

The early thermalization and HBT puzzles at RHIC

Wojciech Florkowski^{1,2} b. 1961

¹ J. Kochanowski University, Kielce, Poland

² Institute of Nuclear Physics, Polish Academy of Sciences, Kraków, Poland

50 Cracow School of Theoretical Physics
Zakopane, June 9 - 19, 2010



RHIC at BNL

Relativistic Heavy Ion Collider at Brookhaven National Laboratory



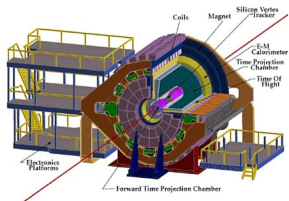
Google Maps: <http://maps.google.com/?ll=40.874649,-72.870598&spx=0.047118,0.079823&z=14>



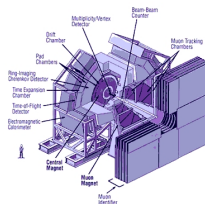
RHIC at BNL

Four experiments

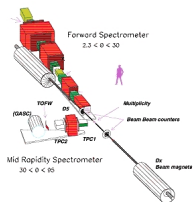
STAR



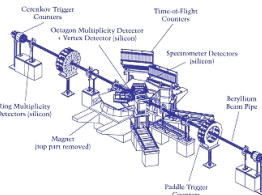
PHENIX



BRAHMS



PHOBOS



Outline

1. Successes and problems of perfect-fluid hydrodynamics at RHIC
 - very good description of one-particle soft hadronic observables: transverse-momentum spectra, elliptic flow
 - problems with two-particle observables (the so called **HBT puzzle**)
 - unphysical (?) very early start of hydrodynamics (later **ET puzzle**)
2. Resolving HBT puzzle (almost done)
 - equation of state
 - initial profiles
 - initial transverse flow
3. Resolving ET problem (not done yet)
 - very strongly interacting matter → AdS/CFT
 - Color Glass Condensate & String Models
 - **this talk**: Interpolation between initial weakly interacting system and later strongly interacting fluid — initial free-streaming or initial transverse-hydrodynamics followed by the perfect-fluid hydrodynamics, Landau matching conditions



Outline

4. Concept of transverse hydrodynamics
 - motivation
 - general formalism
 - description of the RHIC data
5. HBT vs. v_2 puzzle (?)
 - consequences of realistic EOS
 - quark-coalescence picture
 - inclusion of shear and bulk viscosity
- 6 Consequences for the early Universe
 - no dramatic phenomena at the phase transition
 - precise time development at times 5–100 μs
- 7 Conclusions

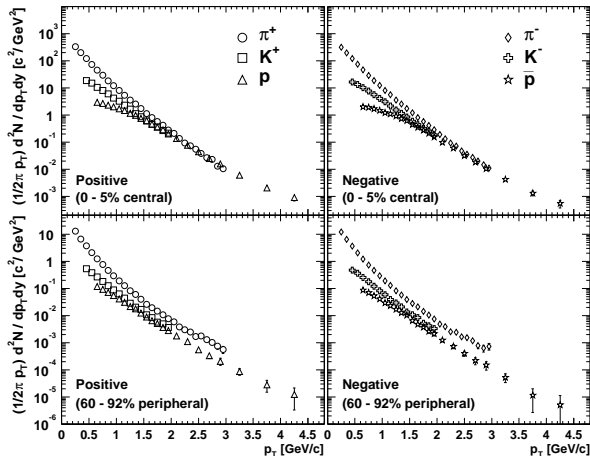


1. Soft hadronic production at RHIC — successes and problems



1.1 Experimental transverse-momentum spectra

effective inverse slopes: $T_{\text{eff}} = T + \frac{1}{2}mv_{\text{hyd}}^2$, different slopes \rightarrow evidence for flow



PHENIX, Phys. Rev. C69, 034909 (2004)



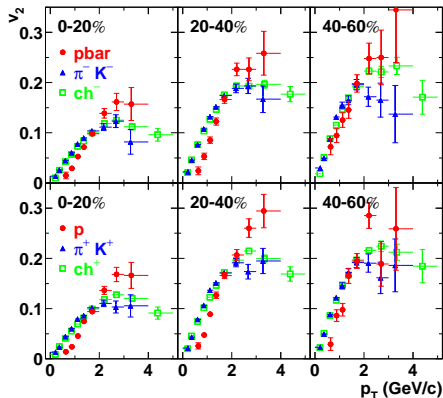
1.2 Experimental elliptic flow

v_2^{exp} agrees with perfect-hydro predictions!

$$\frac{dN}{dyd^2p_{\perp}} = \frac{dN}{2\pi dy p_{\perp} dp_{\perp}} (1 + 2v_2 \cos(2\phi_p))$$



<http://www.phenix.bnl.gov/WWW/software/luxor/ani/ellipticFlow/ellipticSmall1-1.mpg>
Animation by Jeffery Mitchell (Brookhaven National Laboratory)

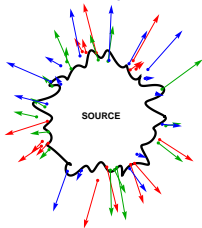


PHENIX,
Phys.Rev.Lett.91,182301(2003)

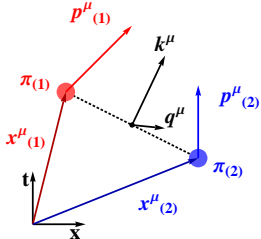


1.3 HBT radii – definitions

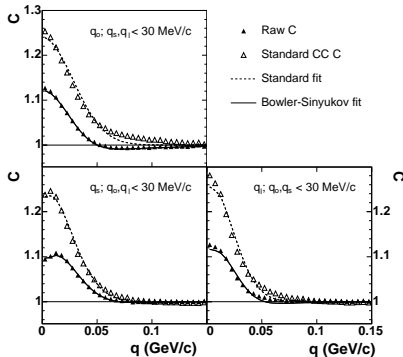
source emits identical pions, $\pi^+\pi^+$, $\pi^-\pi^-$



correlation function \equiv two-particle
distribution function, $C(p_1, p_2) \rightarrow C(k, q)$



three projections of the correlation
functions

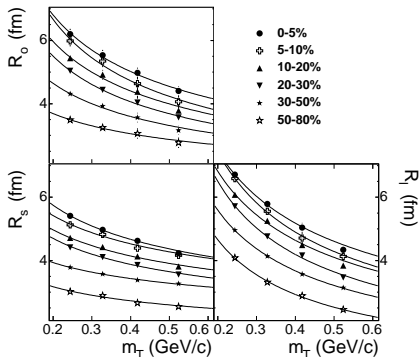


STAR,
Phys.Rev.C71,044906(2005)



1.3 HBT radii – physical interpretation

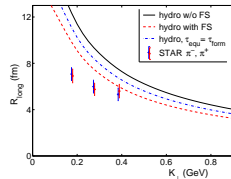
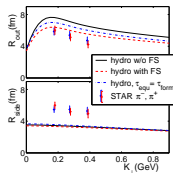
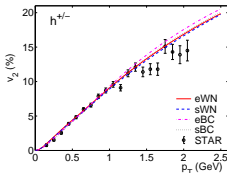
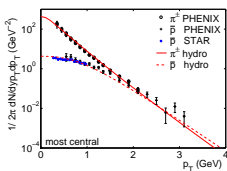
- "Fourier transform"
- HBT radii
 - R_{side} - spatial transverse extension, $R_{side}^2 = \langle \tilde{y}^2 \rangle$
 - R_{out} - spatial transverse extension + emission time, $R_{out}^2 = \langle (\tilde{x} - v_{\perp} \tilde{t})^2 \rangle$
 - R_{long} - longitudinal extension (homogeneity length), $R_{long}^2 = \langle (\tilde{z} - v_{\parallel} \tilde{t})^2 \rangle$
- HBT radii decrease with k_T , a signal of flow again!



