EACH ORDER IN α_s (CONTROLLED BY KINEMATICS)
UV-COLLINEAR DOUBLE LOGS CANCEL IN SINGLET SECTOR $\Rightarrow$ SINGLE LOGS AT EACH ORDER IN α_s LOGS (CONTROLLED BY DYNAMICS: BFKL)
DOUBLE LOGS SURVIVE IN NONSINGLET/VALENCE, BUT POWER SUPPRESSED IN τ

THE NEED FOR RESUMMATION

- AT EACH EXTRA ORDER IN α_s
  - EXTRA $ln^2(1-x)$ (SOFT GLUONS, SOFT LOGS) IN DIAGONAL $q \to q$ AND $g \to g$ RADIATION
  - EXTRA $(1-x)ln^2(1-x)$ OF SIMILAR ORIGIN (SUBLEADING SOFT LOGS), BUT OTHER CONTRIBUTIONS OF SAME ORDER PRESENT
  - EXTRA $ln\frac{1}{x}$ (HIGH ENERGY, SMALL x, BFKL LOGS) IN GLUON SECTOR (GLUON RADIATION FROM GLUONS)
  - EXTRA $xln^2\frac{1}{x}$ (SUBLEADING HIGH ENERGY LOGS) IN NONSINGLET SECTOR
- THESE CONTRIBUTIONS SPOIL THE CONVERGENCE OF THE PERTURBATIVE EXPANSION
- MUST BE SUMMED TO ALL ORDERS (RESUMMATION) WHEN $\alpha_s \ln\frac{1}{x} \sim 1$ OR $\alpha_s \ln(1-x) \sim 1$
- SOFT GLUON RESUMMATION KINEMATICAL:
  - KNOWN IN CLOSED FORM FROM FIXED–ORDER SINCE THE LATE '80 (Sterman 1987; Catani, Trentadue 1989)
  - IMPLEMENTED IN THE MID-'90 (Catani, Mangano, Nason, Trentadue, 1997)
  - FIRST PHENOMENOLOGY AFTER 2000
- SMALL x RESUMMATION DYNAMICAL:
  - KNOWN AT LO SINCE THE LATE '70 (BFKL 1975-78), NLO LATE 90' (Fadin, Lipatov, 1998)
  - IMPLEMENTED IN THE EARLY 2000 (Ciafaloni, Colferai, Salam, Stasto; Altarelli, Ball, s.f.; 1998-2005)
  - FIRST PHENOMENOLOGY NOW

RESUMMATION

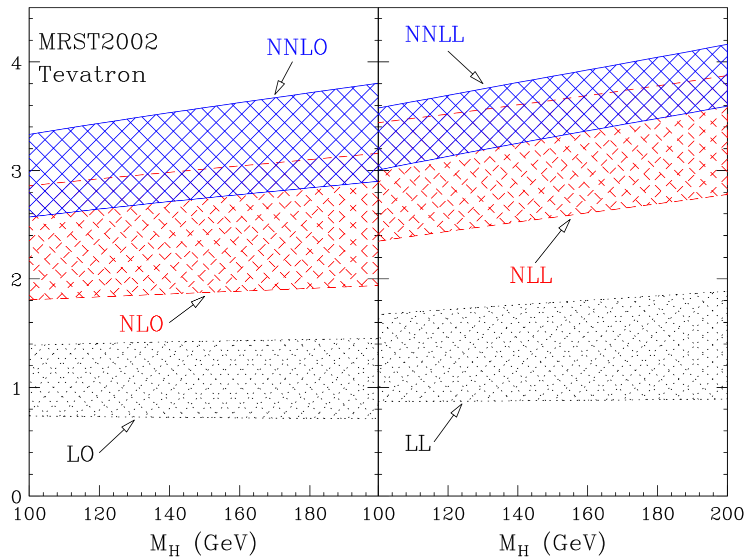
- SOFT GLUONS
- SMALL x
- SMALL x PHENOMENOLOGY?

SOFT GLUONS

THE IMPACT OF SOFT GLUONS

IMPORTANT WHENEVER PARTONIC CM ENERGY CLOSE TO FINAL STATE MASS
NEEDED FOR PERCENT ACCURACY IN HIGGS & TOP PRODUCTION

HIGGS (Catani et al., 2002)



TOP (Cacciari et al, 2008)

$\Delta\sigma_{t\bar{t}}^{NLO}(LHC14) = 11.6\%$

$\Delta\sigma_{t\bar{t}}^{NLO+NLL}(LHC14) = 9.3\%$

WHAT IS THE SOFT SCALE?

FOR HIGGS PRODUCTION AT LHC $\tau = m_h^2/s \sim 10^-4$: IS IT "LARGE"?

- HARD CROSS SECTION CONVOLUTED WITH PDF $\Rightarrow$ PARTONIC ENERGY $\hat{s} \sim < x > s$ SMALLER THAN HADRONIC s
- ARGUMENT CAN BE MADE RIGOROUS USING SADDLE–POINT
- SOFT RESUMMATION MORE IMPORTANT IF $\langle x \rangle << 1 \Rightarrow$ MORE IMPORTANT FOR SEA PROCESSES:
  - HIGGS: GLUON-GLUON
  - DY AT LHC (pp VS, $p\bar{p}$)
- RESUMMATION DETERMINED BY SMALL x GENERIC BEHAVIOUR OF PDFS!

SOFT GLUONS: THEORETICAL PROGRESS

- IR SINGULARITIES TO ALL ORDERS FOR ANY NUMBER OF LEGS CONJECTURED TO BE DETERMINED BY THREE UNIVERSAL ANOMALOUS DIMENSIONS (Becher, Neubert; Gardi, Magnea, 2009)
  PROVEN IN PLANAR LIMIT(Magnea, Dixon, Sterman, 2010) , EXACT RESULTS AVAILABLE IN $\mathcal{N} = 4$ CASE (Alday, 2009)

- TWO LOOP SOFT ANOMALOUS DIMENSIONS COMPUTED $\Rightarrow$ NNLL TOP PRODUCTION (Beneke, Falgari, Schiwnn, 2010)

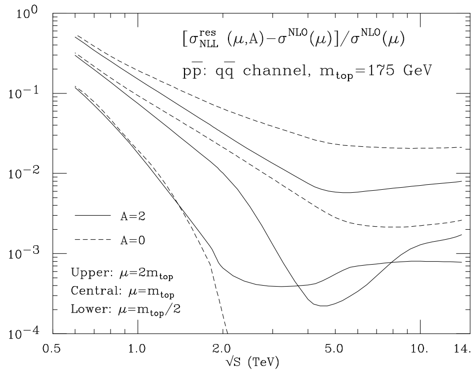
- CLASS OF $1 - x$ POWER-SUPPRESSED TERMS EXPONENTIATED (Laenen, Magnea, Stavenga 2008), CHECKED AT LL (Grunberg, Ravindran, 2009), BASED ON MODIFIED EVOLUTION EQN. $\alpha_s(Q^2(1-x)/x))$ (Dokshitzer, Marchesini, Salam, 2006)

- CONJECTURED EXPONENTIATION OF POWER-SUPPRESSED TERMS FOR PHYSICAL ANOMALOUS DIMENSION (Moch, Vogt 2009)

THE ROLE OF SUBLEADING TERMS

EXAMPLE 1 (OLD): TOP

- SUBLEADING TERMS HELP IN IMPROVING THE RESUMMED-FIXED ORDER MATCHING (CAN CHECK WITH KNOWN FIXED ORDERS)
- IF $\mu_R = \mu_F$ VARIED TOGETHER, NLL UNCERTAINTY ON σ_t
  2% W/O MATCHING TERMS $(A = 0)$,
  7% WITH MATCHING TERMS $(A = 2)$;
  9% UNCERTAINTY IF SCALES VARIED INDEPENDENTLY
  (Cacciari, Frixione, Mangano, Nason, Ridolfi, 1998)



(Bonciani, Catani, Mangano, Nason, 1998)

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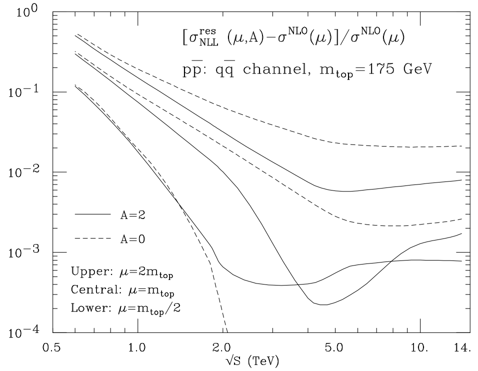


(Bonciani, Catani, Mangano, Nason, 1998)

EXAMPLE 2 (NEW): HIGGS

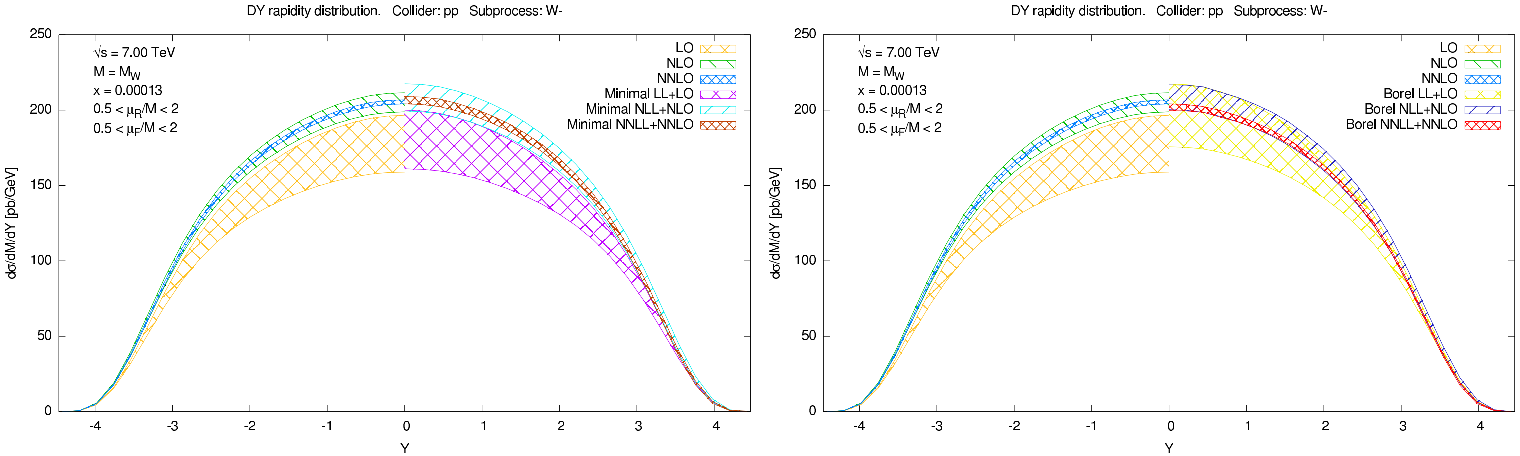
$m_H = 120\ GeV$, LHC 14 TeV, MSTW08NNLO PDFs

$\sigma^{NNLO+NNLL} - \sigma^{NNLO} = 3.4pb$ (6.8%) (de Florian, Grazzini 2009)
$\sigma^{NNLO+NNLL} - \sigma^{NNLO} = .9pb$ (1.8%) (Ahrens, Becher, Neubert, Yang 2009)
RESULT FOR DFG OBTAINED USING `http://theory.fi.infn.it/grazzini/hcalculators.html`

- ALMOST ALL THE DIFFERENCE DUE TO POWER SUPPRESSED TERMS
- RESUMMATION OF π^2 TERMS DONE BY AHRENS ET AL. (Parisi, 1980, Eynck, Laenen, Magnea, 2003)$\Rightarrow$ $\sigma^{NNLO+NNLL+\pi^2} - \sigma^{NNLO+NNLL} = 2.9pb$ (5.8%)

EXAMPLE 3 (NEW): DRELL-YAN, W Z

- RESUMMED SERIES DIVERGES, PRESCRIPTION NEEDED TO SUM IT
- STANDARD (MINIMAL) PRESCRIPTION INVOLVES CANCELLATION WITH CONTRIBUTION FROM UNPHYSICAL REGION; OTHER PRESCRIPTIONS AVAILABLE (E.G. BOREL SUM) (Ridolfi, S.F. et al, 2006-2009), DIFFERENT HO AND MATCHING
- IMPACT OF RESUMMATION COMPARABLE TO NNLO, BUT AMBIGUITY LARGE



(Bonvini, s.f., Ridolfi, prelim)

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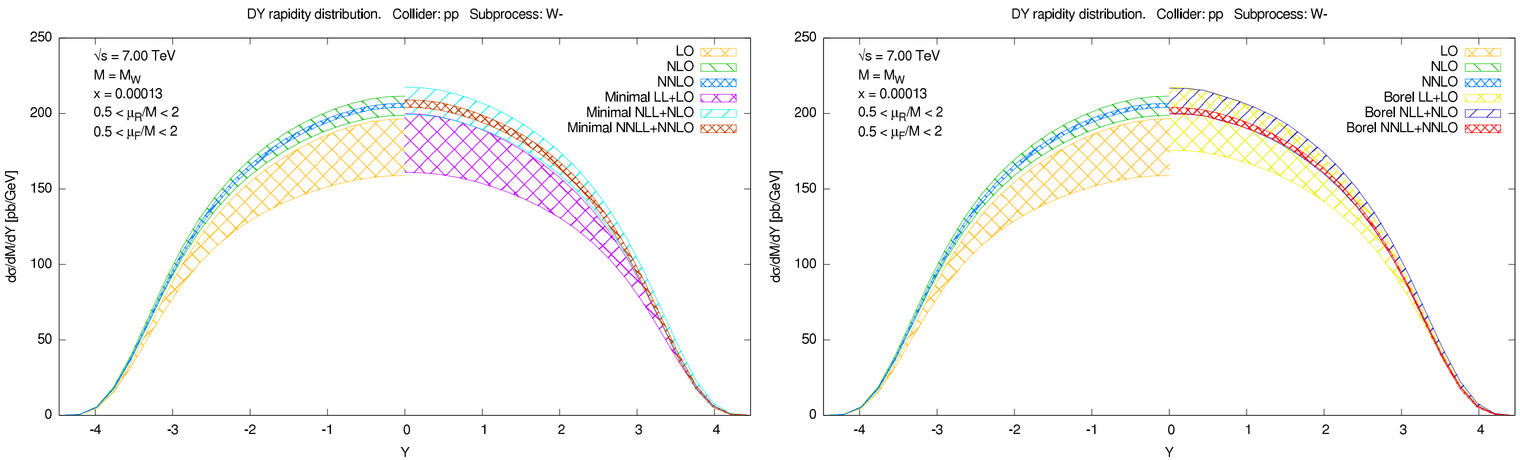


(Bonvini, s.f., Ridolfi, prelim)

DESIDERATA

- EXPLORE THE PHENOMENOLOGICAL IMPLICATIONS OF PRESCRIPTIONS, SUBLEADING TERMS, MATCHING
- TEST AND IMPLEMENT SUBLEADING RESUMMATION ASAP
- NEED RESUMMED PDFS!

