Quantum Anomalies in Noncommutative Geometry

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Outline

① "Standard" anomalies • What are anomalies? Approaches to anomalies The axioms • Example Some results Geometrical approach Noncommutative 2-torus

Conclusions

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- 3 Anomalies on spectral triples
 - Some results
 - Geometrical approach
 - Noncommutative 2-torus

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Heuristic introduction

Classical FT

imposed symmetries

 $\mathcal{L}[A_{\mu}] = \mathcal{L}[A_{\mu}^{g}]$ \downarrow conserved currents

 $D_{\mu}j^{\mu} = 0$

Quantum FT

broken symmetries $Z[A_\mu]
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QFT methods

Feynman diagrams

Path-integrals - the Fujikawa method The non-invariance of the path-integral measure!

 $J[\beta, A_{\mu}] = \mathrm{e}^{-\int_{M} \beta \mathbf{A}}$

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Figure: Triangle diagram appearing in the famous ABJ anomaly computation.

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What are anomalies? Approaches to anomalies

Geometry of anomalies

BRST cohomology

- BRST (non)invariance $s\mathcal{L} = 0$, $sW[A_{\mu}] = \mathbf{A}$
- Nilpotency $s^2 = 0 \Rightarrow s \int_M \nu^a \mathbf{A}^a_{\text{consistent}} = 0$
 - Wess-Zumino consistency conditions
 - Chain of descent equations $\Longrightarrow A^{2n}_{\text{consistent}} \longleftrightarrow A^{2n+2}_{\text{singlet}}$

Index theory

- Singlet anomaly \longleftrightarrow AS index theorem
- Non-abelian anomaly
 - \sim AS index theorem for families of elliptic operators
 - AS index theorem in 2n + 2 dimensions (AGG). The winding number of the fermionic determinant.

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The axioms Example

The axioms of noncommutative geometry

$(\mathcal{A},\mathcal{H},\mathcal{D})$ - spectral triple

- \mathcal{A} dense *-subalgebra of a C^* -algebra, unital
- \mathcal{H} Hilbert space; by GNS construction \exists a faithful representation $\rho(\mathcal{A}) \simeq \mathcal{B}(\mathcal{H})$
- \mathcal{D} the Dirac operator selfadjoint, unbounded
 - \mathcal{D}^{-1} compact
 - $[\mathcal{D}, \rho(a)] \in \mathcal{B}(\mathcal{H})$ for all $a \in A$
- $\gamma \in \mathcal{B}(\mathcal{H})$ chirality selfadjoint, unitary

$$\gamma^2 = \mathrm{id}, \qquad \forall_{a \in \mathcal{A}} \ \gamma a = a\gamma, \qquad \mathcal{D}\gamma = -\gamma \mathcal{D}$$

The \mathbb{Z}_2 -grading of the Hilbert space $\mathcal{H}=\mathcal{H}^+\oplus\mathcal{H}^-$

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A commutative spectral triple

Example 1

Let ${\cal M}$ be compact Riemannian spin manifold, then

- $\mathcal{A} = C^{\infty}(M)$ dense in $C^{0}(M)$
 - $\bullet\,$ unital if M is compact

•
$$\mathcal{H} = L^2(S(M))$$

• representation: $\rho(f)\xi=f(x)\cdot\xi(x)$

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$${\cal D}=\gamma^\mu\partial_\mu$$
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Some results

Moyal plane R⁴_{*}
 (J.M. Gracia-Bondía, C.P. Martín, *Phys. Lett. B* 479 (2000) 321)

 $\mathbf{A} = \alpha \operatorname{Tr}\{T^a, T^b\}T^c + \beta \operatorname{Tr}[T^a, T^b]T^c$

- Geometric approach
 - (D. Perrot, Contemp. Math. 434 (2007) 125.)
 - The role of projections in \mathcal{A} .