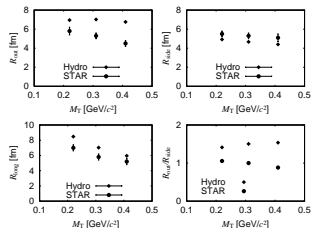
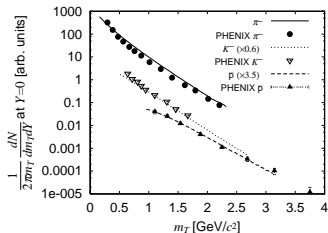
STAR,
Phys.Rev.C71,044906(2005)



1.4 Experiment vs. theory / start of RHIC activity



U.Heinz and P.Kolb, Nucl. Phys. A702, 269 (2002)



T.Hirano, K.Morita, S.Muroya, and C.Nonaka, Phys. Rev. C65, 061902 (2002)



1.5 “Standard Model/Scheme” of heavy-ion collisions

main ingredients of the 2+1 models:

- **initial conditions**, short thermalization time, $\tau_i \leq 1$ fm
 - Glauber model, e.g., initial entropy/energy density is proportional to the linear combination of the wounded-nucleon density and binary-collision density,

$$\sigma_i(\mathbf{x}_\perp) \text{ or } \varepsilon_i(\mathbf{x}_\perp) \propto \rho_{\text{sr}}(\mathbf{x}_\perp) = \frac{1 - \kappa}{2} \bar{w}(\mathbf{x}_\perp) + \kappa \bar{n}(\mathbf{x}_\perp)$$

- Color Glass Condensate
 - initial transverse flow, usually set equal to zero (?)
- **HYDRODYNAMIC STAGE**
 - v_2 data suggest that matter behaves like a perfect fluid **main tool**: **perfect-fluid hydrodynamics** (Shuryak + Teaney, Heinz + Kolb + Huovinen + Ruuskanen + Voloshin, Kolb + Rapp, Hirano + Nara, Bass + Nonaka, ...)
 - hadronization included in the **equation of state**
- **freeze-out**, Cooper-Frye formula
 - freeze-out hypersurface, thermal description of hadron production
 - transition hypersurface, change to a hadronic cascade



1.6 Perfect-fluid hydrodynamics

- energy-momentum conservation law

$$\partial_\mu T^{\mu\nu} = 0$$

- energy-momentum of the perfect fluid

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu - P g^{\mu\nu}$$

ϵ - energy density, P - pressure, u^μ - fluid four-velocity

- mid-rapidity ($|y| \leq 1$) for RHIC

$\mu_B \approx 0$, temperature is the only independent parameter

- 2+1 codes

boost-invariance, equations solved at $z = 0$, solutions for $z \neq 0$ obtained by Lorentz boosts

- 3+1 codes – general codes in 3 spatial dimensions



1.7 RHIC puzzles in soft hadronic sector

both the HBT and ET puzzles are related to the applications of hydrodynamics

HBT: discrepancy between the data and hydrodynamic calculations

- simple parameterizations a la Blast-Wave model very often did a very good job in describing all soft hadronic observables
- dramatic failure of kinetic models

ET: very early starting point for hydrodynamics

- τ_i identified with complete local thermalization time τ_{therm}
- Shuryak, Teaney: $\tau_i = 1$ fm, hadronic cascade
- Heinz, Kolb: $\tau_i = 0.6$ fm
- Cracow: $\tau_i = 0.25$ fm
- Pratt: $\tau_i = 0.2$ fm
- partonic cascade models by inclusion of $2 \rightarrow 3$ and back $3 \rightarrow 2$ processes make $\tau_{therm} \sim 1$ fm.



2. Resolving the HBT puzzle

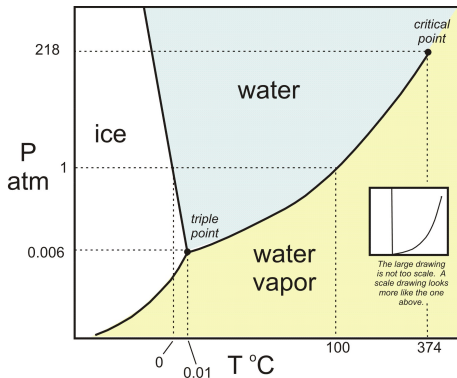


- W. Broniowski, M. Chojnacki, WF, A. Kisiel, PRL **101** (2008) 022301
- S. Pratt, PRL **102** (2009) 232301 (considerations without v_2)
with several improvements done in the hydrodynamic models, the HBT puzzle is practically eliminated (discrepancy at the level of 10%)
 - realistic equation of state (++)
 - early start of hydrodynamics (++)
 - modified initial conditions (+-)
 - shear viscosity included (-+)
 - fluctuations of the initial eccentricity (+-)
 - two-particle method for the correlation functions important (++)
 - Coulomb corrections not important (++)
 - fast freeze-out (+-)

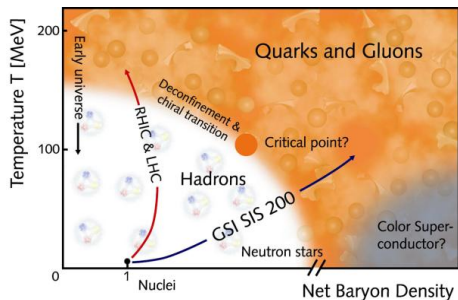


2.1 phase diagrams

- phase diagram for water



- phase diagram for QCD



2.1 modeling the QCD at zero baryon chemical potential

- hadron gas model for low temperatures

input files from **SHARE: Statistical hadronization with resonances**

G. Torrieri, S. Steinke, W. Broniowski, W. Florkowski, J. Letessier, J. Rafelski, Comput. Phys. Commun. **167**, 229 (2005)

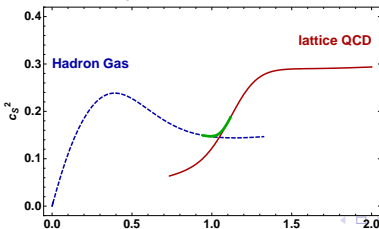
- lattice QCD simulations for large temperatures

based on: Y. Aoki, Z. Fodor, S. Katz, K. Szabo, JHEP **0601**, 089 (2006)

simple parameterization of pressure: T. Biro, J. Zimanyi, Phys.Lett.**B650**, 193 (2007)