CAVEAT

AT THE PERCENT LEVEL, NOT ONLY QCD CORRECTIONS ARE RELEVANT
EXAMPLE: NNLO HIGGS XSECT ($m_H = 120$ GEV, LHC14):
$\sigma^{NNLO} = 47.6pb$ (ABNY) OR $\sigma^{NNLO} = 51.1pb$ (DEFG),
6% DIFFERENCE LIKELY DUE TO EW CORRECTIONS (NOT INCLUDED IN ABNY)

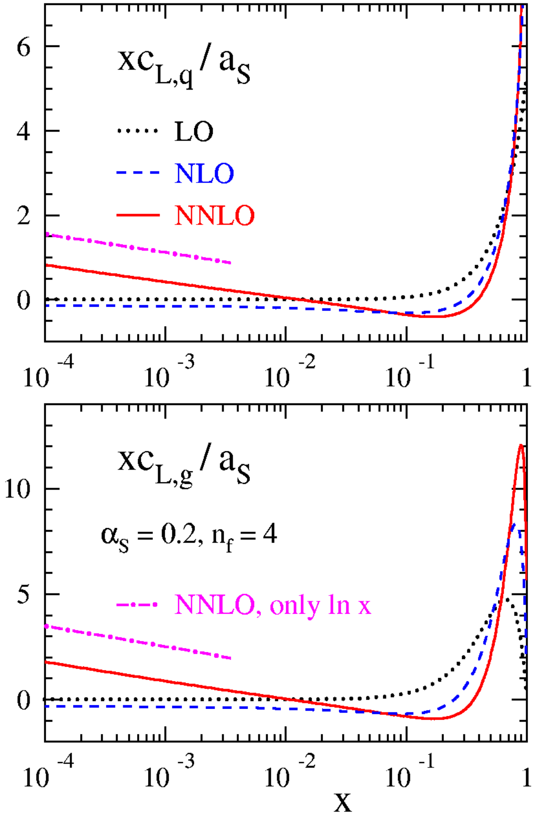
SMALL x

WHY WE SHOULD WORRY ABOUT SMALL X: THE NNLO CORRECTIONS

THEORY

THE COEFFICIENT FUNCTION C_L

(Moch, Vermaseren, Vogt 2005)

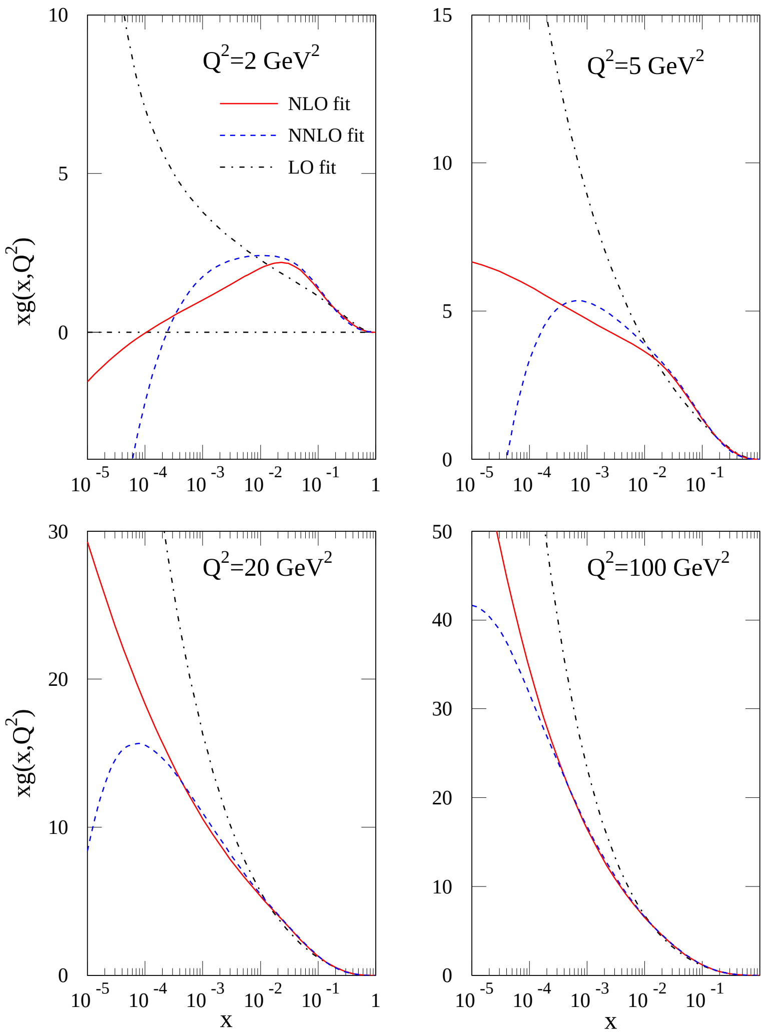


- PERTURBATIOMN THEORY UNSTABLE
- LEADING LOG APPROX POOR

PHENOMENOLOGY

THE BEST–FIT GLUON

(mstw 2008)

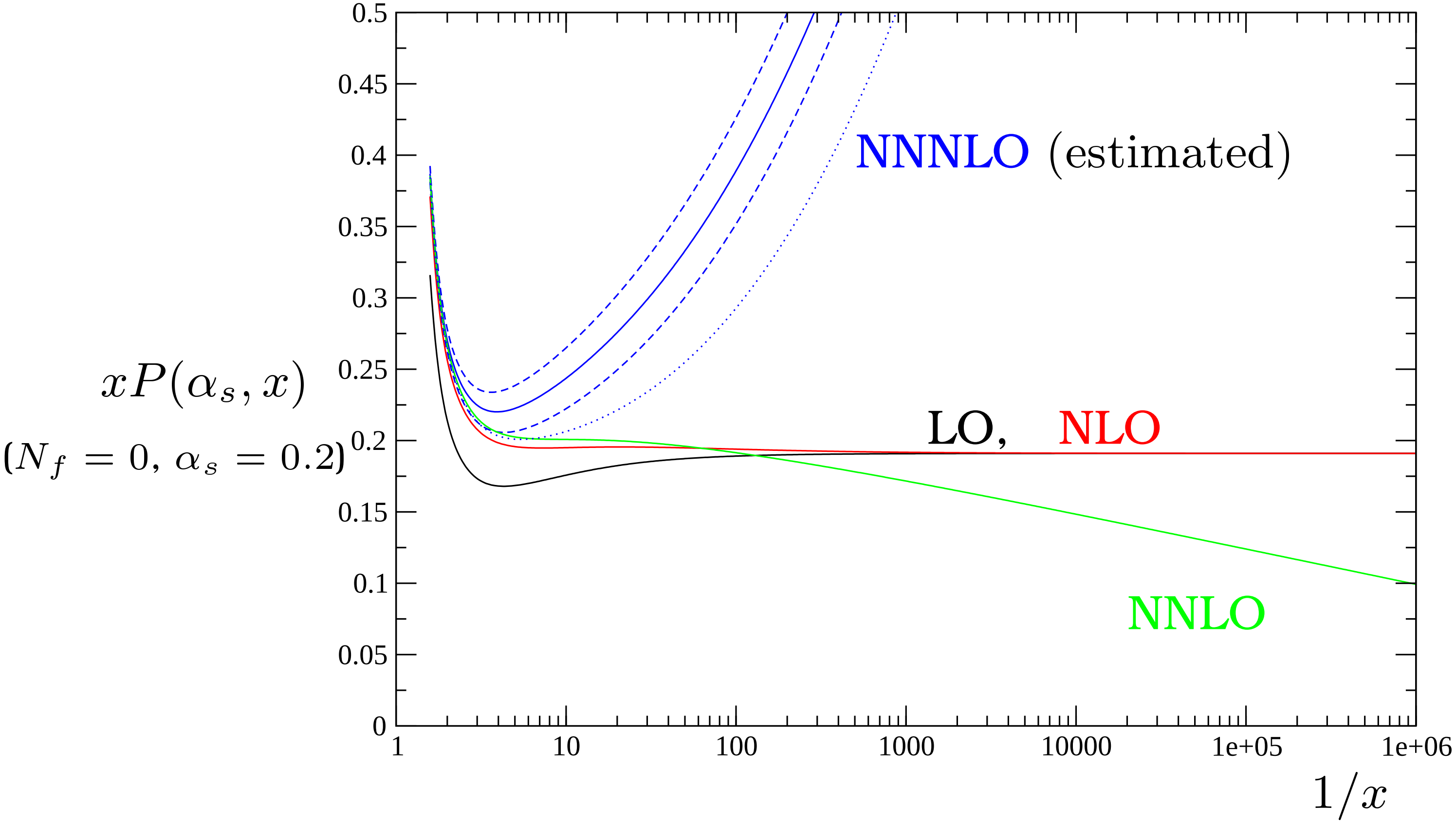


WHY WE SHOULD WORRY ABOUT SMALL X:

PERTURBATIVE INSTABILITY: THE SINGLET SPLITTING FUNCTION

$$xP(\alpha_s, x) \underset{x\to 0}{\sim}$$

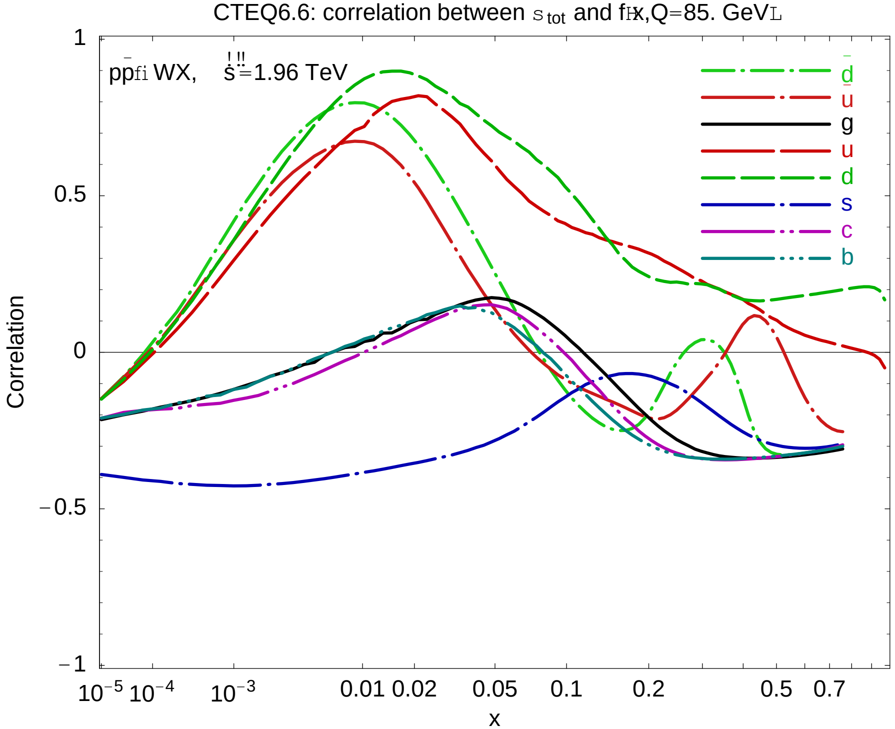
$$\alpha_s c_1^{(1)} + \alpha_s^2 c_2^{(1)} + \alpha_s^3 \left(c_3^{(2)} \ln x + c_3^{(1)} \right) + \alpha_s^4 \left(c_4^{(4)} \ln^3 x + c_4^{(3)} \ln^2 x + c_4^{(2)} \ln x + c_4^{(1)} \right) + \dots$$



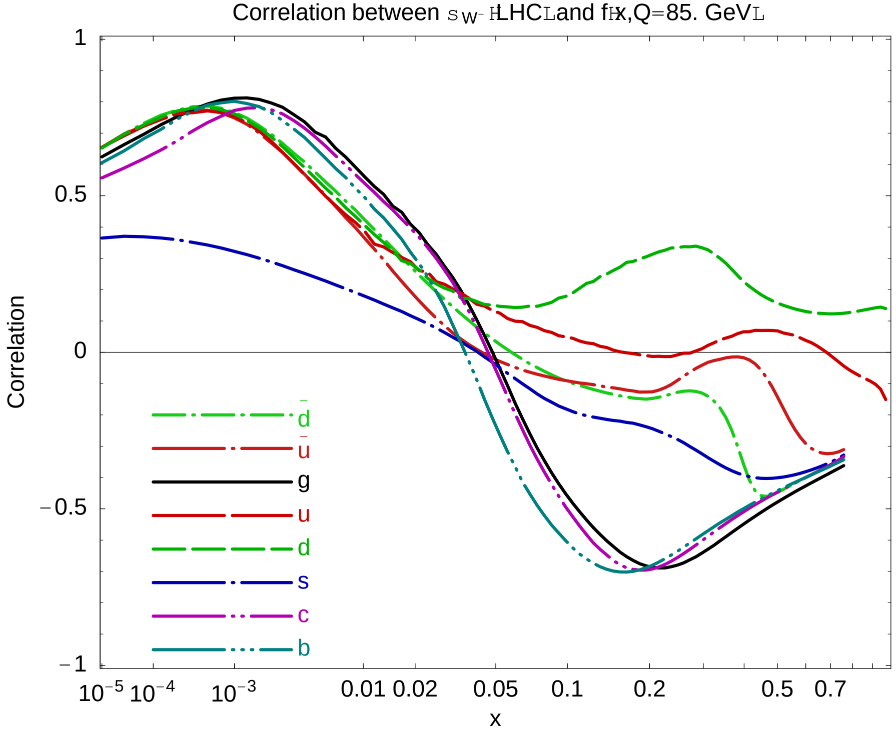
WHY WE SHOULD WORRY ABOUT SMALL X: THE IMPACT AT LHC

CORRELATION BETWEEN PDFS AND THE W TOTAL CROSS SECTION
(CTEQ 2008)

TEVATRON



LHC



UNCERTAINTIES ON SMALL x PDFS PROPAGATE TO INCLUSIVE OBSERVABLES

SMALL x RESUMMATION: WHERE DO WE STAND?

