Long lived oscillons

T. Romańczukiewicz

Jagiellonian University

6 June 2009

Plan

Plan

Oscillon

Nonperturbative, long living, spatially localized classical solution in (nonintegrable) field theories exhibiting almost periodic oscillations in time.
Breather-like metastable state.

Oscillons appear in many (classical) field theories including

- massive nonlinear scalar FT in (1-6)+1 dim
- abelian and non-abelian Higgs model
- bosonic sector of SM

But massive scalar field seems to essential.

Oscillons can be formed from generic initial conditions like collisions of topological defects, thermal fluctuations or gaussian initial data.

Oscillon

Nonperturbative, long living, spatially localized classical solution in (nonintegrable) field theories exhibiting almost periodic oscillations in time.
Breather-like metastable state.

Oscillons appear in many (classical) field theories including

- massive nonlinear scalar FT in (1-6)+1 dim
- abelian and non-abelian Higgs model
- bosonic sector of SM

But massive scalar field seems to essential.

Oscillons can be formed from generic initial conditions like collisions of topological defects, thermal fluctuations or gaussian initial data.

Oscillon

Nonperturbative, long living, spatially localized classical solution in (nonintegrable) field theories exhibiting almost periodic oscillations in time.
Breather-like metastable state.

Oscillons appear in many (classical) field theories including

- massive nonlinear scalar FT in (1-6)+1 dim
- abelian and non-abelian Higgs model
- bosonic sector of SM

But massive scalar field seems to essential.

Oscillons can be formed from generic initial conditions like collisions of topological defects, thermal fluctuations or gaussian initial data.

Oscillon

Nonperturbative, long living, spatially localized classical solution in (nonintegrable) field theories exhibiting almost periodic oscillations in time.
Breather-like metastable state.

Oscillons appear in many (classical) field theories including

- massive nonlinear scalar FT in (1-6)+1 dim
- abelian and non-abelian Higgs model
- bosonic sector of SM

But massive scalar field seems to essential.

Oscillons can be formed from generic initial conditions like collisions of topological defects, thermal fluctuations or gaussian initial data.

Example: oscillon in ϕ^4

Let us consider a very famous ϕ^4 1+1 dimensional model:

$$\phi_{tt} - \phi_{tt} + 2\phi(\phi^2 - 1) = 0$$

and gaussian initial data

$$\phi(x, t = 0) = 1 + ae^{-bx^2}, \quad \dot{\phi}(x, t = 0) = 0$$

If $a \ll 1$ we can (?) linearize around a vacuum $\phi = 1 + u$ obtaining Klein-Gordon equation

$$\ddot{u} - u'' + 4u = 0.$$

It can be shown that

$$u(x = 0, t) = \frac{a}{\sqrt{4\pi b}} \int dk \, e^{-\frac{k^2}{4b}} \cos \sqrt{k^2 + 4}t.$$

in the limit $t \to \infty$ the leading term is

$$u(x = 0, t) = a \cos(2t + \delta)/t.$$

Example: oscillon in ϕ^4

Let us consider a very famous ϕ^4 1+1 dimensional model:

$$\phi_{tt} - \phi_{tt} + 2\phi(\phi^2 - 1) = 0$$

and gaussian initial data

$$\phi(x, t = 0) = 1 + ae^{-bx^2}, \quad \dot{\phi}(x, t = 0) = 0$$

If $a \ll 1$ we can (?) linearize around a vacuum $\phi = 1 + u$ obtaining Klein-Gordon equation

$$\ddot{u} - u'' + 4u = 0.$$

It can be shown that

$$u(x = 0, t) = \frac{a}{\sqrt{4\pi b}} \int dk\, e^{-\frac{k^2}{4b}} \cos\sqrt{k^2 + 4}t.$$

in the limit $t \to \infty$ the leading term is

$$u(x = 0, t) = a\cos(2t + \delta)/t.$$

Example: oscillon in ϕ^4

Let us consider a very famous ϕ^4 1+1 dimensional model:

$$\phi_{tt} - \phi_{tt} + 2\phi(\phi^2 - 1) = 0$$

and gaussian initial data

$$\phi(x, t = 0) = 1 + ae^{-bx^2}, \quad \dot{\phi}(x, t = 0) = 0$$

If $a \ll 1$ we can (?) linearize around a vacuum $\phi = 1 + u$ obtaining Klein-Gordon equation

$$\ddot{u} - u'' + 4u = 0.$$

It can be shown that

$$u(x = 0, t) = \frac{a}{\sqrt{4\pi b}} \int dk \, e^{-\frac{k^2}{4b}} \cos \sqrt{k^2 + 4}t.$$

in the limit $t \to \infty$ the leading term is

$$u(x = 0, t) = a \cos(2t + \delta)/t.$$

Example: oscillon in ϕ^4

Let us consider a very famous ϕ^4 1+1 dimensional model:

$$\phi_{tt} - \phi_{tt} + 2\phi(\phi^2 - 1) = 0$$

and gaussian initial data

$$\phi(x, t = 0) = 1 + ae^{-bx^2}, \quad \dot{\phi}(x, t = 0) = 0$$

If $a \ll 1$ we can (?) linearize around a vacuum $\phi = 1 + u$ obtaining Klein-Gordon equation

