Asymptotic Safety of simple Yukawa systems

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Hierachy problem & Triviality

- Both problems might be solved within the AS scenario.

- The Higgs field is parametrized in terms of a bosonic field ϕ with a Lagrangian

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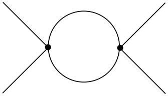
- 1-loop correction to the four-Higgs-boson coupling $\lambda\phi^4$ is represented by the diagram



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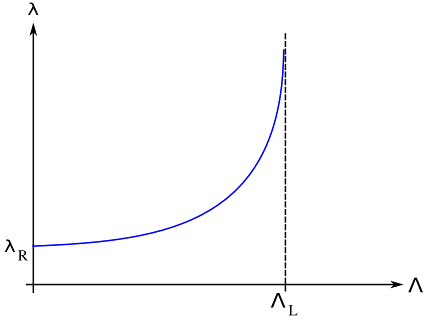

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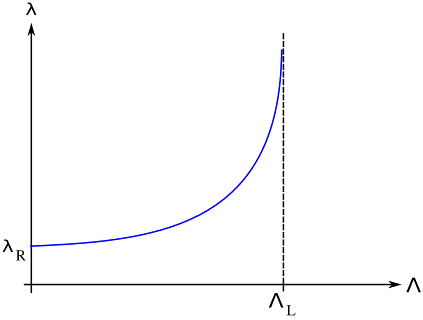

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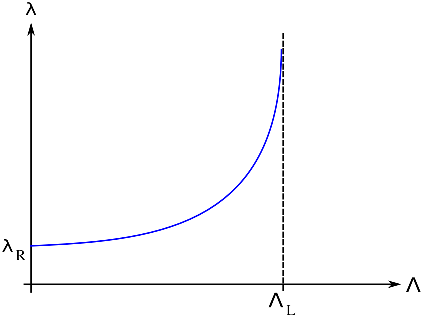

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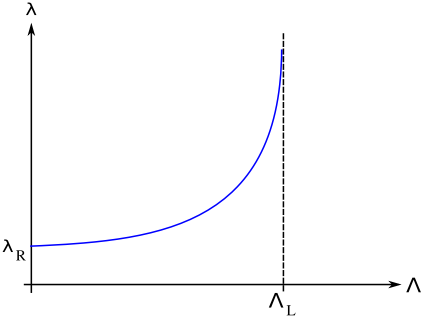

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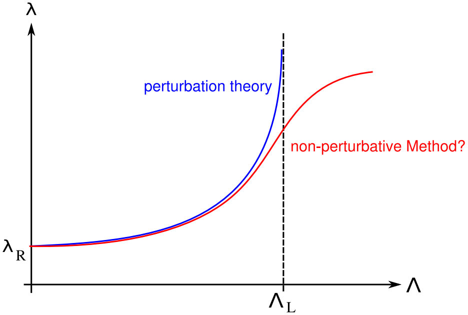

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- Near the Landau pole perturbation theory will loose its validity since λ grows large
- We need a non-perturbative tool to study triviality!

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Effective quantum field theory

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- Expansion in terms of running couplings $g_{i,k}$ and all possible field operators $\mathcal{O}_i$.

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- Dependence of the effective action on the scale k (or more conveniently $t = \text{Log}(k/\Lambda)$) is by definition given by the β-functions of the running couplings:

$$\partial_t \Gamma_k[\chi] = \sum_i \beta_{i,k}\mathcal{O}_i .$$

General notion of Asymptotic Safety

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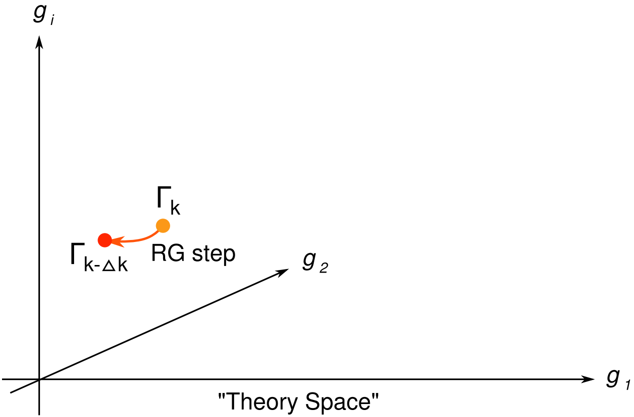