- SMALL x TERMS IN DGLAP RESUMMED TO ALL ORDERS AT THE LEADING AND SUBLEADING LEVEL (BFKL 75-76, Fadin-Lipatov 98)
- SMALL x CORRECTIONS TO HARD CROSS SECTIONS KNOWN AT THE LEADING NONTRIVIAL LEVEL FOR HQ PHOTO– & ELECTROPRODUCTION (Catani, Ciafaloni, Hautmann, 91; DIS (Catani, Hautmann, 94); HQ HADROPRODUCTION (Ball, K.Ellis, 01); GG→HIGGS (Marzani, Ball, Del Duca, s.f., Vicini, 08); DRELL-YAN (Marzani, Ball, 09); ISOLATED PHOTON (Diana, 10)
- TWO ALTERNATIVE APPROACHES TO DGLAP RESUMMATION:
  - SMALL x RESUMMATION OF DGLAP (Altarelli, Ball, s.f., ABF)
  - INCLUSION INTO BFKL OF FIXED-ORDER DGLAP & SUBSEQUENT NUMERICAL DECONVOLUTION OF RESUMMED DGLAP SPLITTING FUNCTION (Ciafaloni, Colferai, Salam, CCS)
- STABLE PERTURBATIVE EXPANSION OF THE RESUMMED DGLAP SPLITTING FUNCTION UP TO NLO WITH $n_f = 0$ (CCS+Stasto 02, ABF 06):
  - DGLAP-BFKL MATCHING THROUGH SUITABLE DOUBLE BFKL+GLAP EXPANSION (Ball, s.f. 95, ABF 2000)
  - COLLINEAR/ANTICOLLINEAR GLUON EMISSION SYMMETRY (Salam 99)
  - RUNNING COUPLING (CCS 99, ABF 01)
- EXTENSION TO HARD COEFFICIENT FUNCTIONS OF SMALL x RUNNING COUPLING RESUMMATION (Ball 08)
- EXTENSION TO $n_f \neq 0$ AND SCHEME–INDEPENDENT MATCHING OF DIS COEFFICIENT FUNCTIONS AND DGLAP EVOLUTION (ABF 09)
- DIS RESUMMED PHENOMENOLOGY (ABF+Rojo 2010+ in progress)

THE FIRST INGREDIENT: DUALITY (fixed coupling)

(T. JAROSZEWICZ, 1982; R. BALL & S.F., 1995)

THE ALTARELLI-PARISI EQN IS AN INTEGRO–DIFFERENTIAL EQUATION $\Rightarrow$ IT CAN BE EQUIVALENTLY VIEWED AS Q^2–EVOLUTION EQUATION FOR x–MOMENTS (usual RG eqn.), OR x–EVOLUTION EQUATION FOR Q^2–MOMENTS(BFKL eqn.)

EVOLUTION IN $t = \ln Q^2$

$$\frac{d}{dt} G(N,t) = \gamma(N,\alpha_s)\; G(N,t)$$

MELLIN x–MOMENTS

$$G(N,t) = \int_0^\infty d\xi\, e^{-N\xi}\; G(\xi,t)$$

EVOLUTION IN $\xi = \ln 1/x$

$$\frac{d}{d\xi} G(\xi,M) = \chi(M,\alpha_s)\; G(\xi,M)$$

MELLIN Q^2–MOMENTS

$$G(\xi,M) = \int_{-\infty}^\infty dt\, e^{-Mt}\; G(\xi,t)$$

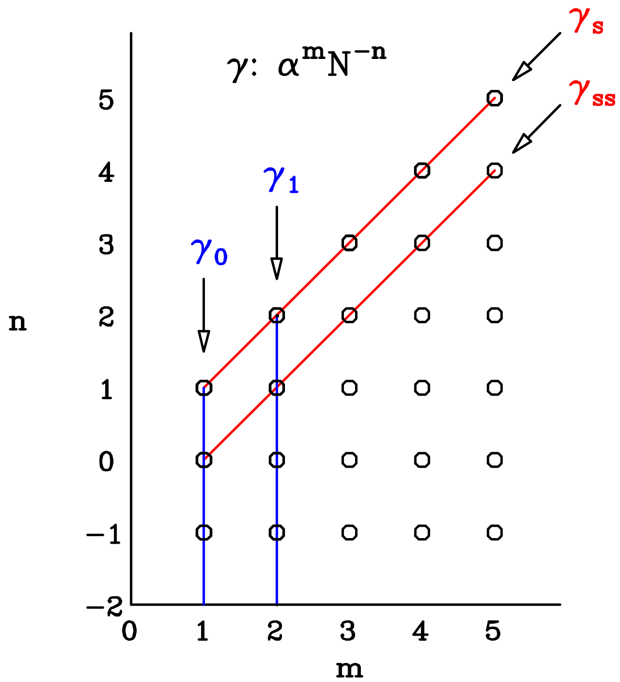
THE TWO EQUATIONS HAVE THE SAME SOLUTIONS PROVIDED THE EVOLUTION KERNELS ARE RELATED BY

$$\chi(\gamma(N,\alpha_s),\alpha_s) = N$$
$$\gamma(\chi(M,\alpha_s),\alpha_s) = M$$

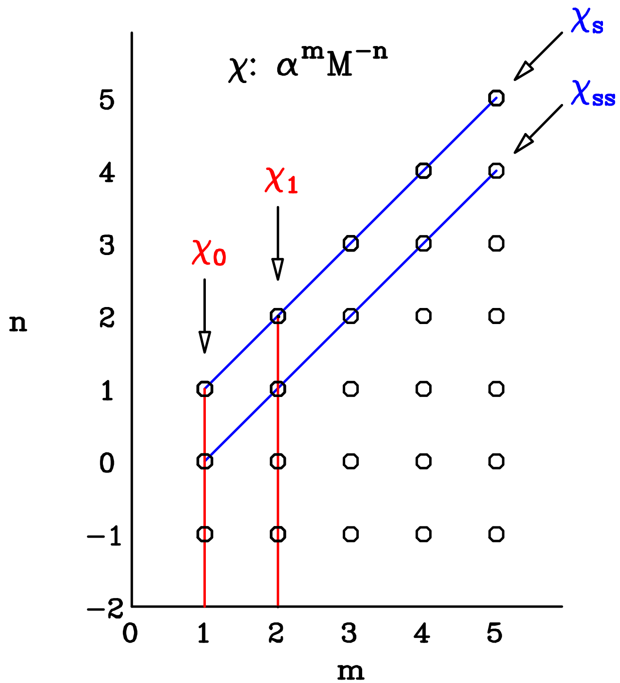
& BOUNDARY CONDITIONS RELATED BY

$$H_0[M] \to G_0(N) = H_0[\gamma(N,\alpha_s)]/\chi'(\gamma(N,\alpha_s))$$

DUAL PERTURBATIVE EXPANSIONS



$\ln Q^2$ EVOLUTION



$\ln 1/x$ EVOLUTION

$$\gamma(N) = \alpha\left(\frac{c_{-1}^{(1)}}{N} + c_0^{(1)} + \ldots\right) + \alpha^2\left(\frac{c_{-2}^{(2)}}{N^2} + \frac{c_{-1}^{(2)}}{N} + \ldots\right)$$

$$\chi(M) = \alpha\left(\frac{\tilde{c}_{-1}^{(1)}}{M} + \tilde{c}_0^{(1)} + \ldots\right) + \alpha^2\left(\frac{\tilde{c}_{-2}^{(2)}}{M^2} + \frac{\tilde{c}_{-1}^{(2)}}{M} + \ldots\right)$$

$$\gamma_s(N) = c_{-1}^{(1)}\frac{\alpha}{N} + c_{-2}^{(2)}\frac{\alpha^2}{N^2} + \ldots$$

$$\chi_s(M) = \tilde{c}_{-1}^{(1)}\frac{\alpha}{M} + \tilde{c}_{-2}^{(2)}\frac{\alpha^2}{M^2} + \ldots$$

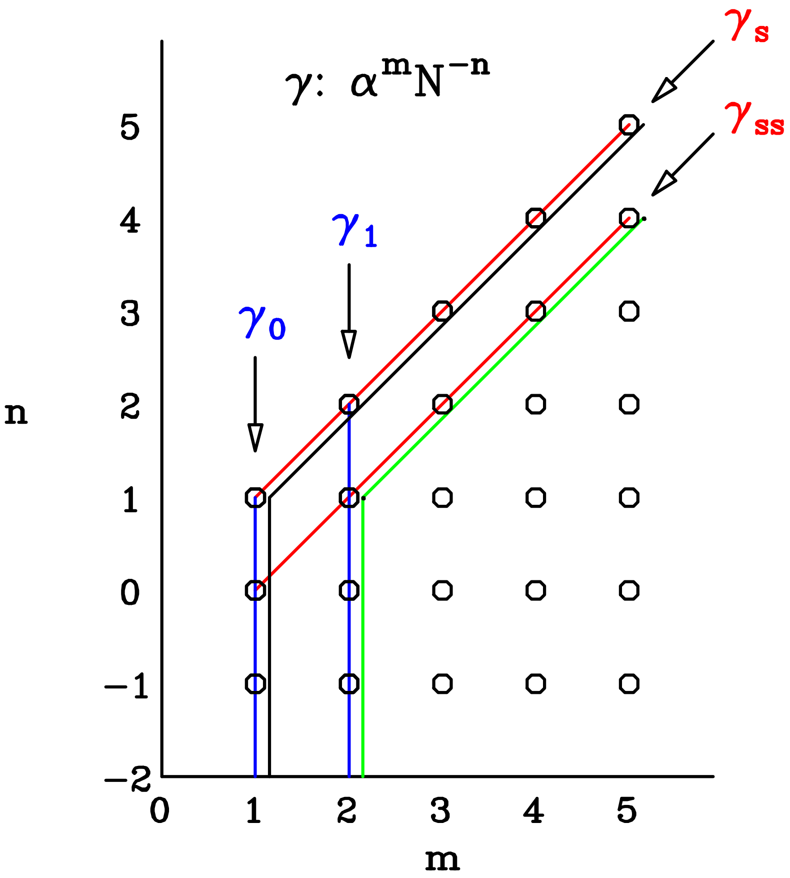
$1/N$ POLES $\Leftrightarrow \ln 1/x$

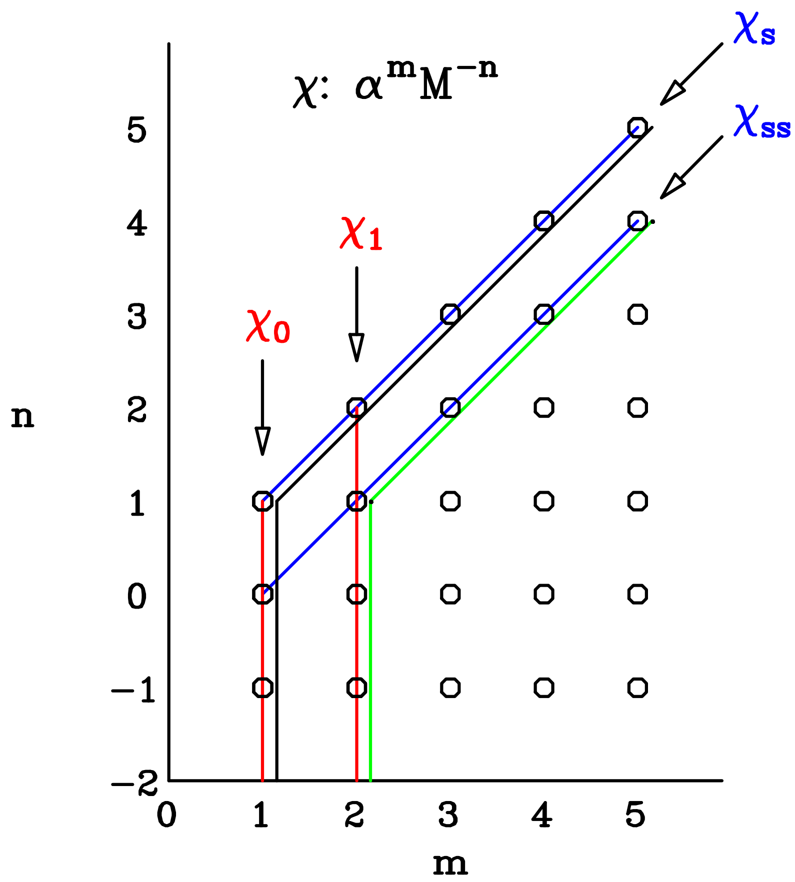
$1/M$ POLES $\Leftrightarrow \ln Q^2$

$$\gamma_0(N) \Longleftarrow\!\!\Longrightarrow \chi_s(\alpha_s/M)$$

$$\gamma_s(\alpha_s/N) \Longleftarrow\!\!\Longrightarrow \chi_0(M)$$

The Double–Leading Expansion





$$\gamma(N,\alpha_s) = \left[\alpha_s\gamma_0(N) + \gamma_s\left(\frac{\alpha_s}{N}\right) - \frac{n_c\alpha_s}{\pi N}\right] \Leftarrow\ \Rightarrow \chi(M,\alpha_s) = \left[\alpha_s\chi_0(M) + \chi_s\left(\frac{\alpha_s}{M}\right) - \frac{n_c\alpha_s}{\pi M}\right]$$

$$+\alpha_s\left[\alpha_s\gamma_1(N) + \gamma_{ss}\left(\frac{\alpha_s}{N}\right) - \alpha_s\left(\frac{e_2}{N^2} + \frac{e_1}{N}\right) - e_0\right] \qquad +\alpha_s\left[\alpha_s\chi_1(M) + \chi_{ss}\left(\frac{\alpha_s}{M}\right) - \alpha_s\left(\frac{f_2}{M^2} + \frac{f_1}{M}\right) - f_0\right]$$

$$+\cdots \qquad\qquad +\cdots$$

Duality holds order–by–order in the double–leading expansion:

the dual of $\chi^{\rm LO}_{\rm DL}$ is $\gamma^{\rm LO}_{\rm DL}$ up to terms of order $\gamma^{\rm NLO}_{\rm DL}$, and conversely

