

# QCD at High Energies

**Jochen Bartels**

*II.Inst.f.Theor.Physik, Univ.Hamburg*

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- Introduction
- Perturbation Theory and applications
  - QCD Reggeon Field Theory
  - Saturation
  - LHC related issues (multiple interactions)
- The Gauge/String Duality
  - High energy scattering of planar amplitudes
  - The (hard) Pomeron in AdS/CFT

*First Cracow School I attended was in 1979. Title of my talk was:*  
'High Energy Behavior of Nonabelian Gauge Theories'  
(massive  $SU(2)$  Higgs model)

*Since then three (?) more visits:*

1987 (analytic study of the nonlinear QCD lattice Hamiltonian)

In the nineties (small  $x$  physics)

2001 'Saturation in deep inelastic scattering'

2010 'High energy QCD'

*(Note: at least one attempt of getting away from the 'high energy problem' - not successful!)*

*Most important: enjoyed collaborations with several colleagues from Krakow*

# Introduction

High energy QCD:

Impact of HERA:

- structure functions at small  $x$
- boost of BFKL, BFKL and QCD reggeon field theory
- saturation
- diffraction

Impact of RHIC → Lectures of Larry MacLerran:

- physics of strong color fields, color glass condensate (saturation) ...
- new formulation of high energy QCD

LHC needs and will stimulate:

- hard scattering (for discovering new physics) in the small- $x$  region
- multiple interactions, interface to nonperturbative physics  
(as background to new physics)
- soft physics (total cross section etc).

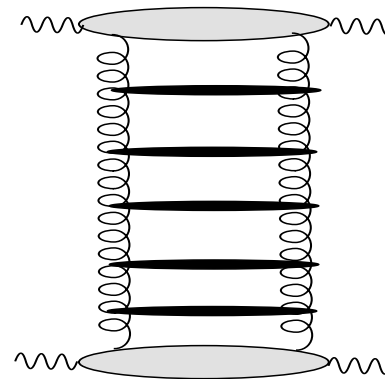
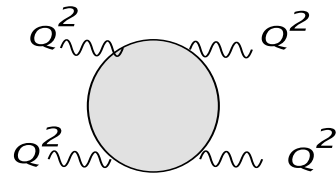
The AdS/CFT duality conjecture:

- $N = 4$  SYM, planar approximation, all order perturbation theory
- Pomeron/graviton duality
- modelling soft physics (e.g. Pomeron, Odderon) → lectures of Stan Brodsky (?)

## Start in pQCD: LO BFKL

Start: leading log-s summation in  $2 \rightarrow 2$  scattering in the limit  $s \rightarrow \infty$ ,  $t$  fixed

$$T(s, t) = \sum_n (\alpha_{em} \alpha_s)^2 (\alpha_s \ln s)^n$$



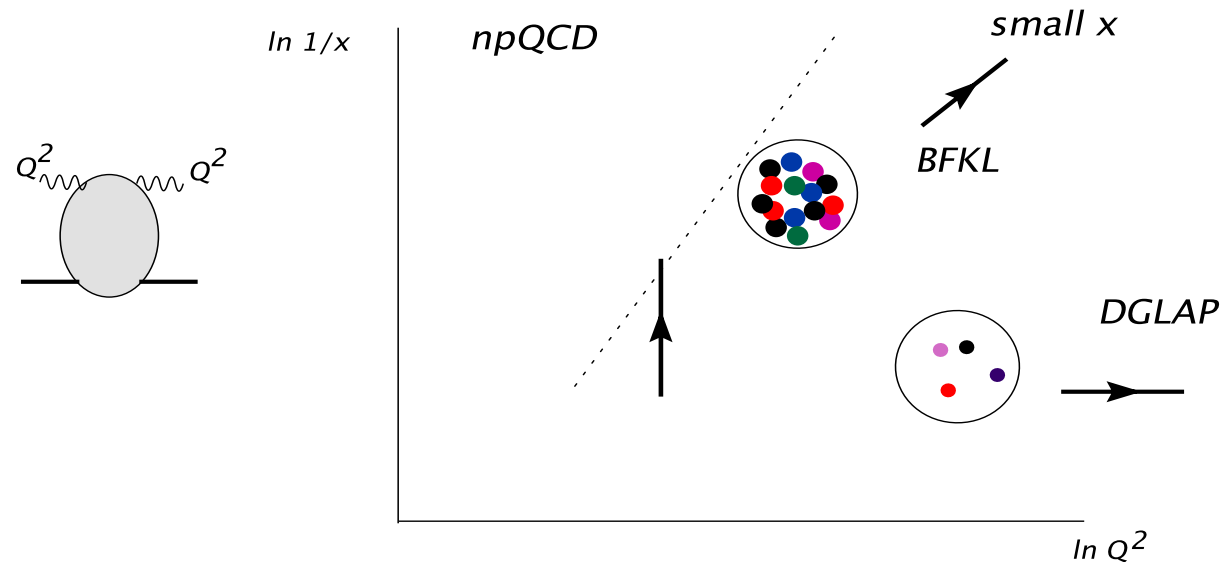
$\sim s^p$

with  $p = \alpha_s \frac{4N_c \ln 2}{\pi}$ . Needs large  $Q^2$  to justify the use of perturbation theory.

All elements (kernel, impact factor) are known in NLO (Fadin, Lipatov; Camicci, Ciafaloni; JB, Chachamis, Giese)

Lots of phenomenology of BFKL in  $ep$ ,  $e^+e^-$ ,  $pp$ .

Validity of BFKL: depends upon environment. E.g. in DIS:



Near the dashed line (saturation line) naive BFKL needs modifications. Different ideas:

a) Set boundary conditions (Lipatov; Kowalski, Lipatov, Ross )

Most interesting consequence: fixed cut in the  $\omega$ -plane turns into string of poles.

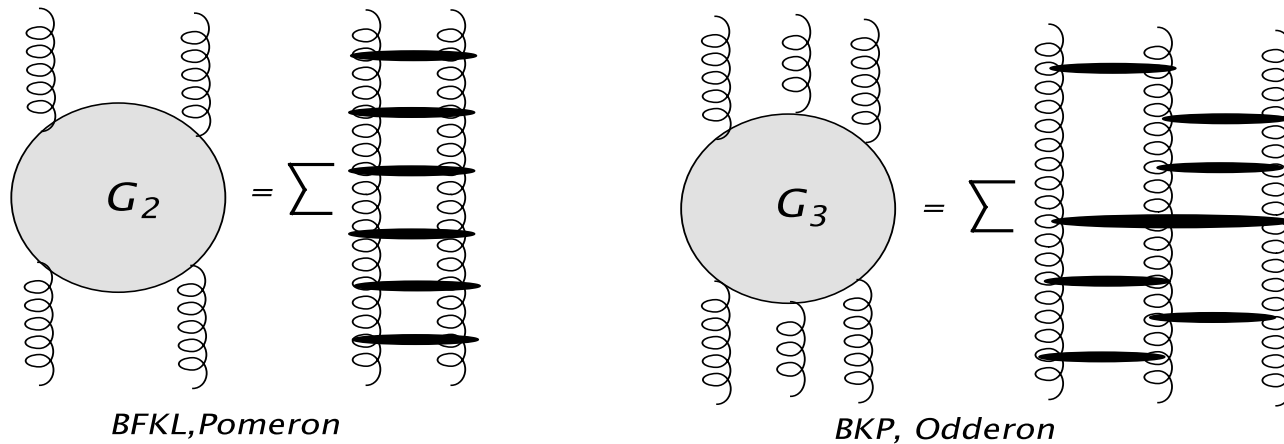
b) Saturation model (GBW), nonlinear evolution equations (Balitsky, Kovchegov)

b) 'Pomeron loops', include elements of QCD reggeon field theory.

## Beyond BFKL: field theory

BFKL is the beginning of a  $2 + 1$  dimensional field theory which describes QCD at high energies (lives rapidity, transverse coordinates).

Define Green's functions of interacting reggeized gluons:  $G_\omega(\vec{r}_1, \dots, \vec{r}_n; \vec{r}'_1, \dots, \vec{r}'_n)$

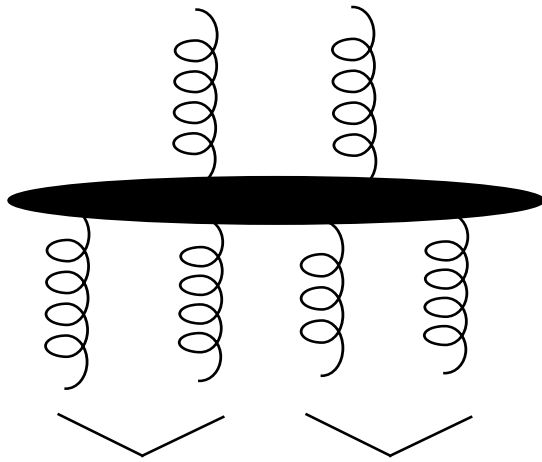


Evolution in time (=rapidity): Schroedinger-like equations  $\partial_y G = \sum K_{BFKL} \otimes G$ .

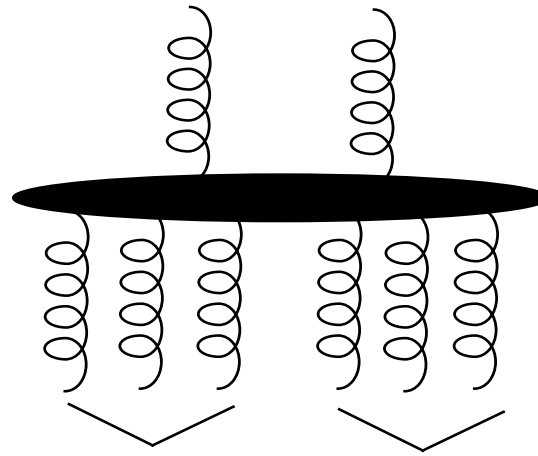
Solution: eigenvalues of the kernel. Known for  $n = 2$ ,  $n = 3$ .