- cross-over phase transition, M. Chojnacki, Acta Phys. Pol. **38** (2007) 3249

thermodynamic variables change suddenly at T_c but smoothly ,
the sound velocity does not drop to zero



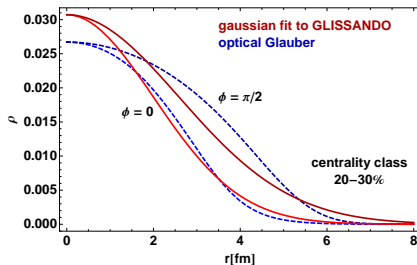
2.2 Initial conditions

Nuclear matter profiles play an important role

- most of the approaches use the Glauber model or Color Glass Condensate,
- W. Broniowski et al., PRL **101** (2008) 022301, Gaussian profiles (Gaussian approximation to Glauber)

$$\frac{dN}{dxdy} \sim \exp\left(-\frac{x^2}{2a^2} - \frac{y^2}{2b^2}\right)$$

the widths a and b determined from GLISSANDO, W. Broniowski et al., Comput. Phys. Commun. **180** (2009) 69



2.3 THERMINATOR

THERMINATOR

THERMal heavy-ion generATOR

Adam Kisiel, Tomasz Taluć, Wojciech Broniowski, Wojciech Florkowski

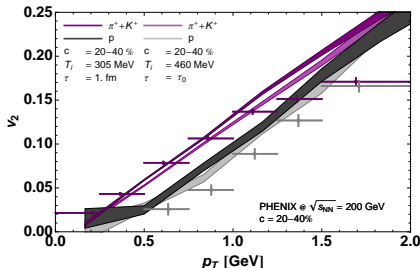
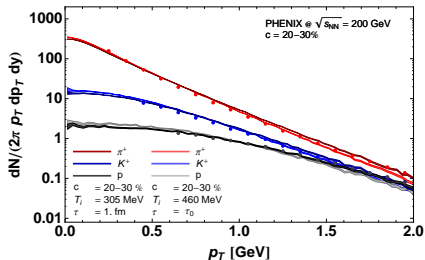
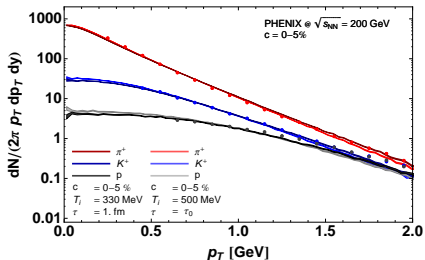
A.Kisiel, T.Taluc, W.Broniowski, W.Florkowski, Comput.Phys.Commun.174:669-687, 2006.

<http://www.ifj.edu.pl/dept/no4/nz41/therminator/therminator.html>

- Cooper-Frye formula used with the freeze-out hypersurface ($T_f = \text{const}$)
- Monte-Carlo code used for particles generation and decays
- THERMINATOR 2, M. Chojnacki, in preparation



2.4 Results for the spectra and v_2



2.4 Results for femtoscopy

A. Kisiel, WF, and W. Broniowski, Phys. Rev. **C73**, 064902 (2006)

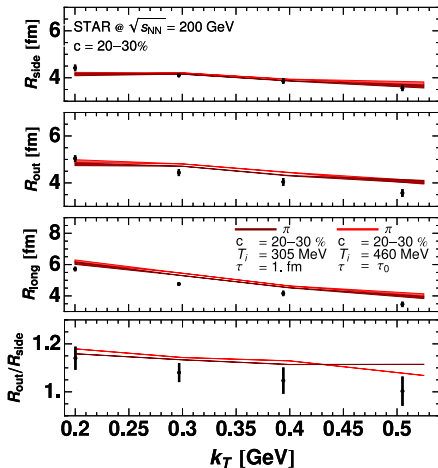
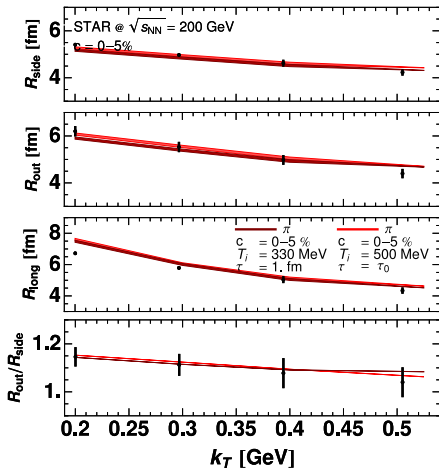
- two-particle method used to calculate the correlation functions (procedure mimics closely the experimental situation),
- the wave function calculated in the pair rest frame (PRF) includes Coulomb (option)
- correlation function fitted in the Bertsch-Pratt coordinates ($k_T, q_{out}, q_{side}, q_{long}$) with Bowler-Sinyukov correction (option)

$$C(\vec{q}, \vec{k}) = (1-\lambda) + \lambda K_{coul}(q_{inv}) \left[1 + \exp\left(-R_{out}^2 q_{out}^2 - R_{side}^2 q_{side}^2 - R_{long}^2 q_{long}^2\right) \right],$$

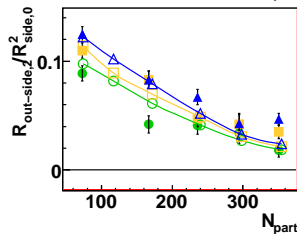
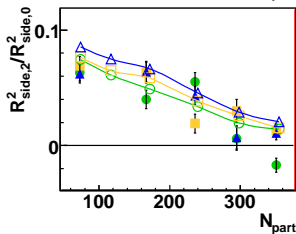
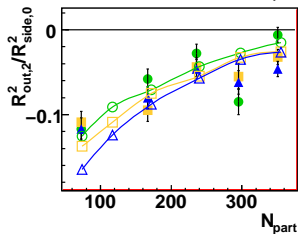
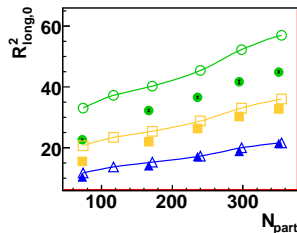
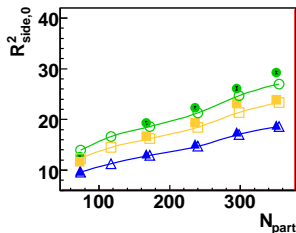
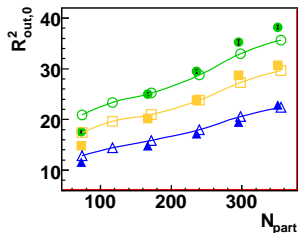
- HBT radii ($R_{out}, R_{side}, R_{long}$) obtained from the fit and compared with data



2.4 HBT results



2.4 Oscillations of the HBT radii, Kisiel et al, PRC 79 (2009) 014902



3. Early-thermalization puzzle



2.1 Fluctuating string tension

- apparent thermalization in string models

Bialas Phys. Lett. B466 (1999) 301

Gaussian fluctuations of the string tension can account for “thermal” character of transverse-mass distributions

$$\frac{dn_{\kappa}}{d^2p_{\perp}} \sim e^{-\pi m_{\perp}^2 / \kappa^2}, \quad P(\kappa) = \sqrt{\frac{2}{\pi \langle \kappa^2 \rangle}} \exp\left(-\frac{\kappa^2}{2 \langle \kappa^2 \rangle}\right)$$

$$\int_0^{\infty} d\kappa P(\kappa) \frac{dn_{\kappa}}{d^2p_{\perp}} \sim \exp\left(-m_{\perp} \sqrt{\frac{2\pi}{\langle \kappa^2 \rangle}}\right)$$

$$T = \sqrt{\frac{\langle \kappa^2 \rangle}{2\pi}}, \quad y \approx \eta, P_L \approx 0$$

- similar effects from the fluctuations of color fields in heavy-ion reactions, color-flux-tube models,

WF, Acta Phys. Polon. B35 (2004) 799



2.1 Color glass condensate & Glasma

Larry McLerran et al.