$$\ddot{u} - u'' + 4u = 0.$$

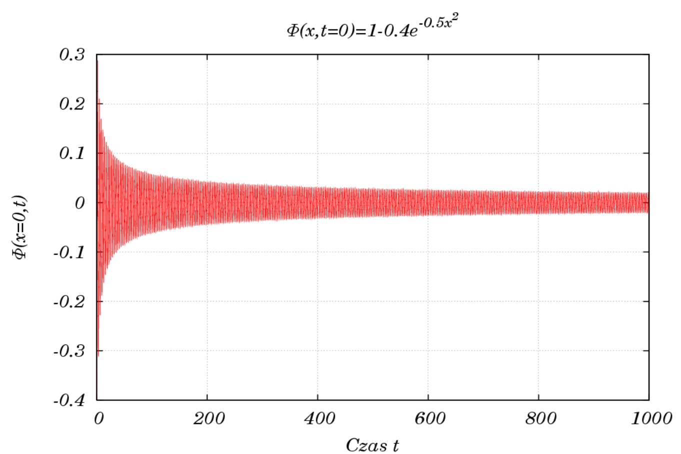
It can be shown that

$$u(x = 0, t) = \frac{a}{\sqrt{4\pi b}} \int dk \, e^{-\frac{k^2}{4b}} \cos \sqrt{k^2 + 4}t.$$

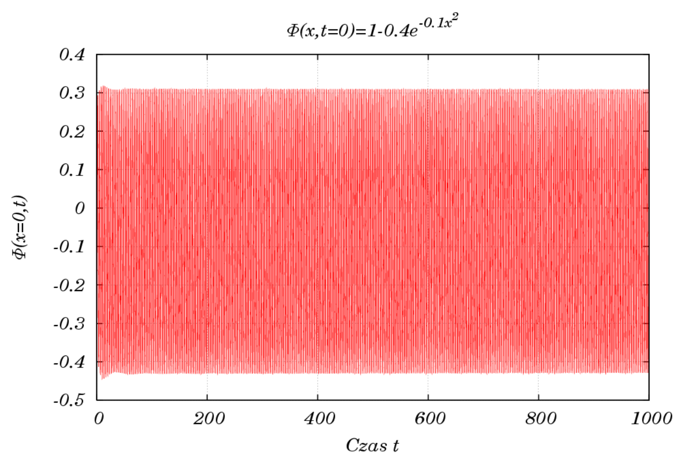
in the limit $t \to \infty$ the leading term is

$$u(x = 0, t) = a\cos(2t + \delta)/t.$$

Numerical simulation in the center $\phi(x=0,t)$ for initial data
$\phi(x,t=0) = 1 - 0.4\exp(-0.5x^2), \dot{\phi}(x,t=0) = 0$
Agreement with solution to linearized (KG) equation



However $\phi(x=0,t)$ for initial data $\phi(x,t=0) = 1 - 0.4\exp(-0.1x^2)$, $\dot{\phi}(x,t=0) = 0$ showed huge discrepancy with linearized theory.
Note: only the width has changed



This is clearly a nonlinear effect. Frequency of the oscillations in the second case was below the mass threshold $\omega = 1.90 < 2.00$.

There is no significant decrease of the amplitude but because ϕ^4 model is nonintegrable the oscillon radiates, but very very slowly.

Oscillons are very similar to breathers known from integrable sine-Gordon model

More analogy to breathers

For initial data $\phi(x, t = 0) = 1 - a\exp(-bx^2)$, $\dot{\phi}(x, t = 0) = 0$ only some pairs (a, b), give long-time almost periodic solutions $\omega < m$

Sine-Gordon breathers:

$$\phi_c(x, t) = -4 \arctan\left[\frac{c}{\sqrt{1-c^2}} \frac{\sin\left(\sqrt{1-c^2}t\right)}{\cosh cx}\right].$$

The less amplitude the higher frequency and wider profile.

This is clearly a nonlinear effect. Frequency of the oscillations in the second case was below the mass threshold $\omega = 1.90 < 2.00$.
There is no significant decrease of the amplitude but because ϕ^4 model is nonintegrable the oscillon radiates, but very very slowly.
Oscillons are very similar to breathers known from integrable sine-Gordon model

More analogy to breathers

For initial data $\phi(x, t=0) = 1 - a\exp(-bx^2)$, $\dot{\phi}(x, t=0) = 0$ only some pairs (a, b), give long-time almost periodic solutions $\omega < m$
Sine-Gordon breathers:

$$\phi_c(x,t) = -4\arctan\left[\frac{c}{\sqrt{1-c^2}}\frac{\sin\left(\sqrt{1-c^2}t\right)}{\cosh cx}\right].$$

The less amplitude the higher frequency and wider profile.