THE SECOND INGREDIENT: EXCHANGE SYMMETRY

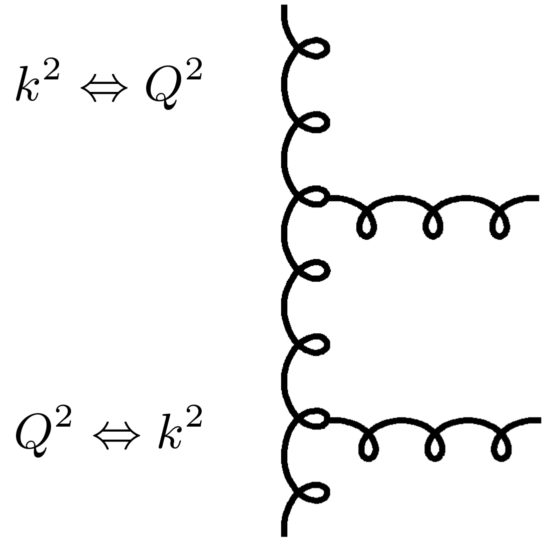
DIAGRAMS FOR $\ln 1/x$ EVOLUTION KERNEL

$$\frac{d}{d\xi} G(\xi, M) = \chi(M, \alpha_s)\; G(\xi, M)$$

$$\chi(\xi, M) = \int_{-\infty}^{\infty} \frac{dQ^2}{Q^2} \left(\frac{Q^2}{k^2}\right)^{-M} \chi(\xi, \frac{Q^2}{k^2})$$

SYMMETRIC UPON INTERCHANGE
OF INITIAL AND FINAL PARTON VIRTUALITIES

$$Q^2 \leftrightarrow k^2 \Leftrightarrow M \leftrightarrow 1 - M$$



COLLINEAR RES. OF $\frac{1}{M}$ POLES $\leftrightarrow$ ANTICOLLINEAR RES. OF $\frac{1}{1-M}$ POLES

SYMMETRY BREAKING

- DIS KINEMATIC VARIABLES $s = \frac{Q^2}{x}$ (small x)
- RUNNING OF THE COUPLING $\alpha_s(Q^2)$

BOTH CAN BE DETERMINED EXACTLY

THE THIRD INGREDIENT: RUNNING COUPLING

- THE RUNNING OF THE COUPLING $\alpha(t) = \alpha_\mu[1 - \beta_0 \alpha_\mu t + \ldots]$
  ($t \equiv \ln \frac{Q^2}{\mu^2}$) IS LEADING LOG Q^2, BUT NEXT–TO–LEADING LOG $\frac{1}{x}$

- UPON M-MELLIN TRANSFORMATION ($\ln x$ EVOLUTION)
  $\alpha_s(t)$ BECOMES AN OPERATOR:

$$\alpha_s(M) = \alpha_{\mu^2}[1 + \beta_0 \alpha_{\mu^2} \frac{d}{dM} + \ldots]$$

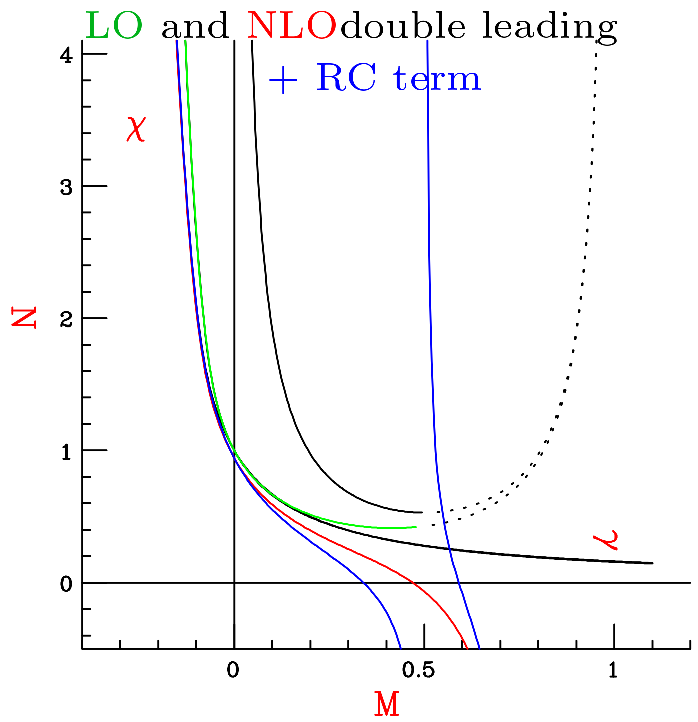
$\Rightarrow$ EVOLUTION EQUATION for $G(N, M)$ with b.c. $H_0(M)$

$$\left(1 - \frac{\alpha_\mu}{N}\right) \chi(M) G(N, M) - H_0(M) = \beta_0 \alpha_\mu \frac{d}{dM} G(N, M)$$

- BAD NEWS: PERTURBATIVE INSTABILITY

NLO R.C. CORRECTION
NOT UNIFORMLY SMALL AS $x \to 0$:

$$\frac{\Delta P_{ss}(\alpha_s, \xi)}{P_s(\alpha_s, \xi)} \underset{\xi \to \infty}{\sim} (\alpha_s \xi)^2$$



Exact asymptotic solution

ASYMPTOTIC BEHAVIOUR CONTROLLED BY

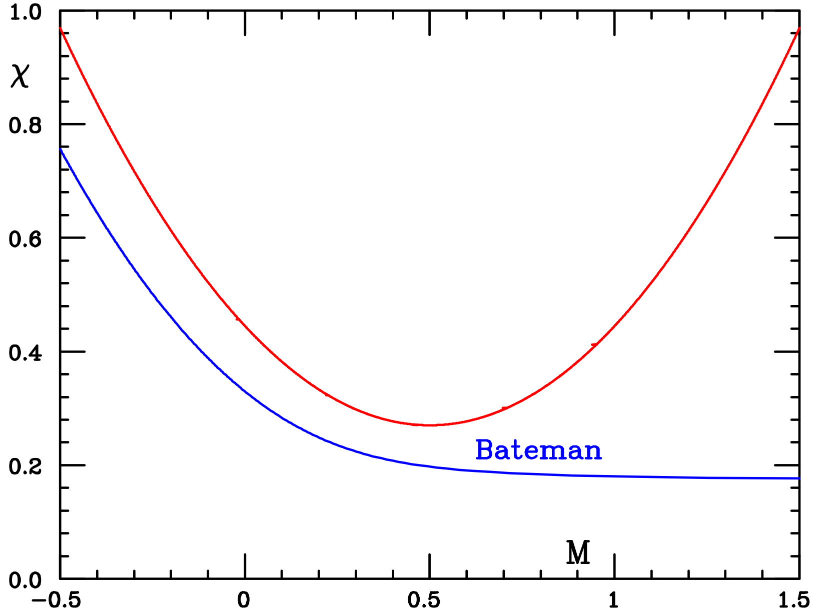
MINIMUM OF $\chi(M) \Leftrightarrow$ RIGHTMOST SING. OF $\gamma(N)$

QUADRATIC KERNEL $\chi_q(\hat{\alpha}_s, M) = [c(\hat{\alpha}_s) + \frac{1}{2}\kappa(\hat{\alpha}_s)(M - M_s)^2]$

CAN SOLVE EXACTLY WITH LINEARIZED $c(\hat{\alpha}_s)$, $\kappa(\hat{\alpha}_s)$
IN TERMS OF BATEMAN FUNCTION $K_\nu(x)$:

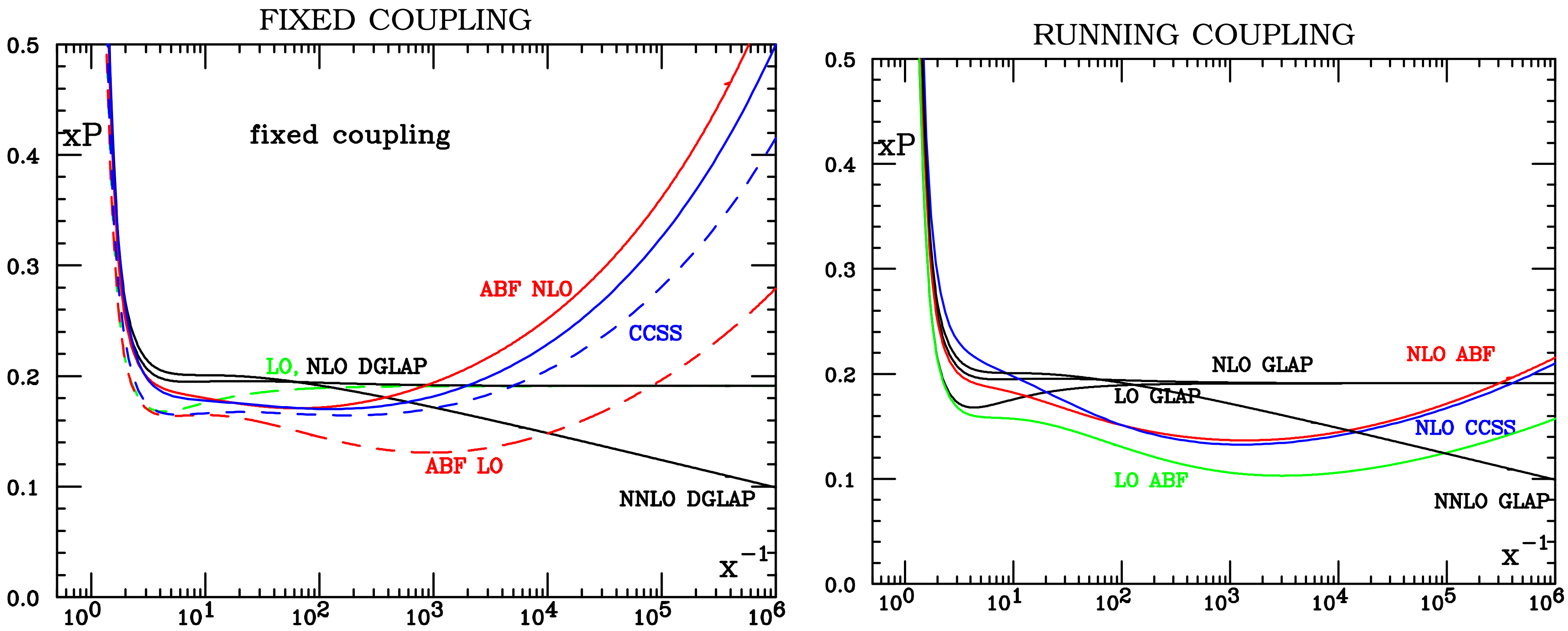
- $G(N,t) \propto K_{2B(\alpha_s,N)}\left[\frac{1}{\beta_0\bar{\alpha}_s(t)A(\alpha_s,N)}\right]$
  A, B DEPEND ON α_s, N TRHOUGH c, κ
- ASYMPTOTIC LEADING LOG SMALL x SERIES RESUMMED
- BRANCH CUT IN γ REPLACED BY SIMPLE POLE

THE EFFECTIVE RESUMMED KERNEL



RESUMMATION: GENERAL FEATURES

THE SPLITTING FUNCTION $(n_f = 0)$



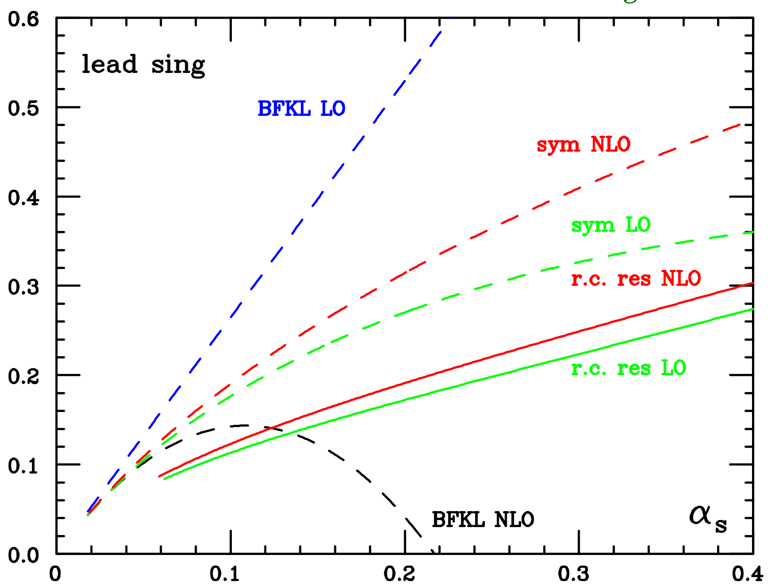
- RESUMMED EXPANSION CONVERGES RAPIDLY
  ESPECIALLY WITH RUNNING COUPLING
- BEHAVIOUR FOR $x < 10^{-2}$ VERY STABLE
- CAREFUL MATCHING OF SMALL x RUNNING COUPLING TERMS REQUIRED
  compare with CCSS $x \sim 0.2$
- DOMINANT QUALITATIVE BEHAVIOUR: DIP
  (SUPPRESSION) OF SPLITTING FUNCTION AT MEDIUM–SMALL x