- **Kovchegov** Nucl. Phys. A830, 395-402, 2009
at early proper times $\tau \ll 1/Q_s$ the classical gluon fields lead to the following energy-momentum tensor (**Lappi,Fukushima**)

$$T^{\mu\nu} \Big|_{\tau \ll 1/Q_s} = \begin{pmatrix} \epsilon(\tau) & 0 & 0 & 0 \\ 0 & \epsilon(\tau) & 0 & 0 \\ 0 & 0 & \epsilon(\tau) & 0 \\ 0 & 0 & 0 & -\epsilon(\tau) \end{pmatrix}$$

at later proper times $\tau \gg 1/Q_s$ both the analytical perturbative approaches (**Kovchegov**) and the full numerical simulations (**Krasnitz**) lead to

$$T^{\mu\nu} \Big|_{\tau \gg 1/Q_s} = \begin{pmatrix} \epsilon(\tau) & 0 & 0 & 0 \\ 0 & \epsilon(\tau)/2 & 0 & 0 \\ 0 & 0 & \epsilon(\tau)/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



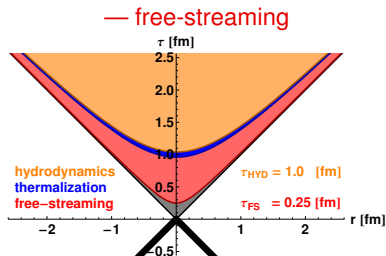
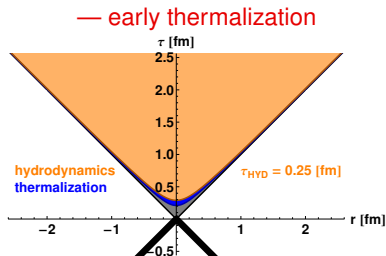
similar effects from viscosity!

$$T^{\mu\nu} = \begin{pmatrix} \varepsilon_3 & 0 & 0 & 0 \\ 0 & P_3 + \frac{2}{3} \frac{\eta}{\tau} & 0 & 0 \\ 0 & 0 & P_3 + \frac{2}{3} \frac{\eta}{\tau} & 0 \\ 0 & 0 & 0 & P_3 - \frac{4}{3} \frac{\eta}{\tau} \end{pmatrix}$$



2.2 Free-streaming, W. Broniowski et al., PRC 80 (2009) 034902

- thermalization requires some time ($\tau \approx 0.25 - 1.0$ fm)
- two scenarios



- model for early stage dynamics
 - free streaming of particles, no interactions
 - sudden thermalization – Landau's matching conditions, $T_{fr. str.}^{\mu\nu} U_\nu = T_{perf. hyd.}^{\mu\nu} U_\nu$
- our results indicate that the two scenarios are equivalent



The early thermalization and HBT puzzles at RHIC

PART II

- free-streaming = free motion, Boltzmann equation without the collision term, at large times the correlation $y = \eta$ builds in (due to pure relativistic kinematics!), $P_L = 0$, another example of the assymetric energy-momentum tensor, Yura Sinyukov et al.
- our opinion: delayed but sudden thermalization is equivalent to the gradual thermalization
- kinetic models: partonic, based on QCD (before hydro stage) & hadronic, UrQMD (after hydro stage)
- no dip in $c_s(T) \rightarrow$ no shock waves — (simple) numerical algorithms for solving hydro work, we do it with Mathematica, later ROOT



4. Transverse hydrodynamics



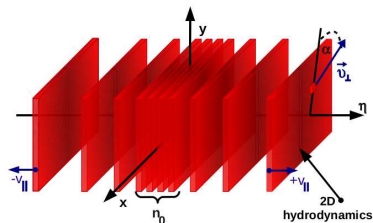
4.1 Transverse-hydro concept

- at early stages only the transverse degrees of freedom are thermalized and described by hydrodynamics
- an earlier formulation of this idea:
 Heinz and S.M.H. Wong Phys. Rev. **C66** (2002) 014907
 Heinz and S.M.H. Wong Nucl. Phys. **A715** (2003) 649
- the new implementation
 Bialas, Chojnacki and Florkowski Phys. Lett. **B661** (2008) 325
 correct description of the p_{\perp} spectra and v_2
 partons identified with pions
 $\langle p_{\perp} \rangle$ is conserved in transverse hydro
 $\langle p_{\perp} \rangle \approx 2\lambda_{\text{slope}} \approx T \sqrt{(1+v)/(1-v)}$
- most recent developments:
 Chojnacki and Florkowski, Acta Phys. Pol. **B39** (2008)
 Ryblewski and Florkowski, Phys. Rev. **C77** (2008) 064906



4.1 Transverse clusters

- superposition of non-interacting transverse clusters
- each cluster is formed by particles moving with the same value of rapidity
- single cluster \Rightarrow 2D hydrodynamics
- $n_0(\eta)$ - density of clusters in rapidity



$$y = \frac{1}{2} \ln \frac{E + p_{\parallel}}{E - p_{\parallel}} \quad \eta = \frac{1}{2} \ln \frac{t + z}{t - z}$$

- standard parameterization of the four-momentum and spacetime coordinate

$$p^{\mu} = (E, \vec{p}_{\perp}, p_{\parallel}) = (m_{\perp} \cosh y, \vec{p}_{\perp}, m_{\perp} \sinh y)$$

$$x^{\mu} = (t, \vec{x}_{\perp}, z) = (\tau \cosh \eta, \vec{x}_{\perp}, \tau \sinh \eta)$$

$$\tau = \sqrt{t^2 - z^2} \quad m_{\perp} = \sqrt{m^2 + p_x^2 + p_y^2}$$



4.1 2D thermodynamics

- non-interacting bosons

$$\Omega_2(T_2, V_2, \mu) = \nu_g T_2 V_2 \int \frac{d^2 p_\perp}{(2\pi)^2} \ln \left(1 - e^{(\mu - m_\perp)/T_2} \right)$$