This is clearly a nonlinear effect. Frequency of the oscillations in the second case was below the mass threshold $\omega = 1.90 < 2.00$.
There is no significant decrease of the amplitude but because ϕ^4 model is nonintegrable the oscillon radiates, but very very slowly.
Oscillons are very similar to breathers known from integrable sine-Gordon model

More analogy to breathers

For initial data $\phi(x, t = 0) = 1 - a\exp(-bx^2)$, $\dot{\phi}(x, t = 0) = 0$ only some pairs (a, b), give long-time almost periodic solutions $\omega < m$

Sine-Gordon breathers:

$$\phi_c(x,t) = -4 \arctan\left[\frac{c}{\sqrt{1-c^2}} \frac{\sin\left(\sqrt{1-c^2}t\right)}{\cosh cx}\right].$$

The less amplitude the higher frequency and wider profile.

This is clearly a nonlinear effect. Frequency of the oscillations in the second case was below the mass threshold $\omega = 1.90 < 2.00$.
There is no significant decrease of the amplitude but because ϕ^4 model is nonintegrable the oscillon radiates, but very very slowly.
Oscillons are very similar to breathers known from integrable sine-Gordon model

More analogy to breathers

For initial data $\phi(x, t = 0) = 1 - a\exp(-bx^2)$, $\dot{\phi}(x, t = 0) = 0$ only some pairs (a, b), give long-time almost periodic solutions $\omega < m$
Sine-Gordon breathers:

$$\phi_c(x, t) = -4 \arctan\left[\frac{c}{\sqrt{1-c^2}} \frac{\sin\left(\sqrt{1-c^2}\,t\right)}{\cosh cx}\right].$$

The less amplitude the higher frequency and wider profile.

This is clearly a nonlinear effect. Frequency of the oscillations in the second case was below the mass threshold $\omega = 1.90 < 2.00$.
There is no significant decrease of the amplitude but because ϕ^4 model is nonintegrable the oscillon radiates, but very very slowly.
Oscillons are very similar to breathers known from integrable sine-Gordon model

More analogy to breathers

For initial data $\phi(x, t = 0) = 1 - a \exp(-bx^2)$, $\dot{\phi}(x, t = 0) = 0$ only some pairs (a, b), give long-time almost periodic solutions $\omega < m$
Sine-Gordon breathers:

$$\phi_c(x, t) = -4 \arctan \left[\frac{c}{\sqrt{1 - c^2}} \frac{\sin\left(\sqrt{1 - c^2} t\right)}{\cosh cx} \right].$$

The less amplitude the higher frequency and wider profile.

This is clearly a nonlinear effect. Frequency of the oscillations in the second case was below the mass threshold $\omega = 1.90 < 2.00$.
There is no significant decrease of the amplitude but because ϕ^4 model is nonintegrable the oscillon radiates, but very very slowly.
Oscillons are very similar to breathers known from integrable sine-Gordon model

More analogy to breathers

For initial data $\phi(x, t = 0) = 1 - a\exp(-bx^2)$, $\dot{\phi}(x, t = 0) = 0$ only some pairs (a, b), give long-time almost periodic solutions $\omega < m$
Sine-Gordon breathers:

$$\phi_c(x, t) = -4 \arctan\left[\frac{c}{\sqrt{1 - c^2}} \frac{\sin\left(\sqrt{1 - c^2}\, t\right)}{\cosh cx}\right].$$

The less amplitude the higher frequency and wider profile.

This is clearly a nonlinear effect. Frequency of the oscillations in the second case was below the mass threshold $\omega = 1.90 < 2.00$.
There is no significant decrease of the amplitude but because ϕ^4 model is nonintegrable the oscillon radiates, but very very slowly.
Oscillons are very similar to breathers known from integrable sine-Gordon model

More analogy to breathers

For initial data $\phi(x, t = 0) = 1 - a\exp(-bx^2)$, $\dot{\phi}(x, t = 0) = 0$ only some pairs (a, b), give long-time almost periodic solutions $\omega < m$
Sine-Gordon breathers:

$$\phi_c(x, t) = -4 \arctan \left[\frac{c}{\sqrt{1 - c^2}} \frac{\sin\left(\sqrt{1 - c^2}\, t\right)}{\cosh cx} \right].$$

The less amplitude the higher frequency and wider profile.

Differences

- Sine-Gordon breathers oscillates periodically with time, oscillons do not.
- Oscillons radiate, loosing their energy ($a \searrow b \searrow, \omega \nearrow < m$)
- In higher dimensions oscillons evolve until they reach some critical amplitude and then they disappear quickly.
- Breathers interact (with other breathers, kinks and wave) elastically whereas oscillons nonelasctically

Differences

- Sine-Gordon breathers oscillates periodically with time, oscillons do not.
- Oscillons radiate, loosing their energy ($a \searrow b \searrow$, $\omega \nearrow < m$)
- In higher dimensions oscillons evolve until they reach some critical amplitude and then they disappear quickly.
- Breathers interact (with other breathers, kinks and wave) elastically whereas oscillons nonelasctically

Differences

- Sine-Gordon breathers oscillates periodically with time, oscillons do not.
- Oscillons radiate, loosing their energy ($a \searrow b \searrow$, $\omega \nearrow < m$)
- In higher dimensions oscillons evolve until they reach some critical amplitude and then they disappear quickly.
- Breathers interact (with other breathers, kinks and wave) elastically whereas oscillons nonelasctically