For general  $n$ : in the limit  $N_c \rightarrow \infty$  the problem is exactly soluble: **integrability (closed chain)**

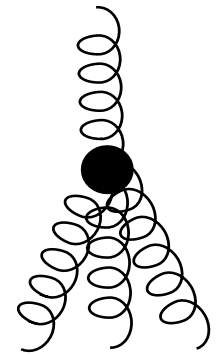
Next step: number changing vertices



*'triple Pomeron'*

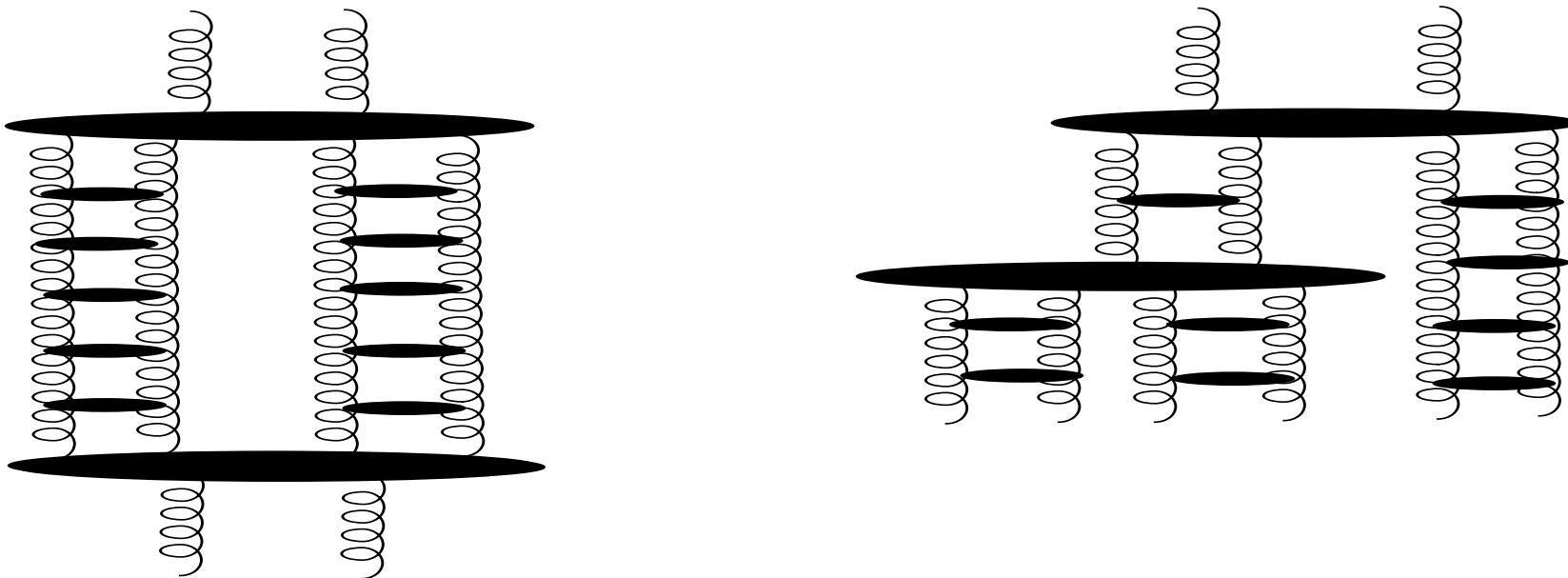


*Pomeron  $\rightarrow$  2 Odderon*



Better understanding of vertices: role of  $d$ -reggeons.  
Can build general Green's functions:



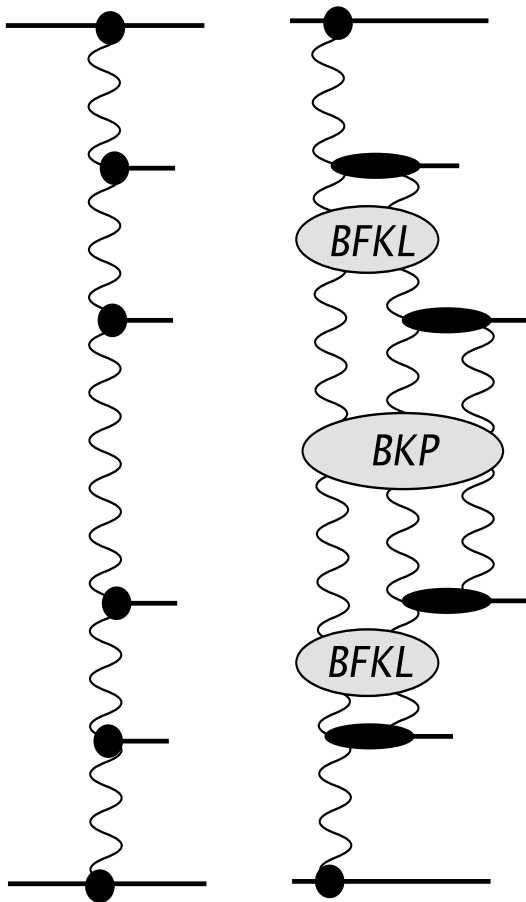


**Möbius invariance:** use complex coordinates  $\rho = r_1 + ir_2$

$$\rho \rightarrow \frac{a\rho + b}{c\rho + d}, \quad \rho^* \rightarrow \frac{a^*\rho^* + b^*}{c^*\rho^* + d^*}$$

Generators are part of Virasoro algebra:  $L_{\pm 1}, L_0$ , conformal bootstrap (at fixed time).

New development: Regge cut and integrability in color octet channel (open chain).



Regge cut in color octet channel appears already in the leading logarithmic approximation:  
double energy discontinuity.

Known since 1980.

Relevant for  $N = 4$  SYM in AdS/CFT (see below).

These octet BKP states are integrable:  
open chains (counterpart the singlet BKP states).  
(Lipatov)

Two comments:

a) What is behind this integrability and Möbius invariance:  
in LLA Regge limit of QCD = Regge limit of  $N = 4$  SYM.  
It is believed that  $N = 4$  SYM is integrable to all orders.

Implication for NLA: Möbius invariance and integrability will be lost in QCD, not in supersymmetric Yang M

b) How to derive interaction terms of QCD reggeon field theory?  
either from energy discontinuities + unitarity (JB)  
or from 'effective action' (Lipatov).

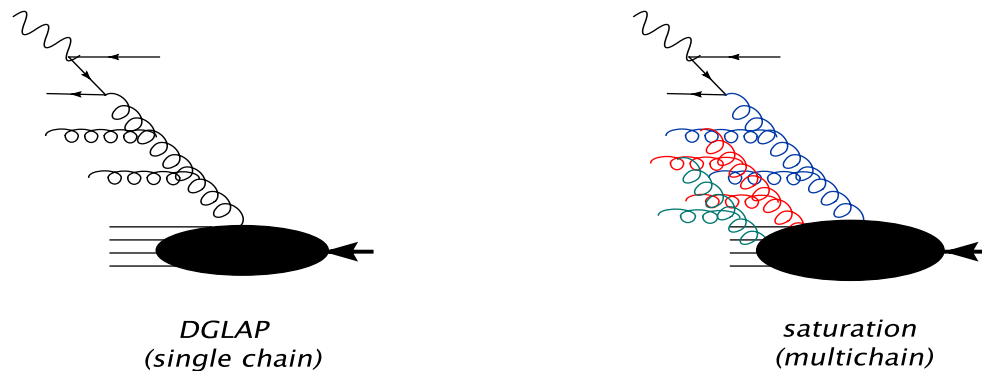
Summary: QCD at very high energies (small- $x$ /Regge):

- 2+1 dimensional field theory
- QCD as 'ugly' relative of 'beautiful'  $N = 4$  SYM:  
inherits integrable structures, dual conformal symmetry.
- nonperturbative physics as 'boundary condition'.

## Saturation in DIS

Saturation at HERA: theory much less controversial than experimental evidence.  
Theory: corrections to QCD parton picture, still within pQCD. Multiple interactions

Saturation (in DIS): at small distances (large  $Q^2$ ) we have DGLAP:



Since gluon density grows with  $1/x$ , somewhere at small  $x$  corrections must become essential: two cascades, gluon annihilation, negative sign:

**'At high density the gluon saturates'**

Saturation scale:  $Q_s(x) = Q_0(1/x)^\lambda$ ,  $\lambda = 0.25 \dots 0.3$ ,  $Q_0$  to be fitted

Formulated in the dipole picture:

$$\sigma_{L,T} = \int dz \int d^2r \psi(Q, r, z)_{L,T} \sigma_{q\bar{q}}(r, x) \psi^*(Q, r, z)_{L,T}$$

Essential quantity: dipole cross section  $\sigma_{q\bar{q}}$ : saturates at an  $x$ -dependent value. Current value of the saturation scale, obtained from  $F_2$ :

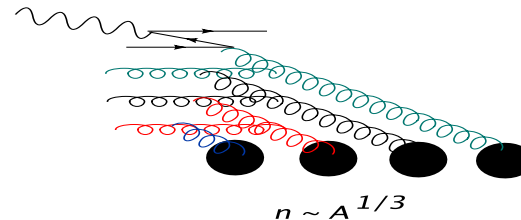
$$Q_s^2 \approx 0.8 \text{ at } x = 10^{-4}, \quad Q_s^2 \approx 3 \text{ at } x = 10^{-6},$$

Future:

LHeC: saturation scale larger than at HERA, **considerable improvement**.

eRHIC: cleaner signal of saturation in  $eA$  collisions:

photons 'sees' the gluons of many nucleons (number of nucleons  $\sim A^{1/3}$ ): gluon density saturates at larger  $x$ -values:  $Q_s^2 \sim A^{1/3}$ .



Comment:

Saturation is a low- $Q^2$ /small  $x$  phenomenon, for the transition region.  
It is not an alternative to DGLAP. At large  $Q^2$  saturation merges into DGLAP.

Gives an answer to a question which DGLAP does not address:  
What are the first corrections to the (leading-twist) parton picture?  
How does the transition to strong interactions start?  
Saturation is the first step in this transition, still within pQCD.

Have experiments seen saturation? What will improve at future machines, e.g. at LHeC?

HERA:

a) flattening of the small- $x$  rise of  $F_2$  of  $xg(x, Q^2)$ .

HERA: not seen. Likely: kinematic region too small, large errors in  $xg(x, Q^2)$ .

LHeC: certainly better chances.

b) Description of  $F_2$  in the transition region:

phenomenological models (GBW), models inspired by nonlinear evolution (BK) equation. Successful description with few parameters.