- gluon dominated systems $\rightarrow \nu_g = 16$
- number of particles not conserved $\rightarrow \mu = 0$

$$n_2 = \frac{\nu_g \pi T_2^2}{12} \quad \epsilon_2 = \frac{\nu_g \zeta(3) T_2^3}{\pi} \quad P_2 = \frac{\epsilon_2}{2} \quad \epsilon_2 + P_2 = T \sigma_2$$



4.1 Energy-momentum tensor

- energy-momentum tensor $T_2^{\mu\nu}$, entropy current S_2^μ , particle current N_2^μ

$$T_2^{\mu\nu} = \frac{n_0}{\tau} [(\varepsilon_2 + P_2) U^\mu U^\nu - P_2 (g^{\mu\nu} + V^\mu V^\nu)]$$

$$N_2^\mu = \frac{n_0}{\tau} n_2 U^\mu \quad S_2^\mu = \frac{n_0}{\tau} s_2 U^\mu$$

- four vectors U^μ and V^μ are defined by

$$U^\mu = (u_0 \cosh \eta, u_x, u_y, u_0 \sinh \eta)$$

$$V^\mu = (\sinh \eta, 0, 0, \cosh \eta)$$

- rest-frame of the fluid element (similar to CGC, glasma)

$$T_2^{\mu\nu} = \frac{n_0}{\tau} \begin{pmatrix} \varepsilon_2 & 0 & 0 & 0 \\ 0 & P_2 & 0 & 0 \\ 0 & 0 & P_2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



4.1 Transverse-hydrodynamics equations

- energy-momentum conservation law

$$\partial_\mu T_2^{\mu\nu} = 0$$

- entropy conservation

$$\partial_\mu S_2^\mu = 0$$

- three-dimensional densities of the transversally thermalized system

$$\varepsilon_3^{\text{tr}} = \frac{n_0}{\tau} \varepsilon_2, \quad \sigma_3^{\text{tr}} = \frac{n_0}{\tau} \sigma_2, \quad \dots$$

- hydrodynamic equations

$$\begin{aligned} U^\mu \partial_\mu (T_2 U^\nu) &= \partial^\nu T_2 + V^\nu V^\mu \partial_\mu T_2 \\ \partial_\mu (\sigma_3^{\text{tr}} U^\mu) &= 0 \end{aligned}$$



4.1 Initial transverse profiles

- $\tau = 0 \rightarrow$ two nuclei pass through each other
- $\tau = \tau_i \rightarrow$ transverse thermalization

initial flow

$$v_i(\vec{x}_\perp) = 0$$

initial profiles, **NOT GAUSSIANS NOW!** (mixed model)

$$\sigma_{2i}(\vec{x}_\perp) = \sigma_2(\tau_i, \vec{x}_\perp) \propto \rho_{sr}(\vec{x}_\perp) = \frac{1 - \kappa}{2} \bar{w}(\vec{x}_\perp) + \kappa \bar{n}(\vec{x}_\perp)$$

$$\varepsilon_{2i}(\vec{x}_\perp) = \varepsilon_2(\tau_i, \vec{x}_\perp) \propto \rho_{sr}(\vec{x}_\perp) = \frac{1 - \kappa}{2} \bar{w}(\vec{x}_\perp) + \kappa \bar{n}(\vec{x}_\perp)$$

initial temperature profile (2D bosons)

$$\varepsilon_2(\tau_i, \vec{x}_\perp) = \frac{\nu g \zeta(3) T_2^3(\tau_i, \vec{x}_\perp)}{\pi}$$

$$\sigma_2(\tau_i, \vec{x}_\perp) = \frac{3}{2} \frac{\nu g \zeta(3) T_2^2(\tau_i, \vec{x}_\perp)}{\pi}$$

$$T_{2i} = T_2(\tau_i, 0)$$



4.2 Landau matching conditions

microscopic view: full isotropisation is expected eventually, gradual process approximated by a delayed step-like transition at $\tau_{tr} = \text{const}$.

- Landau matching conditions

$$T_2^{\mu\nu} U_\nu = T_3^{\mu\nu} U_\nu$$

$$T_3^{\mu\nu} = (\varepsilon_3 + P_3)U^\mu U^\nu - P_3 g^{\mu\nu}$$

- equivalent condition supplemented by the requirement of the entropy growth

$$\varepsilon_3^{\text{tr}} = \frac{n_0}{\tau_{\text{tr}}} \varepsilon_2 = \varepsilon_3,$$

$$\sigma_3^{\text{tr}} = \frac{n_0}{\tau_{\text{tr}}} \sigma_2 \leq \sigma_3,$$

- microscopic approaches:

P. Bozek, Acta Phys. Polon., **B39** (2008) 1375 - dissipative hydro,

η very large in the early stage

B. Zhang, **L.-W. Chen**, **C. M. Ko**, arXiv:0805.0587 - transport theory



4.2 Landau matching conditions

- transverse-hydrodynamics equations are scale invariant — temperature may be multiplied by an arbitrary factor without the change of the flow profile
- the following transformation does not change the 3D energy density and flow, but changes the 3D entropy density in the transverse stage

$$n_0 \rightarrow \lambda n_0, \quad T_2 \rightarrow \lambda^{-1/3} T_2$$

$$\varepsilon_3^{\text{tr}} \rightarrow \varepsilon_3 \quad \sigma_3^{\text{tr}} \rightarrow \lambda^{1/3} \sigma_3^{\text{tr}} \leq \sigma_3 \quad v \rightarrow v$$

- $\lambda \searrow$ as we shall see, the fit to the data favors fewer and hotter clusters (small n_0)



4.2 Landau matching conditions

- transverse-hydrodynamics equations are scale invariant — temperature may be multiplied by an arbitrary factor without the change of the flow profile
- the following transformation does not change the 3D energy density and flow, but changes the 3D entropy density in the transverse stage

$$n_0 \rightarrow \lambda n_0, \quad T_2 \rightarrow \lambda^{-1/3} T_2$$

$$\varepsilon_3^{\text{tr}} \rightarrow \varepsilon_3 \quad \sigma_3^{\text{tr}} \rightarrow \lambda^{1/3} \sigma_3^{\text{tr}} \leq \sigma_3 \quad v \rightarrow v$$

- $\lambda \searrow$ as we shall see, the fit to the data favors fewer and hotter clusters (small n_0)



4.3 Model parameters

Model parameters

n_0	?	overall normalization
T_{2i}	?	initial central temperature of 2D system
T_{3f}	?	freeze-out temperature
τ_i	?	initial proper time
τ_{tr}	?	the 2D \rightarrow 3D transition time
κ	?	admixture of the binary-collision density
μ_B	?	baryon chemical potential
μ_S	?	strangeness chemical potential
μ_{I_3}	?	isospin chemical potential



4.3 Model parameters

Model parameters		
n_0	?	overall normalization
T_{2i}	?	initial central temperature of 2D system
T_{3f}	?	freeze-out temperature
τ_i	?	initial proper time
τ_{tr}	?	the 2D \rightarrow 3D transition time
κ	?	admixture of the binary-collision density
μ_B	28.5 MeV	baryon chemical potential
μ_S	6.9 MeV	strangeness chemical potential
μ_{I_3}	0.9 MeV	isospin chemical potential

chemical potentials μ_B, μ_S, μ_{I_3} , [M.Michalec et al. Acta Phys. Pol. B33 \(2002\) 761](#)