Differences

- Sine-Gordon breathers oscillates periodically with time, oscillons do not.
- Oscillons radiate, loosing their energy ($a \searrow b \searrow$, $\omega \nearrow < m$)
- In higher dimensions oscillons evolve until they reach some critical amplitude and then they disappear quickly.
- Breathers interact (with other breathers, kinks and wave) elastically whereas oscillons nonelasctically

Oscillons usually appear in scalar field theories (*i.e.* D dimensional ϕ^4):

$$\ddot{\phi} - \Delta\phi + 2(\phi^2 - 1) = 0.$$

For $D = 1, 2, 3$ the radiation is very small and one can assume that the solutions are periodic in time:

$$\phi(x,t) = 1 + \sum_n u_n(x) \cos n\Omega t.$$

Just like breathers oscillons posess an oscillating core but they also have radiating tails.

Caution

Solution of this type has infinite energy - there is a standing wave in the whole space.

Oscillons usually appear in scalar field theories (*i.e.* D dimensional ϕ^4):

$$\ddot{\phi} - \Delta\phi + 2(\phi^2 - 1) = 0.$$

For $D = 1, 2, 3$ the radiation is very small and one can assume that the solutions are periodic in time:

$$\phi(x,t) = 1 + \sum_n u_n(x) \cos n\Omega t.$$

Just like breathers oscillons posess an oscillating core but they also have radiating tails.

Caution

Solution of this type has infinite energy - there is a standing wave in the whole space.

Oscillons usually appear in scalar field theories (*i.e.* D dimensional ϕ^4):

$$\ddot{\phi} - \Delta\phi + 2(\phi^2 - 1) = 0.$$

For $D = 1, 2, 3$ the radiation is very small and one can assume that the solutions are periodic in time:

$$\phi(x, t) = 1 + \sum_n u_n(x) \cos n\Omega t.$$

Just like breathers oscillons posess an oscillating core but they also have radiating tails.

Caution

Solution of this type has infinite energy - there is a standing wave in the whole space.

After substitution we obtain

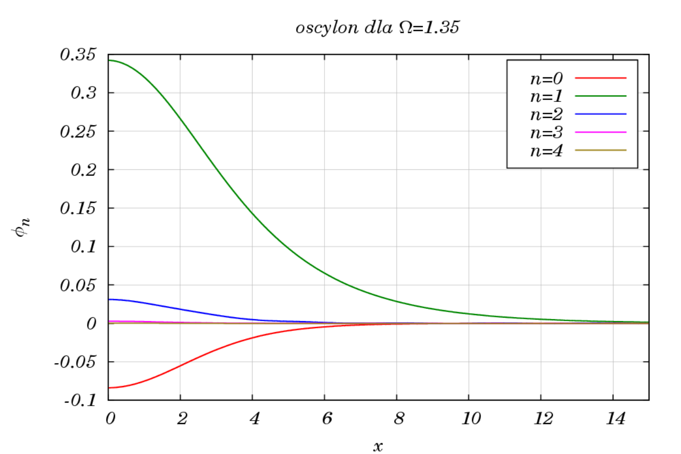
$$\Delta u_n + (n^2\Omega^2 - 4)u_n = F_n(u_0, u_1, \ldots),$$

where

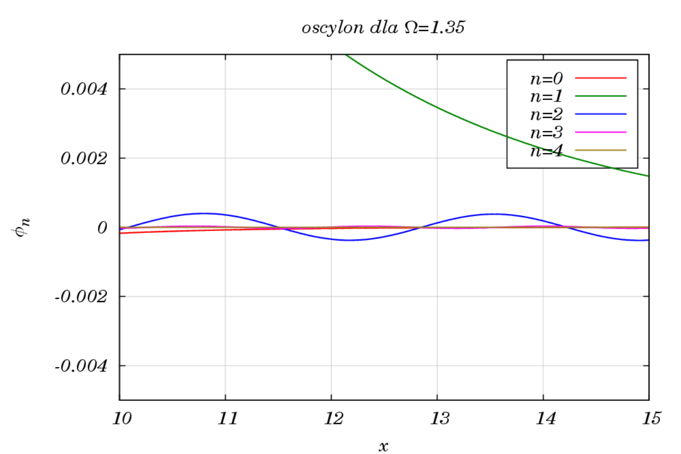
$$F_n(u_0, u_1, \ldots) = 3\sum_{m,p}(\delta_{n,m+p} + \delta_{n,m-p})u_m u_p +$$
$$\frac{1}{2}\sum_{m,p,k}(\delta_{n,m+p+k} + \delta_{n,m-p+k} + \delta_{n,m+p-k} + \delta_{n,m-p-k})u_m u_p u_k.$$

For given $\Omega < 2$ this system can be solved with additional condition of minimizing the outgoing radiation (or asymptotic energy density of the standing wave). In nonintegrable systems this energy density tends to some finite value. In sine-Gordon this tends to 0.