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Perrot's procedure

Assumption: p-summability *i.e.* $\operatorname{Tr}(\mathcal{D}^{-p}) < \infty$, r - biggest integer < p

• The action of the "classical" theory

 $S(\psi, \bar{\psi}) = \langle \bar{\psi}, (Q+A)\psi \rangle, \qquad Q = \mathcal{D} + \gamma m, \qquad A \in \mathcal{B}(\mathcal{H})$

• Path-integral quantization

$$Z(A) = \int d\psi d\bar{\psi} e^{-S(\psi,\bar{\psi},A)} = \det(1+Q^{-1}A)$$
$$W(A) = \operatorname{Tr} \log(1+Q^{-1}A) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \operatorname{Tr}(Q^{-1}A)^n$$

• Regularization - trace extension $\tau(T) = \operatorname{Res}_{z=0} \frac{1}{z} \operatorname{Tr}(T|Q|^{-2z})$

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Perrot's procedure

- BRST operator de Rham differential on S^1 $s : C^{\infty}(S^1) \otimes \mathcal{A} \rightarrow \Omega^1(S^1) \otimes \mathcal{A}.$
- Idempotent loops $g = 1 + (\beta 1)p, \quad C^{\infty}(S^1) \ni \beta = e^{2\pi \operatorname{i} t}, \quad \mathcal{A} \ni p = p^2$
- Note: $(\mathcal{A}, \mathcal{H}, \mathcal{D}) \longleftrightarrow (M_N(\mathcal{A}), \mathcal{H} \otimes \mathbb{C}^N, \mathcal{D} \otimes \mathbf{1})$
- Mauer-Cartan form (ghost field) $\omega = g^{-1}sg = 2\pi \,\mathrm{i}\, p \,\mathrm{d}t$
- The potential is pure gauge $A = g^{-1}Qg Q$

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Perrot's procedure

The anomaly $\mathbf{A}(\omega, A) := s W_{\tau}(A) =$

$$\sum_{n=1}^{r} \frac{(-1)^{n+1}}{n} s \tau (Q^{-1}A)^n + (-1)^r \operatorname{Tr} \left((\omega_+ - Q^{-1}\omega_- Q)(Q^{-1}A)^r \right),$$

Theorem (Perrot)

$$\langle [\mathcal{D}], [p] \rangle = \frac{1}{2\pi i} \oint_{S^1} \mathbf{A}(\omega, A) \in \mathbb{Z},$$

2 $\mathbf{A}(\omega, A)$ is cohomologous, as a one-form, to the sum of residues

$$\widetilde{\mathcal{A}}(\omega, A) := \underset{z=0}{\operatorname{Res}} \frac{1}{z} \operatorname{Tr} \left(\gamma \omega Q^{-2z} \right) + \sum_{n \ge 1} \sum_{k \in \mathbb{N}^n} (-1)^{n+k} c(k) \times \\ \times \underset{z=0}{\operatorname{Res}} \frac{\Gamma(z+n+k)}{\Gamma(z+1)} \operatorname{Tr} \left(\frac{1+\gamma}{2} [\omega, Q] A^{(k_1)} Q A^{(k_2)} Q \dots A^{(k_n)} Q^{-2(z+n+k)} \right).$$

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Results

• The anomaly result $\widetilde{\mathbf{A}}(\omega, A) = \mathbf{A}(\omega, A) =$

 $4\pi \,\mathrm{i}\,\mathrm{d}t \mathop{\mathrm{Res}}_{z=0} \,\mathrm{tr}\left((\beta-1)[\mathcal{D}_{-},p][\mathcal{D}_{+},p] + (2-\beta-\beta^{-1})[\mathcal{D}_{-},p]p[\mathcal{D}_{+},p]\right) Q^{-2z-2}$

• The index result

$$\frac{1}{2\pi i} \oint_{S^1} \mathbf{A}(\omega, A) = 2 \operatorname{Res}_{z=0} \operatorname{tr} \left(- [\mathcal{D}_-, p] [\mathcal{D}_+, p] + 2 [\mathcal{D}_-, p] p [\mathcal{D}_+, p] \right) Q^{-2z-2}$$

• Agreement with Connes-Moscovici

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Projections

$$VU = e^{2\pi i \theta} UV, \qquad f(V) = \sum_{n \in \mathbb{Z}} f_n V^n$$

• Power-Rieffel projection $p = f(V)U + U^{-1}\overline{f}(V^*) + g(V)$



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Projections

$$VU = e^{2\pi i \theta} UV, \qquad f(V) = \sum_{n \in \mathbb{Z}} f_n V^n$$

• Second order projection $p^{(2)} = p_2(V)U^{-2} + p_1(V)U^{-1} + p_0(V) + U\bar{p}_1(V^*) + U^2\bar{p}_2(V^*)$





Conclusions

• Anomaly cancellation condition

- Classically: $Tr\{T^a, T^b\}T^c$
- Moyal plane: $\operatorname{Tr} T^a T^b T^c$
- Generally: ?
- The geometry of anomalies
 - The classical correspondence not clear
 - The role of projections

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