RESUMMATION: GENERAL FEATURES

SMALL x BEHAVIOUR

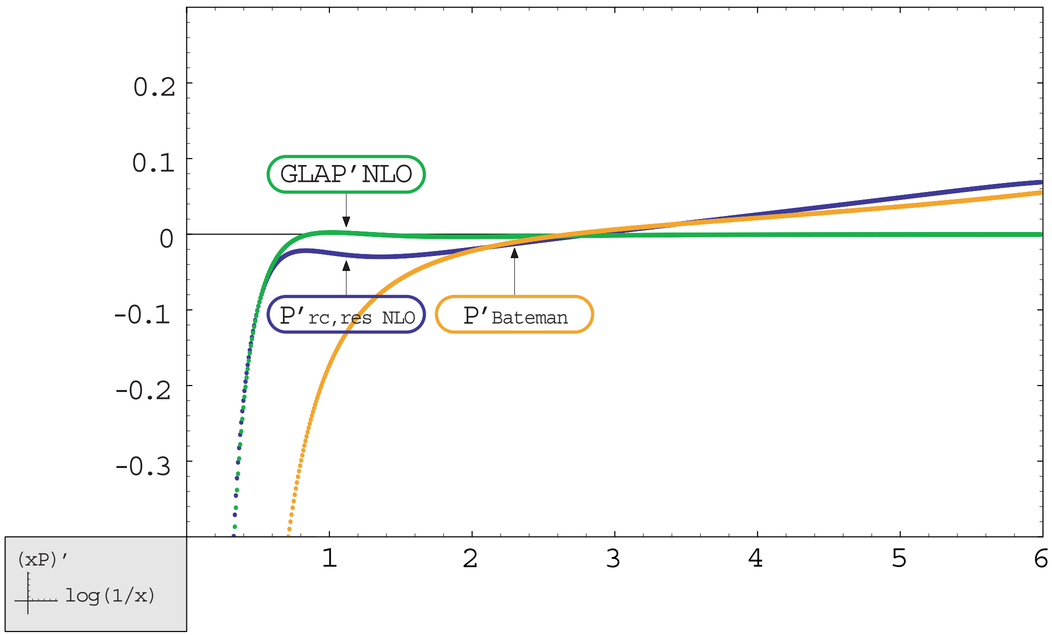
SINGULARITY IN ANOM. DIM. AT $N = \alpha \Rightarrow$ ASYMPT. SMALL-x POWER $G \sim x^{-\alpha}$

SMALL-x POWER VS. α_s



(ABF, 06)

SLOPE OF THE SPLITTING FUNCTION

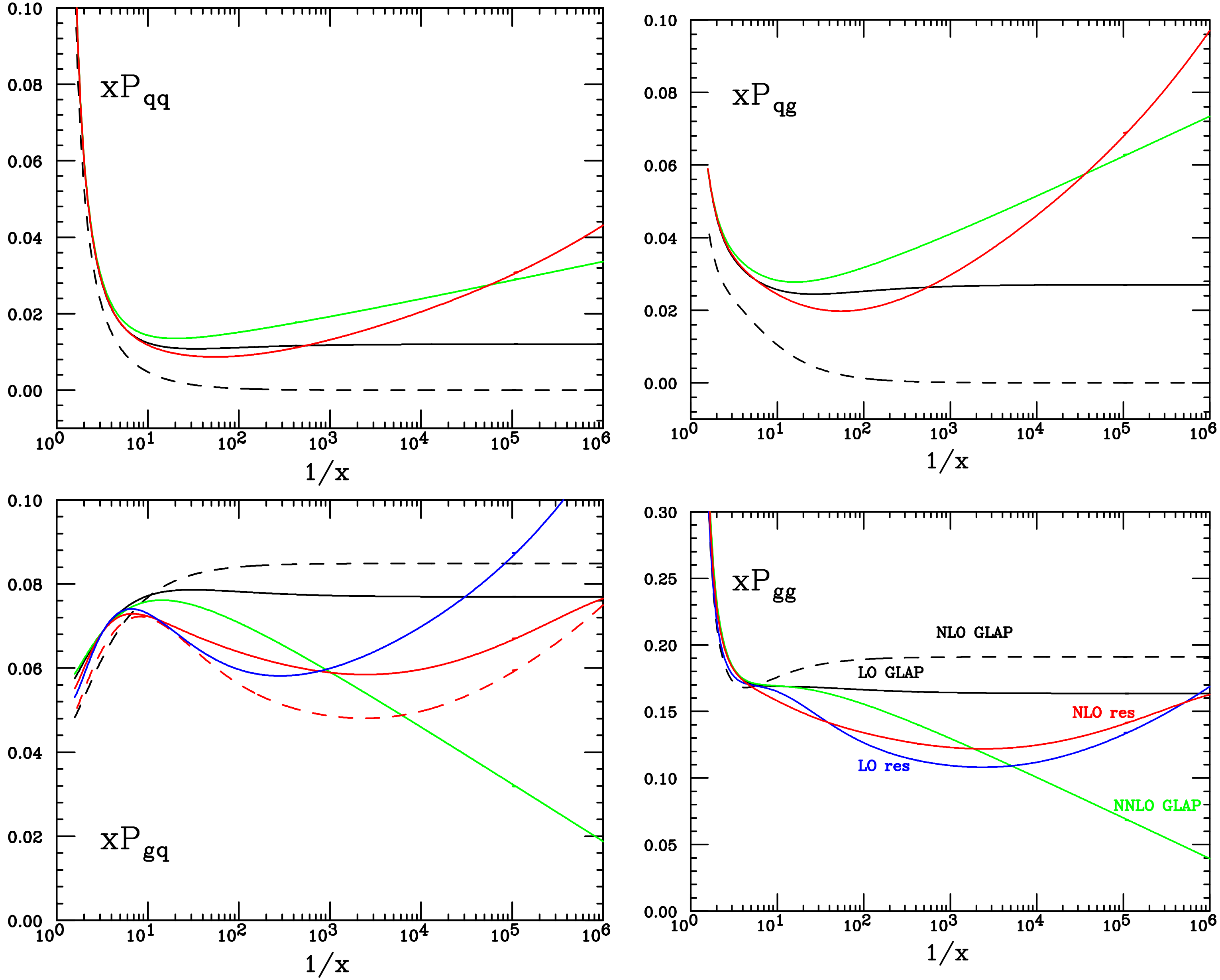


(C.Frugiuele, 07)

- SUBLEADING TERMS (SYM. + R.C.) MANDATORY FOR STABLE PERTURBATIVE EXPANSION
- AT LARGE x ($x \gtrsim 0.2$) SPLITTING FUNCTION COINCIDES WITH NLO GLAP
- SMALL x INTERCEPT & CURVATURE DETERMINE RESUMMED BEHAVIOUR

THE SPLITTING FUNCTION MATRIX

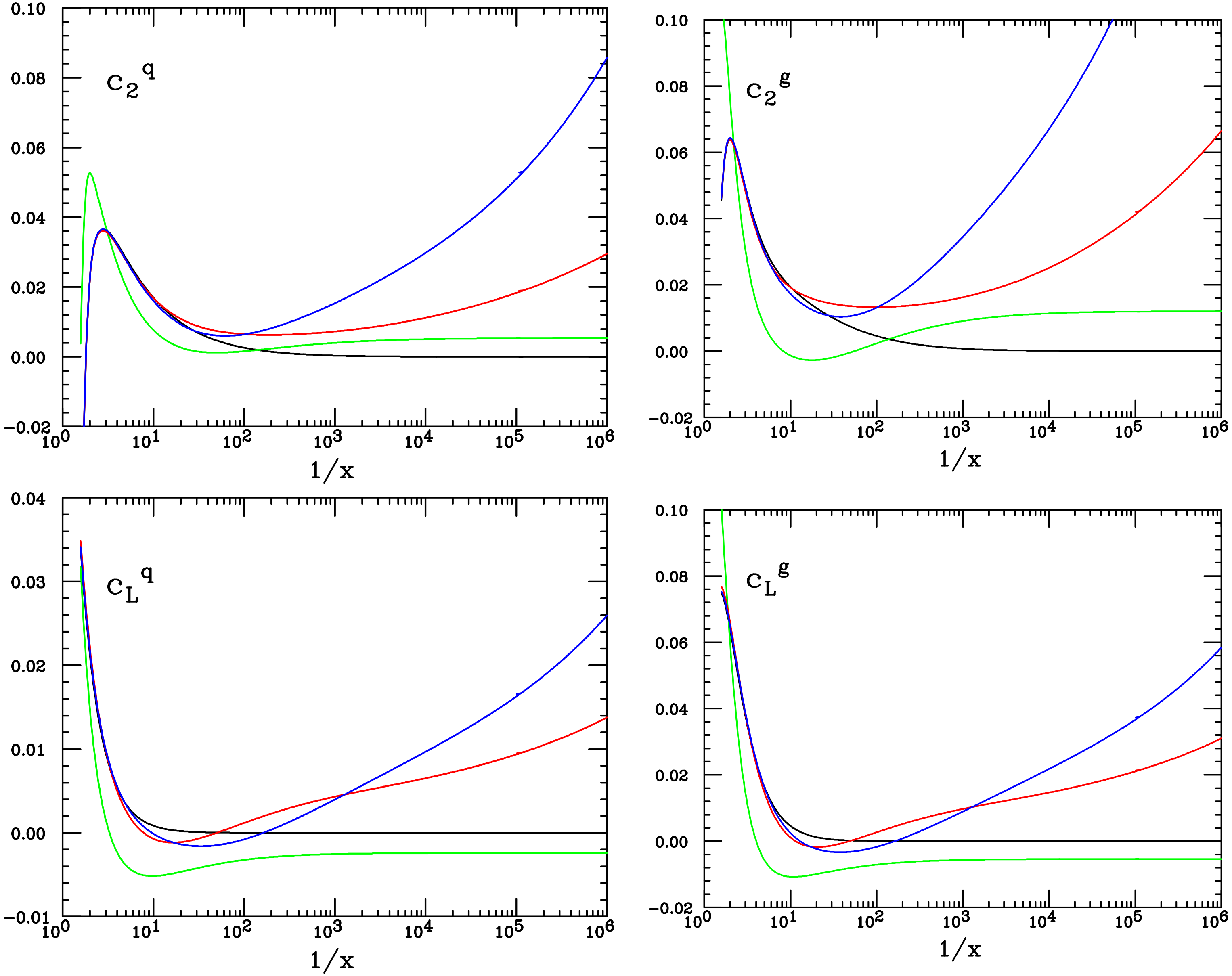
LO (DASH), NLO, NNLO, RESUMMED ($Q_0\overline{\text{MS}}$) RESUMMED ($\overline{\text{MS}}$) $n_f = 4$, $\alpha_s = 0.2$



(ABF, 08)

THE COEFFICIENT FUNCTION MATRIX

NLO, NNLO, RESUMMED ($Q_0\overline{\mathrm{MS}}$) RESUMMED ($\overline{\mathrm{MS}}$) $n_f = 4$, $\alpha_s = 0.2$

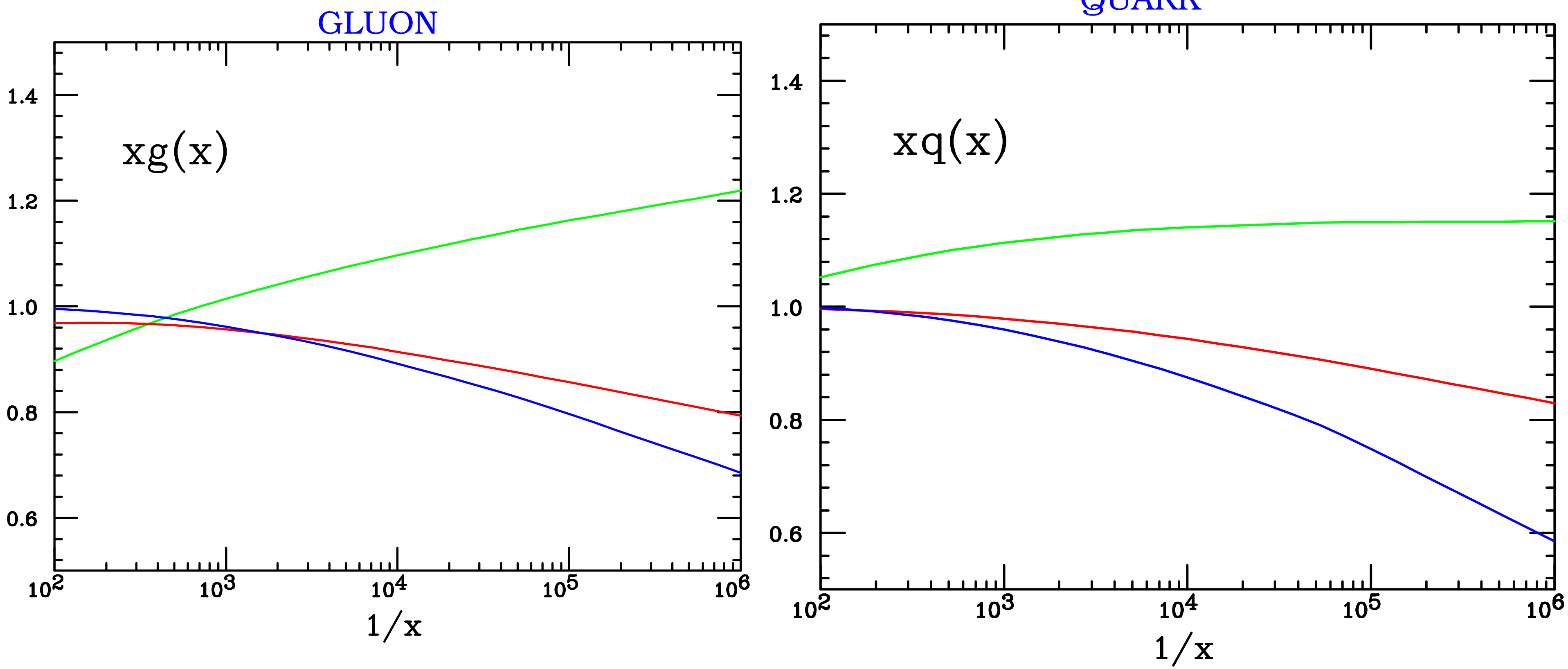


(ABF, 08)

HOW DO THE INITIAL PDFS CHANGE?