Oscillon profiles



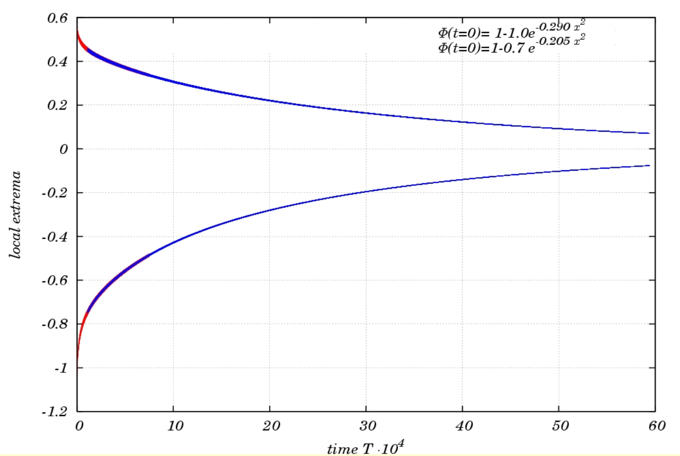
Minimized radiation



Kruskal and Segur showed (and later Forgacs et al proved more rigorously), that in 1+1d ϕ^4 the energy of the oscillon changes with time as

$$\frac{dE}{dt} = -\frac{A}{a} e^{-B/a}$$

so it is beyond all orders of perturbation series



Plan

Oscillons live for relatively long time in comparison to characteristic time in the theory $\sim 1/m$

If some phenomenon lasts much shorter than the lifetime of the oscillon we do not need to consider its radiation and asymptotic stability:

- collisions
- phase transitions
- interaction with waves

In sine-Gordon theory breathers interact with other objects elastically due to the integrability of the theory. Oscillons do not interact elastically.

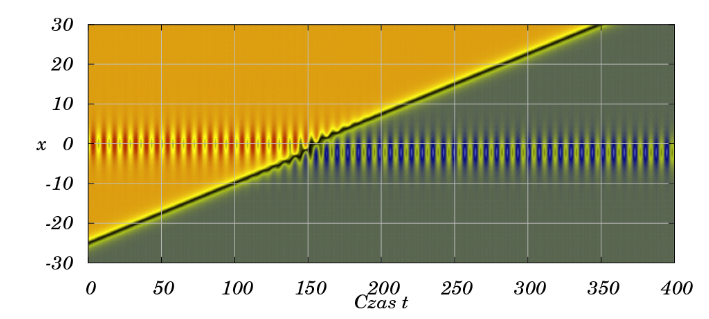
Oscillons live for relatively long time in comparison to characteristic time in the theory $\sim 1/m$

If some phenomenon lasts much shorter than the lifetime of the oscillon we do not need to consider its radiation and asymptotic stability:

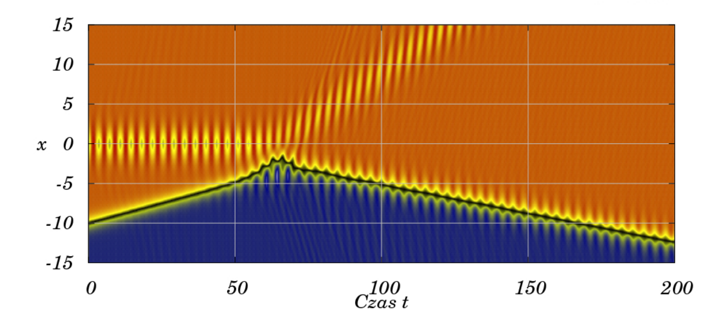
- collisions
- phase transitions
- interaction with waves

In sine-Gordon theory breathers interact with other objects elastically due to the integrability of the theory. Oscillons do not interact elastically.

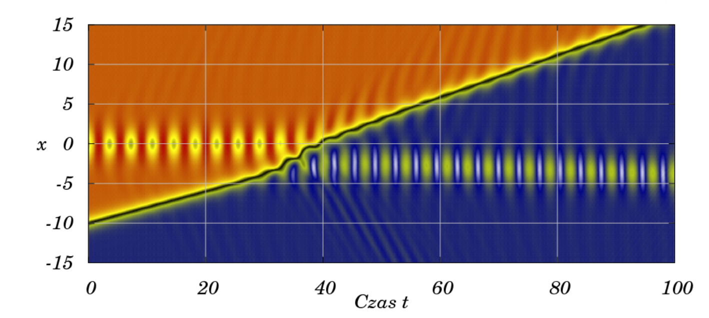
Kink-breather collision in integrable sine-Gordon model.
Soliton is reflected elastically.
No radiation.