Other successful models without saturation.

c) geometric scaling: clear prediction of saturation

$$F_2(x, Q^2) = F_2(Q^2 / Q_s^2(x))$$

Seen in the data.

Within DGLAP: also geometric scaling, at larger  $Q^2$  (Forte).

d) Diffraction: ratio of diffractive over constant cross section

$$\sigma_{diff}^{\gamma^*p} / \sigma_{tot}^{\gamma^*p} \approx const$$

constant with energy (at fixed  $Q^2$ ,  $M^2$ ).

Saturation model (GBW) for diffractive  $q\bar{q}$  production provides simple explanation.

So far: no alternative explanation

In three cases b) - d): saturation provides a 'simple explanation', but, in the cases b) and c), there alternative 'explanations'.

RHIC: → [Larry's lectures](#) .

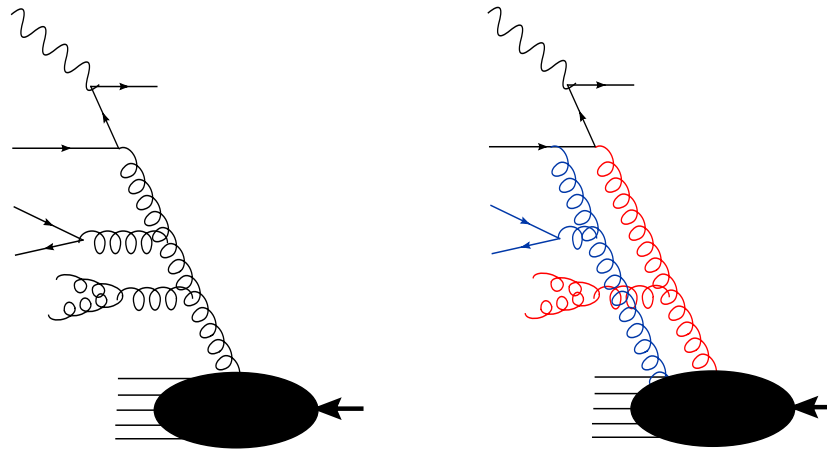
LHeC has better possibilities (larger kinematic region) than HERA:

[saturation line,  \$eA\$  option](#)



Suggestion: measure

inclusive cross sections, correlations (e.g. two-jet) as reliable signal of saturation (multiple interactions):



Correlations in rapidity, angle.

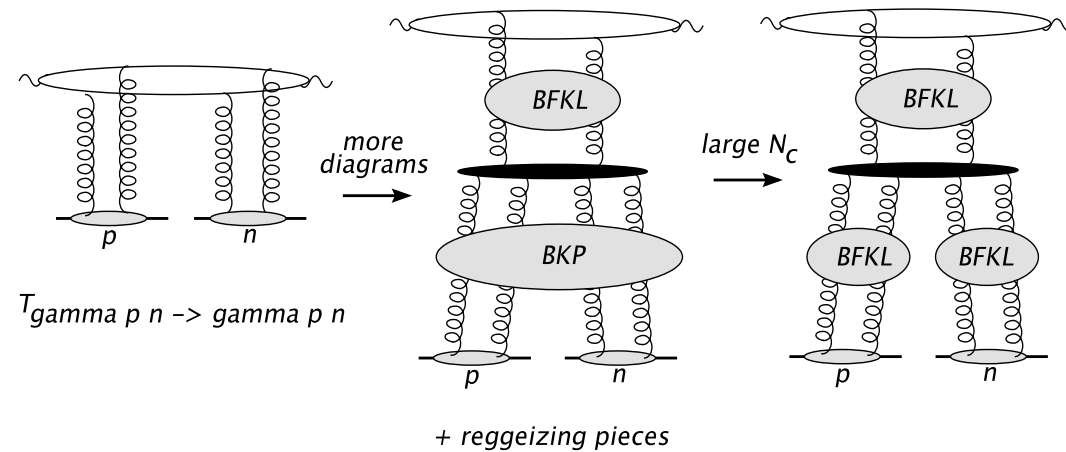
Was difficult at HERA (for larger  $Q^2$ ),  
for LHeC factor 2.0 in  $\ln 1/x$  will help.

## Theory of Saturation in DIS and in $pp$

In the following: use momentum space language.

Alternatives: light cone perturbation theory, color dipole formulation (large  $N_c$ ) in coordinate space.

1)  $\gamma^* p$  scattering: Balitsky-Kovchegov (BK) equation. Framework: scattering on large number of independent color neutral targets (nucleus; proton: view as collection of color neutral pieces).

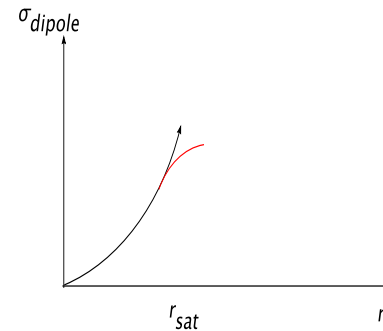


Note: elastic rescattering is included.

Large  $N_c$ , limit of large number of targets: nonlinear BK equation. Determines dipole cross section  $\sigma_{q\bar{q}}(r,$

$$\sigma_{L,T} = \int dz \int d^2r \psi(Q, r, z)_{L,T} \sigma_{q\bar{q}}(r, x) \psi^*(Q, r, z)_{L,T}$$

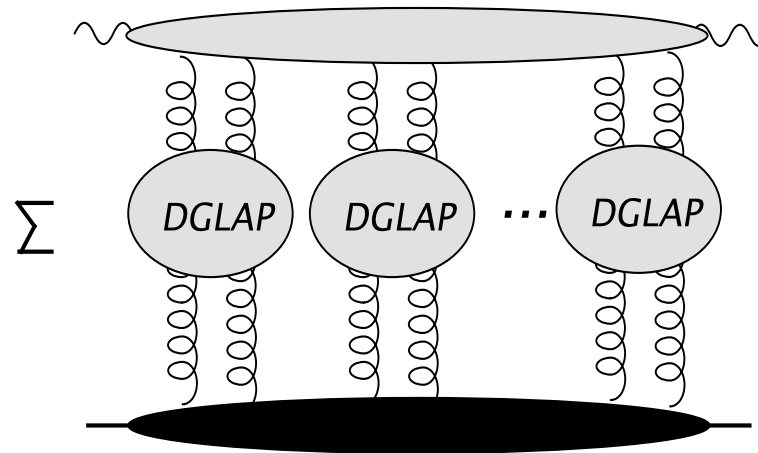
Saturation as a consequence of nonlinearity: flattening at large dipole sizes:



Large- $r$  behavior of integrand depends upon  $\sigma_{q\bar{q}}(r, x)$  and photon wave function.

Two comments on twist expansion:

a) BK dipole cross section does not allow 'twist expansion' ('leading twist shadowing').  
Closer to twist expansion: GBW model

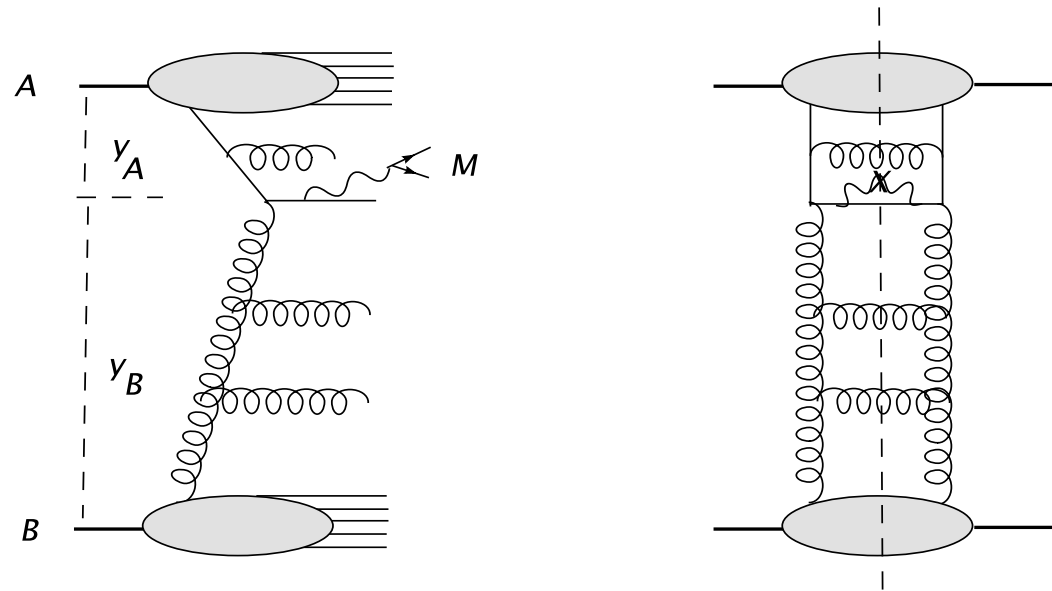


Leading log fan diagrams in BK equation 'wipe out' the higher twist poles.

b) Miraculous cancellation of higher twist in  $F_2 = F_T + F_L$  (JB,K.Golec-Biernat,L.Motyka).