KEEP F_2 & F_L FIXED AT $Q_0 = 5$ GeV
COMPUTE $K(x) \equiv q^{\rm new}(x,Q_0^2)/q^{\rm NLO}(x,Q_0^2);\ g^{\rm new}(x,Q_0^2)/g^{\rm NLO}(x,Q_0^2)$

NNLO, RESUMMED $Q_0\overline{\rm MS}$, RESUMMED $\overline{\rm MS}$



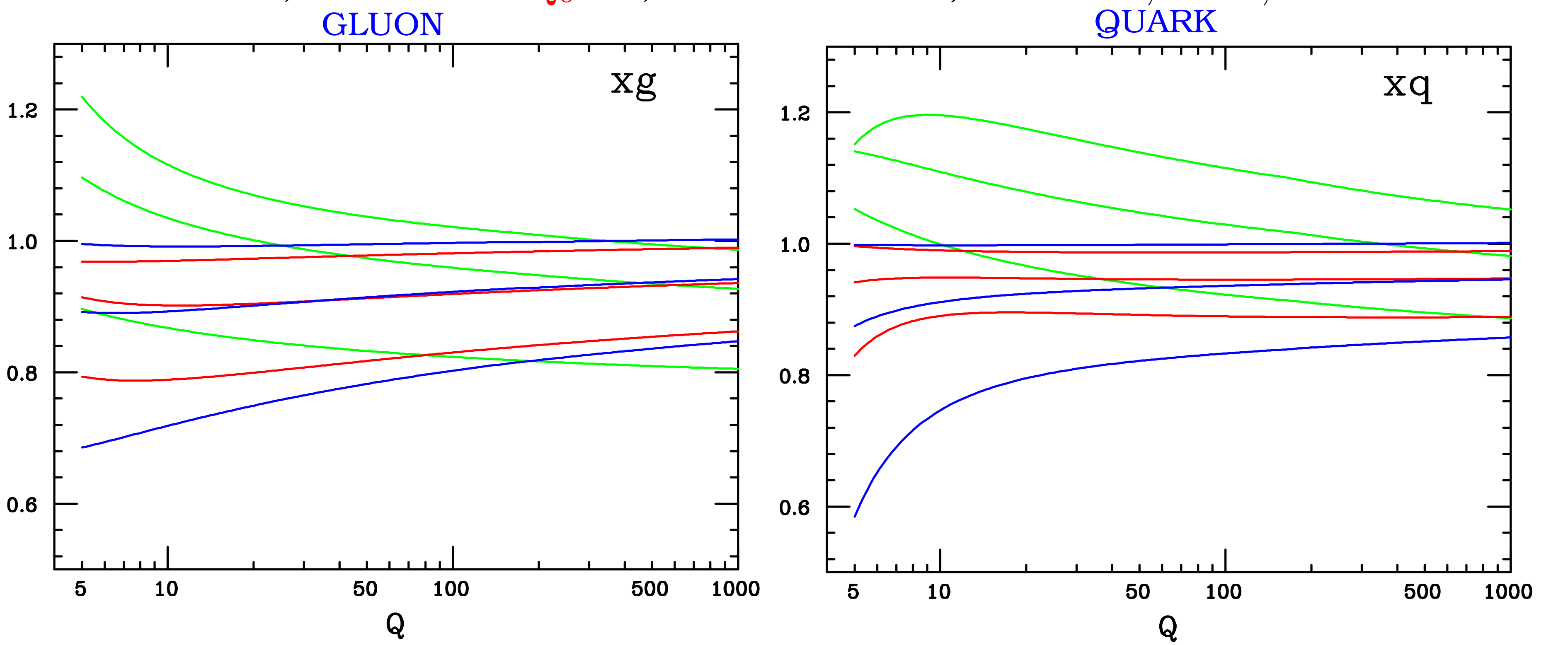
(ABF, 08)

- EFFECT OF RESUMMATION COMPARABLE TO NNLO
- RESUMMED SUPPRESSION DUE TO LARGER COEFFICIENT FUNCTIONS
- SCHEME DEPENDECE REASONABLE (LARGELY CANCELS BETWEEN HARD COEFFN. & SPLITTING FUNCTION)

HOW DO PDFS CHANGE WITH SCALE?

KEEP F_2 & F_L FIXED AT $Q_0 = 5$ GEV
COMPUTE $K(Q) \equiv q^{\rm new}(x,Q^2)/q^{\rm NLO}(x,Q^2);\ g^{\rm new}(x,Q^2)/g^{\rm NLO}(x,Q^2)$

NNLO, RESUMMED $\mathrm{Q_0}\overline{\mathrm{MS}}$, RESUMMED $\overline{\mathrm{MS}}$; $x = 10^{-2},\ 10^{-4},\ 10^{-6}$



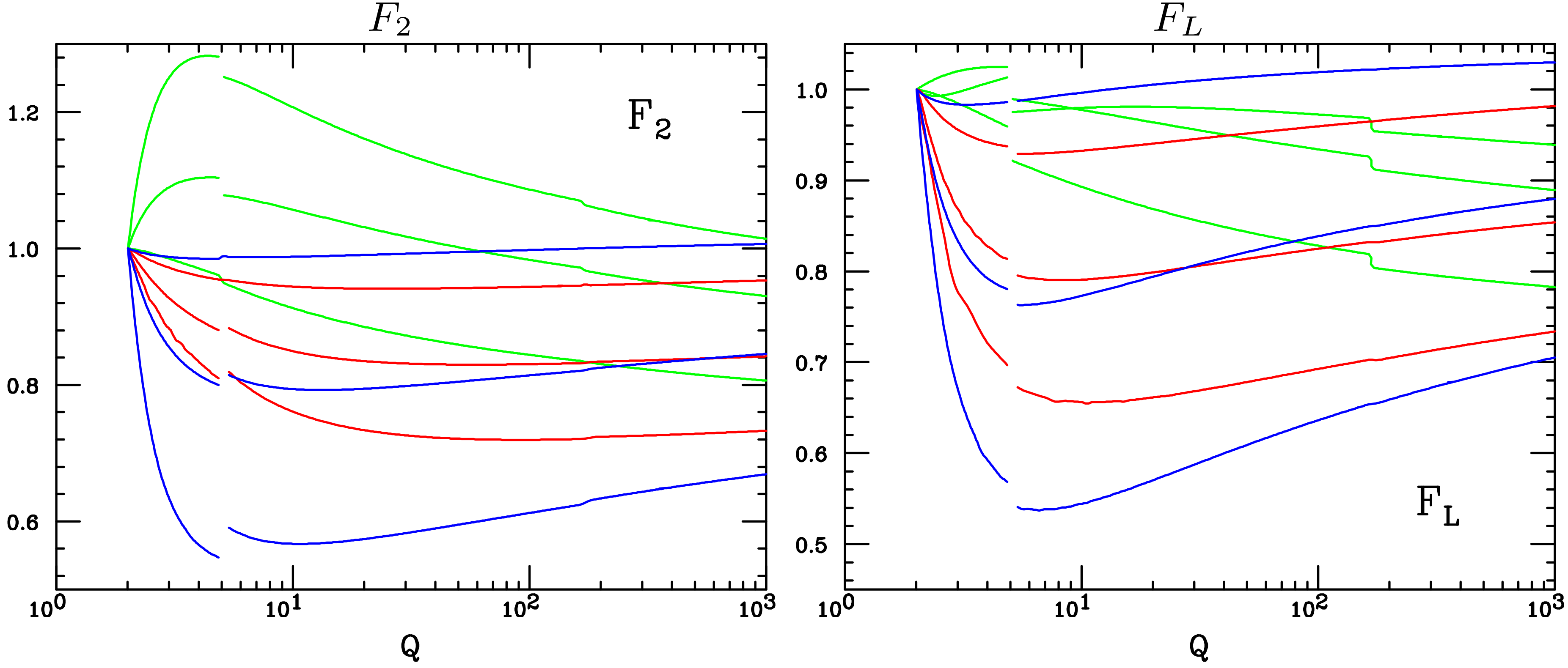
(ABF, 08)

- EVOLUTION WASHES OUT THE DIFFERENCES

THE EFFECT ON PHYSICAL OBSERVABLES

KEEP F_2 & F_L FIXED AT $Q_0 = 2$ GeV
COMPUTE $K(Q) \equiv F_2^{\text{new}}(x,Q^2)/F_2^{\text{NLO}}(x,Q^2);\ F_L^{\text{new}}(x,Q^2)/F_L^{\text{NLO}}(x,Q^2)$

NNLO, RESUMMED $Q_0\overline{\text{MS}}$, RESUMMED $\overline{\text{MS}}$; $x = 10^{-2},\ 10^{-4},\ 10^{-6}$



(ABF, 08)

- EFFECT OF RESUMMATION COMPARABLE TO NNLO
- RESUMMED SUPPRESSION DUE TO DIP IN EVOLUTION & PDF SUPPR. @ LOW SCALE
- SCHEME DEPENDENCE SMALLER THAN FOR PDFs
- EVOLUTION WASHES OUT THE DIFFERENCES

SMALL x RESUMMATION AT LHC?

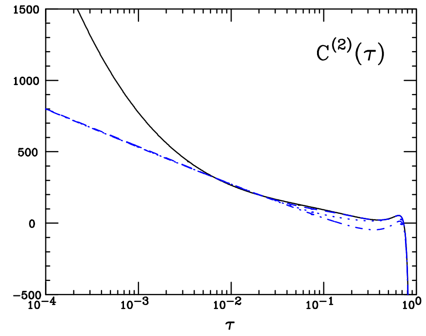
HIGGS PRODUCTION

- PARTONIC CROSS SECTION HAS DOUBLE LOGS AS $x \to 0$ IF $m_t \to \infty$, BUT SIMPLE LOGS FOR FINITE m_t
- ONLY $m_t \to infty$ KNOWN EXACTLY BEYOND NLO
- LEADING SMALL x RESUMMATION KNOWN TO ALL ORDERS FOR FINITE m_t (Marzani et al. 2008)
- CAN MATCH KNOWN SMALL x BEHAVIOUR TO LARGE x EXPANSION TO GIVE "OPTIMAL" NNLO (Harlander et al., 2009)

DRELL-YAN

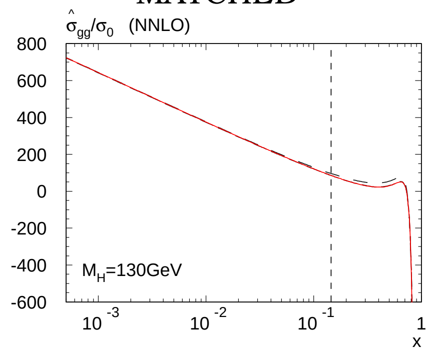
RESUMMATION EFFECTS ESTIMATED TO BE SUBSTANTIALLY LARGER THAN NNLO ($\sim 15\%$ VS. FEW PERCENT)

FINITE VS INFINITE M_t



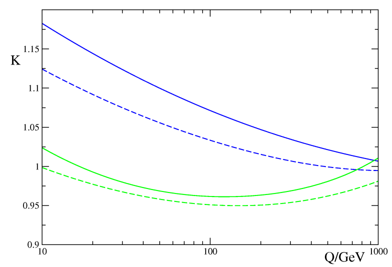
(Marzani, Ball, Del Duca, s.f., Vicini, 2008)

MATCHED



(Harlander, Mantler, Marzani, Ozeren, 2009)

DY RES/NLO VS. NNLO/LO



(Marzani, Ball, 2009)

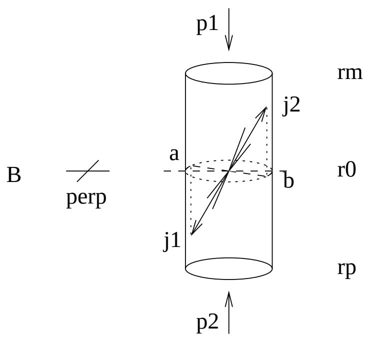
SMALL x PHENOMENOLOGY?

SMALL x RESUMMATION
WHERE IS IT?

- HERA DATA HAVE TAUGHT US THAT THE EFFECT OF HIGH-ENERGY LOGS IS SMALL
- WE ARE SLOWLY REALIZING THAT THIS MIGHT BE ALWAYS THE CASE

MUELLER-NAVELET JETS

- CLASSIC PROCESS TO SEARCH FOR ENERGY (BFKL) LOGS
- AS RAPIDITY GAP GROWS, EXPECT XSECT TO GROW AND AZIMUTHAL CORRELATION TO DECORRELATE DUE TO GLUON RADIATION



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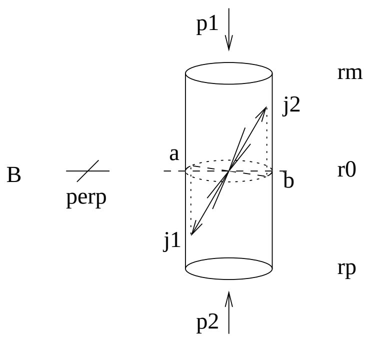
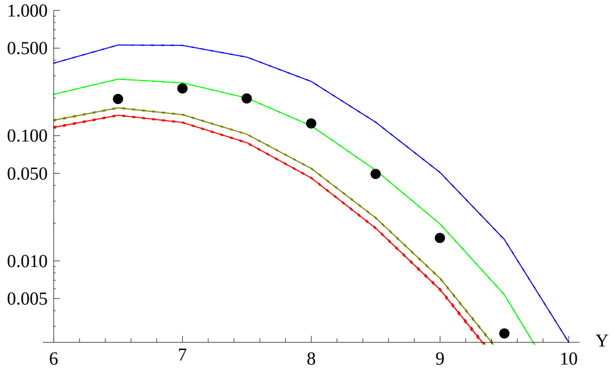
MUELLER-NAVELET JETS

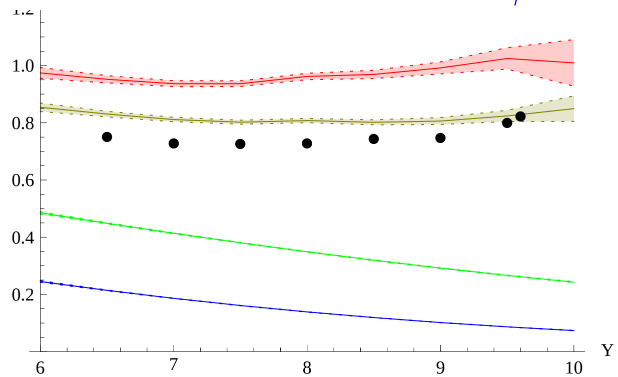
- CLASSIC PROCESS TO SEARCH FOR ENERGY (BFKL) LOGS
- AS RAPIDITY GAP GROWS, EXPECT XSECT TO GROW AND AZIMUTHAL CORRELATION TO DECORRELATE DUE TO GLUON RADIATION
- PROCESS COMPUTED RECENTLY TO FULL NLLx
  (Colferai, Schwennsen, Szymanowski, Wallon, 2010)
  NLO CORRECTIONS VERY LARGE,
  BRING NLLx "BFKL" & NLLQ^2 "DGLAP" RESULTS FOR CROSS SECTION TOWARDS AGREEMENT & RESTORE AZIMUTHAL CORRELATIONS $\Rightarrow$ NO LARGE NFKL EFFECTS