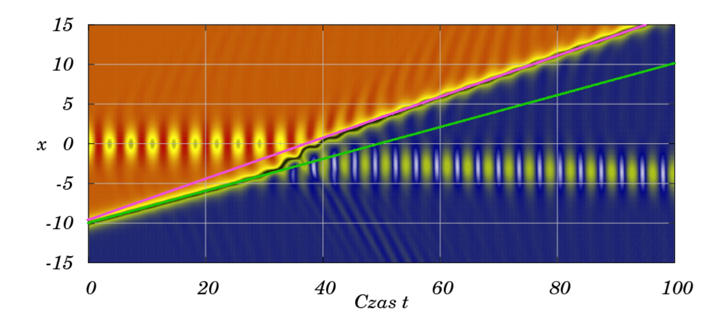
Kink-oscillon collision for small velocities $v = 0.1$.
Kink is reflected from the oscillon.
Excitation of the kink and some radiation visible.



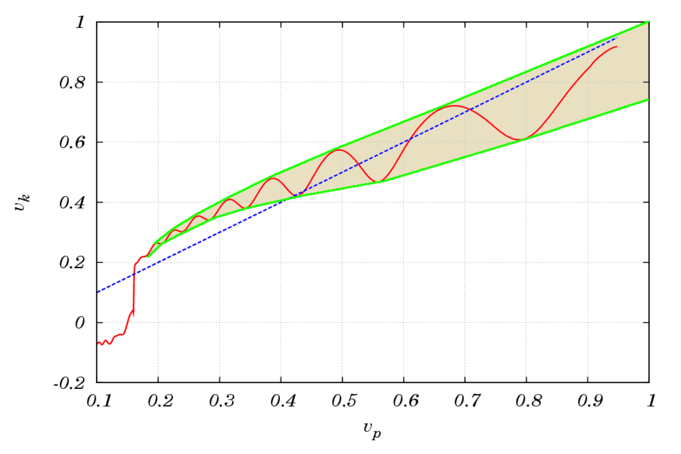
Kink-oscillon collision for large velocities $v = 0.2$.
Kink could have a larger velocity after the collision.
Energy contained in the oscillations can be transformed into the kinetic energy.



Kink-oscillon collision for large velocities $v = 0.2$.
Kink could have a larger velocity after the collision.
Energy contained in the oscillations can be transformed into the kinetic energy.



Final velocity as a function of initial velocity of the kink



This could be qualitatively described by effective theory using only few degrees of freedom:

- position of the kink $X(t)$
- position of the oscillon $Y(t)$
- for small amplitudes we can assume that only the basic frequency dominates so we can neglect the rest of them

$$\phi(x,t) = \psi\left(x - X(t)\right) + A_1(t)\Phi\left(x - Y(t)\right) + \text{higher harmonics} + \text{radiation}.$$

Substitution to the lagrangian gives

$$\mathcal{L} = \mathcal{L}_k + \mathcal{L}_{osc} + \mathcal{L}_{int},$$

where

$$\mathcal{L}_{osc} = \frac{m}{2}\left(\dot{A}^2 - \left(4 + \frac{M}{m}\right)A^2\right) - \gamma_3 A^3 - \gamma_4 A^4 + \frac{1}{2}(A^2 M + M_0)\dot{Y}^2,$$

For slow kink:

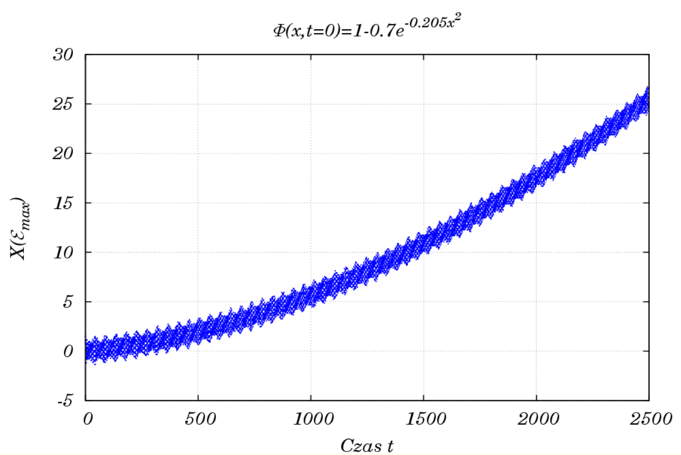
$$\mathcal{L}_k = -\frac{1}{2}M_k\dot{X}^2$$

and the interaction part:

$$\mathcal{L}_{int} = \int \mathrm{d}x \;\; A\dot{Y}\dot{X}\Phi'\Psi' - \dot{A}\dot{X}\Phi\Psi' - 3A^2\Phi^2\Psi^2 - 2A^3\Phi^3\Psi - 2A\Phi\Psi^3 + 2A\Phi\Psi$$

After integrations we obtain a huge system of equations which surprisingly gives similar results as the full PDE.