- $\ll T_{3f}$
- their effect on the evolution of matter is neglected
- appear **only** in the thermal distribution functions used to generate particles on the freeze-out hypersurface



4.3 Model parameters

Model parameters		
n_0	?	overall normalization
T_{2i}	?	initial central temperature of 2D system
T_{3f}	145 MeV	freeze-out temperature
τ_i	?	initial proper time
τ_{tr}	?	the 2D \rightarrow 3D transition time
κ	?	admixture of the binary-collision density
μ_B	28.5 MeV	baryon chemical potential
μ_S	6.9 MeV	strangeness chemical potential
μ_{I_3}	0.9 MeV	isospin chemical potential

freeze-out temperature

- typical values for freeze-out temperature used are ~ 140 -145 MeV
- control the slope of the spectra



4.3 Model parameters

Model parameters		
n_0	?	overall normalization
T_{2i}	?	initial central temperature of 2D system
T_{3f}	145 MeV	freeze-out temperature
τ_i	0.25 fm	initial proper time
τ_{tr}	1 fm	the 2D \rightarrow 3D transition time
κ	?	admixture of the binary-collision density
μ_B	28.5 MeV	baryon chemical potential
μ_S	6.9 MeV	strangeness chemical potential
μ_{I_3}	0.9 MeV	isospin chemical potential

transverse evolution time interval

- $\tau_{tr} - \tau_i \geq 0.75 \text{ fm} \Rightarrow$ radial flow too strong



4.3 Model parameters

Model parameters

n_0	?	overall normalization
T_{2i}	?	initial central temperature of 2D system
T_{3f}	145 MeV	freeze-out temperature
τ_i	0.25 fm	initial proper time
τ_{tr}	1 fm	the 2D \rightarrow 3D transition time
κ	?	admixture of the binary-collision density
μ_B	28.5 MeV	baryon chemical potential
μ_S	6.9 MeV	strangeness chemical potential
μ_{I_3}	0.9 MeV	isospin chemical potential

fitting procedure performed for n_0, T_{2i}, κ

- we find n_0, T_{2i}, κ
- rescale n_0, T_{2i} to satisfy L.M.C.



4.3 Model parameters

Model parameters

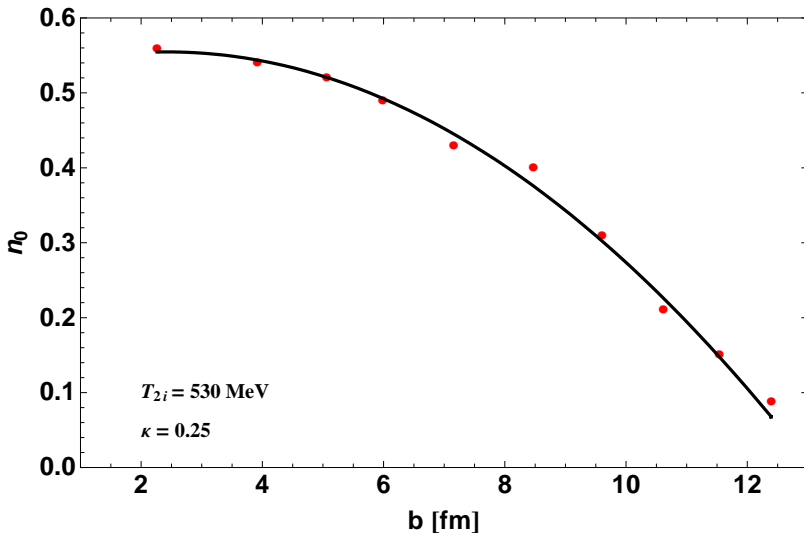
n_0	$n_0(b)$	overall normalization
T_{2i}	530 MeV	initial central temperature of 2D system
T_{3f}	145 MeV	freeze-out temperature
τ_i	0.25 fm	initial proper time
τ_{tr}	1 fm	the 2D \rightarrow 3D transition time
κ	0.25	admixture of the binary-collision density
μ_B	28.5 MeV	baryon chemical potential
μ_S	6.9 MeV	strangeness chemical potential
μ_{I_3}	0.9 MeV	isospin chemical potential

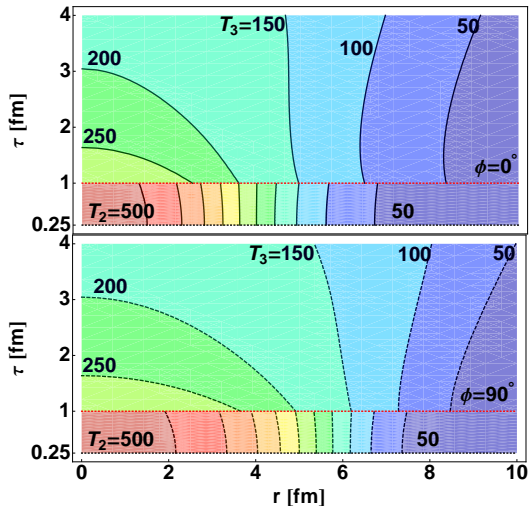
fitting procedure performed for n_0 , T_{2i} , κ

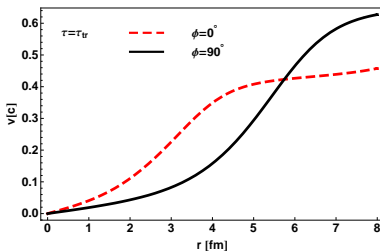
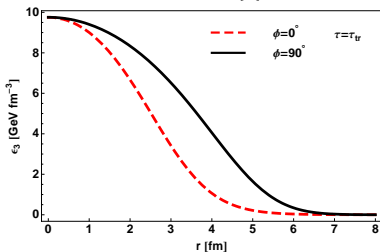
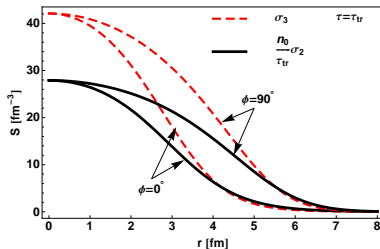
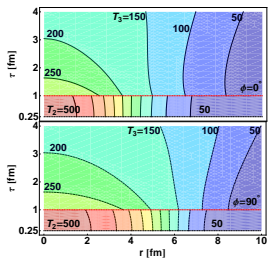
- we find n_0 , T_{2i} , κ
- rescale n_0 , T_{2i} to satisfy L.M.C.