CROSS SECTION

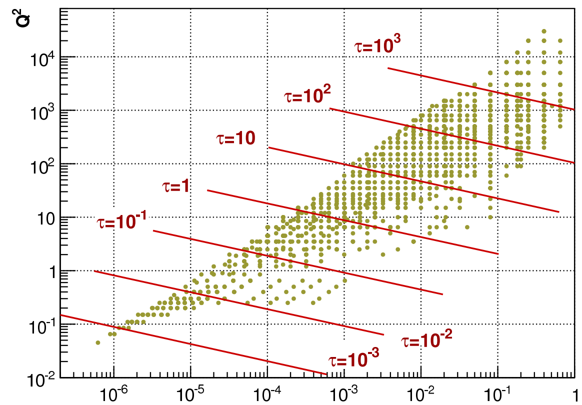


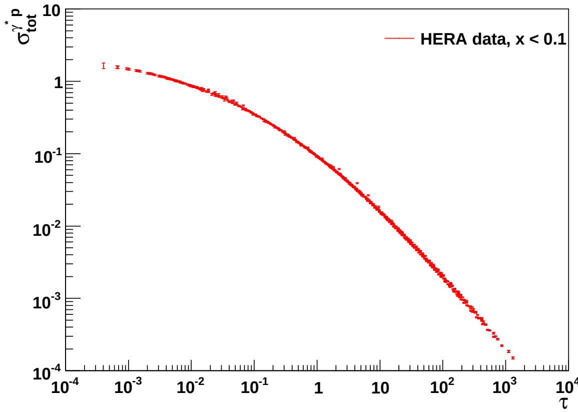
AZIMUTHAL CORRN cos ϕ



LLx BFKL NLLx BFKL LLx vert+ NLL NLxvert+NLL NLO DGLAP

WHAT ABOUT GEOMETRIC SCALING?





- STRUCTURE FUNCTION DATA SCALE W.R. TO $\tau = \frac{Q^2}{Q_0^2}(x/x_0)^\lambda$
  (Staśto, Golec-Biernat, Kwieciński, 2001)
- EVIDENCE FOR NONLINEAR EVOLUTION? (RECOMBINATION, SATURATION,...)
- DOES DGLAP FAIL?? FOR $Q^2 \gtrsim 10$ GEV^2
-

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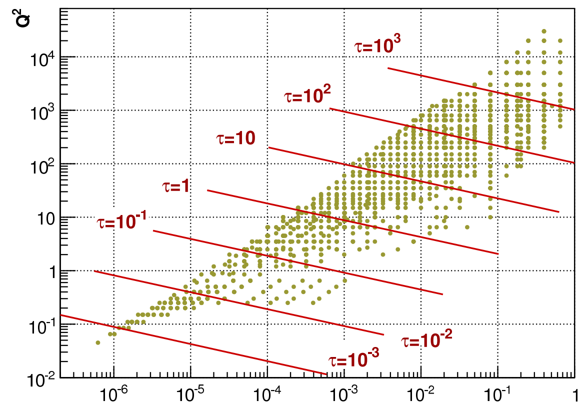

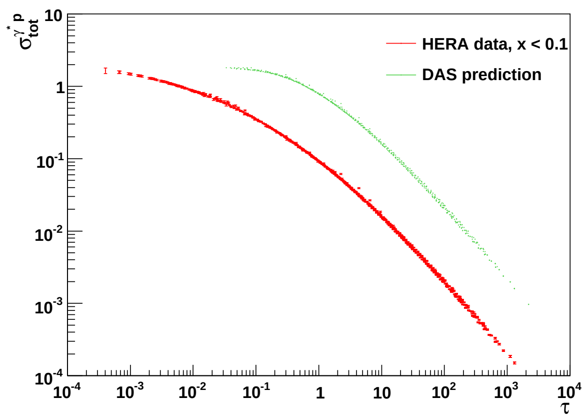

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- BUT DOUBLE–LOG SOLUTION TO LO (LINEAR) DGLAP ("DAS") ALSO SCALES!
  CAN ALSO BE SHOWN ANALYTICALLY
  (Caola, s.f., 2008)

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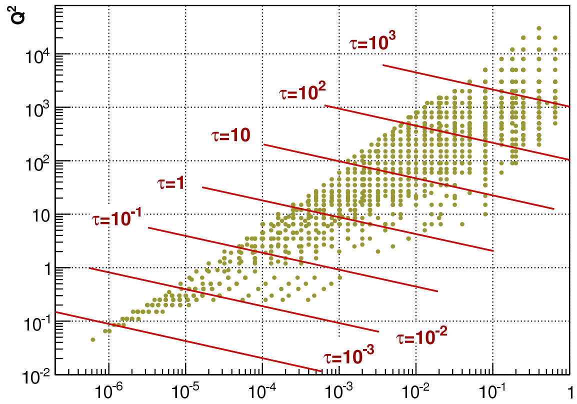

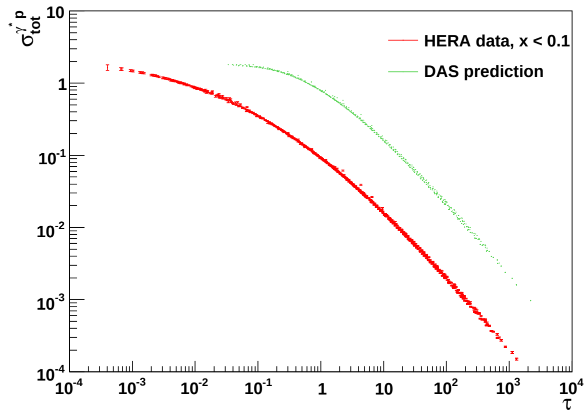

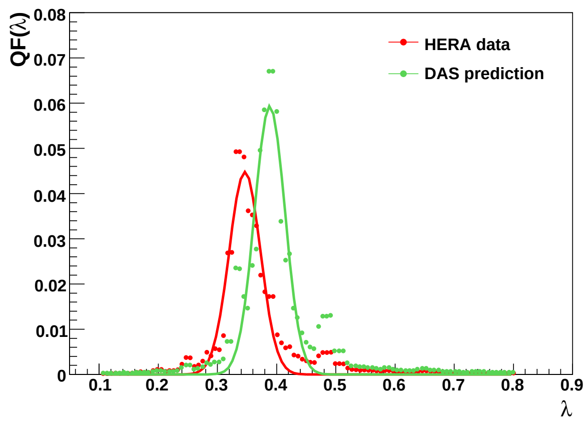

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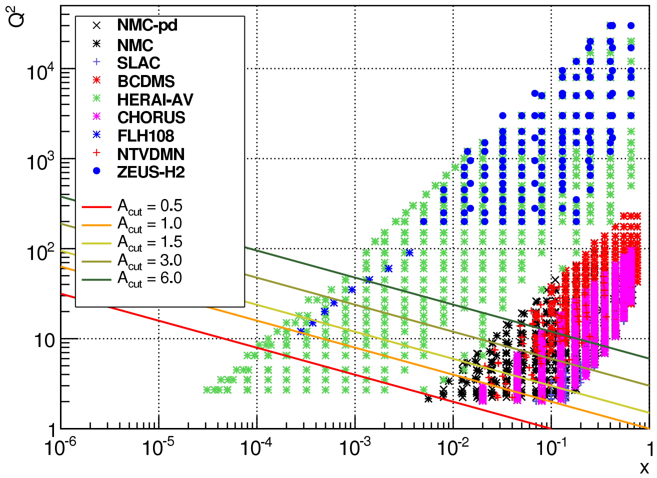
- BUT DOUBLE–LOG SOLUTION TO LO (LINEAR) DGLAP ("DAS") ALSO SCALES!
  CAN ALSO BE SHOWN ANALYTICALLY
  (Caola, s.f., 2008)
- CAN DETERMINE OPTIMAL SCALING FROM "QUALITY FACTOR" ANALYSIS
  (Gélis et al., 2007)
  $\Rightarrow$ OBSERVED λ AGREES WITH "DAS":
  DGLAP PREDICTS GEOMETRIC SCALING
  A FINER TEST NEEDED TO REVAL DEVIATIONS FROM DGLAP!

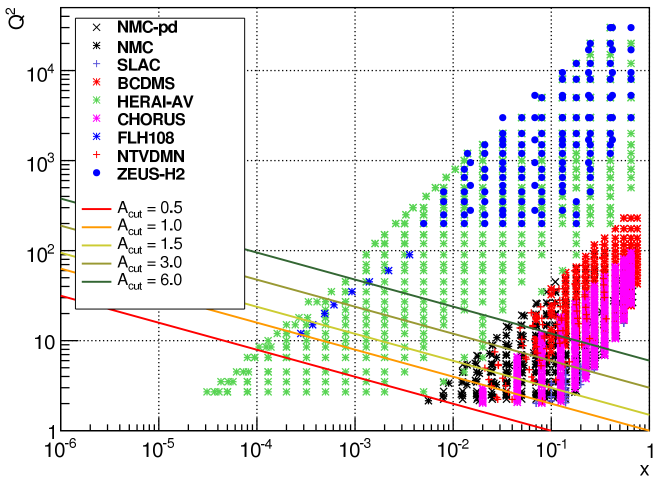
BEYOND DGLAP: TESTING FOR DEVIATIONS



IDEA: (Gélis, 2008, $\Rightarrow$ Caola, s.f. ,Rojo 2010)

- CUT OUT DATA IN THE "DANGEROUS" (SMALL τ) REGION
- DETERMINE PDFs IN THE "SAFE" (LARGE x AND Q^2) REGION
- EVOLVE BACKWARDS AND COMPARE TO DATA

BEYOND DGLAP: TESTING FOR DEVIATIONS



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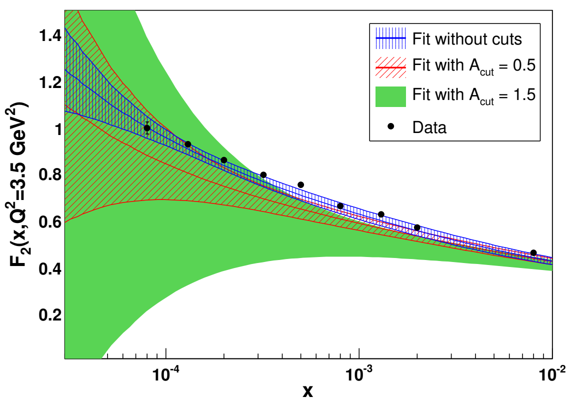
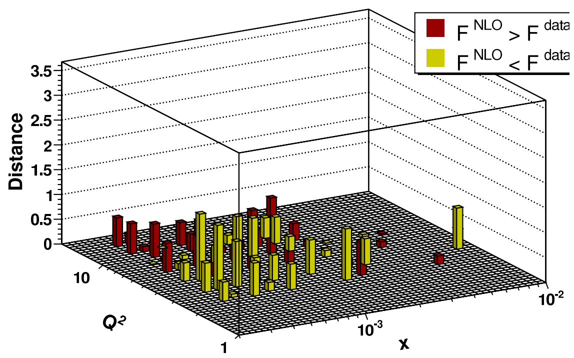
OLD HERA DATA

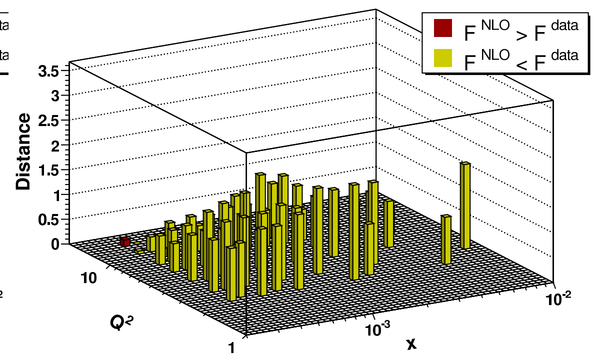
BACKWARD EV. VS DATA



DAT/TH DIST: NO CUT



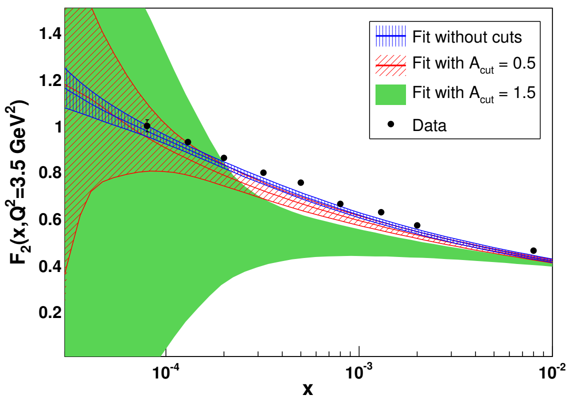
DAT/TH DIST: CUT



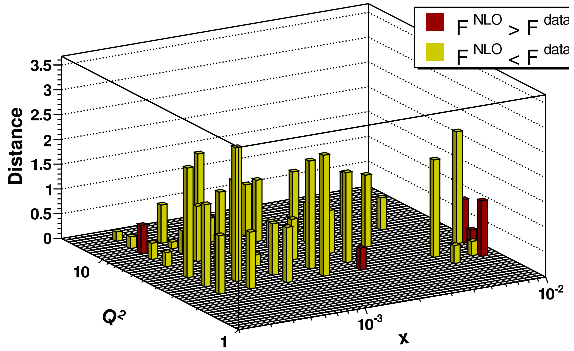
- BACKWARD EVOLVED FIT LIES SYSTEMATICALLY BELOW DATA
- DATA AT LOW x AND Q^2 SHOW LESS EVOLUTION THAN PREDICTED BY NLO DGLAP
- IF LOW x AND Q^2 DATA INCLUDED, THE FIT MANAGES TO COMPENSATE BY READJUSTING THE PDFs

