Non-perturbative low energy amplitudes in non-local chiral quark model

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OUTLINE

- Non-perturbative input to amplitudes for exclusive processes is analyzed within full non-local chiral quark model
- Two examples:
  - Photon Distribution Amplitudes
  - Pion-photon Transition Distribution Amplitudes
- Special attention is paid to the question of inheriting QCD properties by objects calculated in the effective model

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INTRODUCTION

Factorization of the amplitudes for exclusive processes in the presence of the hard scale[1,2,3]

$$\mathcal{M} = (HARD) \otimes (SOFT)$$

$\Rightarrow$ *HARD* part can be calculated in perturbation theory

$\Rightarrow$ *SOFT* part is a subject to the non-perturbative treatment

Examples: $\rightsquigarrow$ Distribution Amplitudes (DA)

$\rightsquigarrow$ Generalized Parton Distributions (GPD)

$\rightsquigarrow$ Transition Distribution Amplitudes (TDA)

How to access the *SOFT* part ?

$\Rightarrow$ extraction from the experiment

$\Rightarrow$ lattice calculations

$\Rightarrow$ low energy effective models

[1] Efremov, Radyushkin; [2] Brodsky, Lepage; [3] Collins, Frankfurt, Strikman

INTRODUCTION (continued...)

SOFT part parametrizes matrix elements of certain non-local quark (gluon) operators on the light-cone, e.g.

$$\langle H' \left| \bar{\psi}(y) \mathcal{O} \psi(x) \right| H \rangle$$

$\Rightarrow$ they should possess properties originating from QCD symmetries (e.g. Lorentz invariance, Ward identities, axial anomaly)

$\Longrightarrow$ it is not obvious that effective models do inherit all QCD symmetries

$\Longrightarrow$ possible problems with correct properties of *SOFT* part in the effective models

CHIRAL QUARK MODEL (χQM)

For simplicity we consider pions only. In order to obtain considered matrix elements we need the model of quark-pion interactions.

At low energy scales spontaneous chiral symmetry breaking (χSB) plays very important role

$\Rightarrow$ the model should incorporate χSB

$\Rightarrow$ there appear constituent quark mass $M \sim 350\,\mathrm{MeV}$

The simplest model is the semi-bosonized Nambu-Jona-Lasinio model[1] (chiral limit)

$$S_{\mathrm{loc}} = \int d^4x\, \bar{\psi}(x)\left(i\not{D} - MU^{\gamma_5}\right)\psi(x)$$

where $U^{\gamma_5}(x) = \exp\left\{\frac{i}{F_\pi}\tau^a\pi^a(x)\gamma_5\right\}$, with $F_\pi = 93\,\mathrm{MeV}$.

- In order to get finite quark loops we need to impose some kind of regularization (but we cannot remove the cutoff parameter at the end)
- However to get correct results for anomalous processes we have to remove regularization
- Particular regularization scheme lacks motivation in terms of QCD...

[1] see e.g. S.P. Klevansky

NON-LOCAL χQM

The most natural way of regularizing quark loops
$\Longrightarrow$ momentum dependent constituent quark mass $M \equiv M(k)$

$$S_{\rm Int} = \int \frac{d^4k\, d^4l}{(2\pi)^8} \bar{\psi}(k)\sqrt{M(k)}U^{\gamma_5}(k-l)\sqrt{M(l)}\psi(l)$$

where usually one defines $M(k) = M\,F^2(k)$, and $F(0) = 1$, $F(k \to \infty) \to 0$.
This action was "derived" from QCD instanton vacuum theory, with Euclidean analytical expression for $M(k)$[1].

Problem:
momentum dependent mass $\Rightarrow$ naive vector current $\bar{\psi}\gamma^\mu\psi$ is not conserved
$\Rightarrow$ local vertex γ^μ has to be replaced by the non-local one Γ^μ
The precise form of the vertex is unconstrained and has to be modeled[2,3,4].
One of the simplest solution is

$$\Gamma^\mu(k,p) = \gamma^\mu - \frac{k^\mu + p^\mu}{k^2 - p^2}(M(k) - M(p))$$

The concrete model is specified by giving $M(k)$ and the form of the vertices.

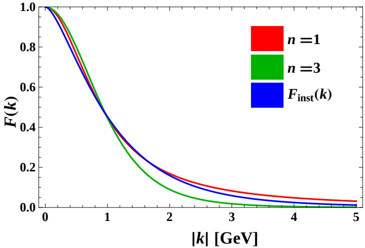
[1]Diakonov, Petrov; [2]Bowler, Birse; [3]B. Holdom, R. Lewis; [4]A. Bzdak, M. Praszalowicz

ONE LOOP CALCULATIONS

As the mass dependence on momentum we take

$$F(k) = \left(\frac{-\Lambda^2}{k^2 - \Lambda^2 + i\epsilon} \right)^n$$

- calculations in Euclidean as well as Minkowski space
- "analytical" solutions



The ansatz above leads to set of poles in the complex plane. Using some tricks we can express the loop integral with N propagators as

$$\sum_{i_1,\ldots,i_N}^{4n+1} f_{i_1} \ldots f_{i_N} \eta_{i_1}^{M_1} \ldots \eta_{i_N}^{M_N} \int \frac{d^D\kappa}{(2\pi)^D} g(\kappa, \eta_{i_1}, \ldots, \eta_N)$$

where g is a function containing only N poles, η_i are solutions of $z^{4n+1} + z^{4n} - (M/\Lambda)^2 = 0$ and f_i are some numbers composed from η_i.