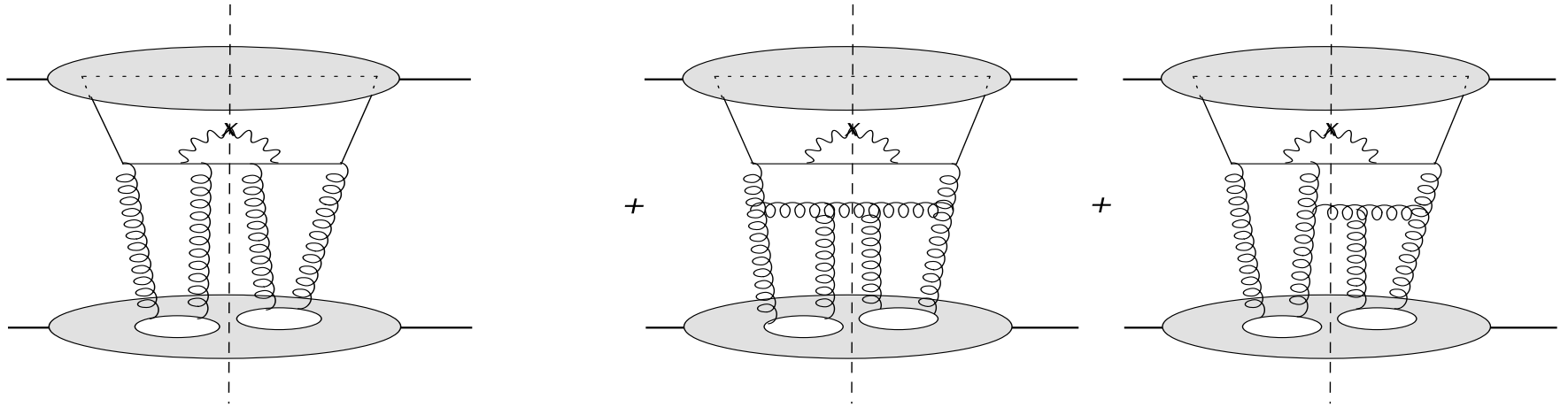
2)  $pp$  scattering at the LHC:

One of the promising places to look for saturation is Drell-Yan (at low  $M(e^+e^-)$ ) in the forward direction



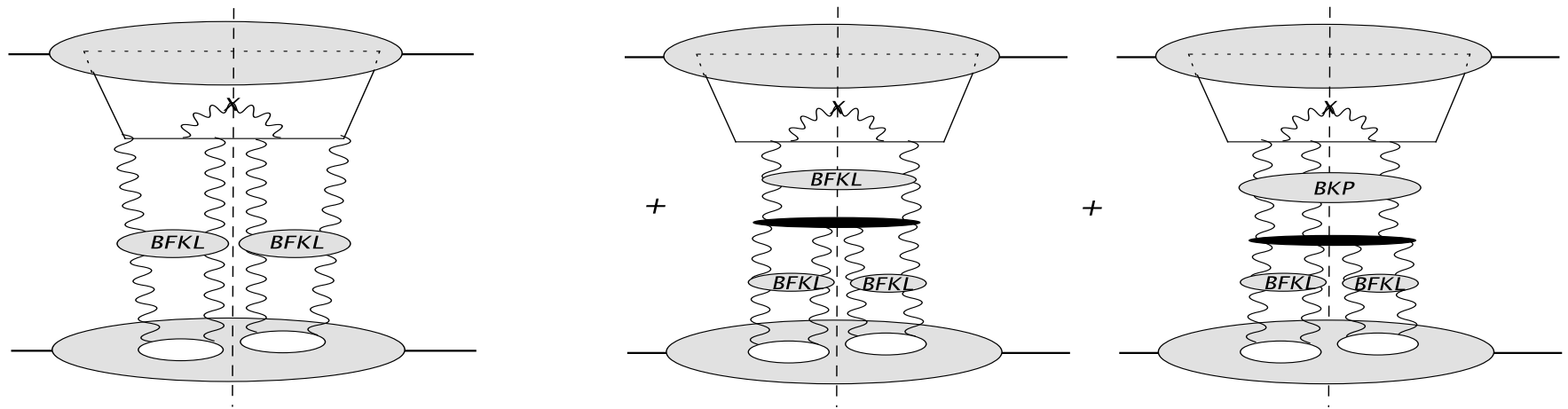
Could we use the dipole cross section from DIS? Unfortunately not, saturation in  $pp$  is more complicated (JB,L.Motyka). Find two essential differences:

Use the same assumption as in DIS: lower proton = collection of independent color singlet sources.



After summing many more diagrams: find two cases:

a) for gluon momenta of the order  $\mathcal{O}(M)$

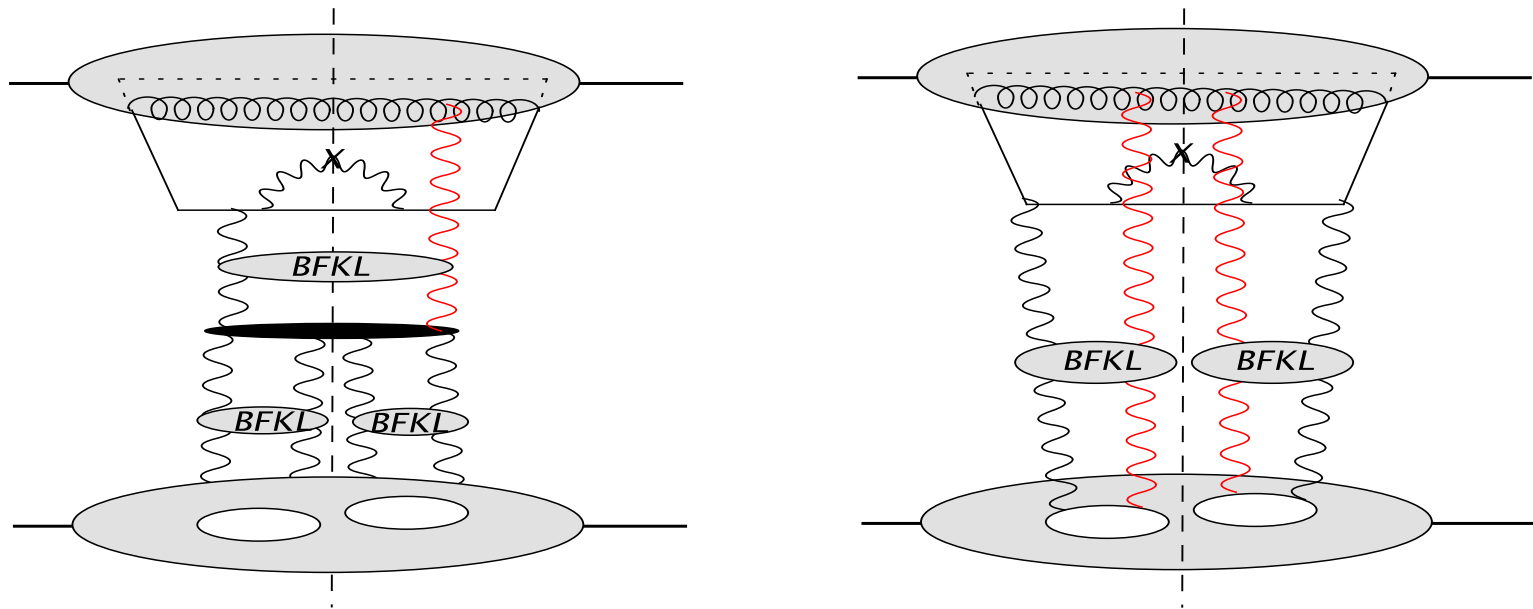


'quark inside the upper proton acts like a virtual photon of size  $\mathcal{O}(1/M)$ '

But: need two, three, four gluon correlators (Gelis, Venugopalan)

The same complication as in inclusive jet cross section (JB, M.Salvadore, G.P.Vacca)

b) for gluon momenta  $k$  less than  $\mathcal{O}(M)$ :

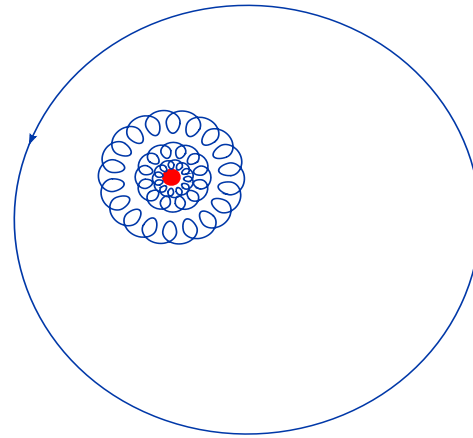


'quark + gluons inside the upper proton form a system of size  $\mathcal{O}(1/k) > \mathcal{O}(1/M)$ .

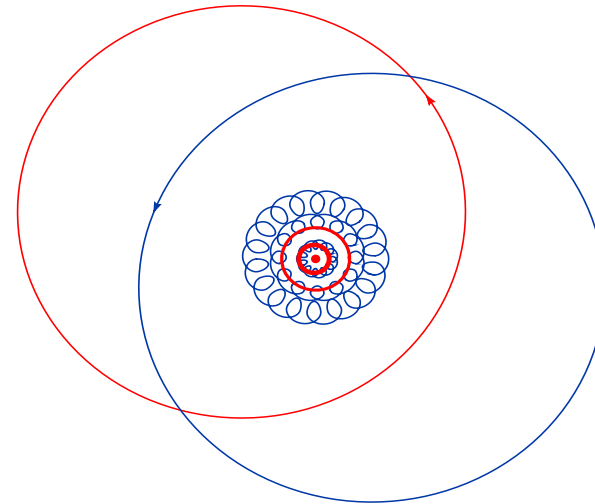
'For  $M$  near  $Q_s(x_B)$  and  $k < M$  saturation in  $pp$  might look different from DIS'.  
May also apply to  $pA$ .



Comparison in the transverse plane: DIS vs.  $pp$  (Drell-Yan near forward region):



*DIS ('photon rest frame')*

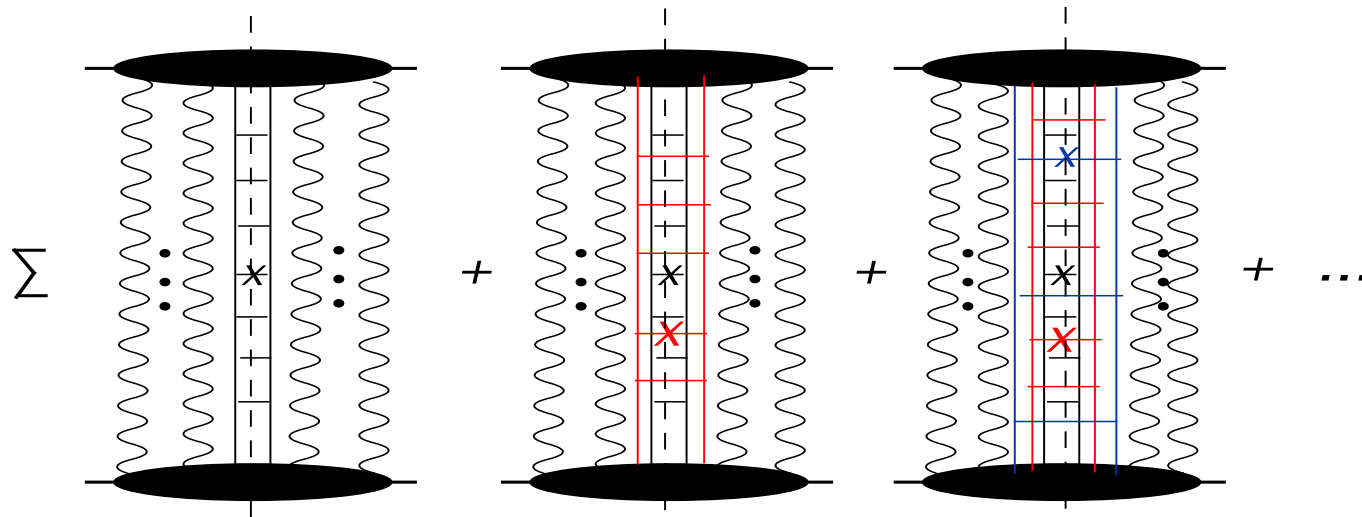


*$pp$  (rest frame of Drell-Yan photon)*

Red (forward) proton will influence the saturation inside the blue proton, attempt to quantify.