Motion of the oscillon under influence of a traveling wave ($\Omega \simeq 1.7$, $\omega = 3.3$, $a = 0.12$)



This phenomenon can be understood in the following way:
A wave ξ with certain frequency ω hits an oscillon which oscillates with frequencies $0, \Omega, 2\Omega, \ldots$. Due to the nonlinear interaction (ϕ^2, ϕ^3) modes with frequencies $\omega_{nm} = n\omega + m\Omega$, $n, m \in \mathbb{Z}$ appear.
Suppose that Ω/ω is not a rational number (to avoid resonances). For small amplitude of the wave we can linearize the equation obtaining

$$\xi_{tt} - \xi_{xx} + [V_0(x) + V_1(x)\cos\Omega t + V_2(x)\cos 2\Omega t + \cdots]\,\xi = 0$$

The most dominating part is V_1 so neglecting the rest we obtain

$$\xi_{tt} - \xi_{xx} + V_1(x)\cos\Omega t\xi = 0$$

The solution can be sought in the following form:

$$\xi = \sum_m \xi_m(x)e^{i(\omega+m\Omega)t}$$

which leads to the following set of equations:

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}x^2} - (\omega + m\Omega)^2\right]\xi_m + \frac{1}{2}V_1(x)\left(\delta_{n,m+1} + \delta_{n,m-1}\right)\xi_n = 0$$

which can be solved with two-point boundary conditions (for some large $|x|$).

- We want only one wave going towards the oscillon (for frequency ω), and rest should be going outwards.
- What we could expect is that the most dominating waves will have frequencies $\omega \pm \Omega$, but if $\omega < \Omega + m$ than $\omega - \Omega < m$ so the wave with such freqency cannot propagate.
- It is easier to create a wave which propagate in the same direction as the initial wave.
- But wave with frequency $\Omega + \omega$ has more momentum than wave with frequency ω with the same energy. If the energy is conserved (which is true) this could lead to a surplus of momentum on the further side of the oscillon.
- This creates a force which pushes the oscillon towards the source of radiation - an example of the negative radiation pressure

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}x^2} - (\omega + m\Omega)^2\right]\xi_m + \frac{1}{2}V_1(x)\left(\delta_{n,m+1} + \delta_{n,m-1}\right)\xi_n = 0$$

which can be solved with two-point boundary conditions (for some large $|x|$).

- We want only one wave going towards the oscillon (for frequency ω), and rest should be going outwards.
- What we could expect is that the most dominating waves will have frequencies $\omega \pm \Omega$, but if $\omega < \Omega + m$ than $\omega - \Omega < m$ so the wave with such freqency cannot propagate.
- It is easier to create a wave which propagate in the same direction as the initial wave.
- But wave with frequency $\Omega + \omega$ has more momentum than wave with frequency ω with the same energy. If the energy is conserved (which is true) this could lead to a surplus of momentum on the further side of the oscillon.
- This creates a force which pushes the oscillon towards the source of radiation - an example of the negative radiation pressure

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}x^2} - (\omega + m\Omega)^2\right] \xi_m + \frac{1}{2} V_1(x) \left(\delta_{n,m+1} + \delta_{n,m-1}\right) \xi_n = 0$$

which can be solved with two-point boundary conditions (for some large $|x|$).

- We want only one wave going towards the oscillon (for frequency ω), and rest should be going outwards.
- What we could expect is that the most dominating waves will have frequencies $\omega \pm \Omega$, but if $\omega < \Omega + m$ than $\omega - \Omega < m$ so the wave with such freqency cannot propagate.
- It is easier to create a wave which propagate in the same direction as the initial wave.
- But wave with frequency $\Omega + \omega$ has more momentum than wave with frequency ω with the same energy. If the energy is conserved (which is true) this could lead to a surplus of momentum on the further side of the oscillon.
- This creates a force which pushes the oscillon towards the source of radiation - an example of the negative radiation pressure

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}x^2} - (\omega + m\Omega)^2\right]\xi_m + \frac{1}{2}V_1(x)\left(\delta_{n,m+1} + \delta_{n,m-1}\right)\xi_n = 0$$

which can be solved with two-point boundary conditions (for some large $|x|$).

- We want only one wave going towards the oscillon (for frequency ω), and rest should be going outwards.
- What we could expect is that the most dominating waves will have frequencies $\omega \pm \Omega$, but if $\omega < \Omega + m$ than $\omega - \Omega < m$ so the wave with such freqency cannot propagate.
- It is easier to create a wave which propagate in the same direction as the initial wave.
- But wave with frequency $\Omega + \omega$ has more momentum than wave with frequency ω with the same energy. If the energy is conserved (which is true) this could lead to a surplus of momentum on the further side of the oscillon.
- This creates a force which pushes the oscillon towards the source of radiation - an example of the negative radiation pressure

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}x^2} - (\omega + m\Omega)^2\right]\xi_m + \frac{1}{2}V_1(x)\left(\delta_{n,m+1} + \delta_{n,m-1}\right)\xi_n = 0$$

which can be solved with two-point boundary conditions (for some large $|x|$).