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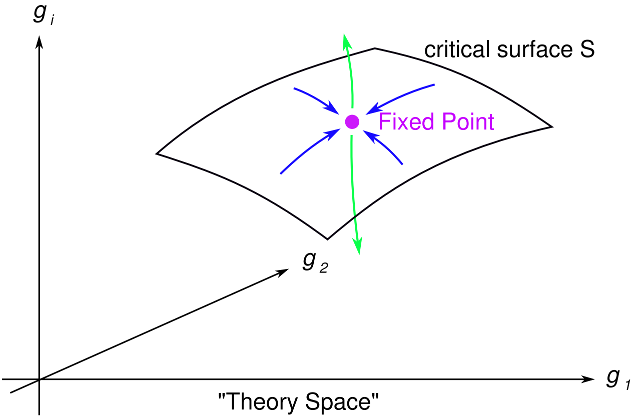

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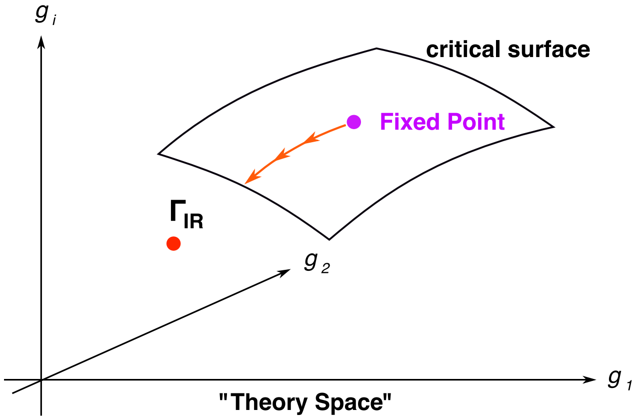

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- RG computation will show how large the Θ_I are at a NGFP.
- If all of them are small $\ll 1$ then then the hierarchy problem is solved.

Flow equation

- We will need a non-perturbative method to compute the flow (β-functions) in theory space.

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- Use exact renormalization group equations (ERGE) derived from Path-Integral representation (Wetterich '93)

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}\mathsf{STr}\{[\Gamma_k^{(2)}[\Phi] + R_k]^{-1}(\partial_t R_k)\}, \quad \partial_t = k\frac{d}{dk}$$

Toy model - Chiral Yukawa system, no gauge bosons

$$\Gamma_k = \int d^dx\Big\{i(\bar\psi^a_L \partial\!\!\!/ \psi^a_L + \bar\psi_R \partial\!\!\!/ \psi_R) + (\partial_\mu \phi^{a\dagger})(\partial^\mu \phi^a) + U_k(\phi^{a\dagger}\phi^a) + \bar h_k \bar\psi_R \phi^a \psi^a_L - \bar h_k \bar\psi^a_L \phi^{a\dagger}\psi_R\Big\}$$

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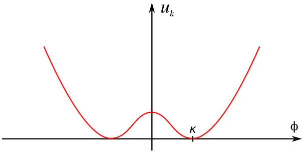
For the phase with spontaneously broken symmetry (SSB), we expand the effective potential around its minimum: $\kappa_k := \tilde\rho_{\mathsf{min}} > 0$,

$$u_k = \frac{\lambda_{2,k}}{2!}(\tilde\rho - \kappa_k)^2 + \frac{\lambda_{3,k}}{3!}(\tilde\rho - \kappa_k)^3 + \ldots$$

$$\kappa,\ \lambda_{n_{\mathsf{max}}},\ \lambda_2 > 0.$$



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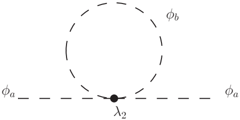

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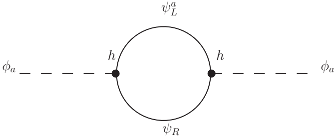

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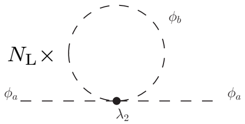

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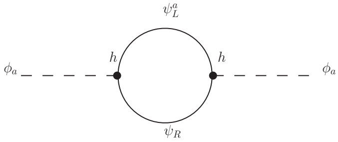

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- The real part of the relevant direction is 1.056 and not anymore 2, so the hierarchy problem is slightly weakened.

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- Choosing $v = 246\text{GeV}$ and $N_\text{L} = 10$ as an example, we find

$$m_{\text{Higgs}} = 0.81 v, \quad m_{\text{top}} = 5.56 v.$$

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- Also gravitational effects can be included: O. Zanusso & R. Percacci and collaborators.