NEW (COMBINED) HERA DATA

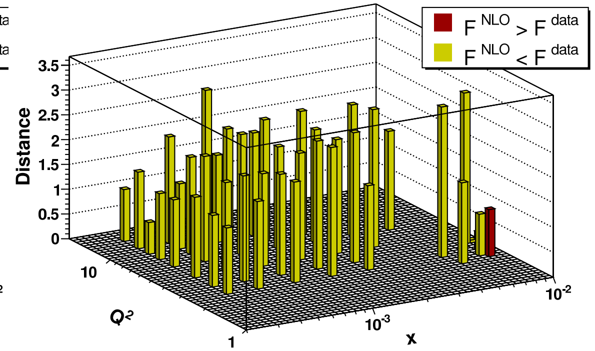
BACKWARD EV. VS DATA



DAT/TH DIST: NO CUT



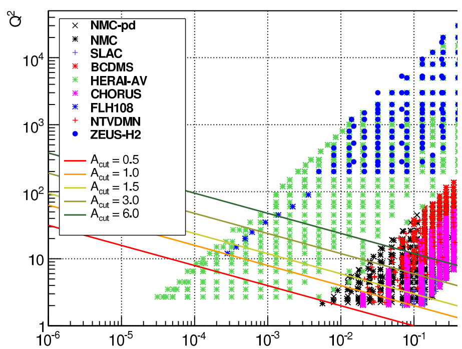
DAT/TH DIST: CUT

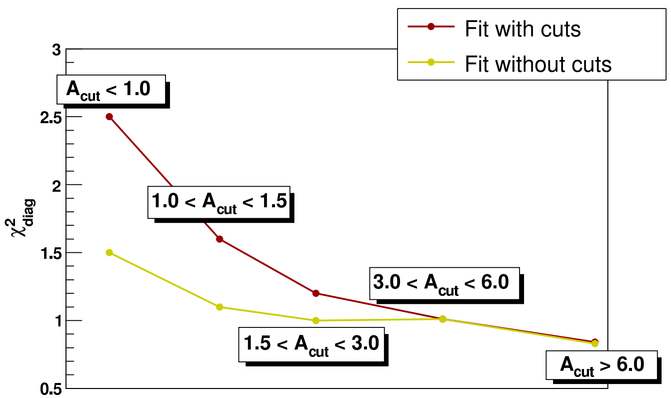


- DATA AT LOW x AND Q^2 SHOW LESS EVOLUTION THAN PREDICTED BY NLO DGLAP
- BACKWARD EVOLVED FIT LIES SYSTEMATICALLY BELOW DATA
- WITH MORE PRECISE DATA, THE FIT NO LONGER MANAGES TO COMPENSATE BY READJUSTING THE PDFS: EVEN FULL FIT LIES BELOW DATA

DETERIORATION IN FIT QUALITY:

χ^2 VS τ SLICES





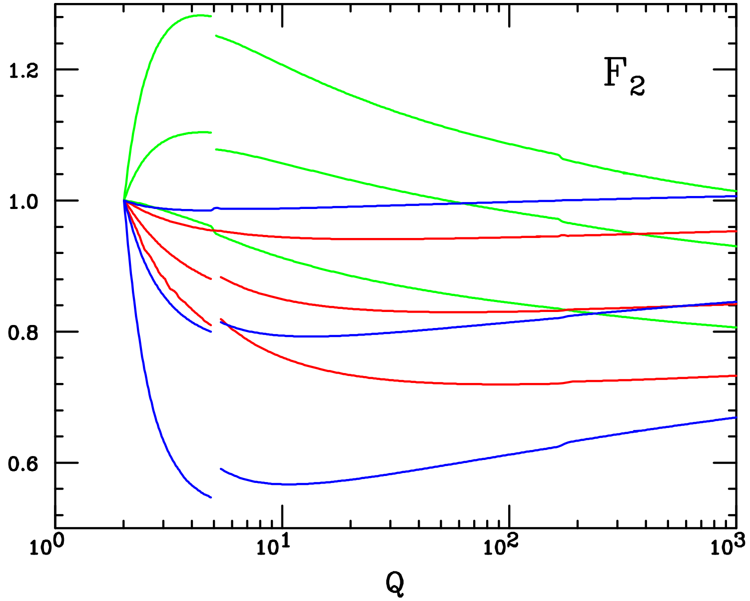
- QUALITY OF UNCUT FIT DETERIORATES IN LOW τ REGIONS
- QUALITY OF CUT FIT INCREASINGLY POOR AS τ DECREASES

DEVIATIONS FROM DGLAP:

SHOULD WE WORRY? (THEORY)

- OBSERVED DEVIATION FROM DGLAP CANNOT BE DUE TO MISSING NNLO TERMS: DATA EVOLVE LESS THAN NLO WHILE NNLO EVOLVES MORE THAN NLO
- PERTURBATIVE RESUMMATION HAS THE RIGHT SIGN AND ROUGH SIZE
- SATURATION OR MORE GENERALLY HIGHER TWIST (POWER SUPPRESSED) EFFECTS MIGHT ALSO REDUCE EVOLUTION
- NOTE: SOUGHT-FOR EVIDENCE IS SUPPRESSED SCALE (Q^2) DEPENDENCE, REGARDLESS OF THE x GROWTH

RES. Q_0, RES. $\overline{\text{MS}}$, NNLO



(ABF, 08)

DEVIATIONS FROM DGLAP:
SHOULD WE WORRY? (PHENOMENOLOGY)

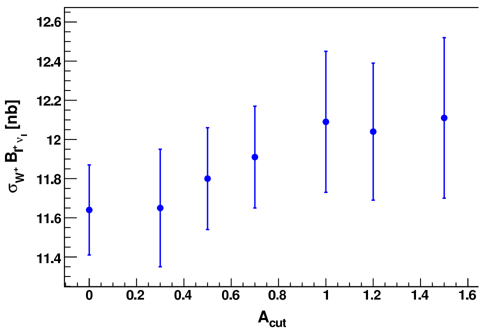
LHC STANDARD CANDLES (14 TEV)

DEPENDENCE ON CUT

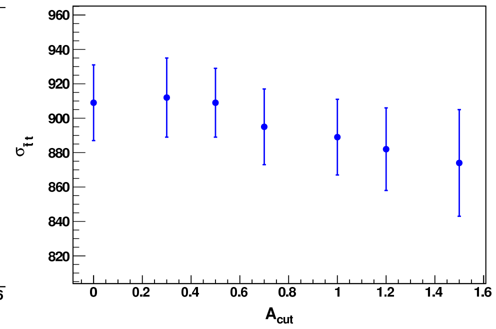
- REMOVING DATA FROM DANGEROUS REGION LEADS TO INCREASED PDF UNCERTAINTIES
- IMPACT ON LHC STANDARD CANDLES MODERATE
- COULD HAVE SOME EFFECT ON LESS INCLUSIVE OBSERVABLES

(Caola, s.f., Rojo, 10)

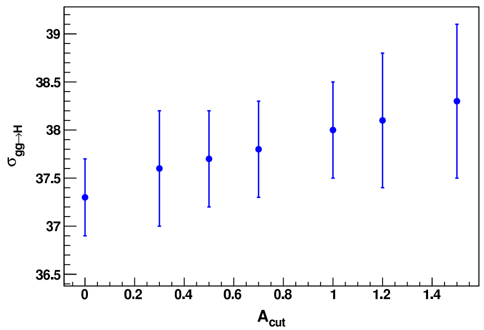
W



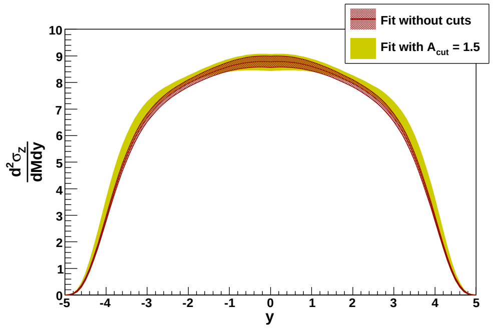
TOP



HIGGS (120 GEV)



Z RAP. DISTN.



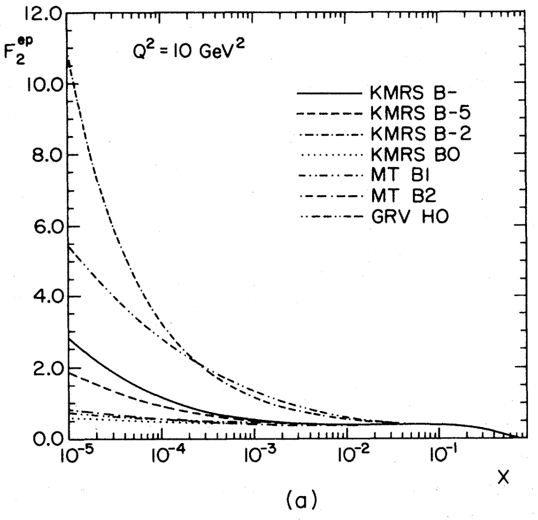
DETERMINATION OF α_s

- α_s FROM SCALING VIOLATION MAY BE BIASED DOWNWARDS (WEAKER EVOLUTION $\rightarrow$ ARTIFICIALLY SMALLER α_s)
- BIAS STRONGER AT NNLO
- MSTW (2010) SEE A CHANGE OF 2 SIGMA FROM NLO TO NNLO $\alpha_s = 0.1202\,^{+0.0012}_{-0.0015}$ TO $\alpha_s = 0.1171 \pm 0.0014$

WHAT'S BEHIND THE CORNER?

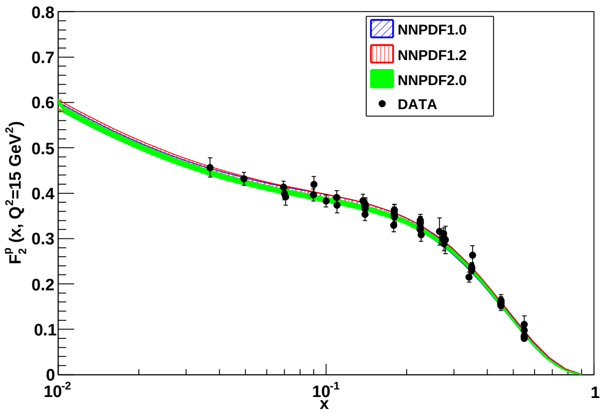
YESTERDAY

F_2 JUST BEFORE THE HERA START (1991)



TODAY

F_2 AND PDF UNCERTAINTIES



- PERTURBATIVE QCD IS READY FOR PRECISION PHYSICS

WHAT'S BEHIND THE CORNER?

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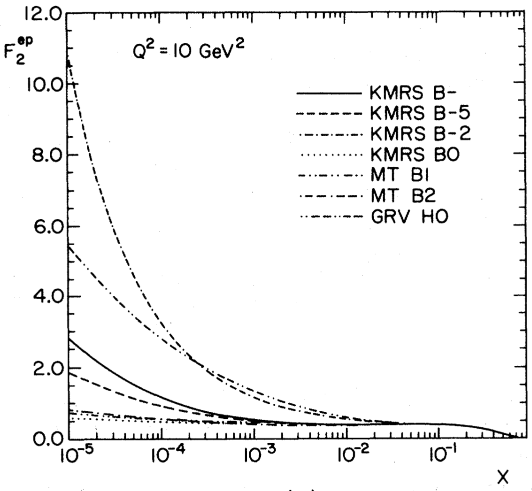
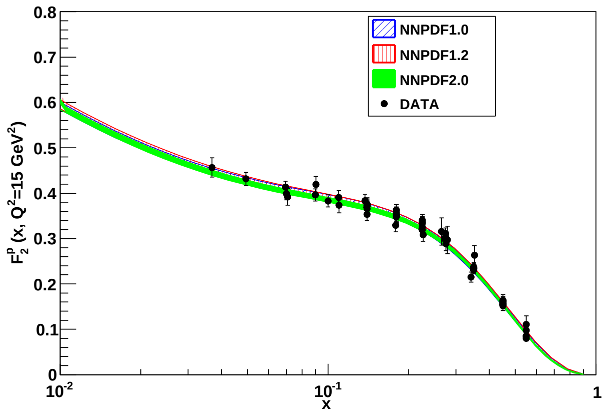


(a)

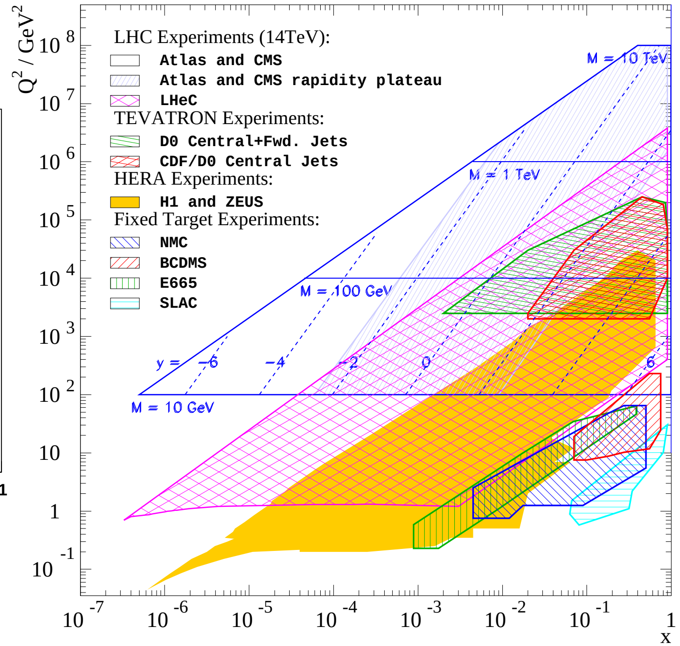
TODAY

F_2 AND PDF UNCERTAINTIES



TOMORROW

THE LHC AND LHeC KINEMATICS



- PERTURBATIVE QCD IS READY FOR PRECISION PHYSICS
- WHAT LIES BEYOND?
  - WE ARE READY TO DISCOVER NEW PHYSICS AT THE LHC
  - WE WILL LIKELY NEED AN LHeC TO STRETCH THE LIMITS OF QCD & EXPLOIT FULLY THE DISCOVERY POTENTIAL OF LHC

CONCLUSION

THIS IS JUST THE BEGINNING!