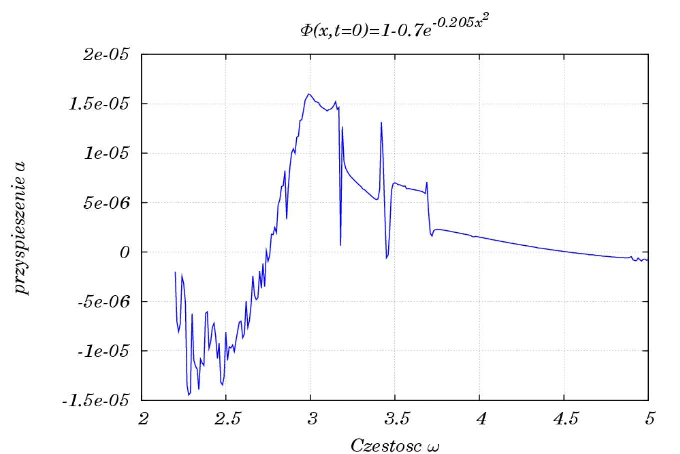
- We want only one wave going towards the oscillon (for frequency ω), and rest should be going outwards.
- What we could expect is that the most dominating waves will have frequencies $\omega \pm \Omega$, but if $\omega < \Omega + m$ than $\omega - \Omega < m$ so the wave with such freqency cannot propagate.
- It is easier to create a wave which propagate in the same direction as the initial wave.
- But wave with frequency $\Omega + \omega$ has more momentum than wave with frequency ω with the same energy. If the energy is conserved (which is true) this could lead to a surplus of momentum on the further side of the oscillon.
- This creates a force which pushes the oscillon towards the source of radiation - an example of the negative radiation pressure

Example numerical results:

n	$A_n(-L)$	$A_n(L)$	$\Delta\dot{P}$
-5	$2.039121 \cdot 10^{-05}$	$1.647402 \cdot 10^{-05}$	$+4.069289 \cdot 10^{-09}$
-4	$3.137356 \cdot 10^{-04}$	$1.195274 \cdot 10^{-03}$	$-1.532133 \cdot 10^{-05}$
-3	$9.974496 \cdot 10^{-19}$	$6.130569 \cdot 10^{-19}$	$+0.000000 \cdot 10^{+00}$
-2	$1.100515 \cdot 10^{-16}$	$9.829189 \cdot 10^{-18}$	$+0.000000 \cdot 10^{+00}$
-1	$2.143040 \cdot 10^{-15}$	$7.542940 \cdot 10^{-16}$	$+0.000000 \cdot 10^{+00}$
0	$4.903725 \cdot 10^{-04}$	$9.990235 \cdot 10^{-01}$	$-1.962688 \cdot 10^{-02}$
1	$1.331028 \cdot 10^{-02}$	$2.815201 \cdot 10^{-02}$	$+1.600781 \cdot 10^{-02}$
2	$2.296707 \cdot 10^{-03}$	$4.740838 \cdot 10^{-04}$	$-2.442669 \cdot 10^{-04}$
3	$6.507891 \cdot 10^{-05}$	$5.759091 \cdot 10^{-06}$	$-3.243351 \cdot 10^{-07}$
4	$6.508629 \cdot 10^{-07}$	$4.948536 \cdot 10^{-08}$	$-4.737035 \cdot 10^{-11}$
5	$6.814760 \cdot 10^{-09}$	$3.024025 \cdot 10^{-10}$	$-7.148810 \cdot 10^{-15}$

Total momentum of the wave is equal to $\Delta\dot{P}_{tot} = -3.87898 \cdot 10^{-3}$.
The oscillon gets $-\Delta\dot{P}_{tot}$ so it moves towards the radiation

Acceleration of the oscillon as a function of frequency (measured from PDE solutions) ($\Omega \simeq 1.7$, $a = 0.12$).



Plan

Oscillons are very common in many field theories and have a long and very interesting life.

- loose energy very slowly (beyond all orders)
- interact nonelastically with other objects
  - small velocities colision with kink - scatter bask
  - fast collisions - oscillating energy is transformed into kinetic energy
- in certain conditions undergo so called negative radiation pressure

Oscillons are very common in many field theories and have a long and very interesting life.

- loose energy very slowly (beyond all orders)
- interact nonelastically with other objects
  - small velocities colision with kink - scatter bask
  - fast collisions - oscillating energy is transformed into kinetic energy
- in certain conditions undergo so called negative radiation pressure

Oscillons are very common in many field theories and have a long and very interesting life.

- loose energy very slowly (beyond all orders)
- interact nonelastically with other objects
  - small velocities colision with kink - scatter bask
  - fast collisions - oscillating energy is transformed into kinetic energy
- in certain conditions undergo so called negative radiation pressure

Oscillons are very common in many field theories and have a long and very interesting life.

- loose energy very slowly (beyond all orders)
- interact nonelastically with other objects
  - small velocities colision with kink - scatter bask
  - fast collisions - oscillating energy is transformed into kinetic energy
- in certain conditions undergo so called negative radiation pressure

Oscillons are very common in many field theories and have a long and very interesting life.

- loose energy very slowly (beyond all orders)
- interact nonelastically with other objects
  - small velocities colision with kink - scatter bask
  - fast collisions - oscillating energy is transformed into kinetic energy
- in certain conditions undergo so called negative radiation pressure

Oscillons are very common in many field theories and have a long and very interesting life.

- loose energy very slowly (beyond all orders)
- interact nonelastically with other objects
  - small velocities colision with kink - scatter bask
  - fast collisions - oscillating energy is transformed into kinetic energy
- in certain conditions undergo so called negative radiation pressure