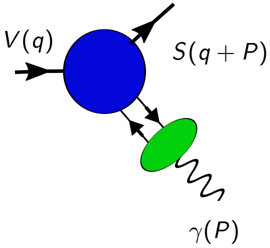
Higher twist light cone amplitudes $\Rightarrow$ delta type singularities in the boundaries of physical support

APPLICATION I

Simplest *SOFT* objects $\Longrightarrow$ Distribution Amplitudes (DA)

Example: radiative vector meson decay $V \to S\gamma$ and Photon DA.
Relevant coordinates are defined by two light-like vectors $n = (1,0,0,-1)$, $\tilde{n} = (1,0,0,1)$. Then we can decompose any vector v as

$$v^\mu = \frac{v^+}{2}\tilde{n}^\mu + \frac{v^-}{2}n^\mu + v_T^\mu$$



General definition:

$$\langle 0 \left| \overline{\psi}(\lambda n)\,\mathcal{O}\psi(-\lambda n) \right| \gamma(P)\rangle \sim F_{\mathcal{O}}\left(P^2\right)\int_0^1 du\, e^{i(2u-1)\lambda P^+}\,\phi_{\mathcal{O}}\left(\boldsymbol{u}, \boldsymbol{P}^2\right)$$

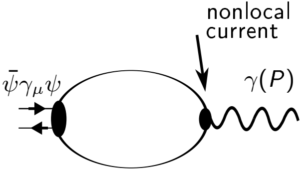
where $\mathcal{O} = \{\sigma^{\mu\nu}, \gamma^\mu, \gamma^\mu\gamma_5\}$ and $F_{\mathcal{O}}$ are relevant form factors.

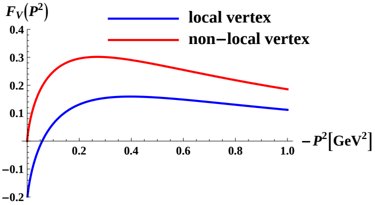
P. Ball, Brown; Arriola, Broniowski, Dorokhov;

As an example consider vector photon DA $\phi_V\left(u, P^2\right)$
$\Rightarrow$ current conservation in QCD: $F_V\left(0\right) = 0$.

NχQM calculations:

- Using full non-local photon-quark vertex and leaving pure QCD vector current operator we recover $F_V\left(0\right) = 0$.
- Diagram in the right has two contributions: hadronic and infinite perturbative part corresponding to electromagnetic ingredient of the photon.
- Amplitudes up to twist-4 in all channels have been calculated.





$M = 350\,\mathrm{MeV},\ \ n = 1$

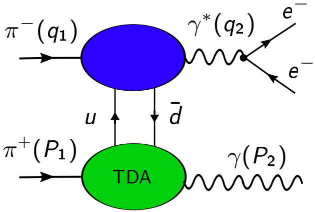
APPLICATION II

More demanding objects to study in Chiral Quark Models:
Transition Distribution Amplitudes appearing for example in $\pi^+\pi^- \to \gamma^*\gamma$ in the forward region[1].

Kinematics:

- high virtuality Q^2 of the upper photon
- low momentum transfer to the lower blob

$$\Delta^2 = (P_2 - P_1)^2 = t \ll Q^2$$



Example: Vector TDA (VTDA)

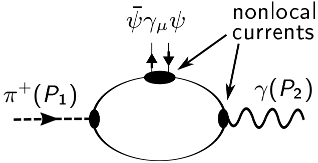
$$\int \frac{d\lambda}{2\pi} e^{i\lambda x p^+} < \gamma(P_2)|\overline{d}(-\frac{\lambda}{2}n)\gamma^\mu u(\frac{\lambda}{2}n)|\pi^+(P_1) > \sim \varepsilon^{\mu\nu\alpha\beta}\varepsilon^*_\nu P_{1\,\alpha} P_{2\,\beta}\, \boldsymbol{V\,(X,\xi,t)}$$

where $\xi = -2\Delta^+/p^+$ with $p = \frac{1}{2}(P_1 + P_2)$ is so called skewedness.

[1] Pire, Szymanowski

Properties of VTDA originating from QCD:

- polynomiality $\int dX\, X^n\, V(X,\xi,t) = a_n \xi^n + a_{n-1}\xi^{n-1} + \ldots + a_0$
- normalization is fixed by axial anomaly $\int dX\, V(X,\xi,t=0) = 1/2\pi^2$



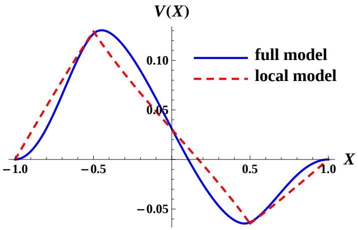
NχQM calculations:

- Polynomiality is satisfied.
- We obtain correct normalization only when both vector currents are non-local.