## LHC related issues: multiple interactions

Event structure can be connected with elements of 'cut QCD reggeon field theory':



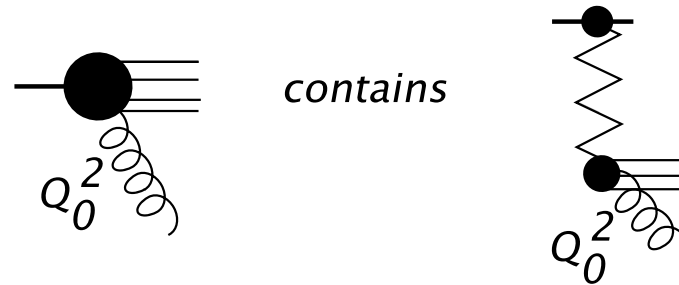
Each cut ladder: computed with standard MC methods.

Uncut 'Pomerons': parametrization of some soft exchange.

Most important: **AGK cutting rules, collinear factorization**

Questions:

where is diffraction (rapidity gaps)? Only inside 'initial conditions' .

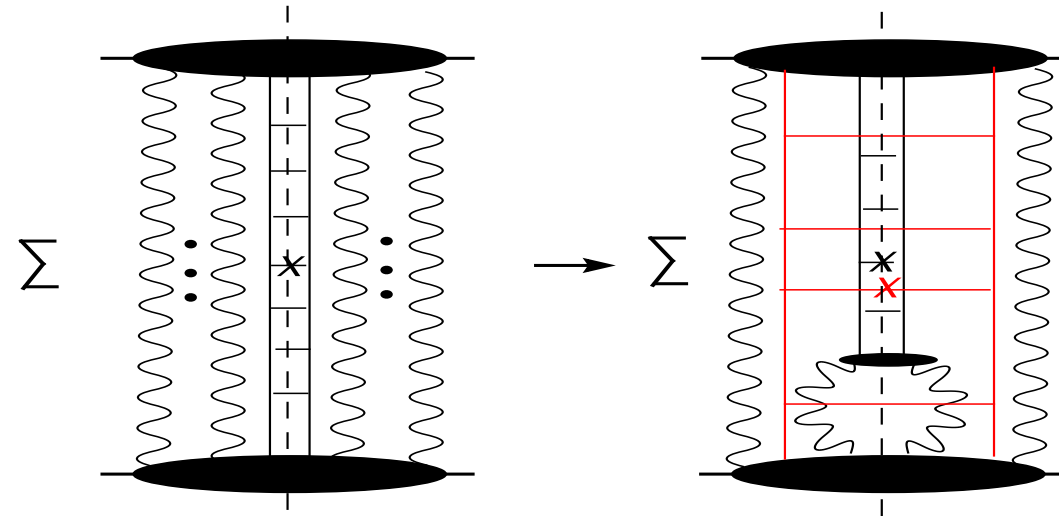


With scale below  $Q_0^2$ , in Monte Carlo not seen as final state.

We know from HERA:

in each chain there are  $\mathcal{O}(10)\%$  events with rapidity gap, scale not always below  $Q_0^2$ .

Improvement: include diffraction into each cut chain:



Second cut ladder tends to fill the gap of the first cut ladder  $\rightarrow$  survival probability.

Direction: 'QCD cut reggeon field theory'.

(Note the difference from discussions 25 - 30 years ago!)

Not discussed: survival probabilities in diffractive final states.

## Conclusions (1)

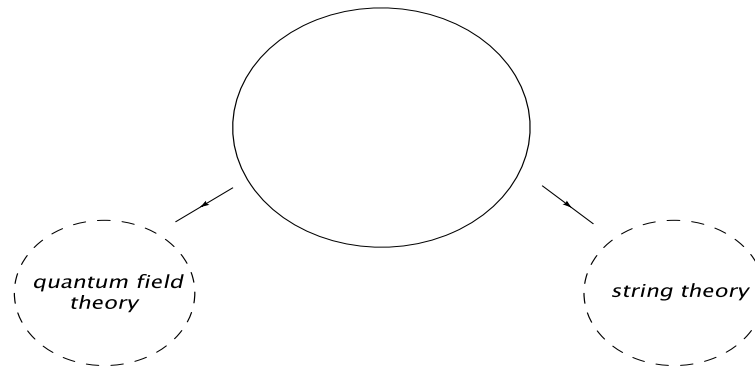
Part A: High energy 'real QCD':

- formal developments
- saturation: DIS -  $pp$
- LHC structure of events

## How much help can we get from the AdS/CFT Duality?

Hypothesis of AdS/CFT correspondence:

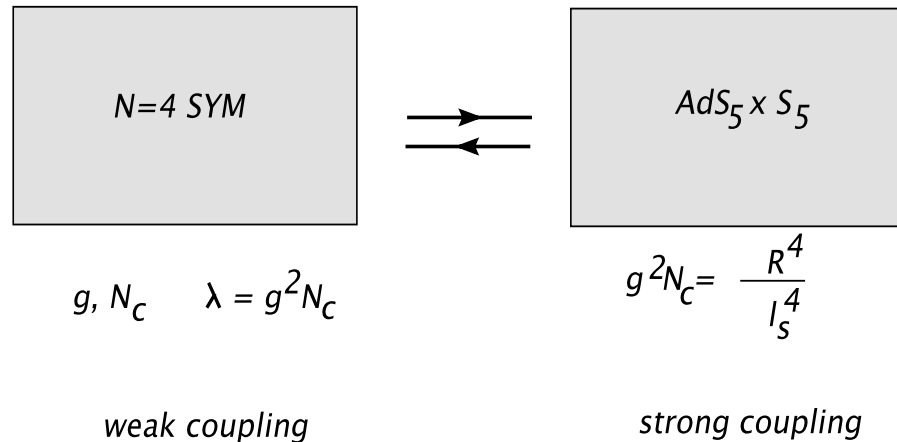
certain quantum field theories and string theories are two different limits of the same theory:



Hopes connected with this conjecture:

- solve quantum field theory beyond perturbation theory
- solve QCD beyond weak coupling (need to know the dual analogue)
- connect string theory with the real world

Frame of this talk is the AdS/CFT correspondence hypothesis applied to  $N = 4$  SYM:



On both sides expansion in  $1/N_c$  (expansion in topology): from  $\lambda = 0$  to  $\lambda = \infty$

$N = 4$  SYM is the most symmetric gauge theory ( $\beta$ -function vanishes).

Differs from QCD (particle content, no running of the coupling constant, no low-energy phase structure)

Hope: theory is soluble (integrable), plays role of 'harmonic oscillator in Quantum mechanics.

Special interest: analyse high energy scattering amplitudes within the AdS/CFT duality.

History: Regge limit stimulated string theory (Veneziano amplitudes),

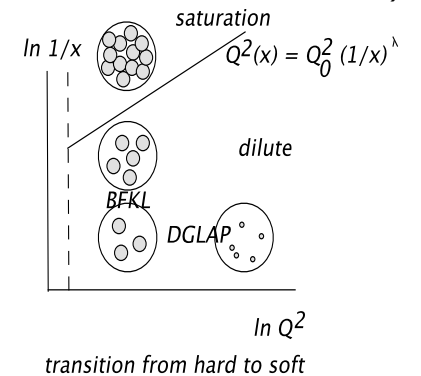
This talk: two lines of investigations

- (a) scattering amplitudes in the planar limit (compare with Veneziano amplitudes).  
 Main interest:  $n$  point amplitudes in  $N = 4$ , guide for multiloop/multileg amplitudes in QCD, BDS formula.  
 Is  $N = 4$  SYM soluble: integrability?
- (b) Vacuum exchange (Pomeron, cylinder):  
 gauge theory side:  
 (Soft) Pomeron in hadron-hadron scattering is non-perturbative: need methods other than pQCD.  
 But: (Soft) Pomeron is also sensitive to low-energy features of QCD (slope  $\alpha'$ : chiral dynamics).

Hard Pomeron: in scattering of small-size projectiles (virtual photon)

Transition in deep inelastic scattering (saturation, unitarization)

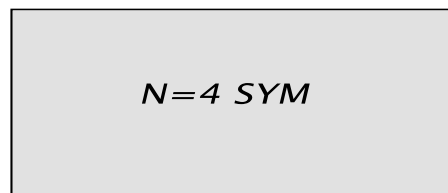
String side: hard Pomeron, unitarization, deep inelastic scattering on a plasma.



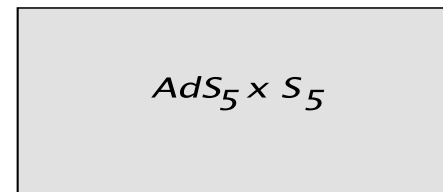
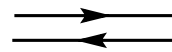


## Planar scattering amplitudes at high energies

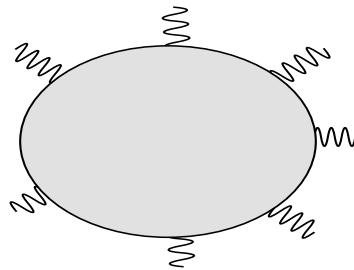
$N = 4$ , MHV amplitudes. Duality:



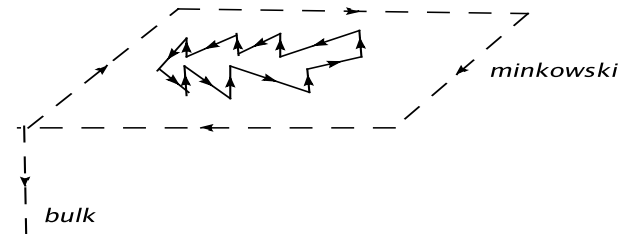
$g, N_c \quad \lambda = g^2 N_c$



$g^2 N_c = \frac{R^4}{l_s^4}$



Compute planar amplitudes



Compute minimal surfaces  
Amplitude has exponential form

"amplitude/Wilson loop correspondence"

Gauge theory side: enormous activity in two loop calculations, (also beyond MHV).  
Recent support for the amplitude/Wilson loop correspondence.