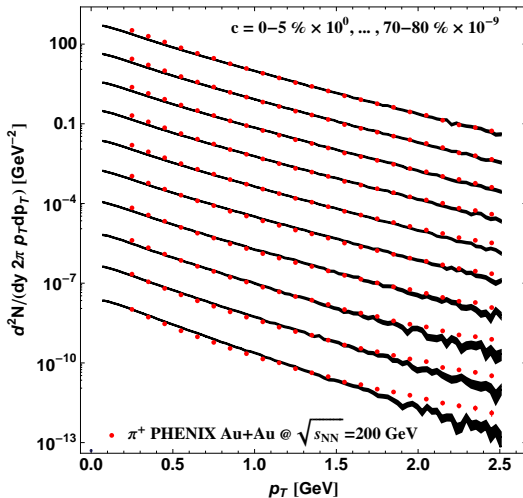


4.3 Centrality dependence of n_0

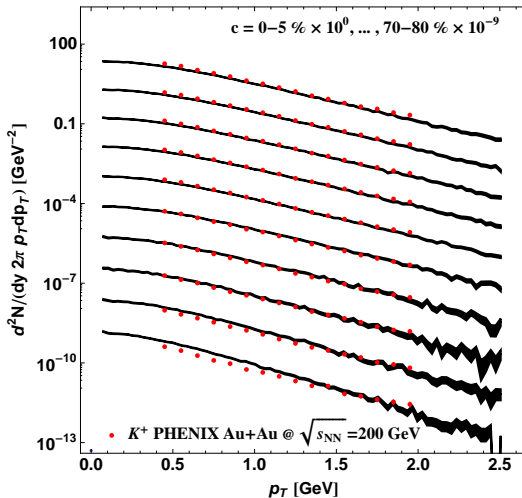


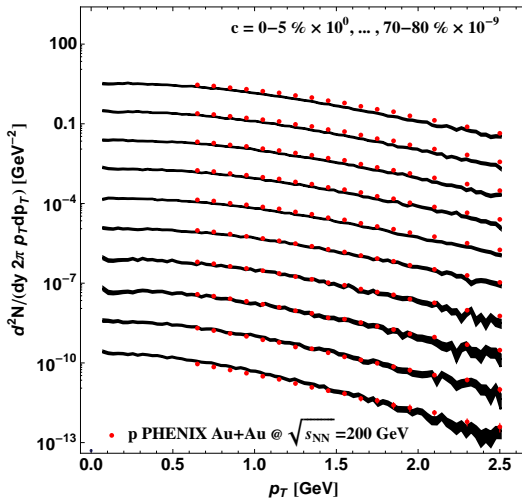
4.3 2D \rightarrow 3D transition (c=20-30%)

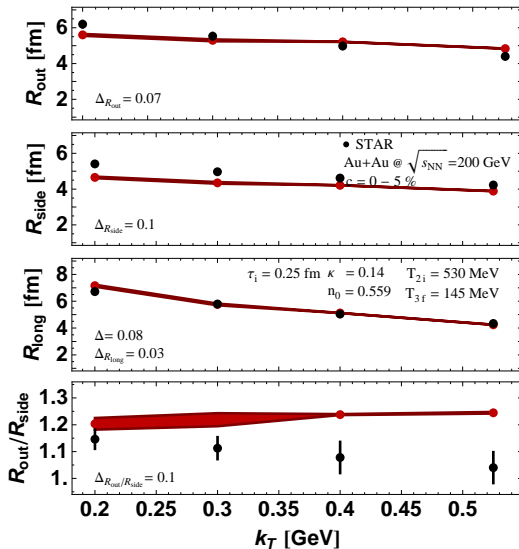
4.3 2D \rightarrow 3D transition (c=20-30%)

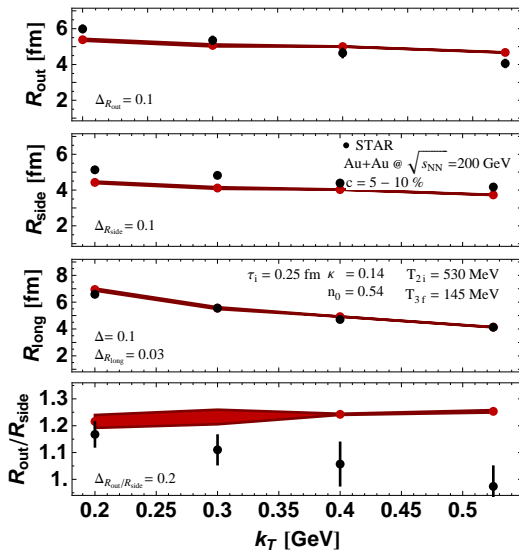
π^+ p_{\perp} -spectra

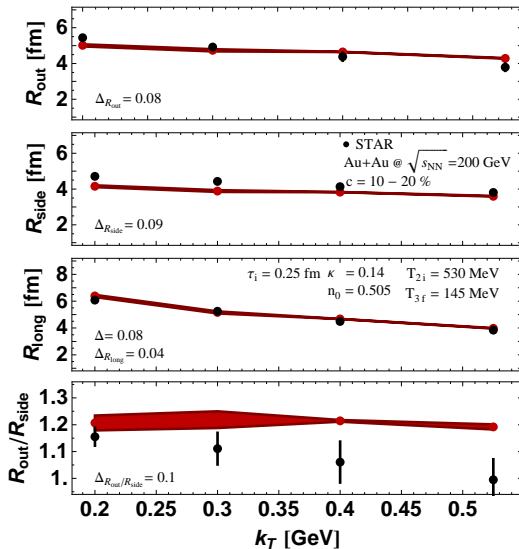
K^+ p_{\perp} -spectra

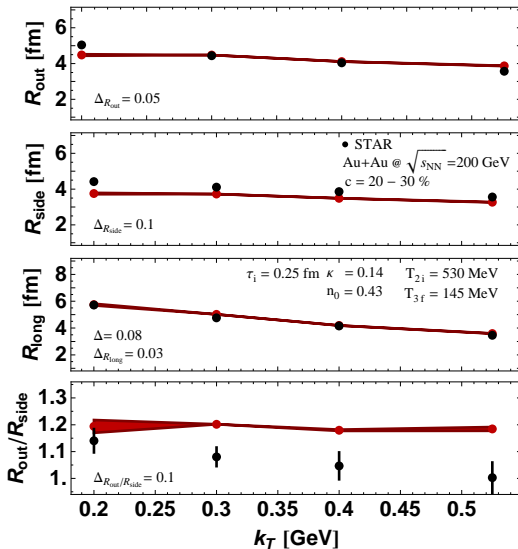


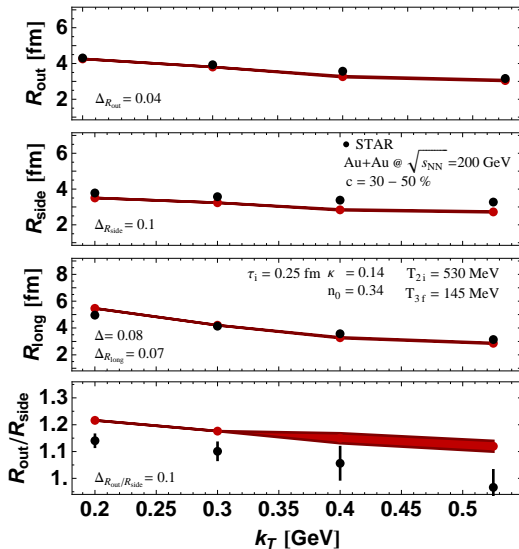
proton p_{\perp} -spectra

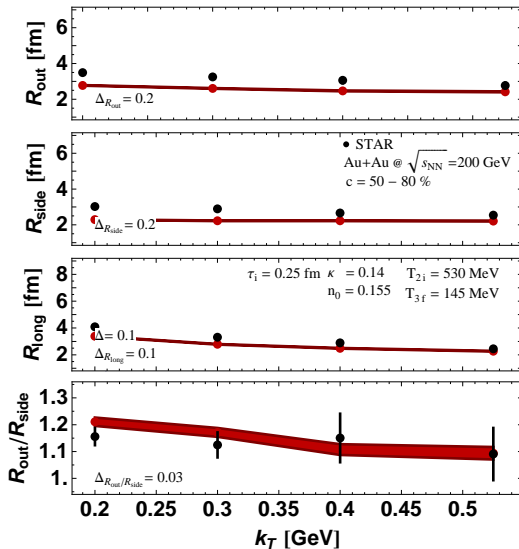
π HBT ($c=0-5\%$)