VTDA is related to pion-photon transition form factor

$$\int dX\, V(X,\xi,t) \sim F_{\pi\gamma}(t)$$

controlling $\gamma^*\gamma \to \pi^0$ reaction.



$M = 350\,\mathrm{MeV},\ n = 1,\ \xi = 0.5$

New BaBar data are available (29 May) which cast some new light on pion Distribution Amplitudes...

$\Rightarrow$ this is currently under investigation...

SUMMARY

- Non-local chiral quark model allows for analyzing low energy matrix elements
  - However, before using in real processes they have to be evolved (scale of effective models is low) - not discussed
- In order to make calculations consistent we have to use modified currents
  - The form of the full currents is not restricted and has to be modelled
  - However, in general it is not clear yet which currents we should modify and when
  - Case of full axial current is more difficult - not discussed
- First analysis of pion-photon Transition Distribution Amplitudes in non-local model

BACKUP

PHOTON DA DEFINITIONS

- tensor channel

$$\left\langle 0 \left| \overline{\psi}(\lambda n)\, \sigma^{\alpha\beta} \psi(-\lambda n) \right| \gamma(P,\varepsilon) \right\rangle = i^2 e \langle \bar{\psi}\psi \rangle F_T(P^2)$$

$$\Bigg\{ \left(\varepsilon_T^{\alpha} \tilde{n}^{\beta} - \varepsilon_T^{\beta} \tilde{n}^{\alpha} \right) \frac{P^+}{2} \chi_m \int_0^1 du\, e^{i\xi\lambda P^+}\, \phi_T(u, P^2)$$

$$+ \frac{1}{2P^+} \left(\tilde{n}^{\alpha} n^{\beta} - \tilde{n}^{\beta} n^{\alpha} \right) \varepsilon^+ \int_0^1 du\, e^{i\xi\lambda P^+}\, \psi_T(u, P^2)$$

$$+ \frac{1}{P^+} \left(\varepsilon_T^{\alpha} n^{\beta} - \varepsilon_T^{\beta} n^{\alpha} \right) \int_0^1 du\, e^{i\xi\lambda P^+}\, h_T(u, P^2) \Bigg\},$$

- vector channel

$$\langle 0 \left| \overline{\psi}(\lambda n)\, \gamma^{\mu} \psi(-\lambda n) \right| \gamma(P,\varepsilon) \rangle = i e f_{3\gamma} F_V(P^2)$$

$$\Bigg\{ \frac{1}{2} \tilde{n}^{\mu} \varepsilon^+ \int_0^1 du\, e^{i\xi\lambda P^+}\, \phi_V(u, P^2)$$

$$+ \varepsilon_T^{\mu} \int_0^1 du\, e^{i\xi\lambda P^+}\, \psi_V(u, P^2) - \frac{1}{2} \frac{P^2}{(P^+)^2} n^{\mu} \varepsilon^+ \int_0^1 du\, e^{i\xi\lambda P^+}\, h_V(u, P^2) \Bigg\},$$

- axial channel

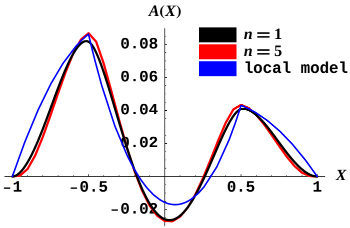
$$\left\langle 0 \left| \overline{\psi} (\lambda n) \gamma^{\mu} \gamma_5 \psi (-\lambda n) \right| \gamma (P, \varepsilon) \right\rangle = i \frac{1}{2} e f_{3\gamma} F_A \left(P^2 \right)$$

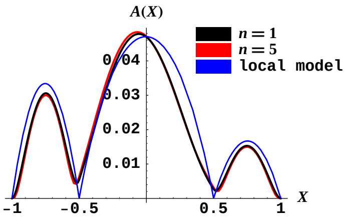
$$\epsilon_{\mu\nu\alpha\beta} \varepsilon_T^{\nu} \tilde{n}^{\alpha} n^{\beta} P^{+} \lambda \int_0^1 du \, e^{i\xi\lambda P^{+}} \phi_A \left(u, P^2 \right),$$

$\xi = 2u - 1$, $\left\langle \bar{\psi}\psi \right\rangle$ is quark condensate, χ_m is the magnetic susceptibility of the quark condensate.

AXIAL TDA

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x p^+} \left\langle \gamma\left(P_2, \varepsilon\right) \middle| \overline{d}\left(-\frac{\lambda}{2} n\right) \gamma^\mu \gamma_5 u\left(\frac{\lambda}{2} n\right) \middle| \pi^+\left(P_1\right) \right\rangle$$
$$= \frac{ie}{2\sqrt{2} F_\pi p^+} P_2^\mu \left(q \cdot \varepsilon^*\right) A\left(X, \xi, t\right) + \dots$$





LEADING TWIST PHOTON DA