String theory side: minimal surfaces are hard to compute, a few cases are known ([Alday, Maldacena](#)).

Most remarkable: Bern-Dixon-Smirnow (BDS) formula for planar  $n$ -gluon scattering amplitude:

Remove color factors, factor out tree amplitude, IR singular:

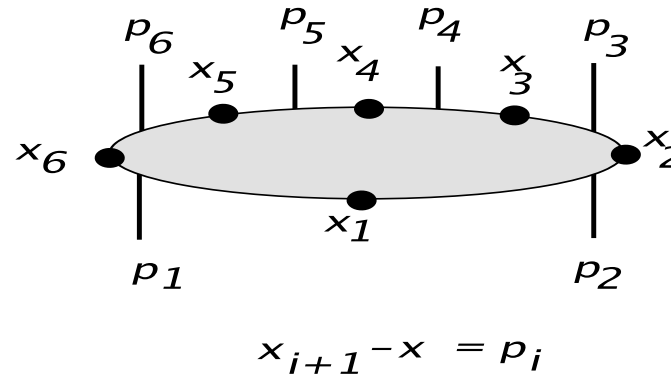
$$\begin{aligned} & \text{tr}(T^{a_1} \dots T^{a_n}) + \text{noncycl.perm}, \quad A_n = A_n^{\text{tree}} \cdot M_n(\epsilon) \\ \ln M_n &= \sum_l a^l \left[ \left( f^{(l)}(\epsilon) I_n(l\epsilon) + F_n(0) \right) + C^{(l)} + E_n^{(l)}[\epsilon] \right] \\ a &= \frac{N_c \alpha}{2\pi} (4\pi e^{-\gamma})^\epsilon, \quad d = 4 - 2\epsilon \end{aligned}$$

Based upon: universality of IR singularities (=poles in  $\epsilon$ ), and 1-loop calculation.

Several tests ([Alday, Maldacena](#); [Drummond, Korchemsky, Sokatchev](#); [JB, Lipatov, Sabio-Vera](#)):  
partly successful ( $n \leq 5$ , partly disagreement  $n \geq 6$ ).

Holy grail: [find the correction to the BDS formula](#)

Another ingredient: dual conformal symmetry:



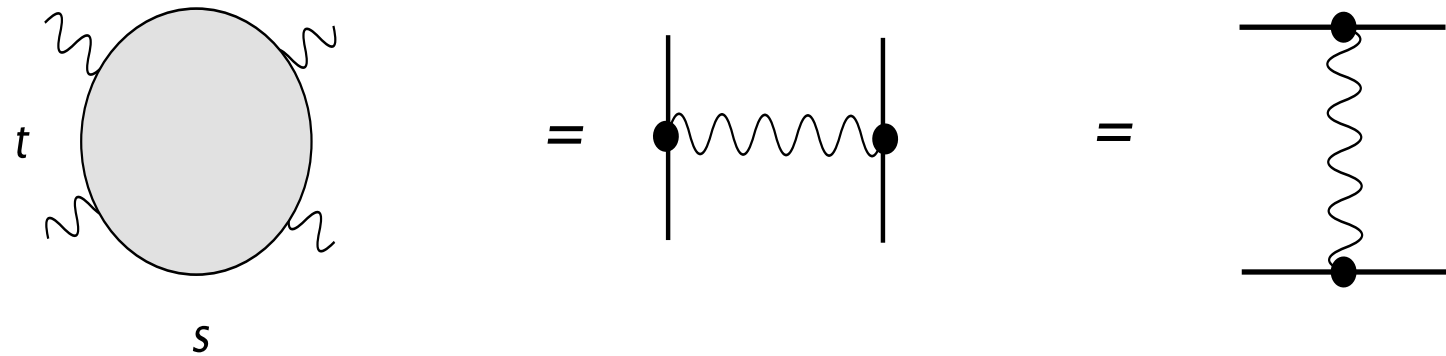
Invariance under (dual) conformal transformations of momenta  $p_i$ .  
 Present believe: in unphysical region (all invariants are negative)

$$M_n \sim \exp[\ln M_n^{BDS} + \text{remainder function}]$$

Remainder function: vanishes for  $n = 4, 5$ , should depend upon anharmonic ratios.  
 In physical regions: amplitudes may be more complicated.

For the remainder of this section: the high energy limit (Regge limit) of BDS formula:

Four-point function:

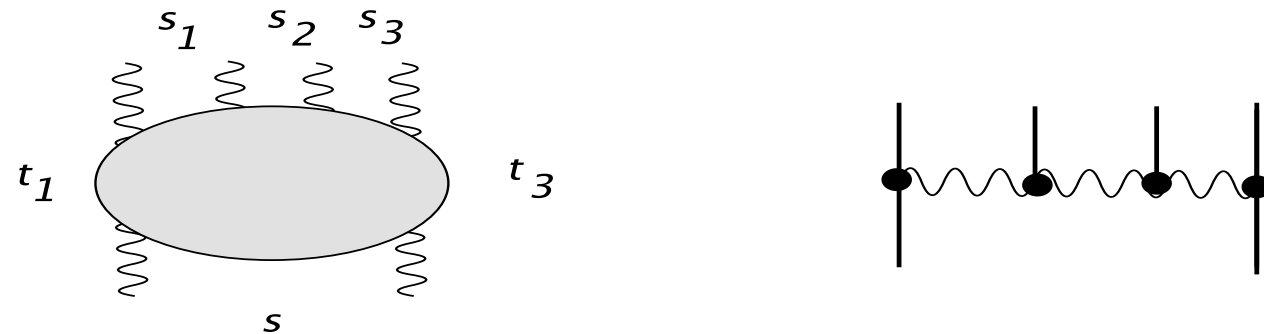
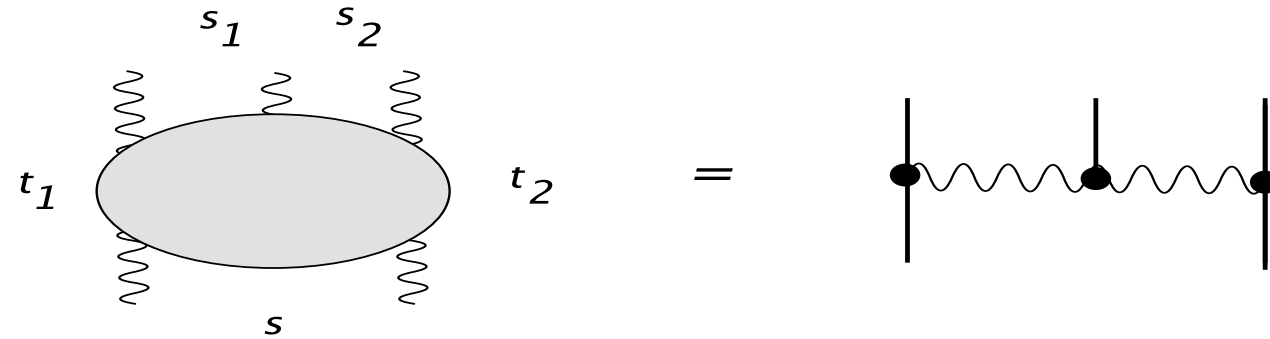


$$A_4(s, t) = \Gamma(t) \left( \frac{-s}{\mu^2} \right)^{\omega(t)} \Gamma(s) = \Gamma(s) \left( \frac{-t}{\mu^2} \right)^{\omega(s)} \Gamma(t)$$

All order gluon trajectory function, vertex function.

Comparison with Veneziano amplitude  $B_4(s, t)$ .

Five, six point functions:

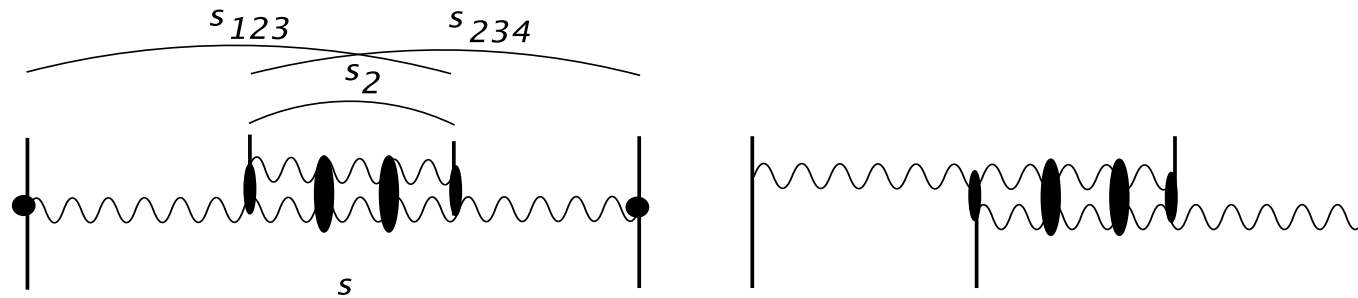


Same trajectory, vertex function, production vertex:

all seems to be consistent. But for  $n \geq 6$ :

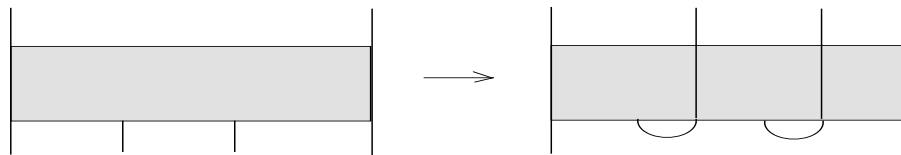
Analytic structure, unphysical vs unphysical kinematic regions: scattering amplitudes = functions of several complex-valued variables: Steinmann relations

Comparison with leading-log calculations in QCD (JB, Lipatov, Sabio-Vera):  
 disagreement for  $2 \rightarrow 4, 3 \rightarrow 3, \dots$ :  
 piece is missing (beyond one loop) (absent also in Veneziano amplitudes).