π HBT ($c=5-10\%$)

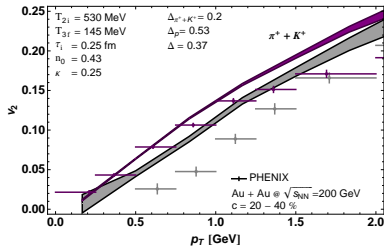
π HBT ($c=10-20\%$)

π HBT ($c=20-30\%$)

π HBT ($c=30-50\%$)

π HBT ($c=50-80\%$)

v_2 (c=20-30%)



protons v_2 is too large by about 50%

possible reasons:

- lack of the hadronic interactions in the final state
- neglecting the viscous effects (inclusion of the bulk viscosity,

P.Bożek Phys.Rev.C81:034909,2010)

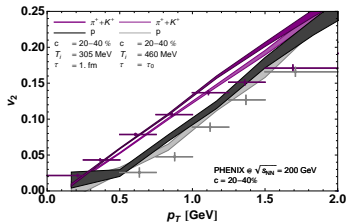


5. HBT vs v_2 puzzle?

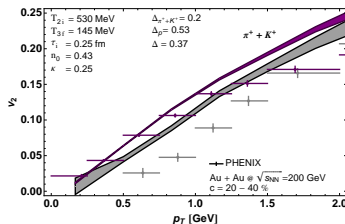


5.1 Proton v_2 for realistic EOS is too large!

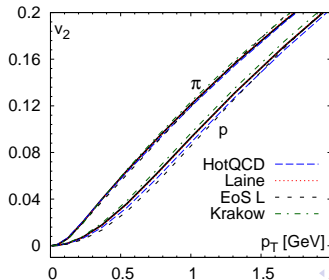
Gaussian initial conditions



transverse hydrodynamics

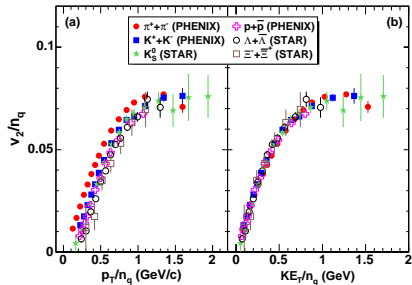
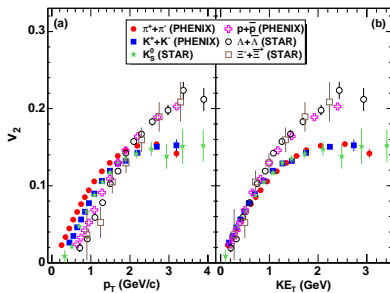


Huovinen and Petreczky, Nucl. Phys. A837 (2010) 26



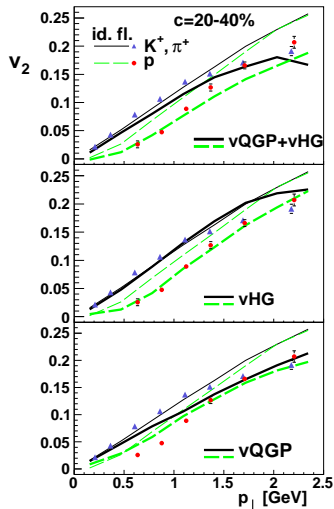
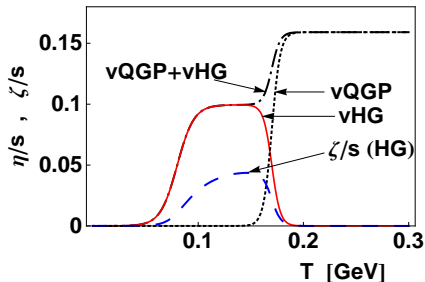
5.2 v_2 scaling

PHENIX, PRL98 (2007) 162301



5.3 Inclusion of the shear and bulk (!) viscosity

P. Bozek, PRC81 (2010) 034909,
 ζ lowers T_f to 135 MeV



6. QCD phase transition in the early Universe

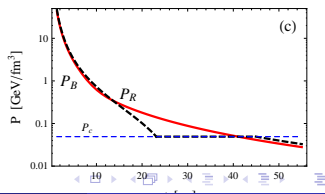
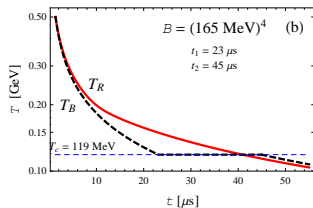
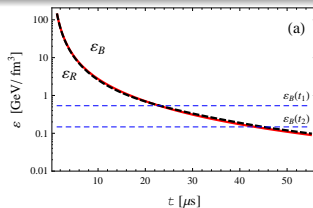
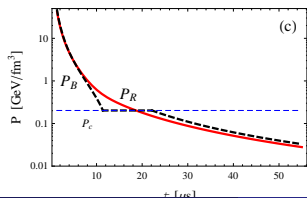
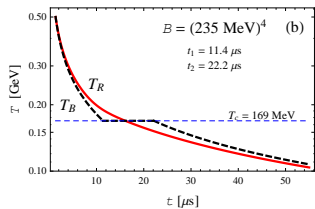
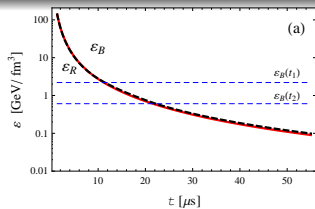


$$\frac{d\varepsilon_R}{dt} = -3\sqrt{\frac{8\pi G\varepsilon_R}{3}}(\varepsilon_R + P_R)$$

$$\left[c_s^{-2}\sigma + 3\sigma_{ew} \right] \frac{dT_R}{dt} = -3\sqrt{\frac{8\pi G(\varepsilon + \varepsilon_{ew})}{3}}(\varepsilon + \varepsilon_{ew} + P + P_{ew})$$



6.1 Energy density evolution



7. Conclusions



- the inclusion of the realistic EOS and the bulk viscosity is the most attractive solution of the HBT- v_2 puzzle
- the shear and shear viscosities are small — perfect fluid behavior confirmed
- the finite-state rescattering negligible
- the modified early dynamics helps to circumvent the early thermalization problems