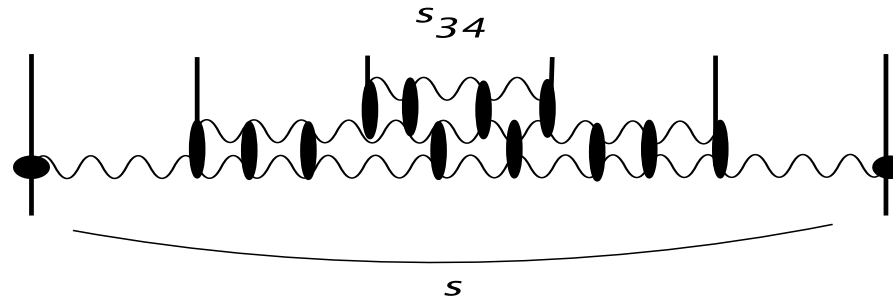
Visible in energy discontinuity or in another physical region:

$s, s_2 > 0, s_{123}, s_{234} < 0$ :



Special feature of this extra piece: [integrability](#).

Go to multi-leg amplitudes  $n > 8$ , e.g.

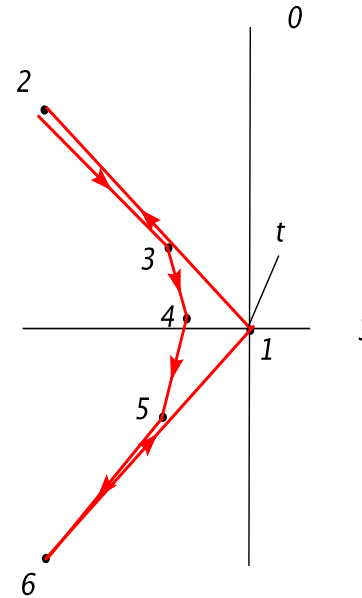
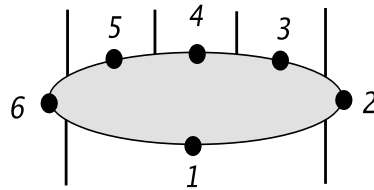


This Regge-cut piece, again, is visible in (double) energy discontinuities or in special physical regions.  
Dependence upon  $s_{34}$ :

$$A_8 \sim s_{34}^{-E_3}, \quad \text{where} \quad H_{3,open}\psi = E_3\psi$$

is the lowest energy of the BKP Hamiltonian describing the rapidity evolution in the  $t_3$  channel.  
In the planar limit the  $t_3$  channel is in a octet state: open chain  
 $H_{3,open}$  is [integrable](#) ([Lipatov](#)).

Back to contours on the string side: in the Regge limit there are characteristic spike



Surfaces are known for special cases, not for general  $n$ .

Analytic continuation of kinematic regions  $\leftrightarrow$  deformations of contours and minimal surfaces.

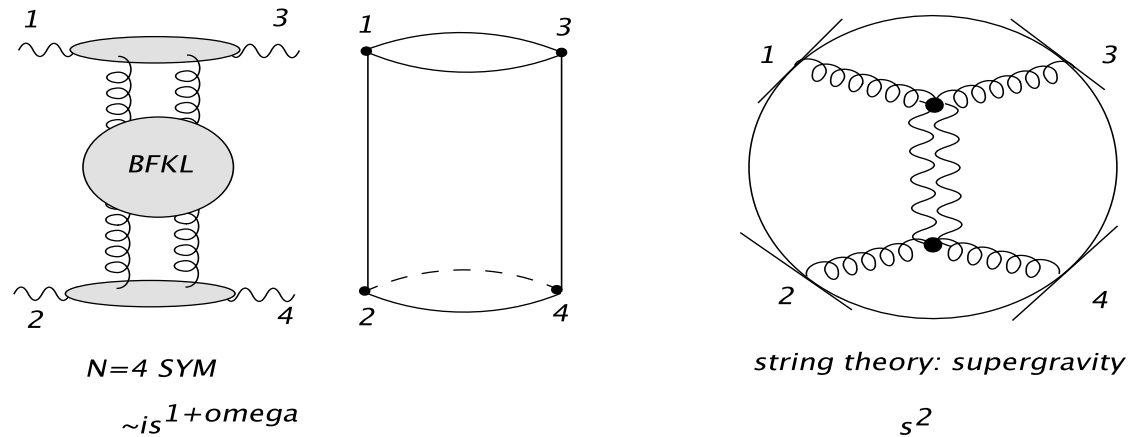
Study of these deformations might provide some guidance.



# The (Hard) Pomeron in AdS/CFT

A. Duality of BFKL and graviton:

Testing ground: correlator of  $R$ -currents (global  $SU(4)$  symmetry): analogue of  $\gamma^* \gamma^*$ -scattering in QCD.  
 Elastic scattering:  $\langle R_{\mu_1}(x_1) R_{\mu_2}(x_2) R_{\mu_3}(x_3) R_{\mu_4}(x_4) \rangle$ . Use dictionary.



Basic message: BFKL in  $N = 4$  SYM is dual to the graviton in  $AdS_5$

The **weak coupling side** : the BFKL amplitude

$$A(s, t) = is \int \frac{d\omega}{2\pi i} \left( \frac{s}{kk'} \right)^\omega \Phi_1(Q_A^2, k, q - k) \otimes G_\omega(k, q - k; k', q - k') \otimes \Phi_2(Q_B^2, k', q - k')$$

Impact factors (for scalar currents) (**Balitski; Cornalba et al.**), characteristic BFKL function (**Lipatov et al**) known in NLO:

$$G_\omega(k, q - k; k', q - k') \sim \frac{1}{\omega - \chi(n, \nu)}$$

Impact factors for  $R$ -currents in  $N = 4$  SYM known in LO.

Connection between small  $x$ -limit and short distance limit (DIS):  
leading twist anomalous dimension near  $\omega = j - 1 \approx 0$

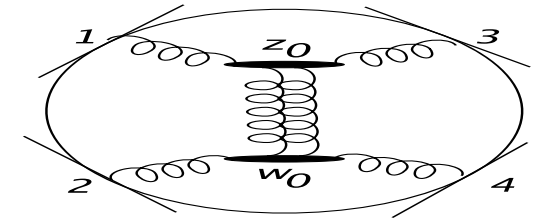
$$A(s, t = 0) \sim \frac{is}{Q^2} \int \frac{d\omega}{2\pi i} \left( \frac{s}{Q_1^2} \right)^\omega \int \frac{d\nu}{2\pi i} \left( \frac{Q_1^2}{Q_2^2} \right)^{i\nu + \omega/2} \Phi_1(n, \nu) \frac{1}{\omega - \chi(\nu, 0)} \Phi_2(n, \nu)$$

The strong coupling side: graviton exchange

The leading term (in  $1/\lambda$ ) is given by supergravity (Witten diagram).

Calculation (Kotanski et al) gives:

$$I^{\text{GR}} = \frac{1}{4} \int \frac{d^4 z dz_0}{z_0} \int \frac{d^4 w dw_0}{w_0} T_{(13)\mu\nu}(z) G_{\mu\nu;\mu'\nu'}(z, w) T_{(24)\mu'\nu'}(w).$$



Fouriertransform, high energy limit, polarization vectors, helicity structure of the exchanged graviton:

$$\frac{2p_{2;\mu}p_{1;\mu'}}{s} \frac{2p_{2;\nu}p_{1;\nu'}}{s}$$

leads to

$$\mathcal{A}_{\lambda_1\lambda_2\lambda_3\lambda_4}^{\text{GR}}(s, t) = s^2 \int dz_0 dw_0 \Phi_{\lambda_1\lambda_3}(|\vec{p}_1|, |\vec{p}_3|; z_0) \Sigma(|\vec{p}_1 + \vec{p}_3|, z_0, w_0) \Phi_{\lambda_2\lambda_4}(|\vec{p}_2|, |\vec{p}_4|; w_0).$$

Comparison with gauge theory side: 'Impact factors', integral over fifth coordinate analogous to transverse momentum.

Cannot see in Witten diagram: reggeization of the graviton.  $j = 2 \rightarrow j = 2 - \frac{2}{\sqrt{\lambda}} + \mathcal{O}(\lambda)$ .  
More general (Lipatov et al, Polchinski et al, Brower et al): existence of function  $j(\nu, \lambda)$

$$1 + \chi(\nu, \lambda) < j(\nu, \lambda) < 2 - \frac{4 + \nu^2}{2\sqrt{\lambda}} + \dots$$

Diffusion in  $\ln z$  (Brower et al.).

Result for  $\gamma^* \gamma^*$  scattering

- intercept: function  $j(\nu, \lambda)$  interpolates between weak and strong coupling:  $1 < j(\nu, \lambda) < 2$ .  
We know the first two corrections for  $\lambda \rightarrow 0$ , first correction at  $\lambda \rightarrow \infty$ .  
Connection with anomalous dimension.
- impact factor: we know the first term at  $\lambda \rightarrow 0$ , the first term at  $\lambda \rightarrow \infty$ .

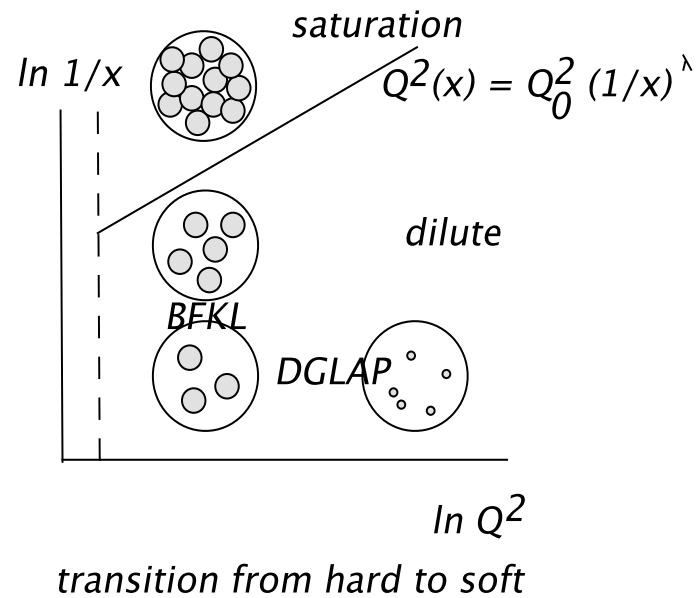
Wanted: representation which can be used for both weak and strong coupling  
(Cornalba et al, Banks et al.: Regge limit in conformal field theory).

B. First steps towards unitarization:

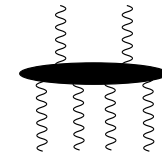
Problem worse than BFKL: single graviton  $\sim s^2$ , double graviton  $\sim s^3, \dots$

Need to go beyond planar (large- $N_c$  limit): cannot separate the limits  $s \rightarrow \infty$  and  $N_c \rightarrow \infty$ .

Is eikonalization the correct answer? QCD experience does not support eikonalization:

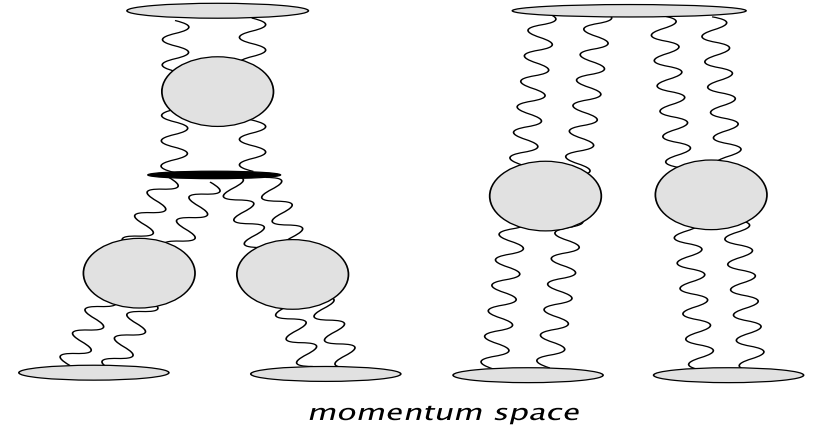
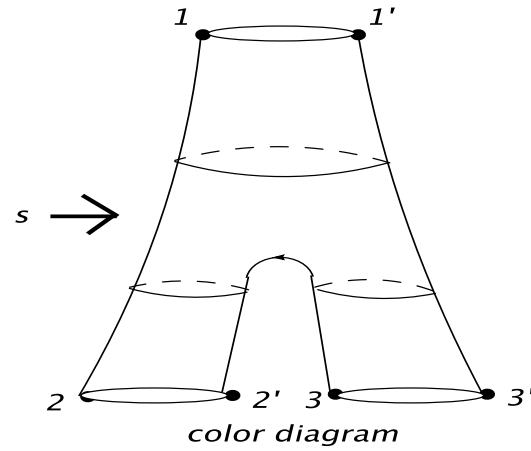


Near the saturation region:  
vital role of the triple Pomeron vertex

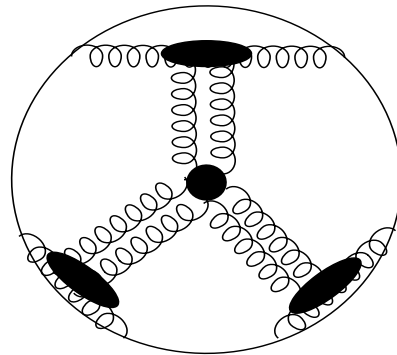


Is there anything similar on the string side: triple graviton vertex.

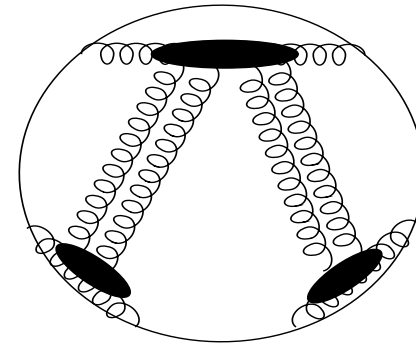
On the gauge theory side: pair-of pants topology



On the string theory side:



*triple graviton vertex vanishes:  
need string theory calculation*



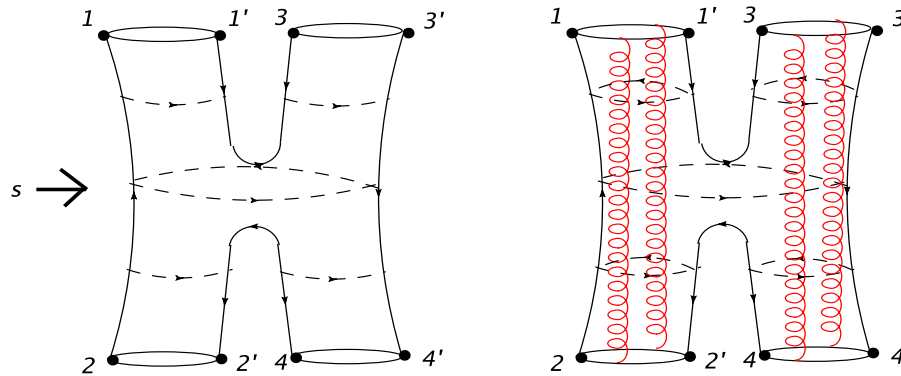
*compute new impact factor  
(leading order in  $1/\Lambda$ )*

Needs string calculation.

C.Integrability (cf. anomalous dimensions for evolution under dilatation):

Important feature of BFKL: generalize from 2 to  $n > 2$  gluons,  
 (LO) Hamiltonian of BKP states is **integrable** for large  $N_c$ .

Where to find large- $N_c$  BKP states: in multi-leg amplitudes, e.g. eight point correlator for  $4 \rightarrow 4$ :  
 (such an amplitude is not quite academic: heavy ion collisions)



$$A_8 \sim s^{1-E}, \quad \text{where} \quad H_{4,closed}\psi = E\psi$$

$E$  is the lowest eigenvalue of the energy spectrum of the 4 gluon BKP Hamiltonian (closed chain).

## First steps towards 'real physics'

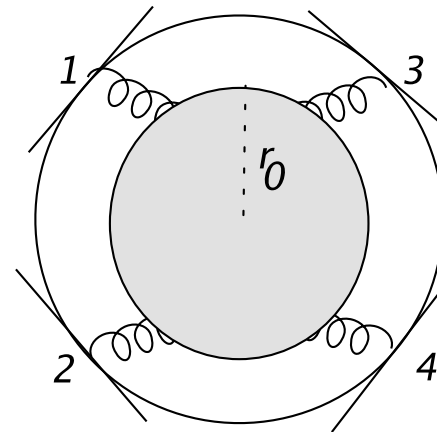
A 'soft' Pomeron in a 'confining' theory (Brower et al):

Observation: 'soft' Pomeron comes from larger values of fifth coordinate  $z_0$ . (smaller  $r$ ):

Modify the  $AdS_5 \times W$ : boundary  $\rightarrow$  scale.

Compute glueball, continue in  $t$ .

Obtain slope parameter.



Questions:

how to connect this soft 'Pomeron' with the hard Pomeron (=reggeized graviton)?

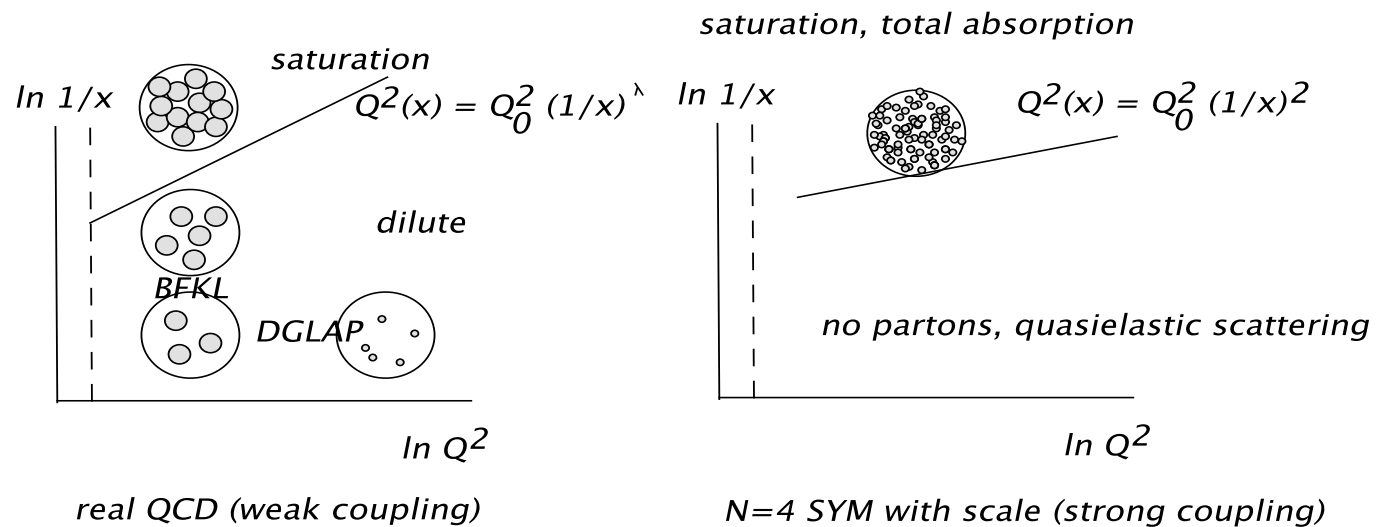
Is there 'saturation'?



# Deep Inelastic scattering (Polchinski et al; Mueller,Hatta,Iancu)

Goal: deep inelastic scattering for all  $x$ .

Framework:  $N = 4$  DIS on hot plasma, or DIS on dilaton field



Most striking results:

- no partons at finite  $x$
- saturation line  $Q_s^2 \sim (T/x)^2$  (multiple graviton exchange).

## Conclusions

Talk was about 'real QCD', and on some recent attempts to make contact with string theory.

Part A: real QCD

- we have already rather detailed knowledge of pQCD at high energies
- saturation (observed) = first step of seeing substantial changes in pQCD
- needed: QCD at the LHC, match on to nonperturbative strong interactions

Part B: new insight from AdS/CFT:

We are at the beginning of exciting investigations

- compute scattering amplitudes in planar  $N = 4$  SYM:  
Regge limit plays a special role
- BFKL/graviton duality  
many open questions: e.g. what is the unitarity content of the reggeizing graviton?